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SIGNAL & SYSTEM

EC-EE

Date of Test : 18/04/2025

ANSWER KEY ➤

1.	(b)	7.	(a)	13.	(b)	19.	(a)	25.	(a)
2.	(c)	8.	(d)	14.	(d)	20.	(d)	26.	(c)
3.	(b)	9.	(a)	15.	(d)	21.	(a)	27.	(d)
4.	(b)	10.	(b)	16.	(b)	22.	(a)	28.	(b)
5.	(d)	11.	(c)	17.	(a)	23.	(d)	29.	(c)
6.	(a)	12.	(c)	18.	(b)	24.	(d)	30.	(c)

DETAILED EXPLANATIONS

1.   (b)

Given, $x(t) = \dfrac{\sin(10\pi t)}{\pi t}$

Taking Fourier transform

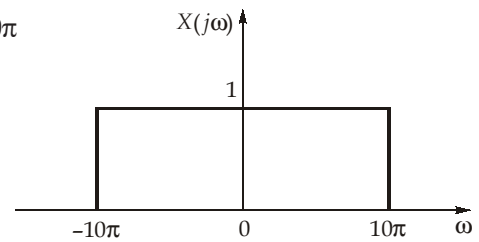$$X(j\omega) = \begin{cases} 1 \; ; & |\omega| \le 10\pi \\ 0 \; ; & |\omega| > 10\pi \end{cases}$$

or

$\therefore$ The maximum frequency 'ω_m' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,

$$\dfrac{2\pi}{T_s} > 2\omega_m$$

$$\dfrac{2\pi}{T_s} > 20\pi$$

$\therefore$ $T_s < \dfrac{1}{10}$

2.   (c)

We know that,

$$a_n = \dfrac{2}{T}\int_T x(t)(\cos\omega_0 nt)\,dt = \dfrac{2}{T}\int_T x(t)\left[\dfrac{e^{j\omega_0 nt} + e^{-j\omega_0 nt}}{2}\right]dt$$

$$= \dfrac{1}{T}\left[\int_T x(t)e^{j\omega_0 nt}\,dt + \int_T x(t)e^{-j\omega_0 nt}\,dt\right]$$

$$a_n = C_n + C_{-n} \quad \text{[By the definition of exponential Fourier series coefficient].}$$

3.   (b)

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt = \int_{-\infty}^{\infty}[f(t)\cos\omega t - jf(t)\sin\omega t]\,dt$$

$$= \int_{-\infty}^{\infty} f(t)\cos\omega t\,dt - j\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt$$

$f(t) \Rightarrow$ even signal
$f(t)\cos\omega t \Rightarrow$ even signal
$f(t)\sin\omega t \Rightarrow$ odd signal

$$\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt = 0$$

$$\int_{-\infty}^{\infty} f(t)\cos\omega t\,dt = 2\int_0^{\infty} f(t)\cos\omega t\,dt$$

$\therefore$ $F(\omega) = 2\int_0^{\infty} f(t)\cos\omega t\,dt$

4.   (b)

Given, $y(t) = \dfrac{1}{2}x\left(\dfrac{t}{2}\right) * \delta(t-4) = \dfrac{1}{2}x\left(\dfrac{t-4}{2}\right) = \dfrac{1}{2}x\left(\dfrac{1}{2}t-2\right)$

at $t = 2$; $y(2) = \dfrac{1}{2}x\left(\dfrac{1}{2}(2)-2\right) = \dfrac{1}{2}x(-1)$

Slope of given signal $x(t)$ is 2.

$$x(-1) = -2$$

$$y(2) = \dfrac{1}{2}(-2) = -1$$

5.   (d)

The given sequence of $x[n]$ is finite duration. Hence, the region of convergence is $0 < |z| < \infty$.

