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ESE 2023

Main Exam Detailed Solutions

Mechanical Engineering

PAPER-I

EXAM DATE : 25-06-2023 | 09:00 AM to 12:00 PM

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ANALYSIS

Mechanical Engineering Paper-I
ESE 2023 Main Examination

Sl.	Subjects	Marks
1.	Thermodynamics	72
2.	Fluid mechanics	52
3.	Heat Transfer	84
4.	IC Engines	56
5.	Refrigeration and Air-Conditioning	52
6.	Hydraulic Machines	40
7.	Power Plant Engineering	72
8.	Renewable Sources of Energy	52
		Total 480

Scroll down for detailed solutions

SECTION : A

Q.1 **(a)** **(i)** Differentiate between rotational and irrotational flows. Can there be any possibility of having zones possessing characteristics of both rotational and irrotational flows?

(ii) If the expression for the stream function is described by $y = x^3 - 3xy^2$, determine whether the flow is rotational or irrotational. Further, find out the correct expression of the velocity potential function of the following two, considering the flow is irrotational:

1. $\phi = y^3 - 3x^2y$
2. $\phi = -7x^3y$

[6+6=12 marks : 2023]

Solution:

(i)

A flow is said to the rotational if the fluid particles, while moving in the direction of flow, rotate about their own axes. If the fluid particles while in motion do not rotate about their axes, then the flow is irrotational.

The motion of the fluid particles in an irrotational flow can be compared with the motion of the carriages of a giant wheel, in which each carriage is suspended at its toe from the periphery of the wheel. As the wheel revolves, the carriages maintain their vertical position while moving along the circular path.

The flow of all practical fluids are truly rotational because of the tangential and shear stresses due to viscous action. But for simplicity of analysis it is possible to assume the flow to be irrotational in some practical situations by assuming that the viscosity of the fluid has only little significance in regions away from the solid boundary.

In the case of an ideal fluid, shear stresses are zero due to the assumption of zero viscosity. Hence, the flow of an ideal fluid along a straight path is purely irrotational.
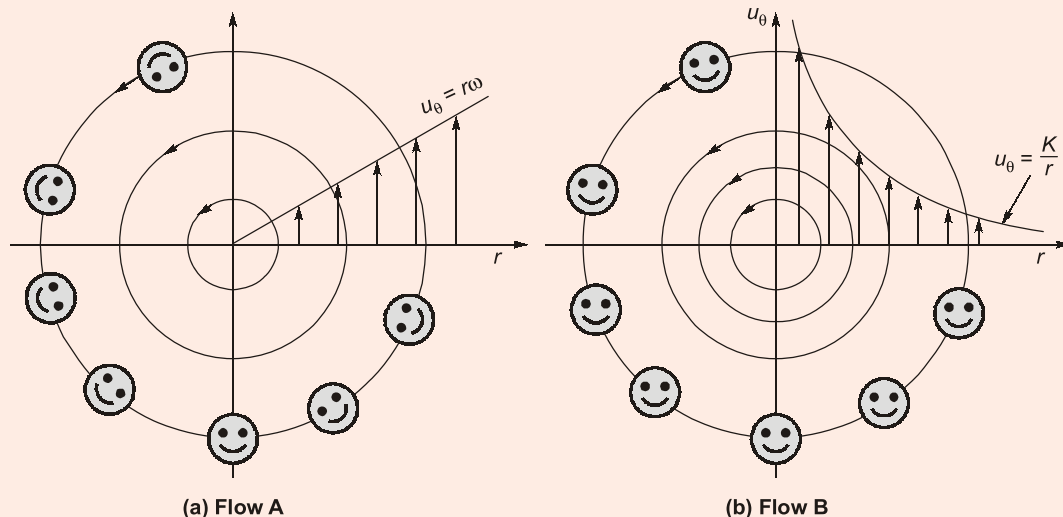
(a) Flow A   **(b) Flow B**

Streamlines and velocity profiles for (a) Flow A, solid-body rotation and (b) Flow B, a line vortex. Flow A is rotational, but flow B is irrotational everywhere except at the origin.

Now, regarding the possibility of having zones possessing characteristics of both rotational and irrotational flows, it is important to note that these two types of flows are fundamentally distinct. In a single flow field, it is not possible to have simultaneous regions with both rotational and irrotational characteristics.

However, it is possible to have transitions or boundaries between rotational and irrotational flow regions within a flow field. These boundaries are known as "vorticity layers" or "vortex sheets." At these interfaces, there can be a sudden change in flow behavior, with vorticity being concentrated or diffused.

In certain situations, such as near solid surfaces or in the presence of complex flow phenomena, there can be regions where the flow exhibits characteristics of both rotational and irrotational behavior. These transitional zones typically occur in areas of flow separation, recirculation, or flow reattachment. However, it's important to note that even in these transitional zones, the flow behavior will predominantly lean towards one characteristic (rotational or irrotational) rather than being equally balanced.

In summary, while it is possible to have transitional regions or boundaries between rotational and irrotational flows, it is not possible to have zones that possess equal characteristics of both types of flows simultaneously within a single flow field.

(ii)

For a possible flow, stream function must satisfy continuity equation and for being irrotational, it must satisfy laplace equation.

$$\psi = x^3 - 3xy^2$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = \frac{\partial}{\partial x}(+6xy) + \frac{\partial}{\partial y}\left(3x^2 - 3y^2\right)$$

$$= 6y - 6y = 0 \quad \text{[Continuity satisfied, flow is possible]}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial}{\partial x}\left(3x^2 - 3y^2\right) + \frac{\partial}{\partial y}(-6xy)$$

$$= 6x - 6x = 0 \quad \text{[Flow field is irrotational]}$$

Velocity potential function corresponding to the stream function, $\psi = x^3 - 3xy^2$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y}$$

$$\Rightarrow \qquad \frac{\partial \phi}{\partial x} = -6xy$$

$$\Rightarrow \qquad \phi = -3x^2y + f(y) + C_1 \qquad \qquad \text{... (i)}$$

and, $$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$\Rightarrow \qquad \frac{\partial \phi}{\partial y} = -(3x^2 - 3y^2)$$

$$\Rightarrow \qquad \phi = -3x^2y + y^3 + f(x) + C_2 \qquad \qquad \text{... (ii)}$$

From equation (i) and (ii),

$$\phi = y^3 - 3x^2y + C$$

Hence, from the options given below, first option i.e. $\phi = y^3 - 3x^2y$ is the correct velocity potential function.

Velocity potential function from the options:

1. $\phi = y^3 - 3x^2y$

For a possible flow velocity potential function must satisfy continuity equation and must be irrotational flow.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial}{\partial x}(-6xy) + \frac{\partial}{\partial y}\left(3y^2 - 3x^2\right)$$

$$= 6y - 6y = 0 \quad [\text{Continuity satisfied, flow is possible}]$$

2. $\phi = -7x^3y$

For a possible flow velocity potential function must satisfy continuity equation and must be irrotational flow.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial}{\partial x}\left(-21x^2y\right) + \frac{\partial}{\partial y}\left(-7x^3\right)$$

$$= -42xy \neq 0 \quad [\text{Continuity not satisfied, flow is not possible}]$$

End of Solution

Q.1 (b) A refrigerated truck whose dimensions ae 12 m × 2.5 m × 3 m is to be precooled from 30°C to an average temperature of 5°C. The construction of the truck is such that a transmission heat gain occurs at the rate of 90 W/°C. If the ambient temperature is 30°C, determine how long it will take for a system with a refrigeration capacity of 10 kW to precool this truck. The density of air may be taken as 1.2 kg/m³ and its specific heat at average temperature of 17.5°C is C_P = 1.0 kJ/kg-°C. State the assumptions, if any.

[12 marks : 2023]

Solution:
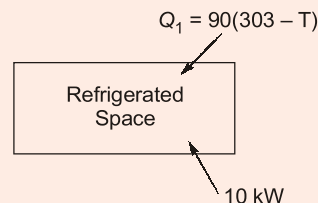
Given: V = 12 m × 2.5 m × 3 m, T_1 = 30°C, T_2 = 5°C, Q_{gain} = 90 W/°C, T_∞ = 30°C, RC = 10 kW, ρ = 1.2 kg/m³, C_P = 1.0 kJ/kg°C

$$M = \rho V = 1.2 \times 12 \times 2.5 \times 3 = 108 \text{ kg}$$

By using first law, $\dot{Q} = \dfrac{du}{dt} \quad [\because W = 0]$

or $90 \times 10^{-3}(303 - T) - 10 = mC_V \dfrac{dT}{dt}$

Where, $C_V = C_P - R = 1 - 0.287 = 0.713 \text{ kJ/kgK}$

$$27.27 - 0.09T - 10 = 108 \times 0.713 \times \frac{dT}{dt}$$

or $0.224 - 1.168 \times 10^{-3} = \dfrac{dT}{dt}$

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$$\therefore \qquad \int_{o}^{t} dt = \int_{303}^{278} \frac{dT}{0.224 - 1.168 \times 10^{-3} \times T}$$

$\therefore$ On solving we get,

$$t = 217.98 \text{ sec}$$

or, $\qquad t = 3.63 \text{ min}$ **Ans.**

End of Solution

Q.1 (c) An engine oil flows through a copper tube of 1 cm internal diameter and 0.02 cm wall thickness at the flow rate of 0.1 kg/s. Consider that the temperature of the oil at the entry is 30°C. If the oil is heated to 50°C by steam condensing at atmospheric pressure, calculate the length of the copper tube. The properties of the oil are as follows:
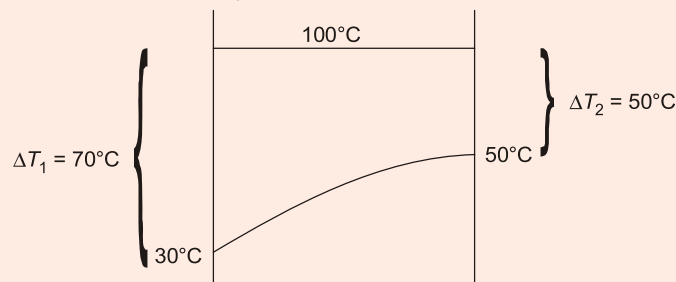
C_P = 1964 J/kg-K, ρ = 876 kg/m³, k = 0.144 W/m-K,

μ = 0.210 N-s/m², Pr = 2870

[12 marks : 2023]

Solution:

Given: $d = 1$ cm $= 0.01$ m, $t = 0.02$ cm, $D = d + 2t = 0.01 + \dfrac{2 \times 0.02}{100} = 0.0104$ m,

$\dot{m} = 0.1 \text{ kg/s}$, $C_P = 1964$ J/kg-K, $\rho_{oil} = 876$ kg/m³, $k = 0.144$ W/m-K, $\mu = 0.210$ N-s/m², Pr = 2870.

$$(\Delta T)_{\text{LMTD}} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \frac{70 - 50}{\ln\left(\dfrac{70}{50}\right)} = 59.44°\text{C}$$

$$\dot{m} = \rho_{oil} \times A \times V_{oil}$$

or, $\qquad V_{oil} = \dfrac{0.1 \times 4}{876 \times \pi \times d^2} = \dfrac{0.1 \times 4}{876 \times \pi \times 0.01^2}$

$$= 1.45346 \text{ m/s}$$

$$\text{Re} = \frac{\rho V d}{\mu} = \frac{876 \times 1.45346 \times 0.01}{0.210} = 60.630 < 2000$$

So, flow is laminar.

