2017

MADE EASY WORKBOOK

Detailed Explanations of Try Yourself Questions

Electrical Engineering

Control System

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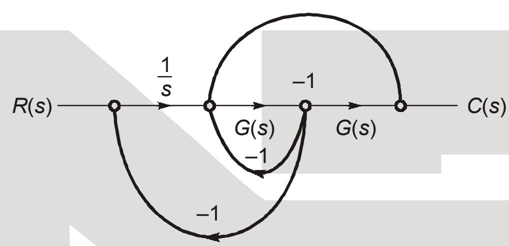
1 Mathematical Models and Block Diagram

Detailed Explanation of Try Yourself Questions

T1 : Solution

Drawing SFG of the above



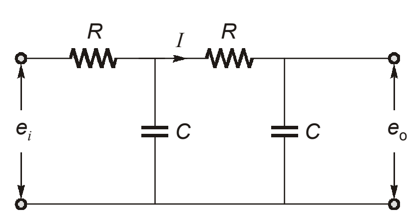
Here, $P_1 = \frac{G^2(s)}{s}$; $L_2 = -G(s)$; $L_1 = -G^2(s)$; $L_3 = -\frac{G(s)}{s}$

$$\frac{C(s)}{R(s)} = \frac{\frac{G^2(s)}{s}}{1-\left(-G^2(s)-\frac{G(s)}{s}-G(s)\right)}$$

$$= \frac{G^2(s)}{s+sG^2(s)+G(s)+sG(s)}$$

Put $G(s) = s$,
$$\frac{C(s)}{R(s)} = \frac{s^2}{s+s^3+s+s^2} = \frac{s^2}{s^3+s^2+2s} = \frac{s}{s^2+s+2}$$

T2 : Solution



$$E_o(s) = \frac{1}{sC} I(s) \qquad \ldots(i)$$

$$I(s) = \frac{E_i(s)}{\left[R + \dfrac{\left(R + \dfrac{1}{sC}\right) \times \dfrac{1}{sC}}{\left(R + \dfrac{1}{sC} + \dfrac{1}{sC}\right)}\right]} \times \frac{\dfrac{1}{sC}}{\left(R + \dfrac{1}{sC} + \dfrac{1}{sC}\right)}$$

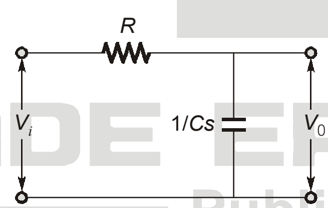
(Using current division rule)

$$= \frac{E_i(s)}{\dfrac{R + \dfrac{1}{sC}}{sCR + 2} + R} \times \frac{1}{sCR + 2} = \frac{E_i(s)}{\left(R + \dfrac{1}{sC}\right) + R(sCR + 2)}$$

$$E_0(s) = \frac{\dfrac{1}{sC} \times E_i(s)}{\dfrac{(1 + RSC) + SCR(SCR + 2)}{SC}}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{S^2C^2R^2 + 3\,SCR + 1} = \frac{1}{S^2T^2 + 3ST + 1}$$

T3 : Solution



$$\frac{V_0}{V_i} = \frac{1/Cs}{R + \dfrac{1}{Cs}} = \frac{1}{RCs + 1}$$

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2 Time Response Analysis

Detailed Explanation of Try Yourself Questions

T1 : Solution

$$G(s) = \frac{k}{s(s+p)}$$

Now, the closed loop system

$$T(s) = \frac{K}{s^2 + sp + K}$$

∴ Comparing it with standard equation

$$K = \omega_n^2$$

$$2\xi\omega_n = p$$

$$t_s = \frac{4}{\xi\omega_n}$$

⇒ $\xi\omega_n = 1$

∴ $p = 2$

now, $e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.1$

$$\xi = 0.537$$

∴ $\omega_n = \frac{p}{2\xi} = 1.69$

∴ $K = \omega_n^2 = 2.85$

T2 : Solution

Taking Laplace transform we get

$$X(s) = \frac{1}{(s^2 + 6s + 5)} . 12\left[\frac{1}{s} - \frac{1}{(s+2)}\right]$$

