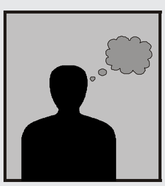
2017

MADE EASY

WORKBOOK



Detailed Explanations of Try Yourself Questions

Electrical Engineering

Electromagnetic Theory

MADE EASY Publications

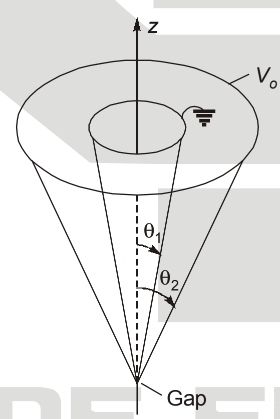
1 Static Electric Fields

Detailed Explanation of Try Yourself Questions

T1 : Solution

Consider the coaxial cone of figure, where the gap serves as an insulator between the two conducting cones. Here V depends only on θ, so Laplace's equation coordinates becomes

$$\nabla^2 V = \frac{1}{r^2 \sin\theta} \frac{d}{d\theta}\left[\sin\theta \frac{dV}{d\theta}\right] = 0$$



Potential $V(\theta)$ due to conducting cones.

Since $r = 0$ and $\theta = 0, \pi$ are excluded, we can multiply by $r^2 \sin\theta$ to get

$$\frac{d}{d\theta}\left[\sin\theta \frac{dV}{d\theta}\right] = 0$$

Integrating once gives

$$\sin\theta \frac{dV}{d\theta} = A$$

or

$$\frac{dV}{d\theta} = \frac{A}{\sin\theta}$$

Integrating this results in

$$V = A\int \frac{d\theta}{\sin\theta} = A\int \frac{d\theta}{2\cos\theta/2\,\sin\theta/2} = A\int \frac{1/2\sec^2\theta/2\,d\theta}{\tan\theta/2} = \int \frac{d(\tan\theta/2)}{\tan\theta/2}$$

$$= A\,ln(\tan\theta/2) + B$$

We now apply the boundary conditions to determine the integration constants A and B.

$$V(\theta = \theta_1) = 0 \rightarrow 0 = A\,ln(\tan\theta_1/2) + B$$

or $$B = -A\,ln(\tan\theta_1/2)$$

Hence $$V = A\,ln\left[\frac{\tan\theta/2}{\tan\theta_1/2}\right]$$

Also $$V(\theta = \theta_2) = V_o \rightarrow V_o = A\,ln\left[\frac{\tan\theta_2/2}{\tan\theta_1/2}\right]$$

or $$A = \frac{V_o}{ln\left[\frac{\tan\theta_2/2}{\tan\theta_1/2}\right]}$$

Thus, $$V = \frac{V_o\,ln\left[\frac{\tan\theta/2}{\tan\theta_1/2}\right]}{ln\left[\frac{\tan\theta_2/2}{\tan\theta_1/2}\right]}$$

$$E = -\nabla V = -\frac{1}{r}\frac{dV}{d\theta}a_\theta = -\frac{A}{r\sin\theta}a_\theta = -\frac{V_o}{r\sin\theta\,ln\left[\frac{\tan\theta_2/2}{\tan\theta_1/2}\right]}a_\theta$$

Taking $\theta_1 = \pi/10$, $\theta_2 = \pi/6$, and $V_o = 50$ gives

$$V = \frac{50\,ln\left[\frac{\tan\theta/2}{\tan\pi/20}\right]}{ln\left[\frac{\tan\pi/12}{\tan\pi/20}\right]} = 95.1\,ln\left[\frac{\tan\theta/2}{0.1584}\right]V$$

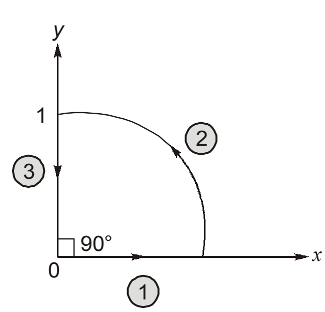
$$E = -\frac{95.1}{r\sin\theta}a_\theta \text{ V/m}$$

T2 : Solution

$$\overrightarrow{dl_1} = d\rho\,\hat{a}_\rho$$

$$\overrightarrow{dl_2} = \rho\,d\phi\,\hat{a}_\phi$$

$$\overrightarrow{dl_3} = d\rho\,\hat{a}_\rho$$



$$\oint_C \vec{E}\cdot\overrightarrow{dl} = \int_0^1 4\rho\sin\phi\,\hat{a}_\rho\cdot d\rho\,\hat{a}_\rho\Big|_{\phi=0} + \int_0^{\pi/2} 4\rho\sin\phi\,\hat{a}_\rho\cdot\rho\,d\phi\,\hat{a}_\phi\Big|_{\rho=1} + \int_1^0 4\rho\sin\phi\,\hat{\rho}\cdot d\rho\,\hat{a}_\rho\Big|_{\phi=90°}$$

$$= -4$$

T3 : Solution

$\vec{E}$ at (3, –2, 1) due to 10 μC/m² is

$$\frac{\rho_s}{2\epsilon_0}\hat{n} = -\frac{10}{2\epsilon_0}(-\hat{x}) = \frac{10\hat{x}}{2\epsilon_0}$$

$\vec{E}$ due to 20 μC/m² is

$$\frac{\rho_s}{2\,\epsilon_0}\hat{n} = \frac{20}{2\,\epsilon_0}(-\hat{z}) = -\frac{20}{2\,\epsilon_0}\,\hat{z}$$

Total,
$$\vec{E} = \frac{5\hat{x}}{\epsilon_0} - \frac{10}{\epsilon_0}\hat{z} = \frac{5}{\epsilon_0}(\hat{x} - 2\hat{z})\ \mu\text{V/m}$$