So, we know that for constant wall temperature.

$$\overline{\text{Nu}} = 3.66$$

$$\frac{\overline{h}d}{k} = 3.66$$

or,
$$\bar{h} = \frac{3.66 \times 0.144}{0.01} = 52.704 \text{ W}/m^2 - K$$

$$Q = \dot{m}_o \times C_P \times (50 - 30) = \bar{h}A \times (\Delta T)_{LMTD}$$

$$0.1 \times 1964 \times 20 = 52.704 \times A \times 59.44$$

or,
$$A = 1.25386 \text{ m}^2$$

$$\pi DL = 1.25386$$

or,
$$L = \frac{1.25386}{\pi \times 0.0104} = 38.3765 \text{ mtr.}$$
Ans.

End of Solution

Q.1 **(d)** Explain the mechanism of NO_x formation and also the methods for its reduction in stationary gas turbine engines.

[12 marks : 2023]

Solution:

NO_x **Formation:** The oxides of nitrogen are the predominant emissions from stationary gas turbine engines and the one that is controlled by the standards. The most prevalent NO_x emission are nitric oxide, NO, and nitrogen dioxide, NO_2. Nitric oxide is the one mainly formed in the combustion chamber. Factors that influence the amount of NO formed are:

(i)   Peak temperature,

(ii)   Percentage of excess air,

(iii)   Pressure,

(iv)   Residence time at peak temperature and

(v)   Fuel bound nitrogen,

The peak temperature is attained when the fuel is burned with the stoichiometric (chemically correct) amount of air. Higher the temperature of the air at the inlet to the combustion chamber, higher the resulting equilibrium adiabatic flame temperature.

Burning the fuel with excess air lowers the maximum temperature but increases the availability of oxygen and nitrogen in the products of combustion. It is known that for a fixed air supply temperature and combustion chamber pressure, the amount of NO formed for equilibrium conditions increases from 0% excess air to 30% excess air, then starts to decrease even though the adiabatic equilibrium flame temperature decreases continuously. It is a known fact that increasing the combustion temperature, pressure, increase the equilibrium adiabatic flame temperature but decreases the amount of NO formed.

The preceding discussion assumes that equilibrium has been reached. The next important thing is to determine the rate at which at the products will reach equilibrium. The basic mechanism presently used to predict the formation of NO had its origin in the work of Zeldovich and coworkers around 1946. The reader is referred to Wark and Warner for a development of one mechanism that can be used to determine the rate at which NO is formed. The equation developed by Wark and Warner is

$$(1 - Y)^{c+1} (1 + Y)^{c-1} = e^{-Mt}$$

where,
$$Y = \frac{NO}{[NO]_e}$$

$$c = \frac{\left(2.1 \times 10^4\right)\left[x_{N_2}\right]^{0.5} e^{-7750/T}}{T\left[x_{O_2}\right]^{0.5}}$$

$$M = \frac{\left(5.4 \times 10^{15}\right)\left[p\right]^{0.5}\left[x_{N_2}\right]^{0.5} e^{-58330/T}}{T}$$

In the above equations:

$\quad\quad\quad$ [NO] = Concentration of NO

$\quad\quad\quad$ $[NO]_e$ = Concentration of NO at equilibrium

$\quad\quad\quad$ x_{N_2} = Mole fraction of N_2 in the products

$\quad\quad\quad$ x_{O_2} = Mole fraction of O_2 in the products

$\quad\quad\quad$ T = Temperature, K

$\quad\quad\quad$ p = Pressure, atm

NO_x Reduction in Stationary Engines: It is known that the higher the temperature and longer the gases are at that temperature, more nitric oxide is formed. NO_x is the main pollutant from stationary gas turbine engines.

Prior to NO_x emission controls, gas turbine engine combustion chambers were designed so that the fuel-air ratio in the primary zone was approximately the stoichiometric value; that is, the percent excess air in the primary zone was 0%. This resulted in maximum temperature. The maximum temperature can be reduced by designing the combustion chamber so that the primary zone either operates fuel rich (insufficient air for complete combustion) or fuel lean (excess air). Both of these conditions can result in increased smoke (fuel rich) or increased carbon monoxide and total hydrocarbon emissions (fuel lean). Several methods can be used to reduce NO_x emissions such as water or steam injection or staged combustion or selective catalytic reduction.

The most commonly used method of controlling NO_x emissions is with water or steam injection into the primary zone of the combustion chamber. The water (or steam) injected acts as a heat sink, resulting in a lower maximum temperature, thereby reducing the amount of NO_x formed. The rate at which water is injected is approximately 50% of the fuel flow. Steam, rates are usually 100-200% of the fuel flow.

Staged combustion is currently being tested by a number of manufacturers. It provides a way of achieving NO emission levels of 25 ppmvd or less at 15% oxygen without using water or steam injection. Most of the systems being tested use a two-stage premixed combustor for use with natural gas. The resulting mixture is lean so the amount of NO_x is low.

Selective catalytic reduction involves injecting ammonia into the gas turbine engine exhaust steam. The exhaust gases then pass over a catalyst where the NO_x reacts with the ammonia (NH_3), oxygen (O_2) and nitrogen, (N_2) to form water, (H_2O), and nitrogen (N_2). When combined with water or steam injection, it is reported that NO_x levels of 10 ppm or less can be achieved. One major disadvantage is that the reaction is very much temperature dependent. For a vanadium pentoxide type catalyst, the exhaust gas temperature range for best operation is 600-750°F. For this reason, the selective catalytic reduction method for reducing NO_x emission is limited to combined cycles only.

End of Solution

Q.1 (e) (i) Why are higher heat transfer rates experienced in dropwise condensation than in film condensation?

(ii) Distinguish between nucleate boiling and film boiling.

[6+6=12 marks : 2023]

Solution:

(i)

There are two distinct forms of condensation which are observed: film condensation and dropwise condensation. In film condensation, the condensate wets the surface and forms a liquid film on the surface that slides down under the influence of gravity. In dropwise condensation, the condensed vapor forms droplets on the surface instead of a continuous film and surface is covered by countless drops of varying diameters.

In dropwise condensation, the small droplets that form at the nucleation sites on the surface grow as a result of continued, coalesce into large droplets, and slide down when reach a certain size, clearing the surface and exposing it to the vapor. There is no liquid film in this case to resist heat transfer. As a result, with dropwise condensation, heat transfer coefficients can be achieved that are more than 10 times larger than those associated with film condensation. Large heat transfer coefficients enable designers to achieve a specified heat transfer rate with a smaller surface area.

(ii)
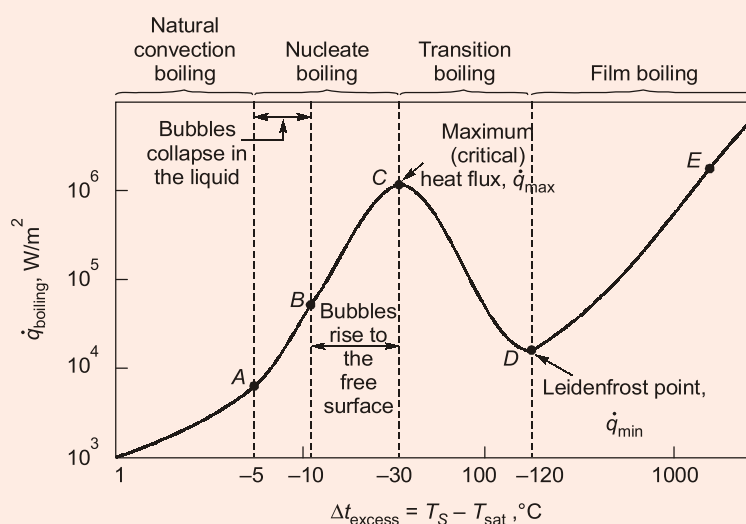
Typical boiling curve for water at 1 atm

Nucleate Boiling: The nucleate boiling can be represented between points A and C. Point A is referred to as the onset of nucleate boiling (ONB). The bubbles form at an increasing rate at an increasing number of nucleation sites as we move along the boiling curve towards point C.

The nucleate boiling regime can be separated into two distinct regions. In region A-B, ($5°C \leq \Delta T_{excess} \leq 10°C$), isolated bubbles are formed at various preferential nucleation sites on the heated surface. In region B-C ($10°C \leq \Delta T_{excess} \leq 30°C$), the heater temperature is further increased, and bubbles form at such great rates at such a large number of nucleation sites that they form numerous continuous columns of vapor in the liquid. These bubbles move all

the way up to the free surface, where they break up and release their vapor content. The large heat fluxes obtainable in this region are caused by the combined effect of entrainment and evaporation.

Film boiling: In this, region, the heater surface is completely covered by a continuous stable vapor film. Point D, where the heat flux reaches a minimum is called Leidenfrost point. The presence of a vapor film between the heater surface and the liquid is responsible for the low heat transfer rates in the film boiling region. The heat transfer rate increases with increasing excess temperature as a result of heat transfer from the heated surface to the liquid through the vapor film by radiation, which becomes significant at high temperatures.

End of Solution

Q.2 (a) (i) Find the distance from the pipe wall at which the local velocity is equal to the average velocity for turbulent flow in pipe.

(ii) Distinguish between hydrodynamically smooth and rough boundaries.