6.   (a)

$$r_{xx}(k) = \sum_{k=-\infty}^{\infty} x[n]\,x[n-k]$$

$$\sum_{k=-4}^{k=4} r_{xx}(k) = x[n]\,x[n+3] + x[n]\,x[n+2] + x[n]\,x[n+1] + x[n]\,x[n] + x[n]\,x[n-1]$$

$$+ x[n]\,x[n-2] + x[n]\,x[n-3]$$

$$= -2 - 5 + 2 + 10 + 2 - 5 - 2 = 0$$

7.   (a)

By Parsevals theorem,

Energy of a signal $x[n]$ is, $E = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi}\left|X(e^{j\omega})^2\right|d\omega$

where $X(e^{j\omega})$ is discrete time Fourier transform of $x[n]$,

So, $\dfrac{\sin(2n)}{\pi n} \xrightarrow{\text{DTFT}} X(e^{j\omega}) = \begin{cases} 1; & |\omega| \le 2 \\ 0; & \text{otherwise} \end{cases}$

$\therefore$ Energy, $E = \dfrac{1}{2\pi}\displaystyle\int_{-2}^{2} 1 \cdot d\omega = \dfrac{1}{2\pi}[4]$

$\therefore$ $E = \dfrac{2}{\pi} = 0.64$ J
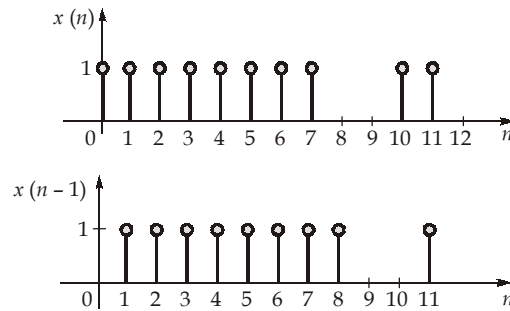
8.   (d)

Given, DFT signal $x[n]$

$$\{4, 1+2j, 2-2j, 3+j, -3, 3-j, 2+2j, 1-2j\}$$

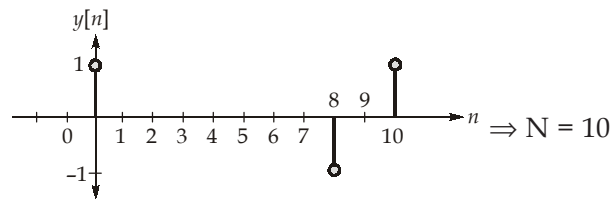$$\sum_{n=0}^{7}(-1)^n x[n] = x[0] - x[1] + x[2] - x[3] + x[4] - x[5] + x[6] - x[7]$$

(or)

$$= X\left[\dfrac{8}{2}\right] = X[4] = -3$$

9. **(a)**

Subtracting the two signal, we get

$\Rightarrow$ N = 10

10. **(b)**

For a causal system, $h(t) = 0$ for $t < 0$

Given, $x(t) = 0$ for $t < -1$

$\therefore$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t - \tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t - \tau)\, d\tau$$

$$= \int_{0}^{\infty} h(\tau)\, x(t - \tau)\, d\tau \quad \because h(\tau) = 0 \text{ for } \tau < 0$$

$$= \int_{0}^{t+1} h(\tau)\, x(t - \tau)\, d\tau \quad \because x(t - \tau) = 0 \text{ for } t - \tau < -1$$

11. **(c)**

We know that the Laplace transform of

$$\cos(at)\,u(t) = \frac{s}{s^2 + a^2}$$

$\therefore$ $$\cos(\pi t)\,u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function $x(t)$ can be written as,

$$= \cos\pi t[u(t) - u(t - 1)]$$
$$= \cos(\pi t)u(t) - \cos\pi t\, u(t - 1)$$
$$= \cos\pi t\,u(t) - \cos\pi(t - 1 + 1)u(t - 1)$$
$$= \cos\pi t\,u(t) - \cos[\pi(t - 1) + \pi]\,u(t - 1)$$
$$x(t) = \cos(\pi t)\,u(t) + \cos[\pi(t - 1)]\,u(t - 1)$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2 + \pi^2} + \frac{s\,e^{-s}}{s^2 + \pi^2} \qquad [\because x(t - t_0) = X(s)\cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1 + e^{-s}]}{s^2 + \pi^2}$$

12. **(c)**

Given, sinusoidal pulse

$$z(t) = \begin{cases} e^{j10t} \; ; & |t| < \pi \\ 0 \; ; & |t| > \pi \end{cases}$$

We may express $z(t)$ as the product of a complex sinusoid e^{j10t} and a rectangular pulse $x(t)$.