T4 : Solution

$$\vec{E}_1 = \hat{x} + 2\hat{y} - 2\hat{z}$$

Tangential components are continuous across the boundary ie

$$\vec{E}_{1\tan} = \vec{E}_{2\tan}$$

$\therefore$
$$\vec{E}_2 = \hat{x} + 2\hat{y} + E_{2z}\,\hat{z}$$

Normal components are continuous

$$D_{1z} = D_{2z}$$

$\Rightarrow$
$$\epsilon_1 E_{1z} = \epsilon_2 E_{2z}$$

$$2.(-2) = 4\,E2_z$$

$\Rightarrow$
$$E_{2_z} = -1$$

$\therefore$
$$\vec{E}_2 = \hat{x} + 2\hat{y} - \hat{z}\ \text{V/m}$$

T5 : Solution

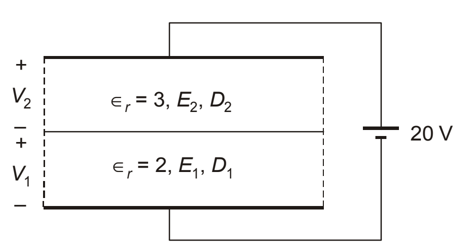
$$\vec{F} = I(\vec{L}\times\vec{B}) = 10\left(2\hat{z}\times 0.02(\hat{y}-\hat{x})\right)$$

$$\frac{\vec{F}}{L} = 0.2\left(\hat{z}\times(\hat{y}-\hat{x})\right) = 0.2(-\hat{x}) - 0.2(\hat{y})$$

$$= -0.2(\hat{x}+\hat{y})\ \text{N/m}$$

T6 : Solution

Let,



$$d = 1\text{ cm}$$
$$A = 100\text{ cm}^2$$

From boundary conditions,

$$D_1 = D_2$$

$$\Rightarrow \quad 2\epsilon_0 E_1 = 3\epsilon_0 E_2$$

$$\Rightarrow \quad E_2 = \frac{2}{3}E_1$$

Also

$$V_2 + V_1 = 20\text{ V}$$

$$\Rightarrow \quad E_2\, d + E_1\, d = 20\text{ V}$$

$$\Rightarrow \quad \left(\frac{2}{3}E_1 + E_1\right)d = 20$$

$$E_1 = \frac{20}{d\left(\frac{5}{3}\right)} = 2000 \times \frac{3}{5} = 1200\text{ V/m}$$

$$D_1 = D_2 = 2\epsilon_0\, E_1 = 21.2\text{ nC/m}^2$$

T7 : Solution

$$|J_c| = |J_d|$$

$$\Rightarrow \quad \sigma = \omega\epsilon$$

$$\Rightarrow \quad \epsilon_0\epsilon_r = \frac{\sigma}{\omega}$$

$$\Rightarrow \quad \epsilon_r = \frac{\sigma}{2\pi f \times \epsilon_0}$$

$$\epsilon_r = \frac{10}{2\pi \times 90 \times 10^9 \times 8.852 \times 10^{-12}}$$

$$= \frac{1}{2\pi \times 9 \times 8.852} \times 1000 = 1.998 \approx 2$$

T8 : Solution

$$\frac{\sigma}{\omega\epsilon} = \tan 2\theta_\eta = \tan 60° = 1.732$$

$$|\eta| = 240 = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} = \frac{\frac{120\pi}{\sqrt{\epsilon_r}}}{(1+3)^{1/4}}$$

$$\Rightarrow \quad \epsilon_r = \frac{\pi^2}{8} = 1.234$$

Complex permittivity

$$\epsilon_c = \epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right) = 1.234 \times \frac{10^{-9}}{36\pi}(1 - j1.732)$$

$$= (10.91 - j18.9) \times 10^{-12}\text{ F/m}$$

$$\therefore \quad x = 10.91$$

$$y = -18.9$$

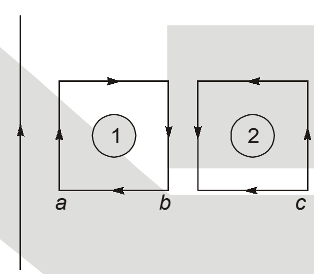
■■■■

2 Static Magnetic Fields

Detailed Explanation *of* Try Yourself Questions

T1 : Solution

(c)



Since the field due to infinite wire is $\propto \frac{1}{r}$.

So, flux from loop 1 will be proportional to $\int_a^b \frac{1}{r} dr$ and flux from loop 2 will be proportional to $\int_b^c \frac{1}{r} dr$.

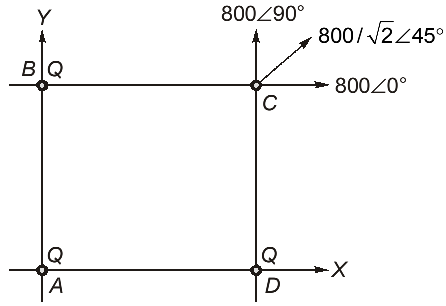
So, $$\int_a^b \frac{1}{r} dr = \int_b^c \frac{1}{r} dr$$

$\Rightarrow$ $$ln(b) - ln(a) = ln(c) - ln(b)$$

$$2\, ln(b) = ln(ac)$$

$\Rightarrow$ $$b = \sqrt{ac}$$

T2 : Solution



Since distance of B and D from C is same so field due to D will be $800\angle 0°$ and distance of A from C is $\sqrt{2}$ times distance of B from C so field due to A will be $\frac{800}{\sqrt{2}}\angle 45°$.

Total field due to changes at A, B, C is,

$$= \left(800\angle 0 + 800\angle 90° + \frac{800}{\sqrt{2}}\angle 45°\right)$$

$$= 1697\angle 45° \text{ V/m}$$

■■■■