[12+8=20 marks : 2023]

Solution:

(i)

Velocity distribution for turbulent flow is given by,

$$\frac{u-v}{V^*} = 5.75\log_{10}\left(\frac{y}{R}\right)+3.75$$

For local velocity (u) = Average velocity (v)

$$0 = 5.75\log_{10}\left(\frac{y}{R}\right)+3.75$$

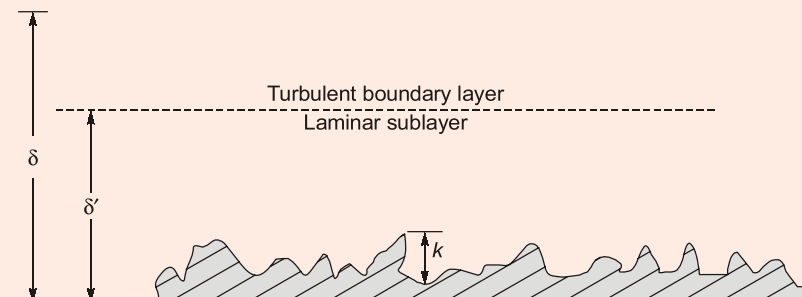$$\Rightarrow \quad \log_{10}\left(\frac{y}{R}\right) = -\frac{3.75}{5.75}$$

$$\Rightarrow \quad \frac{y}{R} = 0.222754$$

$$\Rightarrow \quad y = (0.222754)R$$

At a distance of $(0.222754)R$, the local velocity is equal to the average velocity for turbulent flow in pipe.

(ii)

Hydrodynamically smooth boundary: When the average depth k of the surface irregularities is less than laminar sub-layer of the surface δ. The eddy which is formed outside of the laminar sub-layer tries to penetrate in the laminar sub-layer boundary.

Hydrodynamically rough boundary: When the average depth k of the surface irregularities is greater than laminar sub-layer of surface δ. The eddy which is formed outside of the laminar sub-layer penetrates the laminar sub-layer.
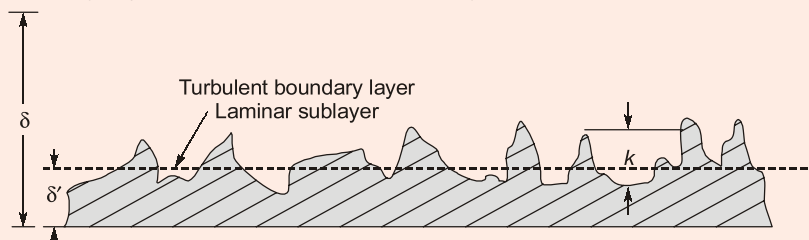
End of Solution

Q.2 **(b)** **(i)** In a closed system, 3 kg of air at initial conditions of 400 kPa and 90°C adiabatically expands until its volume is 2.5 times the initial volume and temperature becomes equal to that of surroundings. If the conditions of the surroundings are 100 kPa and 25°C, determine the following for this process:

1. The maximum work
2. The change in availability
3. The irreversibility

(ii) Prove that for an ideal gas, the slope of an isochoric line on the T-s diagram is more than that of the isobaric line.

[15+5=20 marks : 2023]

Solution:

(i)

Given: Mass of air, m = 3 kg, Initial pressure, P_1 = 400 kPa, Initial temperature, T_1 = 90°C = 363 K, Final volume, V_2 = 2.5 V_1, Final temperature, T_2 = 25°C = 298 K = T_o, Ambient pressure, P_o = 100 kPa.

From the property relation,

$$Tds = dU + Pdv$$

The entropy change of air between the initial and final states is

$$\int_1^2 ds = \int_1^2 mC_V \frac{dT}{T} + \int_1^2 \frac{mRdv}{T}$$

$\Rightarrow$

$$s_2 - s_1 = mC_V \ln\frac{T_2}{T_1} + mR\ln\frac{V_2}{V_1}$$

1. **Maximum work:**

$$W_{max} = (u_2 - U_1) - T_o(S_1 - S_2)$$

$$= m\left[C_V(T_1 - T_2) + T_o\left(C_V \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}\right)\right]$$

$$= 3\left[0.718(363 - 298) + 298\left(0.718\ln\frac{298}{363} + 0.287\ln 2.5\right)\right]$$

$$= 248.458 \text{ kJ} \qquad\qquad \text{Ans.}$$

2. The change in availability:

$$CAE = \phi_1 - \phi_2$$
$$= (U_1 - U_2) - T_o(S_1 - S_2) + P_o(V_1 - V_2)$$
$$= W_{max} + P_o(V_1 - V_2) = 248.458 + 100 \times (V_1 - 2.5V_1)$$
$$= 248.458 - 100 \times 1.5\ V_1 = 248.458 - 150 \times \frac{mRT_1}{P_1}$$
$$= 248.458 - 150 \times \frac{3 \times 0.287 \times 363}{400} = 131.254\ kJ \qquad \textbf{Ans.}$$
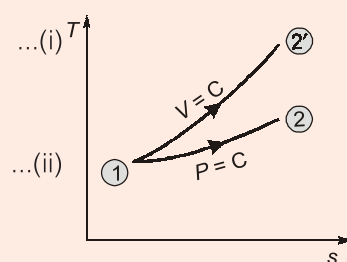
3. The irreversibility:

$$I = T_o(\Delta S)_{universe}$$
$$(\Delta S)_{universe} = (\Delta S)_{sys} + (\Delta S)_{surr.}$$

For adiabatic process, $(\Delta S)_{surr.} = 0$

$\therefore$ $\qquad (\Delta S)_{universe} = (\Delta S)_{sys}$

So, $\qquad I = T_o(\Delta S)_{sys} = 298 \times \left[mC_V \ln\frac{T_2}{T_1} + mR\ln\frac{V_2}{V_1} \right]$

$$= 298 \times \left[3 \times 0.718\ln\frac{298}{363} + 3 \times 0.287\ln 2.5 \right]$$
$$= 108.448\ kJ \qquad \textbf{Ans.}$$

(ii)

Writing the two TdS equations

$$TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp \qquad\qquad ...(i)$$

$$TdS = C_v dT + T\left(\frac{\partial p}{\partial T}\right)_V dV \qquad\qquad ...(ii)$$

At constant pressure Eq. (i) reduces to

$$T(dS)_p = C_p(dT)_p - 0$$

$\Rightarrow$ $\qquad \left(\frac{\partial T}{\partial S}\right)_p = \frac{T}{C_p}$ (slope of constant pressure line on T-s diagram)

$$...(iii)$$

At constant volume Eq. (ii) reduces to

$$T(dS)_v = C_v(dT)_v$$

$\Rightarrow$ $\qquad \left(\frac{\partial T}{\partial S}\right)_v = \frac{T}{C_v}$ (slope of constant volume line on T-s diagram)

$$...(iv)$$

$$\frac{(\partial T/\partial S)_v}{(\partial T/\partial S)_p} = \frac{C_p}{C_v} \text{ as } C_p > C_v \Rightarrow \frac{C_p}{C_v} > 1 \Rightarrow \left(\frac{\partial T}{\partial S}\right)_v > \left(\frac{\partial T}{\partial S}\right)_p$$

End of Solution

Q.2 (c) A square plate heater (15 cm × 15 cm) is inserted between two slabs. Slab A is 2 cm thick (k = 50 W/m-°C) and slab B is 1 cm thick (k = 0.2 W/m-°C). The outside heat transfer coefficients on side of A and side of B are 200 W/m²-°C and 50 W/m²-°C respectively. The temperature of surrounding air is 25°C. If the rating of heater is 1 kW, find the

(i) Maximum temperature of the system;

(ii) Outer surface temperature of two slabs.

Assume steady-state heat flow. [20 marks : 2023]

Solution:

Q_{heater} = 1 kW

A = 0.15 × 0.15 = 0.0225 m²
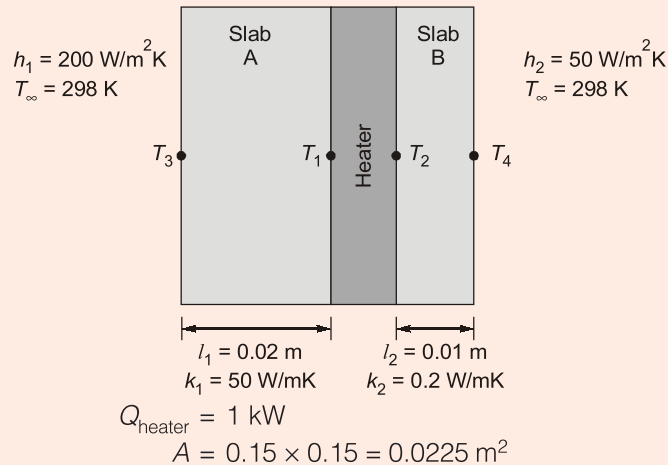
Heat transfer through slab-A:

$$Q_1 = \frac{T_1 - T_3}{\left(\dfrac{l_1}{k_1 A}\right)} = \frac{T_3 - T_\infty}{\left(\dfrac{1}{h_1 A}\right)} = \frac{T_1 - T_\infty}{\left(\dfrac{l_1}{k_1 A}\right) + \left(\dfrac{1}{h_1 A}\right)}$$

$$= \frac{T_1 - T_3}{\left(\dfrac{0.02}{50 \times (0.0225)}\right)} = \frac{T_3 - 298}{\left(\dfrac{1}{200 \times (0.0225)}\right)} = \frac{T_1 - 298}{\left(\dfrac{0.02}{50 \times (0.0225)}\right) + \left(\dfrac{1}{200 \times (0.0225)}\right)}$$

$$Q_1 = \frac{T_1 - T_3}{0.017778} = \frac{T_3 - 298}{0.222222} = \frac{T_1 - 298}{0.24} \quad \dots \text{(i)}$$

Heat transfer through slab-B:

$$Q_2 = \frac{T_2 - T_4}{\left(\dfrac{l_2}{k_2 A}\right)} = \frac{T_4 - T_\infty}{\left(\dfrac{1}{h_2 A}\right)} = \frac{T_2 - T_\infty}{\left(\dfrac{l_2}{k_2 A}\right) + \left(\dfrac{1}{h_2 A}\right)}$$

$$= \frac{T_2 - T_4}{\left(\dfrac{0.01}{0.2 \times (0.0225)}\right)} = \frac{T_4 - 298}{\left(\dfrac{1}{50 \times (0.0225)}\right)} = \frac{T_2 - T_\infty}{\left(\dfrac{0.01}{0.2 \times (0.0225)}\right) + \left(\dfrac{1}{50 \times (0.0225)}\right)}$$

$$Q_2 = \frac{T_2 - T_4}{2.222222} = \frac{T_4 - 298}{0.888888} = \frac{T_2 - 298}{3.111111} \quad \dots \text{(ii)}$$

$$Q_{heater} = Q_1 + Q_2$$

$$1 \times 10^3 = \frac{T_1 - 298}{0.24} + \frac{T_2 - 298}{3.111111}$$

Assuming $T_1 = T_2$ because heater plate is very thin. So, temperature on both sides of heater will be same.

$$1 \times 10^3 = \frac{T_1 - 298}{0.24} + \frac{T_1 - 298}{3.111111}$$

$$\Rightarrow \qquad T_1 = T_2 = 520.81 \text{ K} = 247.81°\text{C}$$

The maximum temperature is 520.81 K i.e. 247.81°C

Substituting the value of T_1 in equation (i):