$$X(s) = \frac{12}{s(s+5)(s+1)}.12\left[\frac{1}{s}-\frac{1}{(s+2)}\right]$$

Now, using final value theorem

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

$$\therefore \quad X(s) = \lim_{s\to 0}\left[\frac{12}{(s+5)(s+1)}-\frac{12s}{(s+5)(s+1)(s+3)}\right]$$

$$= \frac{12}{5} = 2.4$$

T3 : Solution

To find the impulse response let us difference the response.

$$c'(t) = 12\,e^{-10t} - 12e^{-60t}$$

tanking inverse laplace transform we get

$$C'(s) = \frac{600}{(s+10)(s+60)}$$

$$C'(s) = \frac{600}{s^2+70s+600}$$

$\therefore$ $c'(s)$ is the impulse response thus comparing it with the standard equation.

$$2\xi\omega_n = 70$$

$$\omega_n = \sqrt{600}$$

$$\therefore \quad \xi = 1.428 \approx 1.43$$

T4 : Solution

Since real port of the given second order equation is at –0.602 thus they can be considered as dominant poles.

Thus $\quad t_p = \dfrac{\pi}{\omega_d}$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_n = \sqrt{2.829} = 1.6819$$

and $\quad 2\xi\omega_n = 1.204$

$$\xi = \frac{1.204}{2\times 1.6819} = 0.3579$$

$$\therefore \quad \omega_d = 1.6819\sqrt{1-\xi^2}$$

$$\omega_d = 1.577$$

$$\therefore \quad t_p = 1.999 \approx 2$$

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3 Stability

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given, $|\text{Open loop transfer function}| < 1$

i.e. $|G(s)\,H(s)| < 1$

or, $|G(j\omega)\,H(j\omega)| < 1$

or, $|X| < 1$

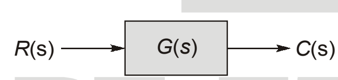
$\therefore$ Gain margin, $GM = \dfrac{1}{|X|}$ i.e. G.M. > 1

$\therefore$ Gain margin (in dB) > 0

Since gain margin of the given system is positive, therefore, the system is always stable.

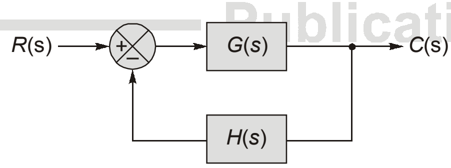
T2 : Solution

With negative feedback, the system stability will increase. In open loop system.



The gain of the system is $G(s)$.

Where as in closed loop system



The closed loop gain of the systems $\dfrac{G(s)H(s)}{1+G(s)H(s)}$ hence it is divided by $1 + G(s)\,H(s)$, in closed loop system with negative feedback gain decreases.

T3 : Solution

The correct sequence of steps needed to improve system stability is to use negative feedback, reduce gain and insert deviation action.

T4 : Solution

$$1 + G(s) = 0$$

$$\Rightarrow \quad s\tau_1\left[1+s(\tau_1+\tau_2)+s^2\tau_1\tau_2\right]+K = 0$$

$$s^3\tau_1^2\tau_2 + s^2\tau_1(\tau_1+\tau_2)+s\tau_1+K = 0$$

Using R-H criteria

s^3	$\tau_1^2\tau_2$	τ_1
s^2	$\tau_1(\tau_1+\tau_2)$	K
s^1	$\dfrac{\left[\tau_1^2(\tau_1+\tau_2)-K\tau_1^2\tau_2\right]}{\tau_1(\tau_1+\tau_2)}$	
s^0	K	