Let,

$$x(t) = \begin{cases} 1 \; ; & |t| < \pi \\ 0 \; ; & |t| > \pi \end{cases}$$

Fourier transform of $x(t)$ is $X(j\omega)$

$$\therefore \qquad X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt = \int_{-\pi}^{\pi} 1 \cdot e^{-j\omega t} \, dt = \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-\pi}^{\pi}$$

$$= -\frac{1}{j\omega} \left[e^{-j\pi\omega} - e^{+j\omega\pi} \right] = \frac{e^{j\omega\pi} - e^{-j\omega\pi}}{j\omega} = \frac{2}{\omega} \left[\frac{e^{j\omega\pi} - e^{-j\omega\pi}}{2j} \right]$$

$$\therefore \qquad X(j\omega) = \frac{2}{\omega} \sin(\omega\pi)$$

By using frequency shift property of Fourier transform, we get,

$$z(t) = e^{j10t} \cdot x(t) \xleftrightarrow{\;\text{FT}\;} X(j(\omega - 10))$$

$$\therefore \qquad z(t) \xleftrightarrow{\;\text{FT}\;} \frac{2}{\omega - 10} \sin\big((\omega - 10)\pi\big)$$

13. **(b)**

Given, $\qquad X(s) = \log(s + 2) - \log(s + 3)$

Differentiating both the sides with respect to s

$$\frac{d}{ds} X(s) = \frac{1}{s+2} - \frac{1}{s+3} \qquad\qquad \text{...(i)}$$

From the properties of Laplace transform, we know that,

$$t x(t) \longleftrightarrow -\frac{d}{ds} X(s)$$

Thus equation (i) can be written as,

$$-t x(t) = \left[e^{-2t} - e^{-3t} \right] u(t)$$

or,

$$x(t) = \left[\frac{e^{-3t} - e^{-2t}}{t} \right] u(t)$$

14. **(d)**

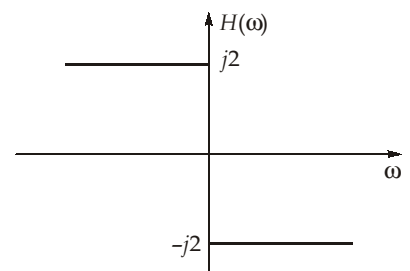By redrawing the given frequency response, we get,

We can write $H(\omega) = -j2\,\text{sgn}(\omega)$

We know that,

For $\qquad \text{sgn}(t) \xleftrightarrow{\;\text{FT}\;} \dfrac{2}{j\omega}$

By duality property

$$\frac{2}{jt} \xleftrightarrow{\;\text{FT}\;} 2\pi\, \text{sgn}(-\omega)$$

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} -2\pi \operatorname{sgn}(\omega)$$

$$\frac{2}{\pi t} \xleftrightarrow{\text{FT}} -j2\operatorname{sgn}(\omega)$$

or $\qquad = 2(\pi t)^{-1}$

15. (d)

Given $\qquad x(t) = \sin(150\pi t)$

Time period, $\qquad T = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{150\pi} = \dfrac{1}{75}$ sec

3 time periods $= 3 \times T = 3 \times \dfrac{1}{75} = \dfrac{1}{25}$ sec

$\therefore$ The signal sampled at a rate of five samples is $\dfrac{1}{25}$ sec

So, 1 sample in $\dfrac{1}{125}$ sec $= T_s$ [sampling interval]

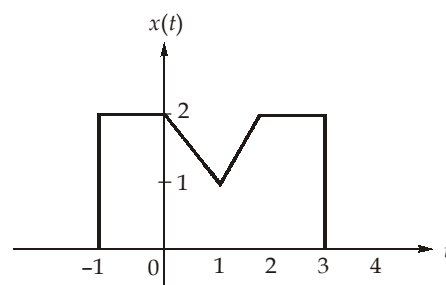$\therefore$ Sampling frequency $= f_s = \dfrac{1}{T_s} = 125$ samples/sec

also, $\qquad$ Nyquist rate $= f_N = 2f_m = 2 \times 75 \qquad\qquad [\because \omega_m = 150\pi \Rightarrow f_m = 75 \text{ Hz}]$
$\qquad\qquad\qquad\qquad\qquad = 150$ samples/sec