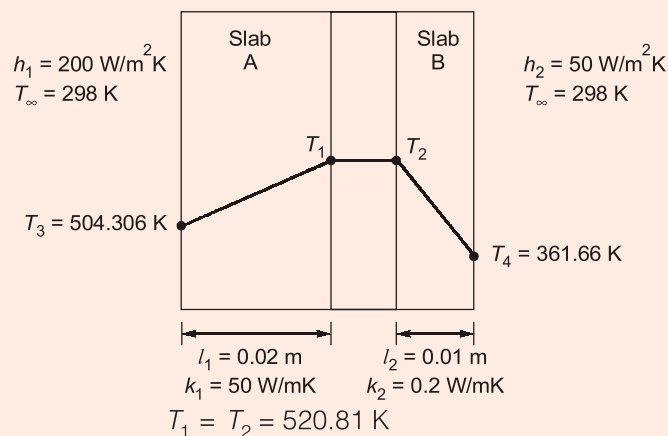$$\frac{520.81 - T_3}{0.017778} = \frac{T_3 - 298}{0.222222} = \frac{520.81 - 298}{0.24}$$

$$\Rightarrow \qquad T_3 = 504.306 \text{ K} = 271.306°\text{C}$$

Substituting the value of T_2 in equation (ii):

$$\frac{520.81 - T_4}{2.222222} = \frac{T_4 - 298}{0.888888} = \frac{520.81 - 298}{3.111111}$$

$$\Rightarrow \qquad T_4 = 361.66 \text{ K} = 88.66°\text{C}$$

The outer temperature towards slab- A is 504.306 K i.e. 271.306°C and towards slab-B is 361.66 K i.e. 88.66°C.

$$h_1 = 200 \text{ W/m}^2\text{K}$$
$$T_\infty = 298 \text{ K}$$

Slab A

Slab B

$$h_2 = 50 \text{ W/m}^2\text{K}$$
$$T_\infty = 298 \text{ K}$$

$$T_1$$
$$T_2$$

$$T_3 = 504.306 \text{ K}$$

$$T_4 = 361.66 \text{ K}$$

$$l_1 = 0.02 \text{ m}$$
$$k_1 = 50 \text{ W/mK}$$

$$l_2 = 0.01 \text{ m}$$
$$k_2 = 0.2 \text{ W/mK}$$

$$T_1 = T_2 = 520.81 \text{ K}$$

End of Solution

Q.3 (a) A centrifugal pump discharges 2000 litres of water developing a head of 20 m when running at 300 r.p.m. The impeller diameter at the outlet and outlet flow velocity are 1.5 m and 3.0 m/s respectively. If the blade are set back at an angle of 30° at the outlet, determine the-
   (i) Manometric efficiency;
   (ii) Power required by the pump;
   (iii) Minimum speed to start the pump if the inner diameter is 750 mm.

[20 marks : 2023]

Solution:

Given: $Q = 2000$ litres/s, $H_m = 20$ m, $N = 300$ rpm, $d_2 = 1.5$ m, $r_2 = 0.75$ m, $V_{F_2} = 3$ m/s, $\phi = 30°$

(i) **Manometric efficiency (η_m):**

$$\eta_m = \frac{gH_m}{u_2 V_{w_2}}$$

$$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 1.5 \times 300}{60} = 23.562 \text{ m/s}$$

$$V_{w2} = u_2 - V_{F_2} \times \cot\phi$$

$$= (23.562) - 3 \times \cot(30°) = 18.3658 \text{ m/s}$$

$$\eta_m = \frac{9.81 \times 20}{(23.562) \times (18.3658)} = 0.4534 = 45.34\%$$

(ii) Power requirement:

$$\eta_o = \eta_m = 45.34\%$$

$$\text{Power required} = \frac{\text{Output power of motor}}{\eta_o}$$

$$= \frac{\rho g Q H}{\eta_o} = \frac{1000 \times 9.81 \times \left(\frac{2000}{1000}\right) \times 20}{(0.4534)}$$

$$= 865460.96 \text{ W} = 865.46096 \text{ kW}$$

(iii) Minimum starting speed of pump:

$$r_1 = \frac{750}{2} = 375 \text{ mm} = 0.375 \text{ m}$$

$$\frac{2\pi N_{min}}{60} = \sqrt{\frac{2gH_m}{\left(r_2^2 - r_1^2\right)}}$$

$$N_{min} = \frac{60}{2\pi}\sqrt{\frac{2gH_m}{\left(r_2^2 - r_1^2\right)}} = \frac{60}{2\pi} \times \sqrt{\frac{2 \times 9.81 \times 20}{(0.75)^2 - (0.375)^2}}$$

$$= 291.235 \text{ rpm}$$

End of Solution

Q.3 (b) Air flows at 12 m/s past a smooth rectangular flat plate 0.4 m wide and 3 m long. Assuming that the transition occurs at Re = 5.5×10^5, calculate the total drag force when:

(i) the flow is parallel to the length of the plate;

(ii) the flow is parallel to the width of the plate

Assume:

Density of air, ρ = 1.24 kg/m³

Kinematic viscosity, ν = 0.15 stokes   [20 marks : 2023]

Solution:

Given; U_∞ = 12 m/s, L = 3 m, b = 0.4 m, $(Re)_{cr} = 5.5 \times 10^5$, ρ = 1.24 kg/m³, ν = 0.15 stokes = 0.15×10^{-4} m²/s

(i) Flow is parallel to length of the plate:

Plate Reynolds number, $Re_L = \dfrac{U_\infty L}{\nu} = \dfrac{12 \times 3}{0.15 \times 10^{-4}} = 24 \times 10^5$

$$Re_L > (Re)_{cr}$$

Reynolds number, $(Re)_{cr} = \dfrac{U_\infty x}{\nu}$

$$\Rightarrow \qquad 5.5 \times 10^5 = \frac{12 \times x}{0.15 \times 10^{-4}}$$

$$\Rightarrow \qquad x = 0.6875 \text{ m}$$

Boundary layer is laminar over initial length x and for remaining length of $(L-x)$, it is turbulent. The friction drag over an initial length x over which the boundary layer remains laminar is:

$$(F_{DF})_{\text{laminar}} = \frac{1.328}{\sqrt{Re_L}} \times \frac{1}{2}\rho U_\infty^2 x \times b$$

$$= \frac{1.328}{\sqrt{5.5\times 10^5}} \times \frac{1}{2} \times 1.24 \times 12^2 \times (0.6875) \times 0.4 = 0.04396 \text{ N}$$

The friction drag of turbulent boundary layer over the rear portion of the plate is:

$$(F_{DF})_{\text{turbulent}} = \left[\frac{0.074L}{(Re_L)^{1/5}} - \frac{0.074x}{(Re_x)_{cr}^{1/5}}\right] \times \frac{1}{2}\rho U_\infty^2 \times b$$

$$= \left[\frac{0.074\times 3}{(24\times 10^5)^{1/5}} - \frac{0.074\times 0.6875}{(5.5\times 10^5)^{1/5}}\right] \times \frac{1}{2} \times 1.24 \times 12^2 \times 0.4$$

$$= 0.29068 \text{ N}$$

$$(F_{DF})_{\text{total}} = (F_{DF})_{\text{laminar}} + (F_{DF})_{\text{turbulent}}$$
$$= 0.04396 + 0.29068 = 0.33464 \text{ N}$$

(ii) Flow is parallel to width of the plate:

Plate Reynolds number, $\quad Re_b = \dfrac{U_\infty b}{\nu} = \dfrac{12\times 0.4}{0.15\times 10^{-4}} = 3.2\times 10^5$

$$Re_b < (Re)_{cr}$$

$\Rightarrow \quad$ Boundary layer is laminar.

Average friction drag coefficient for the laminar boundary layer is given by:

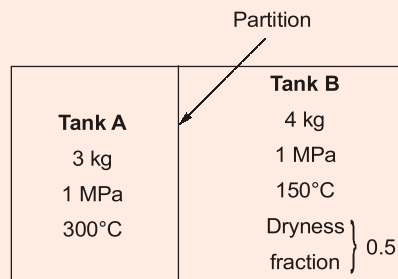$$C_f = \frac{1.328}{\sqrt{Re_b}} = \frac{1.328}{\sqrt{3.2\times 10^5}} = 2.3476\times 10^{-3}$$

Friction drag, $\qquad F_{DF} = C_f \times \dfrac{1}{2}\rho U_\infty^2 \times \text{ Area of plate}$

$$= \left(2.3476\times 10^{-3}\right) \times \frac{1}{2} \times 1.24 \times 12^2 \times (3\times 0.4) = 0.25151 \text{ N}$$

End of Solution

Q.3 (c) Two tanks, tank A and tank B, are separated by a partition as shown in the figure. Tank A contains 3 kg of steam at 1 MPa and 300°C. Tank B contains 4 kg of saturated liquid-vapour mixture at 150°C with a dryness fraction of 0.5.

The partition is removed and two fluids are allowed to mix until the thermal equilibrium and mechanical equilibrium are acquired. If the pressure of the final state is 300 kPa, determine:

(i) The temperature of the final state;

(ii) The quality of the steam at final state;

(iii) The amount of heat lost from the tanks.