$$\Rightarrow \quad K > 0\,;\ \tau_1 > 0\,;\ \tau_2 > 0$$

Also,

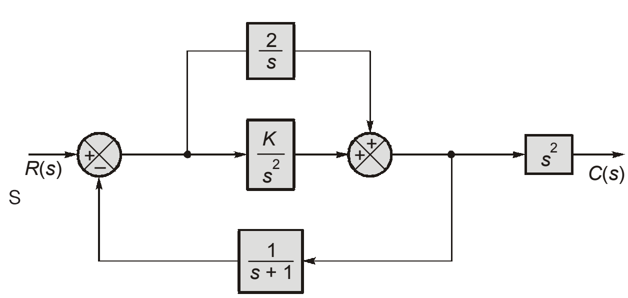
$$\frac{\tau_1(\tau_1+\tau_2)-K\tau_2}{(\tau_1+\tau_2)} > 0$$

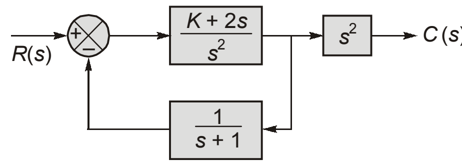
$$K\tau_2\tau_1 < \tau_1(\tau_1+\tau_2)$$

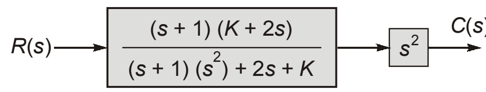
$$\Rightarrow \quad K < \left(1+\frac{\tau_1}{\tau_2}\right)$$

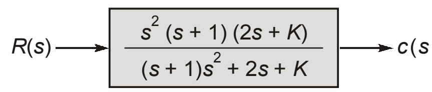
$0 < K < \left(1+\dfrac{\tau_1}{\tau_2}\right)$; [$\tau_1 > 0$ and $\tau_2 > 0$ and this is the only possible case.]

T5 : Solution









$$\frac{C(s)}{R(s)} = \frac{s^2(s+1)(2s+K)}{s^2(s+1)+2s+K}$$

at $\quad K = 2$

$$\frac{C(s)}{R(s)} = \frac{s^2(s+1)^2}{(s^2+2)(s+1)}$$

Thus poles at $\pm j\sqrt{2}$ and one at –1.

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4 Root Locus Technique

Detailed Explanation of Try Yourself Questions

T1 : Solution

Characteristic equation is given as

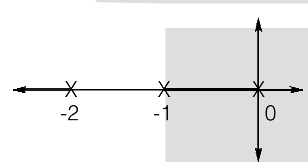
$$1 + G(s)\,H(s) = 0$$

On comparing this characteristic equation with the equation given in problem, we have

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+2)}$$

P = Number of open loop poles = 3 = number of branches on root locus

Z = 0 = Number of branches terminating at zeros.



Angle of Asymptotes: The $P - Z$ branches terminating at infinity will go along certain straight lines.

$$\text{Number of asymptotes} = P - Z = 3 - 0 = 3$$

$$\theta = \frac{180°(2q+1)}{P-Z} \qquad q = 0, 1, 2 \ldots$$

$$\theta_1 = \frac{180 \times (2 \times 0 + 1)}{3} = 60°$$

$$\theta_2 = \frac{180°(2 \times 1 + 1)}{3} = 180°$$

$$\theta_3 = \frac{180°(2 \times 2 + 1)}{3} = 300°$$

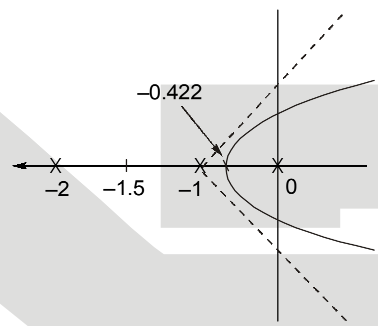
Centroid: It is the interpection point of the asymptotes on the real axis. It may or may not be a part of root locus.

$$\text{Centroid} = \frac{\Sigma \text{ Real part of open loop poles } - \Sigma \text{ Real part of open loop zeros}}{P - Z}$$

$$= \frac{0-1-2}{3} = -1$$

Centroid → (–1, 0)

Break-away or break-in points: These are those points on whose multiple roots of the characteristic equation occur.