$\therefore \qquad$ The ratio, $\dfrac{f_s}{f_N} = \dfrac{125}{150} = \dfrac{5}{6} = 0.83$

16. (b)

Given,

By the definition of Fourier transform,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t} \, d\omega$$

at $t = 0$,

$$x(0) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} X(j\omega)\, d\omega$$

$$\therefore \qquad \int\limits_{-\infty}^{\infty} X(j\omega)\, d\omega = 2\pi\, x(0) = 2\pi(2) = 4\pi \approx 12.57$$

17. **(a)**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!} z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2z}\right)^n}{n!}$$

$$X(z) = 1 + \frac{\frac{1}{2z}}{1!} + \frac{\left(\frac{1}{2z}\right)^2}{2!} + \frac{\left(\frac{1}{2z}\right)^3}{3!} + \dots$$

$$X(z) = e^{1/2z}$$

$$X(1) = e^{1/2} = \sqrt{e} = 1.648 \approx 1.65$$

18. **(b)**

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N} nK}$$

$$g[n] = x[n-2]_{\text{mod N}}$$

$$G[k] = e^{-j\frac{2\pi}{N}(2)k} X[k]$$

$$G[1] = e^{-j\frac{2\pi}{4}(2)1} X[1] = e^{-j\pi} X[1]$$

$$G[1] = -X[1] = -7$$

19. **(a)**

Given,
$$X(z) = \frac{10 - 8z^{-1}}{2 - 5z^{-1} + 2z^{-2}}$$

$$= \frac{2}{(2 - z^{-1})} + \frac{4}{(1 - 2z^{-1})}$$

$$X(z) = \frac{2z}{2z-1} + \frac{4z}{z-2}$$

$$X(z) = \frac{z}{\left(z - \frac{1}{2}\right)} + \frac{4z}{(z-2)}$$

Since, ROC includes unit circle,

$\therefore$ ROC of $X(z)$ is $\dfrac{1}{2} < |z| < 2$

$$x[n] = \left(\dfrac{1}{2}\right)^n u[n] - 4(2^n)u[-n-1]$$

$\therefore \qquad\qquad x(1) = \dfrac{1}{2} = 0.5$

20. **(d)**

Given: $\qquad\qquad x(n) = \left\{-2, -3, \underset{\uparrow}{-5}, 3, 2\right\}$

Using Parseval's Theorem,

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \dfrac{1}{2\pi} \int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega$$

$$\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-2}^{2} |x(n)|^2 = (-2)^2 + (-3)^2 + (-5)^2 + (3)^2 + (2)^2 = 51$$

$$2\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = 4\pi \times 51 = 204\pi$$

$$2\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = 204\pi$$

21. **(a)**

Given, $\qquad$ signal $x[n] = (0.5)^{2n} u[n] = \left(\dfrac{1}{2}\right)^{2n} u(n) = \left(\dfrac{1}{4}\right)^n u(n)$

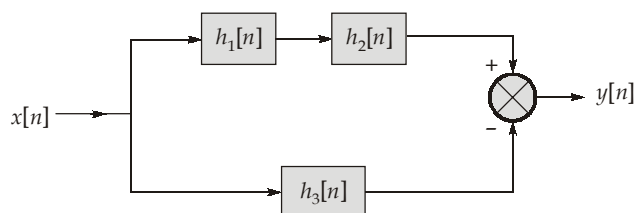From the definition of DTFT, the signal can be written as,

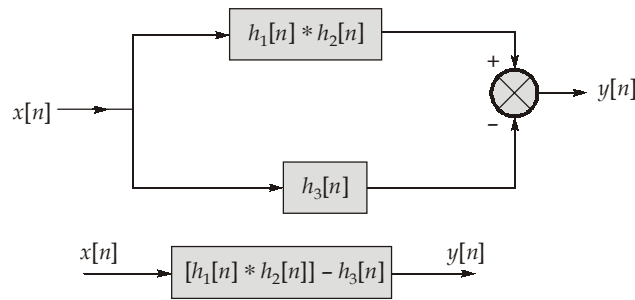$$X(e^{j\omega}) = \dfrac{1}{1 - \dfrac{1}{4}e^{-j\omega}}$$

$$X(e^{j\omega}) = \dfrac{4}{4 - e^{-j\omega}}$$

at $\omega = \pi$, $\qquad X(e^{j\pi}) = \dfrac{4}{4 - \cos\pi} = \dfrac{4}{4-(-1)} = \dfrac{4}{5} = 0.8$