Partition

Tank A	Tank B
3 kg	4 kg
1 MPa	1 MPa
300°C	150°C
	Dryness fraction 0.5

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Steam Table

T	P = 200 kPa (120.23)				P = 300 kPa (133.55)				P = 400 kPa (143.63)			
	v	u	h	s	v	u	h	s	v	u	h	s
900	2.70643	3854.5	4395.8	9.4565	1.80406	3854.2	4395.4	9.2691	1.35288	3853.9	4395.1	9.1361
1000	2.93740	4052.5	4640.0	9.6563	1.95812	4052.3	4639.7	9.4689	1.46847	4052.0	4639.4	9.3360
1100	3.16834	4257.0	4890.7	9.8458	2.11214	4256.8	4890.4	9.6585	1.58404	4256.5	4890.1	9.5255
1200	3.39927	4467.5	5147.3	10.0262	2.26614	4467.2	5147.1	9.8389	1.69958	4467.0	5146.8	9.7059
1300	3.63018	4683.2	5409.3	10.1982	2.42013	4683.0	5409.0	10.0109	1.81511	4682.8	5408.8	9.8780

T	P = 500 kPa (151.86)				P = 600 kPa (158.85)				P = 800 kPa (170.43)			
	v	u	h	s	v	u	h	s	v	u	h	s
Sat.	0.37489	2561.2	2748.7	6.8212	0.31567	2567.4	2756.8	6.7600	0.24043	2576.8	2769.1	6.6627
200	0.42492	2642.9	2855.4	7.0592	0.35202	2638.9	2850.1	6.9665	0.26080	2630.6	2839.2	6.8158
250	0.47436	2723.5	2960.7	7.2708	0.39383	2720.9	2957.2	7.1816	0.29314	2715.5	2950.0	7.0384
300	0.52256	2802.9	3064.2	7.4598	0.43437	2801.0	3061.6	7.3723	0.32411	2797.1	3056.4	7.2372
350	0.57012	2882.6	3167.6	7.6328	0.47424	2881.1	3165.7	7.5463	0.35439	2878.2	3161.7	7.4088
400	0.61728	2963.2	3271.8	7.7937	0.51372	2962.0	3270.2	7.7078	0.38426	2959.7	3267.1	7.5715
500	0.71093	3128.4	3483.8	8.0872	0.59199	3127.6	3482.7	8.0020	0.44331	3125.9	3480.6	7.8672
600	0.80406	3299.6	3701.7	8.3521	0.66974	3299.1	3700.9	8.2673	0.50184	3297.9	3699.4	8.1332
700	0.89691	3477.5	3926.0	8.5952	0.74720	3477.1	3925.4	8.5107	0.56007	3476.2	3924.3	8.3770
800	0.98959	3662.2	4157.0	8.8211	0.82450	3661.8	4156.5	8.7367	0.61813	3661.1	4155.7	8.6033
900	1.08217	3853.6	4394.7	9.0329	0.90169	3853.3	4394.4	8.9485	0.67610	3852.8	4393.6	8.8153
1000	1.17469	4051.8	4639.1	9.2328	0.97883	4051.5	4638.8	9.1484	0.73401	4051.0	4638.2	9.0153
1100	1.26718	4256.3	4889.9	9.4224	1.05594	4256.1	4889.6	9.3381	0.79188	4255.6	4889.1	9.2049
1200	1.35964	4466.8	5146.6	9.6028	1.13302	4466.5	5146.3	9.5185	0.84974	4466.1	5145.8	9.3854
1300	1.45210	4682.5	5408.6	9.7749	1.21009	4682.3	5408.3	9.6906	0.90758	4681.8	5407.9	9.5575

T	P = 1.00 MPa (179.91)				P = 1.20 MPa (187.99)				P = 1.40 MPa (195.07)			
	v	u	h	s	v	u	h	s	v	u	h	s
Sat.	0.19444	2583.6	2778.1	6.5864	0.16333	2588.8	2784.8	6.5233	0.14084	2592.8	2790.0	6.4692
200	0.20596	2621.9	2827.9	6.6939	0.16930	2612.7	2815.9	6.5898	0.14302	2603.1	2803.3	6.4975
250	0.23268	2709.9	2942.6	6.9246	0.19235	2704.2	2935.0	6.8293	0.16350	2698.3	2927.2	6.7467
300	0.25794	2793.2	3051.2	7.1228	0.21382	2789.2	3045.8	7.0316	0.18228	2785.2	3040.4	6.9533
350	0.28247	2875.2	3157.7	7.3010	0.23452	2872.2	3153.6	7.2120	0.20026	2869.1	3149.5	7.1359
400	0.30659	2957.3	3263.9	7.4650	0.25480	2954.9	3260.7	7.3773	0.21780	2952.5	3257.4	7.3025
500	0.35411	3124.3	3478.4	7.7621	0.29463	3122.7	3476.3	7.6758	0.25215	3121.1	3474.1	7.6026

Steam Table

Temp. °C T	Pressure kPa, MPa P	Specific Volume, m³/kg		Internal Energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg-K		
		Sat. Liquid v_f	Sat. Vapour v_g	Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
105	0.12082	0.001047	1.4194	440.00	2072.3	2512.3	440.13	2243.7	2683.8	1.3629	5.9328	7.2958
110	0.14328	0.001052	1.2102	461.12	2057.0	2518.1	461.27	2230.2	2691.5	1.4184	5.8202	7.2386
115	0.16906	0.001056	1.0366	482.28	2041.4	2523.7	482.46	2216.5	2699.0	1.4733	5.7100	7.1832
120	0.19853	0.001060	0.8919	503.48	2025.8	2529.2	503.69	2202.6	2706.3	1.5275	5.6020	7.1295
125	0.2321	0.001065	0.77059	524.72	2009.9	2534.6	524.96	2188.5	2713.5	1.5812	5.4962	7.0774
130	0.2701	0.001070	0.66850	546.00	1993.9	2539.9	546.29	2174.2	2720.5	1.6343	5.3925	7.0269
135	0.3130	0.001075	0.58217	567.34	1977.7	2545.0	567.67	2159.6	2727.3	1.6869	5.2907	6.9777
140	0.3613	0.001080	0.50885	588.72	1961.3	2550.0	589.11	2144.8	2733.9	1.7390	5.1908	6.9298
145	0.4154	0.001085	0.44632	610.16	1944.7	2554.9	610.61	2129.6	2740.3	1.7906	5.0926	6.8832
150	0.4759	0.001090	0.39278	631.66	1927.9	2559.5	632.18	2114.3	2746.4	1.8417	4.9960	6.8378
155	0.5431	0.001096	0.34676	653.23	1910.8	2564.0	653.82	2098.6	2752.4	1.8924	4.9010	6.7934
160	0.6178	0.001102	0.30706	674.85	1893.5	2568.4	675.53	2082.6	2758.1	1.9426	4.8075	6.7501
165	0.7005	0.001108	0.27269	696.55	1876.0	2572.5	697.32	2066.2	2763.5	1.9924	4.7153	6.7078
170	0.7917	0.001114	0.24283	718.31	1858.1	2576.5	719.20	2049.5	2768.7	2.0418	4.6244	6.6663
175	0.8920	0.001121	0.21680	740.16	1840.0	2580.2	741.16	2032.4	2773.6	2.0909	4.5347	6.6256
180	1.0022	0.001127	0.19405	762.08	1821.6	2583.7	763.21	2015.0	2778.2	2.1395	4.4461	6.5857
185	1.1227	0.001134	0.17409	784.08	1802.9	2587.0	785.36	1997.1	2782.4	2.1878	4.3586	6.5464
190	1.2544	0.001141	0.15654	806.17	1783.8	2590.0	807.61	1978.8	2786.4	2.2358	4.2720	6.5078
195	1.3978	0.001149	0.14105	828.36	1764.4	2592.8	829.96	1960.0	2790.0	2.2835	4.1863	6.4697
200	1.5538	0.001156	0.12736	850.64	1744.7	2595.3	852.43	1940.7	2793.2	2.3308	4.1014	6.4322
205	1.7230	0.001164	0.11521	873.02	1724.5	2597.5	875.03	1921.0	2796.0	2.3779	4.0172	6.3951
210	1.9063	0.001173	0.10441	895.51	1703.9	2599.4	897.75	1900.7	2798.5	2.4247	3.9337	6.3584
215	2.1042	0.001181	0.09479	918.12	1682.9	2601.1	920.61	1879.9	2800.5	2.4713	3.8507	6.3221
220	2.3178	0.001190	0.08619	940.85	1661.5	2602.3	943.61	1858.5	2802.1	2.5177	3.7683	6.2860

Steam Table

Pressure MPa P	Temp. °C T	Specific Volume, m³/kg		Internal Energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg-K		
		Sat. Liquid v_f	Sat. Vapour v_g	Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
0.275	130.60	0.001070	0.65573	548.57	1992.0	2540.5	548.87	2172.4	2721.3	1.6407	5.3801	7.0208
0.300	133.55	0.001073	0.60058	561.13	1982.4	2543.6	561.45	2163.9	2725.3	1.6717	5.3201	6.9918
0.325	136.30	0.001076	0.5620	572.88	1973.5	2546.3	573.23	2155.8	2729.0	1.7005	5.2646	6.9651
0.350	138.88	0.001079	0.5243	583.93	1965.0	2548.9	584.31	2148.1	2732.4	1.7274	5.2130	6.9404
0.375	141.32	0.001081	0.4914	594.38	1956.9	2551.3	594.79	2140.8	2735.6	1.7527	5.1647	6.9174
0.40	143.63	0.001084	0.4625	604.29	1949.3	2553.6	604.73	2133.8	2738.5	1.7766	5.1193	6.8958
0.45	147.93	0.001088	0.4140	622.75	1934.9	2557.6	623.24	2120.7	2743.9	1.8206	5.0359	6.8565
0.50	151.86	0.001093	0.37490	639.66	1921.6	2561.2	640.21	2108.5	2748.7	1.8606	4.9606	6.8212
0.55	155.48	0.001097	0.34270	655.30	1909.2	2564.5	655.91	2097.0	2752.9	1.8972	4.8920	6.7892
0.60	158.85	0.001101	0.31570	669.88	1897.5	2567.4	670.54	2086.3	2756.8	1.9311	4.8289	6.7600
0.65	162.01	0.001104	0.29270	683.55	1886.5	2570.1	684.26	2076.0	2760.3	1.9627	4.7704	6.7330
0.70	164.97	0.001108	0.27290	696.43	1876.1	2572.5	697.20	2066.3	2763.5	1.9922	4.7158	6.7080
0.75	167.77	0.001111	0.25560	708.62	1866.1	2574.7	709.45	2057.0	2766.4	2.0199	4.6647	6.6846
0.80	170.43	0.001115	0.24040	720.20	1856.6	2576.8	721.10	2048.0	2769.1	2.0461	4.6166	6.6627
0.85	172.96	0.001118	0.22270	731.25	1847.4	2578.7	732.20	2039.4	2771.6	2.0709	4.5711	6.6421
0.90	175.38	0.001121	0.21500	741.81	1838.7	2580.5	742.82	2031.1	2773.9	2.0946	4.5280	6.6225
0.95	177.69	0.001124	0.2042	751.94	1830.2	2582.1	753.00	2023.1	2776.1	2.1171	4.4869	6.6040
1.00	179.91	0.001127	0.19444	761.67	1822.0	2583.6	762.79	2015.3	2778.1	2.1386	4.4478	6.5864
1.10	184.09	0.001133	0.17753	780.08	1806.3	2586.4	781.32	2000.4	2781.7	2.1791	4.3744	6.5535
1.20	187.99	0.001139	0.16333	797.27	1791.6	2588.8	798.64	1986.2	2784.8	2.2165	4.3067	6.5233
1.30	191.64	0.001144	0.15125	813.42	1777.5	2590.9	814.91	1972.7	2787.6	2.2514	4.2438	6.4953
1.40	195.07	0.001149	0.14084	828.68	1764.1	2592.8	830.29	1959.7	2790.0	2.2842	4.1850	6.4692
1.50	198.32	0.001154	0.13177	843.14	1751.3	2594.5	844.87	1947.3	2792.1	2.3150	4.2198	6.4448
1.75	205.76	0.001166	0.11349	876.44	1721.4	2597.8	878.48	1918.0	2796.4	2.3851	4.0044	6.3895
2.00	212.42	0.001177	0.09963	906.42	1693.8	2600.3	908.77	1890.7	2799.5	2.4473	3.8935	6.3408
2.25	218.45	0.001187	0.08875	933.81	1668.2	2602.0	936.48	1865.2	2801.7	2.5034	3.7938	6.2971

[20 marks : 2023]

Solution:

(i)

Given:

Tank A:

Mass, m_A = 3 kg, Initial pressure, $(P_1)_A$ = 1 MPa, Initial temperature, $(T_1)_A$ = 300°C = 573 K.