$$s(s^2 + 3s + 2) + K = 0$$

$$K = -(s^3 + 3s^2 + 2s)$$

$$\frac{dK}{ds} = -(3s^2 + 6s + 2) = 0$$

$$s = -0.422, -1.577$$

Now verify the valid break-away point

$$K = 0.234 \text{ (valid) at } s = -0.422$$

$$K = \text{negative (not valid) at } s = -1.577$$

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T2 : Solution

$$OLTF = \frac{k}{s(s^2+4s+8)}$$

Poles are at $s_1 = 0$

and $s_{2,3} = \dfrac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$

There are 3 poles and no zero with root loci, all terminating at infinity.

$$\phi = \frac{(2q+1)180°}{P-Z} = 60°, 180°, 300° \text{ for } q = 0, 1, 2 \text{ (angle of asymptotes)}$$

$$\text{Centroid} = \frac{0-2+j2-2-j2}{3} = \frac{-4}{3} = -1.33$$

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$$1+\frac{k}{s(s^2+4s+8)} = 0$$

$$\Rightarrow \quad k = -s(s^2 + 4s + 8) = -(s^3 + 4s^2 + 8s)$$

for break away points, $\dfrac{dk}{ds} = 0$

$$\Rightarrow \quad \frac{dk}{ds} = -(3s^2 + 8s + 8)$$

$$s_{1,\,2} = -\frac{8 \pm \sqrt{64 - 4\times 8\times 3}}{2\times 3}$$

$$= -\frac{8 \pm \sqrt{64-96}}{6} = -1.33 \pm j0.943$$

As $\dfrac{dk}{ds}$ is imaginary, there is no breakaway point from the real axis.

Imaginary axis crossing:

Characteristic equation $= s(s^2 + 4s + 8) + k$

$= s^3 + 4s^2 + 8s + k = 0$

s^3	1	8
s^2	4	k
s^1	$\dfrac{32-k}{4}$	
s^0	k	

From Routh-Hurwitz Criteria:

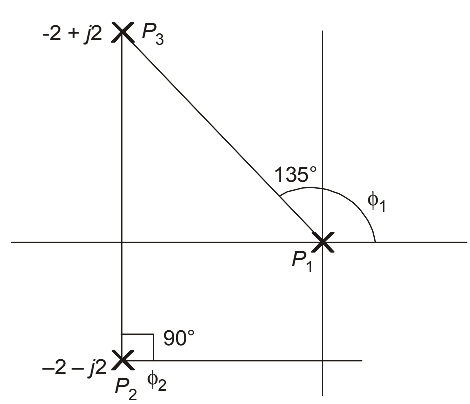
For $k = 32$, the system is marginally stable and beyond $k = 32$ the system becomes unstable.

Hence, $4s^2 + k = 4s^2 + 32$

$$s = j2\sqrt{2} = j\omega$$

$$\omega = 2\sqrt{2} = 2.83$$

The root locus cuts the imaginary axis at $\pm j2.83$.



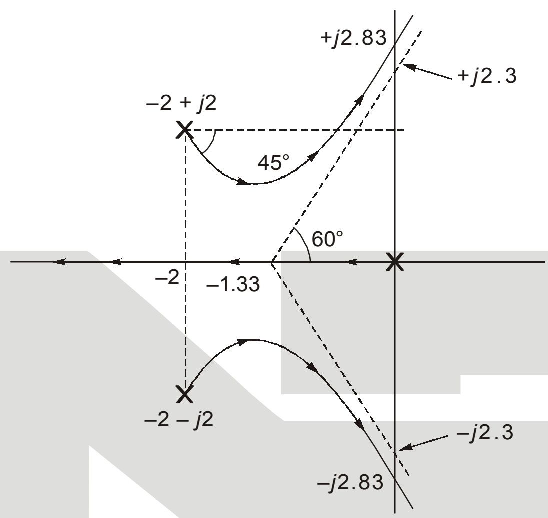
$$\phi_1 = 180° - \tan^{-1}\left(\frac{2}{2}\right) = 135°$$

$$\phi_2 = 90°$$
$$\Sigma\phi_p = \phi_1 + \phi_2 = 135° + 90° = 225°$$
$$\phi = \Sigma\phi_2 - \Sigma\phi_p = 0 - 225° = -225°$$