22. **(a)**

Given,

From the given figure, overall impulse response,

$\therefore \qquad h[n] = [h_1[n] * h_2[n]] - h_3[n]$

$\qquad\qquad\qquad = [u[n+2] * \delta[n-2]] - [u[n] - u[n-2]]$

$\qquad\qquad\qquad = u[n] - u[n] + u[n-2] \qquad\qquad \{\therefore\ x[n] * \delta[n - n_0] = x[n - n_0]\}$

$\qquad\qquad h[n] = u[n-2]$

at $n = 2$; $\qquad h[2] = u[2-2] = u[0] = 1$

23. **(d)**

Given, $\qquad\qquad h(n) = \left(\dfrac{7}{8}\right)^n u(n)$

Taking DTFT

$$H(e^{j\omega}) = \frac{1}{1 - \left(\dfrac{7}{8}\right)e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{1}{1 - \dfrac{7}{8}\cos\omega + j\dfrac{7}{8}\sin\omega}$$

$$H(e^{j\omega}) = \frac{1}{\left(1 - \dfrac{7}{8}\cos\omega\right) + j\left(\dfrac{7}{8}\sin\omega\right)}$$

Let half power frequency bandwidth be ω_0

$$\left|H(e^{j\omega})\right|\Big\|_{w=\omega_0} = \frac{1}{\sqrt{2}}H(e^{j\omega})\Big|_{\max\,(\omega=0)}$$

$$\frac{1}{\sqrt{\left(1 - \dfrac{7}{8}\cos\omega_0\right)^2 + \left(\dfrac{7}{8}\sin\omega_0\right)^2}} = \frac{1}{\sqrt{2}} \times 8$$

$$1 + \left(\dfrac{7}{8}\cos\omega_0\right)^2 - \dfrac{7}{4}\cos\omega_0 + \left(\dfrac{7}{8}\sin\omega_0\right)^2 = \frac{1}{32}$$

$$1 + \left(\dfrac{7}{8}\right)^2 - \dfrac{7}{4}\cos\omega_0 = \frac{1}{32}$$

$$\cos\omega_0 = \frac{111}{64} \times \frac{4}{7}$$

$$\omega_0 = 0.1337 \text{ radians/sec}$$

24. **(d)**

Given: $x(n) = 3n^3(3)^n\, u(n)$

$$Y(z) = -\frac{d}{dz}X(z)$$

We know, from the differentiation property of z-transform,

$$nx(n) \xrightarrow{\;\;Z.T\;\;} -z\frac{dX(z)}{dz}$$

Using time shifting property of z-transform,

$$(n-1)x(n-1) \xrightarrow{\;\;Z.T\;\;} \frac{-z^{-1}z\,dX(z)}{dz}$$

$$y(n) = (n-1)x(n-1) \xrightarrow{\;\;\;\;} \frac{-dX(z)}{dz} = Y(z)$$

$$y(n) = (n-1)(3(n-1)^3\, 3^{n-1})u(n-1)$$
$$y(n) = (n-1)^4(3)^{n+1-1}\, u(n-1)$$
$$y(n) = (n-1)^4\,(3)^n u(n-1)$$

25. **(a)**

The Fourier series representation of signal $x(t)$ is,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where, $\omega_0 = \dfrac{2\pi}{T} = \dfrac{2\pi}{\left(\dfrac{\pi}{8}\right)} = 16\ \text{rad/sec}$

When $x(t)$ is passed through given low-pass filter,

the output is, $y(t) = \displaystyle\sum_{n=-\infty}^{\infty} C_n H(jn\omega_0)e^{jn\omega_0 t}$

∴ The largest value of 'n' for which $y(t)$ is non-zero is given by,

$$n\omega_0 \leq 170$$
$$16n \leq 170$$
$$n \leq 10.6$$

∴ maximum harmonics present at the output of filter is 10.