Tank B:

Mass, m_B = 4 kg, Initial temperature, $(T_1)_B$ = 150°C = 423 K

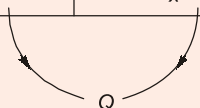Dryness fraction, x_B = 0.5

Final pressure after mixing = 300 kPa

We take the contents of both the tanks as system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$(\delta Q) = (dU)$$

$$\Rightarrow \qquad \delta Q = (dU)_A + (dU)_B$$

$$\delta Q = [m(u_2 - u_1)]_A + [m(u_2 - u_1)]_B \quad \dots \text{(i)}$$

The properties of steam in both the tanks at the initial states are:

For tank A:

At $(P_1)_A$ = 1 MPa and $(T_1)_A$ = 300°C,

$$(v_1)_A = 0.25799 \text{ m}^3/\text{kg}$$
$$(u_1)_A = 2793.7 \text{ kJ/kg}$$

For tank B:

At $(T_1)_B$ = 150°C and $(x_1)_B$ = 0.5,

$$v_f = 0.001091 \text{ m}^3/\text{kg}, \ v_g = 0.39248 \text{ m}^3/\text{kg}$$
$$u_f = 631.66 \text{ kJ/kg } u_{fg} = 1927.4 \text{ kJ/kg}$$

$$\therefore \qquad (v_1)_B = v_f + (x_1)_B \, v_{fg}$$
$$= 0.001091 + 0.5 \times (0.39248 - 0.001091)$$
$$= 0.19679 \text{ m}^3/\text{kg}$$

$$(u_1)_B = u_f + (x_1)_B \, u_{fg}$$
$$= 631.66 + 0.5 \times 1927.4$$
$$= 1595.36 \text{ kJ/kg}$$

The total volume and the total mass of the system:

$$V_T = V_A + V_B = m_A(v_1)_A + m_B(v_1)_B$$
$$= 3 \times 0.25799 + 4 \times 0.19679$$
$$= 1.56113 \text{ m}^3$$

$$m_T = m_A + m_B = 3 + 4 = 7 \text{ kg}$$

Specific volume at the final state:

$$v_2 = \frac{V_T}{m_T} = \frac{1.56113}{7} = 0.2230 \text{ m}^3/\text{kg}$$

So, at the final state, $P_2 = 300$ kPa and $v_2 = 0.2230$ m³/kg

(i) The temperature of the final state:

At $P_2 = 300$ kPa and $v_2 = 0.2230$ m³/kg, $T_2 = T_{sat} = 133.52°$ **Ans.**

(ii) The quality of the steam at the final state:

At $P_2 = 300$ kPa and $T = 133.52°C$

$$v_f = 0.001073 \text{ m}^3/\text{kg}, \ v_g = 0.60582 \text{ m}^3/\text{kg}$$

$\therefore$ $v_2 = v_f + x_2 v_{fg}$

$\Rightarrow$ $0.2230 = 0.001073 + x_2 \times (0.60582 - 0.001073)$

$\Rightarrow$ $x_2 = 0.367$ **Ans.**

(iii) The amount of heat lost from the tanks:

At $P_2 = 300$ kPa and $T = 133.52°C$

$u_f = 561.11$ kJ/kg, $u_{fg} = 1982.1$ kJ/kg

$\therefore$ $u_2 = 561.11 + 0.367 \times 1982.1 = 1288.540$ kJ/kg

From equation (i), $\delta Q = [m(u_2 - u_1)]_A + [m(u_2 - u_1)]_B$

                      $= [3 \times (1288.54 - 2793.7)] + [4 \times (1288.54 - 1595.36)]$

                      $= -5742.76$ kJ

$\therefore$ Heat lost $= 5742.76$ K   **Ans.**

End of Solution

Q.4 **(a)** A truncated cone has top and bottom diameters of 10 cm and 20 cm respectively, and a height of 10 cm. Calculate the shape factor between the top surface and the side, and also the shape factor between the side and itself. Use the figure showing the radiation shape factor for radiation between two parallel coaxial disks:
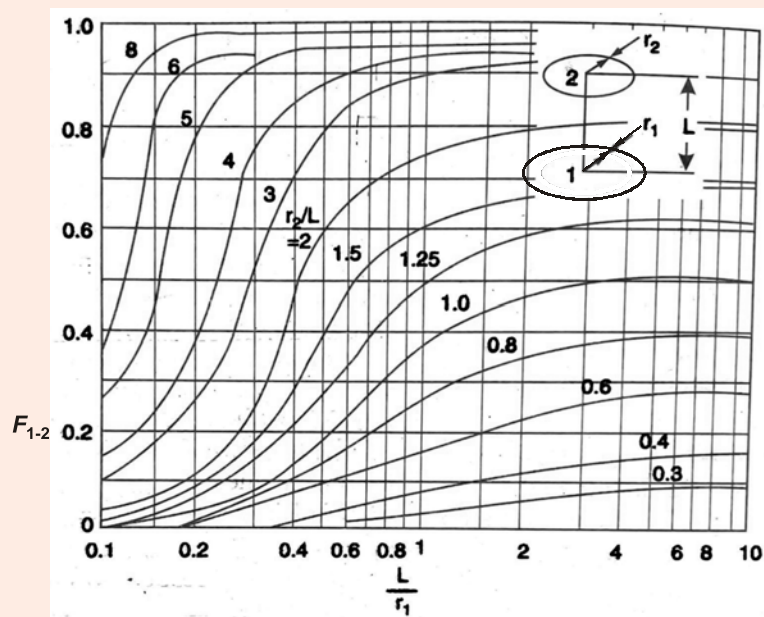
Radiation shape factor for co-axial parallel plates

[20 marks : 2023]

Solution:

Given: $h = L = 10$ cm, $d_1 = 20$ cm, $r_1 = 10$ cm, $d_2 = 10$ cm, $r_2 = 5$ cm, $F_{23} = ?$, $F_{33} = ?$
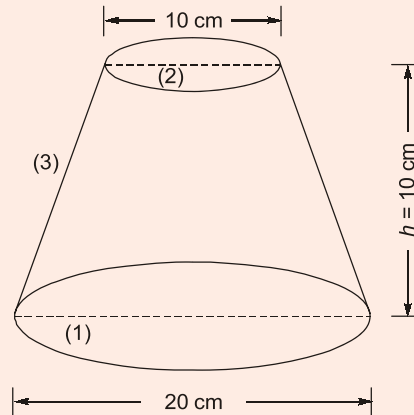
Curve surface area of the frustum:

$$A = \pi(r_2 + r_1)\sqrt{(r_1 - r_2)^2 + h^2}$$

$$= \pi(5 + 10)\sqrt{(10 - 5)^2 + 10^2} = 75\pi\sqrt{5}$$

$$= 526.8611 \text{ cm}^2$$

$$\frac{r_2}{L} = \frac{5}{10} = 0.5 \text{ and } \frac{L}{r_1} = \frac{10}{10} = 1$$

From the given graph, the shape factor F_{12} corresponding to $\left(\dfrac{r_2}{L} = 0.5 \text{ and } \dfrac{L}{r_1} = 1\right)$ is approximately 0.1.

$$F_{12} = 0.1$$

By reciprocity theorem:

$$A_1 F_{12} = A_2 F_{21}$$

$$\Rightarrow \qquad F_{21} = \frac{A_1}{A_2} \times F_{12} = \frac{\pi \times (10)^2}{\pi \times (5)^2} \times 0.1 = 0.4$$

By summation rule,

$$F_{11} + F_{12} + F_{13} = 1$$
$$0 + 0.1 + F_{13} = 1 \qquad [\because F_{11} = 0]$$
$$\Rightarrow \qquad F_{13} = 0.9$$

Also,

$$F_{21} + F_{22} + F_{23} = 1$$
$$0.4 + 0 + F_{23} = 1 \qquad [\because F_{22} = 0]$$
$$\Rightarrow \qquad F_{23} = 0.6$$

By reciprocity theorem:

$$F_{23} = \frac{A_2}{A_3} \times F_{23} = \frac{\pi(5)^2}{526.8611} \times (0.6) = 0.08944$$

$$F_{31} = \frac{A_1}{A_3} \times F_{13} = \frac{\pi(10)^2}{526.8611} \times (0.9) = 0.53666$$

By summation rule:

$$F_{31} + F_{32} + F_{33} = 1$$
$$\Rightarrow \qquad F_{33} = 1 - F_{31} - F_{32} = 1 - (0.53666) - (0.08944)$$
$$= 0.3739$$

Hence, the shape factor between the top and side is 0.6 and the shape factor between the side and itself is 0.3739,

End of Solution

Q.4 (b) A Francis turbine supplied through an 8.0 m diameter penstock has the following particulars:

Output power = 65000 kW
Speed = 150 rpm
Hydraulic efficiency = 90%
Flow rate = 120 m³/s
Mean diameter of turbine at entry = 5 m
Mean blade height at entry = 1.5 m
Entry diameter of draft tube = 4.5 m
Velocity in tailrace = 2.5 m/s

The static pressure head in the penstock measured just before entry to the runner is 60 m. The point of measurement is 3.2 m above the level of the tailrace. The loss in the draft tube is equivalent to 30% of the velocity head at entry to it. The exit plane of the runner is 2 m above the tailrace and the flow leaves the runner without swirl. Calculate:

(i) The overall efficiency
(ii) The direction of flow relative to the runner at inlet.
(iii) The pressure head at entry to draft tube.

[20 marks : 2023]

Solution:

Given: Q = 120 m³/s, d = 8 m, d_1 = 5 m, d_2 = 4.5 m, b_1 = 1.5 m, N = 150 rpm, Output power

= 65000 kW, $\dfrac{P}{\rho g}$ = 60 m, z = 3.2 m, η_h = 0.90, z_2 = 2 m, V_e = 2.5 m/s.