Angle of departure,

$$\phi_D = 180 + \phi = 180 - 225° = -45°$$

Root locus of the given system:



T3 : Solution

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+4)}$$

Step-1 Number of open loop poles ;

$$P = 3$$

Number of open loop zeros ; $Z = 0$

Number of branches terminating at infinity

$$= P - Z = 3$$

Step-2 Angle of asymptotes

$$\theta = \frac{(2q+1)180°}{P-Z} \quad \text{where } q = 0, 1, 2$$

$$\theta_1 = \frac{180°}{3} = 60°$$

$$\theta_2 = \frac{3 \times 180°}{3} = 180°$$

$$\theta_3 = \frac{5 \times 180°}{3} = 300°$$

Step-3 Centroid $= \dfrac{\Sigma \text{ real part of open loop poles} - \Sigma \text{ real part of open loop zeros}}{P-Z}$

$$= \frac{(-1-4)-(0)}{3-0} = -\frac{5}{3}$$

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Step-4 Break away point

$$K + s(s^2 + 5s + 4) = 0$$
$$K = -s^3 - 5s^2 - 4s$$
$$\frac{dK}{ds} = -3s^2 - 10s - 4 = 0$$
$$3s^2 + 10s + 4 = 0$$
$$\Rightarrow \quad s_1, s_2 = -0.4648, -2.8685$$

Valid break-away point will be –0.4648

(i) Routh array table

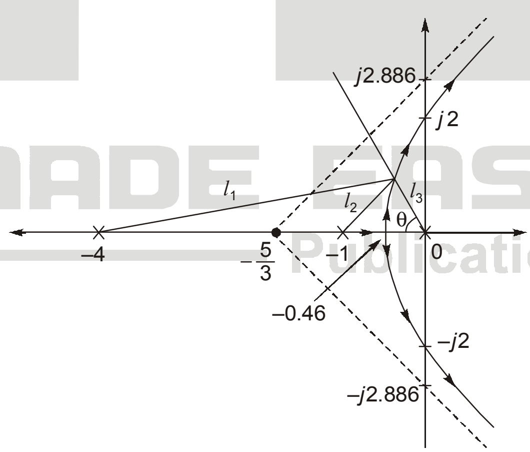
s^3	1	4
s^2	5	K
s^1	$\frac{20-K}{5}$	0
s^0	1	

For system to be stable

$$20 - K > 0 \Rightarrow K < 20$$

For system to be marginally stable.

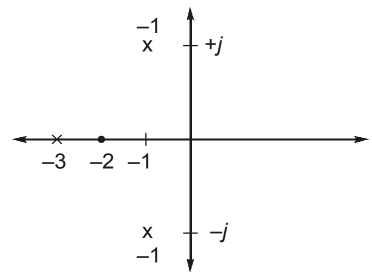
$$K = 20$$
$$A(s) = 5s^2 + 20 = 0$$
$$\Rightarrow \quad s = \pm 2j$$



(ii) To find K,

$$\theta = \cos^{-1}\xi = \cos^{-1}(0.34) = 70.123°$$
$$K = l_1 l_2 l_3$$
$$\text{Gain margin } (GM) = \frac{\text{K(Marginal stability)}}{\text{K(desired)}}$$