26. **(c)**

$$x(t) = \underbrace{4\cos\left(\frac{2\pi}{3}t + 40°\right)}_{x_1(t)} + \underbrace{3\sin\left(\frac{4\pi}{5}t + 20°\right)}_{x_2(t)}$$

$$\omega_1 = \frac{2\pi}{3} \implies T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi/3} = 3$$

$$\omega_2 = \frac{4\pi}{5} \implies T_2 = \frac{2\pi}{4\pi/5} = \frac{5}{2}$$

$$T = \text{LCM of } (T_1, T_2)$$

$$T = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\Rightarrow \qquad T = \frac{3 \times 5}{1} = 15 \text{ sec}$$

27. **(d)**

Given, $x[n]$ is a causal sequence,

$\therefore x[n]$ will be zero for $n < 0$.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

and
$$h[n] = \left(\frac{1}{4}\right)^n u[n]$$

$$h[n] = \begin{cases} \left(\dfrac{1}{4}\right)^n ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$

$\therefore h[0] = 1, \quad h[1] = \dfrac{1}{4}, \, h[2] = \dfrac{1}{16}$

$$y[0] = x[0]\, h[0] + x[1]\, h[-1] + \dots$$

$$\frac{1}{2} = x[0]\,(1) + x[1] \times 0$$

$\therefore \qquad\qquad x[0] = \dfrac{1}{2}$

$$y[1] = x[0]\, h[1] + x[1]\, h[0] + x[2]\, h[-1] + \dots$$

$$1 = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + x[1][1] + x[2][0]$$

$$1 - \frac{1}{8} = x[1]$$

$\therefore \qquad\qquad x[1] = \dfrac{7}{8} = 0.875$

28. **(b)**

Given, $\qquad\qquad h[n] = \left(\dfrac{1}{2}\right)^n u[n]$

Now, we may write $h[n] - Ah[n-1] = \delta[n]$

as $\left(\dfrac{1}{2}\right)^n u[n] - A\left(\dfrac{1}{2}\right)^{n-1} u[n-1] = \delta[n]$

Substitute $n = 1$, we get,

$$\left(\frac{1}{2}\right)^1 u[1] - A\left(\frac{1}{2}\right)^{1-1} u[1-1] = \delta[1]$$

$\Rightarrow \qquad\qquad \dfrac{1}{2} - A = 0 \qquad\qquad (\because \delta[1] = 0)$

$\therefore \qquad\qquad A = \dfrac{1}{2}$

29.   (c)

Using Parseval's theorem, the energy of signal $x(n)$ is,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{N} \sum_{K=0}^{N-1} |X(K)|^2$$

Here, $X(0) = 3, X(1) = 2 + j; X(2) = j$

Using conjugate property of DFT,

$$X(K) = X^*(N - K)$$
$$N = 4$$
$$X(3) = X^*(4 - 3)$$
$$= X^*(1)$$
$$X(3) = 2 - j$$

$$E = \frac{1}{4}\left[\left|X(0)^2\right| + |X(1)|^2 + |X(2)|^2 + |X(3)|^2\right]$$

$$= \frac{1}{4}\left[3^2 + (4+1) + 1 + (4+1)\right]$$

$$= \frac{1}{4}[9 + 5 + 1 + 5]$$

$$E = 5 \text{ Joules}$$

30.   (c)

$$X(z) = \frac{z\left(z - \dfrac{1}{3}\right)}{(z-1)(z+2)}$$

$$\frac{X(z)}{z} = \frac{\left(z - \dfrac{1}{3}\right)}{(z-1)(z+2)}$$

Using partial fraction expansion,

$$\frac{X(z)}{z} = \frac{\dfrac{2}{3}}{3(z-1)} + \frac{\dfrac{7}{3}}{3(z+2)}$$

$$X(z) = \frac{2}{9}\left(\frac{z}{z-1}\right) + \frac{7}{9}\left(\frac{z}{z+2}\right)$$

$$\text{ROC}: |z| > 2$$

∴ $x(n)$ is a right-sided signal.

Taking inverse z-transform of $X(z)$,

$$x(n) = \frac{2}{9}u(n) + \frac{7}{9}(-2)^n u(n)$$

At $n = 2$, $x(2) = \frac{2}{9} + \frac{7}{9}(-2)^2 = \frac{2}{9} + \frac{7}{9} \times 4 = \frac{30}{9}$

$$x(2) = 3.33$$

∎∎∎∎