Velocity of water in penstock:

$$V = \frac{Q}{\dfrac{\pi}{4}d^2} = \frac{120}{\dfrac{\pi}{4} \times 8^2} = 2.387 \text{ m/s}$$

Velocity at entry to the draft tube:

$$V_2 = \frac{Q}{\dfrac{\pi}{4}d_2^2} = \frac{120}{\dfrac{\pi}{4} \times (4.5)^2} = 7.545 \text{ m/s}$$

Heat just before entry to the runner,

$$H_1 = \frac{P}{\rho g} + z + \frac{v^2}{2g} = 60 + 3.2 + \frac{(2.387)^2}{2 \times 9.81} = 63.49 \text{ m}$$

Effective head = H_1 – Kinetic head at tail race

$$H = 63.49 - \frac{(2.5)^2}{2 \times 9.81} = 63.17 \text{ m}$$

Input power = $\rho g Q H = 1000 \times 9.81 \times 120 \times 63.17 = 74363.724 \text{ kW}$

$$\text{Overall efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{65000}{74363.724}$$

$$= 0.874 = 87.4\%$$

Direction of flow relative to runner:

$$\eta_h = \frac{u_1 \times V_{w1}}{gH}$$

$$V_{w1} = \frac{\eta_h \times gH}{u_1} = \frac{\eta_h \times gH}{\left(\frac{\pi d_1 N}{60}\right)} = \frac{0.9 \times 9.81 \times 63.17}{\left(\frac{\pi \times 5 \times 150}{60}\right)} = 14.202 \text{ m/s}$$

From inlet velocity diagram:
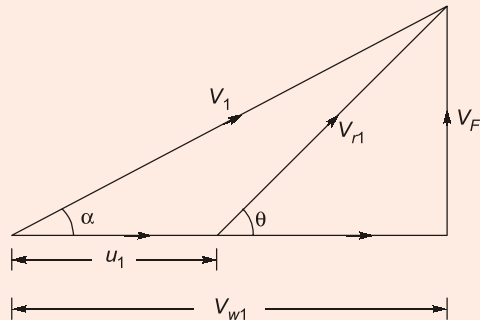
$$\tan\theta = \frac{V_{F_1}}{V_{w1} - u_1} = \frac{\left(\frac{Q}{\pi d_1 b_1}\right)}{(V_{w1} - u_1)} = \frac{\left(\frac{120}{\pi \times 5 \times 1.5}\right)}{14.202 - \left(\frac{\pi \times 5 \times 150}{60}\right)}$$

$\Rightarrow$ $\tan\theta = -0.203166$

$\Rightarrow$ $\theta = (180° - 11.4843°) = 168.516°$

Applying Bernoulli's equation between the entrance to the draft tube and the tail race.

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = 0 + \frac{V_e^2}{2g} + (0.3) \times \frac{V_2^2}{2g}$$

$$\frac{P_2}{\rho g} = Z_2 - (0.7)\frac{V_2^2}{2g} + \frac{V_e^2}{2g} = -2 - (0.7) \times \frac{(7.545)^2}{2 \times 9.81} + \frac{(2.5)^2}{2 \times 9.81}$$

$$= -3.7125 \text{ m}$$

End of Solution

Q.4 (c) Two containers are connected with a pipe having a closed valve. One contains a 5 kg mixture of 62.5% CO_2 and 37.5% O_2 on a mole basis at 30°C and 125 kPa. The second container contains 10 kg of N_2 at 15°C and 200 kPa. The valve in the pipe is opened and gases are allowed to mix. During the mixing process, 100 kJ of heat energy is supplied to the combined tank. Determine the volume of the mixture and write an energy balance equation.

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Molar mass, gas constant, and critical-point properties

Substance	Formula	Molar mass, M kg/kmol	Gas constant, R kJ/kg·K*	Critical-point properties Temperature, K	Critical-point properties Pressure, MPa	Critical-point properties Volume, m³/kmol
Air	—	28.97	0.2870	132.5	3.77	0.0883
Ammonia	NH_3	17.03	0.4882	405.5	11.28	0.0724
Argon	Ar	39.948	0.2081	151	4.86	0.0749
Benzene	C_6H_6	78.115	0.1064	562	4.92	0.2603
Bromine	Br_2	159.808	0.0520	584	10.34	0.1355
n-Butane	C_4H_{10}	58.124	0.1430	425.2	3.80	0.2547
Carbon dioxide	CO_2	44.01	0.1889	304.2	7.39	0.0943
Carbon monoxide	CO	28.011	0.2968	133	3.50	0.0930
Carbon tetrachloride	CCl_4	153.82	0.05405	556.4	4.56	0.2759
Chlorine	Cl_2	70.906	0.1173	417	7.71	0.1242
Chloroform	$CHCl_3$	119.38	0.06964	536.6	5.47	0.2403
Dichlorodifluoromethane (R-12)	CCl_2F_2	120.91	0.06876	384.7	4.01	0.2179
Dichlorofluoromethane (R-21)	$CHCl_2F$	102.92	0.08078	451.7	5.17	0.1973
Ethane	C_2H_6	30.070	0.2765	305.5	4.48	0.1480
Ethyl alcohol	C_2H_5OH	46.07	0.1805	516	6.38	0.1673
Ethylene	C_2H_4	28.054	0.2964	282.4	5.12	0.1242
Helium	He	4.003	2.0769	5.3	0.23	0.0578
n-Hexane	C_6H_{14}	86.179	0.09647	507.9	3.03	0.3677
Hydrogen (normal)	H_2	2.016	4.1240	33.3	1.30	0.0649
Krypton	Kr	83.80	0.09921	209.4	5.50	0.0924
Methane	CH_4	16.043	0.5182	191.1	4.64	0.0993
Methyl alcohol	CH_3OH	32.042	0.2595	513.2	7.95	0.1180
Methyl chloride	CH_3Cl	50.488	0.1647	416.3	6.68	0.1430
Neon	Ne	20.183	0.4119	44.5	2.73	0.0417
Nitrogen	N_2	28.013	0.2968	126.2	3.39	0.0899
Nitrous oxide	N_2O	44.013	0.1889	309.7	7.27	0.0961
Oxygen	O_2	31.999	0.2598	154.8	5.08	0.0780
Propane	C_3H_8	44.097	0.1885	370	4.26	0.1998
Propylene	C_3H_6	42.081	0.1976	365	4.62	0.1810
Sulfur dioxide	SO_2	64.063	0.1298	430.7	7.88	0.1217
Tetrafluoroethane (R-134a)	CF_3CH_2F	102.03	0.08149	374.2	4.059	0.1993
Trichlorofluoromethane (R-11)	CCl_3F	137.37	0.06052	471.2	4.38	0.2478
Water	H_2O	18.015	0.4615	647.1	22.06	0.0560
Xenon	Xe	131.30	0.06332	289.8	5.88	0.1186

*The unit kJ/kg·K is equivalent to kPa·m³/kg·K. The gas constant is calculated from $R = R_u/M$, where $R_u = 8.31447$ kJ/kmol·K and M is the molar mass.

Ideal-gas specific heats of various common gases

At 300 K

Gas	Formula	Gas constant, R kJ/kg·K	c_p kJ/kg·K	c_v kJ/kg·K	k
Air	—	0.2870	1.005	0.718	1.400
Argon	Ar	0.2081	0.5203	0.3122	1.667
Butane	C_4H_{10}	0.1433	1.7164	1.5734	1.091
Carbon dioxide	CO_2	0.1889	0.846	0.657	1.289
Carbon monoxide	CO	0.2968	1.040	0.744	1.400
Ethane	C_2H_6	0.2765	1.7662	1.4897	1.186
Ethylene	C_2H_4	0.2964	1.5482	1.2518	1.237
Helium	He	2.0769	5.1926	3.1156	1.667
Hydrogen	H_2	4.1240	14.307	10.183	1.405
Methane	CH_4	0.5182	2.2537	1.7354	1.299
Neon	Ne	0.4119	1.0299	0.6179	1.667
Nitrogen	N_2	0.2968	1.039	0.743	1.400
Octane	C_8H_{18}	0.0729	1.7113	1.6385	1.044
Oxygen	O_2	0.2598	0.918	0.658	1.395
Propane	C_3H_8	0.1885	1.6794	1.4909	1.126
Steam	H_2O	0.4615	1.8723	1.4108	1.327

Note : The unit kJ/kg·K is equivalent to kJ/kg·°C.

[10 marks : 2023]

Solution:

Given:

Tank 1
62.5% CO_2
37.5% O_2
5 kg
30°C
125 kPa

Tank 2
N_2
10 kg
15°C
200 kPa

$Q = 100$ kJ
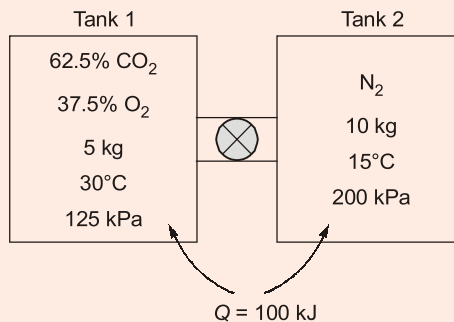
Assumptions:

CO_2, O_2 and N_2 can be treated as an ideal gas and thus the mixture can be also be treated as an ideal gas.

Using the table:

The molar masses of CO_2, O_2 and N_2 are 44.01 kg/kmol, 31.999 kg/kmol and 28.013 kg/kmol respectively. The gas constants for CO_2, O_2 and N_2 are 0.1889 kJ/kg-K, 0.2598 kJ/kg-K and 0.2968 kJ/kg-K respectively.

The molar mass of the mixture in tank 1 is

$$M_1 = x_{CO_2}M_{CO_2} + x_{O_2}M_{O_2}$$
$$= 0.625 \times 44.01 + 0.375 \times 31.999$$
$$= 39.5058 \text{ kg/kmol.}$$

The gas constant in tank 1:

$$R_1 = \frac{\bar{R}}{M_1} = \frac{8.314}{39.5058} = 0.2104 \text{ kJ/kg–K}$$

Volume of tank 1:

$$V_1 = \frac{m_1 R_1 T_1}{P_1} = \frac{5 \times 0.2104 \times (30 + 273)}{125} = 2.55 \text{ m}^3$$

Volume of tank 2:

$$V_2 = \frac{m_2 R_2 T_2}{P_2} = \frac{10 \times 0.2968 \times (15 + 273)}{200} = 4.274 \text{ m}^3$$

So,   Total volume, $V_T = V_1 + V_2 = (2.55 + 4.274) \text{ m}^3 = 6.824 \text{ m}^3$ **Ans.**

The mass in tank 1:

$$m_{CO_2} = n_{CO_2} \times M_{CO_2} = x_{CO_2} \times n_{total} \times M_{CO_2}$$
$$= 0.625 \times \frac{M_{total}}{M_1} \times M_{CO_2} = 0.625 \times \frac{5}{39.5058} \times 44.01$$

$$m_{CO_2} = 3.481 \text{ kg}$$

Similarly, $$m_{O_2} = x_{O_2} \times n_{total} \times M_{O_2}$$
$$= 0.375 \times \frac{5}{39.5058} \times 31.999 = 1.5187 \text{ kg}$$

Let us assume T_f be the final temperature after mixing.