T4 : Solution



$$\phi_d = 180 - (\phi_p - \phi_z)$$

$$= 180° - \left(180 + \tan^{-1}\left(\frac{1}{2}\right) + 90° - 225°\right)$$

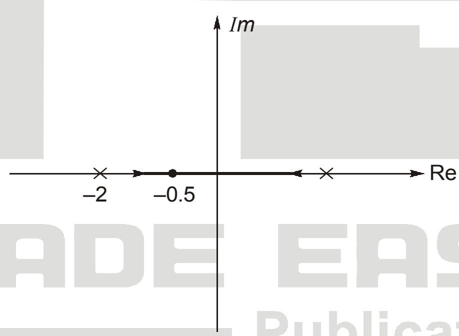
$$= 108.4°$$

T5 : Solution

Given that

$$G(s) = \frac{K}{(s+2)(s-1)}$$

Using root locus method, the break point can be



obtain as

$$\Rightarrow \quad 1 + G(s) = 0$$

$$1 + \frac{K}{(s+2)(s-1)} = 0$$

or $$K = -(s+2)(s-1)$$

$$\frac{dK}{ds} = -2s - 1 = 0$$

or $$s = -0.5$$

To have, both the poles at the same directions

$$|G(s)|_{s=0.5} = 1$$

$$K = 2.25$$

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5 Frequency Response Analysis

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given,

$$G(s) = \frac{K(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$$

$(1 + as)$ is addition of zero to the transfer function whose contribution in slope = +20 dB/decade or –6 dB/octave.

$(1 + bs)$ is addition of pole to the transfer function whose contribution in slope = –20 dB/decade or –6 dB/octave

Observing the change in the slope at different corner frequencies, we conclude that

$$a = \frac{1}{4} \text{ rad/s and } b = \frac{1}{24} \text{ rad/s}$$

From

$$\omega = 0.01 \text{ rad/s to } \omega = 8 \text{ rad/s},$$

$$\text{slope} = -20 \text{ dB/decade}$$

Let the vertical length in dB be y

$$\therefore \quad -20 = \left(\frac{0-y}{\log 8 - \log 0.01}\right)$$

or,

$$-20 = \frac{y}{\log 8 + 2}$$

or,

$$y = 58 \text{ dB}$$

Applying $y = mx + C$ at $\omega = 0.01$ rad/s,

we have:

$$58 = -20 \log 0.01 + C$$

or,

$$C = 58 - 40 = 18$$

Now,

$$C = 20 \log K$$

or,

$$\log K = \frac{18}{20} = 0.9$$

$$\therefore \quad K = \log^{-1}(0.9) = (10)^{0.9} = 7.94$$

$$\therefore \quad \frac{a}{bK} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{24}\right)\times 7.94}$$

$$= \frac{24}{4\times 7.94} = 0.755$$

$$\therefore \quad \frac{a}{bK} = 0.755$$

T2 : Solution

$$\text{OLTF} = G(s) = \frac{1}{(s+2)^2}$$

For unity feedback system,

$$H(s) = 1$$

$$\therefore \quad \text{CLTF} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{(s+2)^2}}{1+\frac{1}{(s+2)^2}}$$

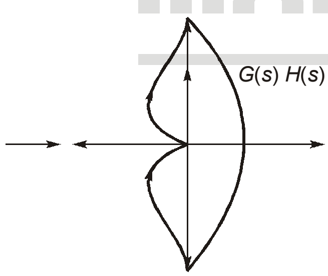
$$= \frac{1}{s^2+4s+5}$$

$\therefore$ Close loop poles will be the roots of $s^2 + 4s + 5 = 0$

i.e. $\quad s = -2 + j$ and $-2 - j$

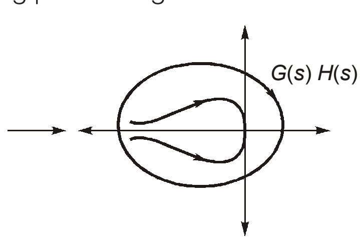
T3 : Solution

$$\frac{1}{s(s+1)}$$



After adding pole at origin

$$\frac{1}{s^2(s+1)}$$



So, nyquist plot of a system will rotate by 90° in clockwise direction.

T4 : Solution

For gain margine we have to find

$$G(s)H(s) = \frac{0.75}{s(1+s)(1+0.5s)}$$

Phase over freqeuncy

$$-180° = -90° - \tan^{-1}(\omega) - \tan^{-1}(0.5 - \omega)$$

$$-90° = \tan^{-1}(\omega) + \tan^{-1}(0.5\,\omega)$$

$$\frac{1.5\omega}{1-0.5\omega^2} = \tan(90°)$$

$$0.5\,\omega^2 = 1$$

$$\omega = \sqrt{2}$$

$$\therefore \quad |G(j\omega)H(j\omega)| = \frac{0.75}{\omega\sqrt{1+\omega^2}\sqrt{1+0.25\omega^2}} = \frac{0.75}{\sqrt{2}\sqrt{1+2}\sqrt{1+0.5}} = \frac{1}{4}$$

$$\therefore \quad \text{Gain margin} = 20\log\frac{1}{|G(j\omega)H(j\omega)|}$$

$$= 20\log 4 = 12$$

T5 : Solution

$$-90° - \tan^{-1}(2\omega) - \tan^{-1}(3\omega) = -180°$$

$$\tan^{-1}(2\omega) + \tan^{-1}(3\omega) = 90°$$

$$\frac{5\omega}{1-6\omega^2} = \tan(90°)$$

$$\therefore \quad 1 - 6\,\omega^2 = 0$$

$$\omega = \frac{1}{\sqrt{6}} = 0.41$$

T6 : Solution

The Bode plot is of type zero system
thus steady state error

$$e_{ss} = \frac{1}{1+K_p}$$

Where K_p = propational error constant

K_p = 40 db

or K_p = 100

$$\therefore \quad e_{ss} = \frac{1}{1+100} = \frac{1}{101} = 0.009$$

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6 Controllers and Compensators

Detailed Explanation of Try Yourself Questions

T1 : Solution

30°

T2 : Solution

The effect of addition of a zero to a transfer function is providing a phase lead.

T3 : Solution

$$G(s) = \frac{\left(1+\frac{s}{4}\right)}{\left(1+\frac{s}{25}\right)}$$

Comparing it with the standard transfer function of phase lead compensator

$$G(s) = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$

$$T = \frac{1}{4}, \quad \alpha T = \frac{1}{25}$$

Now, frequency ω_m occurs at

$$= \sqrt{\frac{1}{\alpha T}\cdot\frac{1}{T}}$$

$$= \sqrt{25\times 4}$$

$$= 10 \text{ rad/sec.}$$

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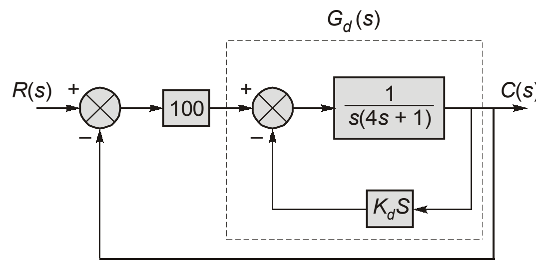
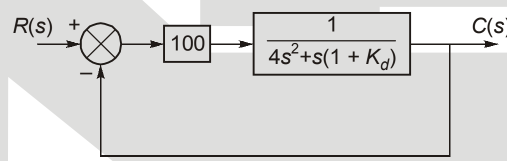
T4 : Solution



Converting the blocked portion into simplified form as

$$G_d(s) = \frac{\frac{1}{s(4s+1)}}{1+\frac{sK_d}{s(4s+1)}} = \frac{1}{4s^2+s(1+K_d)}$$

Now,



Now, simplifying the above block diagram as

$$G(s) = \frac{\frac{100}{4s^2+s(1+K_d)}}{1+\frac{100}{4s^2+s(1+K_d)}}$$

$$= \frac{100}{4s^2+s(1+K_d)+100}$$

$$= \frac{25}{s^2+\frac{s(1+K_d)}{4}+25}$$

Comparing it with standard equation as

$$\omega_n = 5 \text{ rad/sec.}$$

$$2\xi\omega_n = \left(\frac{1+K_d}{4}\right) \qquad \text{Given } \xi = 0.5$$

$$5 = \frac{1+K_d}{4}$$

$\Rightarrow$ $$K_d = 19$$

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7 State Space Analysis

Detailed Explanation of Try Yourself Questions

T1 : Solution

[Ans. : (b)]

T2 : Solution

Given:
$$\dot{x}(t) = Ax(t), \quad x(0) = x_0$$

Taking the Laplace transform

$$sX(s) - x(0) = AX(s)$$
$$[sI - A]\, X(s) = x(0)$$
$$X(s) = [sI - A]^{-1}\, x(0)$$
$$x(t) = \mathcal{L}^{-1}[sI - A]^{-1}\, x(0) \quad \text{...(i)}$$

Conditions given are

For
$$x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$$

For
$$x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad x(t) = \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Using the linearity property in equation (i)

$$K_1\, x_1(t) = \mathcal{L}^{-1}[sI - A]^{-1}\, x_1(0)\, K_1$$
$$K_2\, x_2(t) = \mathcal{L}^{-1}[sI - A]^{-1}\, x_2(0)\, K_2$$

Using the linearity property as

$$K_1\, x_1(t) + K_2\, x_2(t) = \mathcal{L}^{-1}[sI - A]^{-1}\,[K_1 x_1(0) + K_2 x_2(0)] \quad \text{...(ii)}$$

Also
$$X_3(s) = [sI - A]^{-1}\, x_3(0)$$

So,
$$K_1\begin{bmatrix}1\\-1\end{bmatrix}+K_2\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$$
$$\begin{bmatrix}K_1+0K_2\\-K_1+K_2\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$$
$\Rightarrow$
$$K_1 = 3$$
$$K_2 = 8$$
So, from equation (ii), we get $x(t)$
$$x(t) = K_1 x_1(t) + K_2 x_2(t)$$
$$= 3\begin{bmatrix}e^{-t}\\-e^{-t}\end{bmatrix}+8\begin{bmatrix}e^{-t}-e^{-2t}\\-e^{-t}+2e^{-2t}\end{bmatrix}$$
$$= \begin{bmatrix}11e^{-t}-8e^{-2t}\\-11e^{-t}+16e^{-2t}\end{bmatrix}$$

T3 : Solution

Given:
$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$
$$A = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$$
$$[sI-A] = \begin{bmatrix}s & 0\\0 & s\end{bmatrix}-\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}=\begin{bmatrix}s & -1\\0 & s\end{bmatrix}$$
$$[sI-A] = s^2$$
$$\phi(t) = \mathcal{L}^{-1}[sI-A]^{-1}$$
$$= \frac{1}{s^2}\begin{bmatrix}s & 1\\0 & s\end{bmatrix}$$
$$= \mathcal{L}^{-1}\begin{bmatrix}\frac{1}{s} & \frac{1}{s^2}\\0 & \frac{1}{s}\end{bmatrix}=\begin{bmatrix}1 & t\\0 & 1\end{bmatrix}$$

T4 : Solution

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}u$$

$$y = [1 \quad 1 \quad 1]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \; C = [1 \quad 1 \quad 1]$$

Check for controllability:

$$Q_c = [B \; : \; AB \; : \; A^2B]$$

$$= \begin{bmatrix} 0 & 4 & -8 \\ 4 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

For controllable, $|Q_c| \neq 0$

Here, $|Q_c| = 4(0) = 0 \quad \therefore$ Uncontrollable.

Check for observability:

$$Q_o = [C^T \; : \; A^T C^T \; : \; A^{2T} C^T]$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 4 \end{bmatrix}$$

For observable, $|Q_o| \neq 0$

Here $|Q_o| = 1 \quad \therefore$ Observable.

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T5 : Solution

$$\text{Characterstic equation} = \left|(sI - A)^{-1}\right|$$

$$= \begin{bmatrix} 2 & -1 \\ 3 & s+5 \end{bmatrix}^{-1}$$

$$= s(s + 5) + 3$$

$$= s^2 + 5s + 3$$

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