The energy balance equation can be written as:

$$\delta Q = dU + \cancel{\delta W}^{o} \quad \text{[No work interaction]}$$

$$\Rightarrow \quad \delta Q = (dU)_{CO_2} + (dU)_{O_2} + (dU)_{N_2}$$

$$\Rightarrow \quad 100 = [mC_V(T_f - T_1)]_{CO_2} + [mC_V(T_f - T_1)]_{O_2} + [mC_V(T_f - T_2)]_{N_2}$$

Using the table, the values of C_V for CO_2, O_2 and N_2 are 0.657 kJ/kg-K, 0.658 kJ/kg-K and 0.743 kJ/kg-K respectively.

$$\Rightarrow \quad 100 = [3.4815 \times 0.657 (T_f - 30)] + [1.5185 \times 0.658 (T_f - 30)] + [10 \times 0.743 (T_f - 15)]$$

End of Solution

SECTION : B

Q.5 (a) A six-cylinder SI engine operates on a four-stroke cycle. The bore of each cylinder is 75 mm and the stroke is 100 mm. The clearance volume per cylinder is 60 cc. At a speed of 4000 rpm, the fuel consumption is 18 kg/h and the torque developed is 140 Nm. Calculate the
   (i) brake thermal efficiency;
   (ii) relative efficiency on the basis of brake power.
   The calorific value of the fuel can be taken as 45000 kJ/kg.

   [12 marks : 2023]

Solution:

Given : Number of cylinder, $n = 6$; Bore, $d = 75$ mm = 0.075 m; Stroke, $L = 100$ mm = 0.1 m

Clearance volume, $V_C = 60$ cc = 60×10^{-6} m^3

Engine speed, $N = 4000$ rpm

Fuel consumption, $\dot{m}_f = 18$ kg/hr = 5×10^{-3} kg/s

Torque developed, $T = 140$ Nm

Calorific value of the fuel,

$$CV = 45000 \text{ kJ/kg}$$

(i) Brake power, B.P. $= \dfrac{2\pi NT}{60}$

$$\text{B.P.} = \frac{2\pi \times 4000 \times 140}{60} = 58643.063 \text{ W}$$

$$\text{B.P.} = 58.643 \text{ kW}$$

Brake thermal efficiency is given by

$$\eta_{b,th} = \frac{BP}{\dot{m}_f \times CV} \times 100$$

$$\eta_{b,th} = \frac{58.643}{5 \times 10^{-3} \times 45000} \times 100 \simeq 26.1\%$$

(ii) Stroke volume, $V_s = \dfrac{\pi}{4} d^2 l = \dfrac{\pi}{4}(7.5)^2 \times 10 = 441.79$ cc

$$\text{Compression ratio, } r = \frac{V_c + V_s}{V_c} = \frac{60 + 441.79}{60} = 8.36$$

$$\text{Air-standard efficiency, } \eta_{otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8.36^{0.4}} \qquad [\text{Assuming } \gamma = 1.4]$$

$$\eta_{otto} = 0.572 \text{ or } 57.2\%$$

Thus, relative efficiency on the basis of brake power is

$$\eta_{rel} = \frac{\eta_{b,th}}{\eta_{otto}} = \frac{26.1}{57.2} = 0.456 \text{ or } 45.6\%$$

End of Solution

Q.5 **(b)** Draw the T-s and h-s diagrams for stream jet refrigeration system and write the expressions for the following:
   (i)   Nozzle efficiency
   (ii) Entrainment efficiency
   (iii) Compression efficiency

[12 marks : 2023]

Solution:

The working cycle of the steam jet refrigeration system is represented on T-s and h-s chart as shown in figure below.

p_b = Pressure of steam supplied from boiler
p_c = Pressure in the condensor
p_e = Pressure in the evaporator in the flash chamber
ab = Isentropic expansion of steam through nozzle
ab' = Actual expansion of steam through nozzle
a = Condition of steam supplied
c = Condition of water vapour formed in the flash chamber
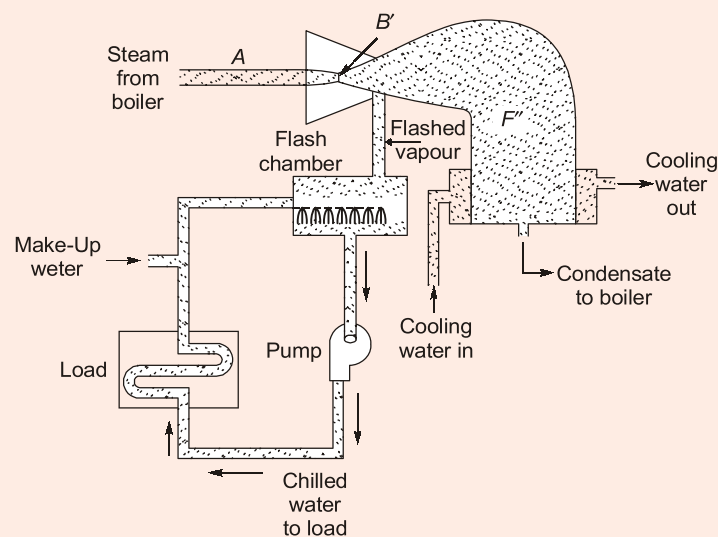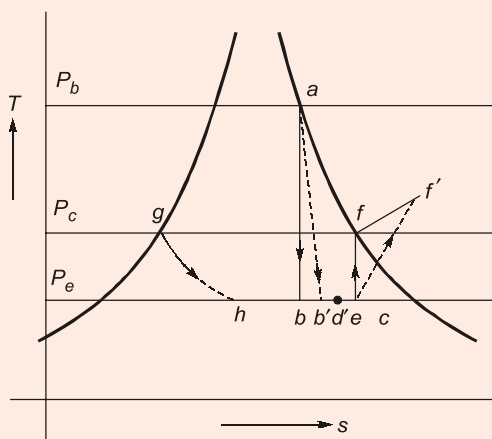
Figure: *T– s* diagram from steam jet refrigeration
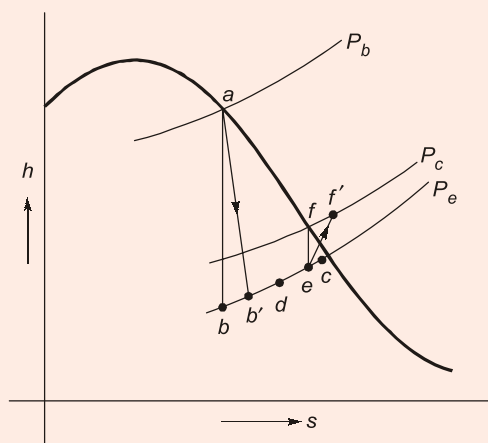
Figure: *H – s* diagram from steam jet refrigeration

b' = Condition of steam coming out of nozzle

d = Condition of steam just before mixing with water vapour

c = Condition of water vapour formed in the evaporator chamber

e = Condition of mixture of steam at d and water vapour at c after mixing and just before starting the compression in the booster ejector

f' = Condition of the mixture entering into the condenser

ef = Isentropic compression in booster ejector

ef' = Actual compression in booster ejector

The actual expansion through nozzle does not follow isentropic process, so that actual drop is taken into account by nozzle efficiency and it is given by

$$\eta_n(\text{Nozzle efficiency}) = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}$$

$$= \frac{h_a - h_b'}{h_a - h_b}$$

The water vapour formed in the flash-chamber has negligible velocity compared with the velocity of the steam coming out of nozzle which is equivalent to $\sqrt{2gJ(h_a - h'_b)}$ m/sec.

The quantity $(h_a - h'_b)$ is equivalent to K.E. of motive steam, now available for entrainment of the vapour in the flash-chamber. (The process of giving the momentum of the water vapour formed in the flash-chamber by high velocity steam is known as entrainment of vapour).

During the entrainment, steam will lose some energy. The process of entrainment is very inefficient and part of original motive force available for compression is reduced and it is taken into account by a factor known as entrainment efficiency and it is given by

$$\eta_e = \frac{(h_a - h_d)}{(h_a - h'_b)}$$

The actual compression of the mixture does not follow the isentropic compression, so it is taken into account by a factor known as compression efficiency and it is given by

$$\eta_c \text{(compression efficiency)} = \frac{h_a - h_e}{h'_f - h_e}$$

End of Solution

Q.5 **(c)** Briefly describe a natural draught cooling tower. Explain why it is hyperbolic in shape.

[12 marks : 2023]

Solution:

In natural draught cooling towers, the flow of air occurs due to the natural pressure head caused by the difference in density between the cold outside air and the hot humid air inside as shown in figure (a) and figure (b).

Thus, the pressure head developed is

$$\Delta p_d = (\rho_0 - \rho_i)gH$$

where, H = height of the tower above the fill, ρ_0 = density of outside air, and ρ_i = density of inside air.
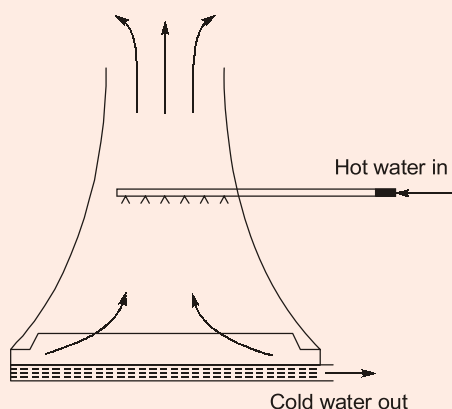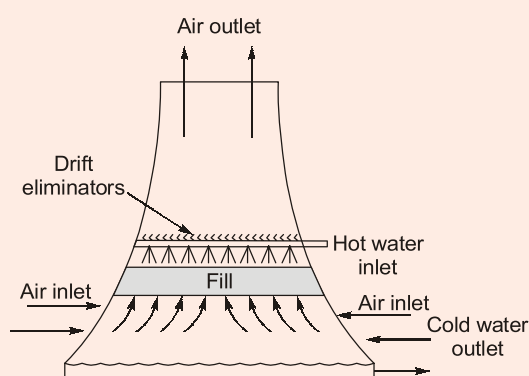
Fig (a): Natural drought cooling tower   **Fig (b):** Counter flow hyperbolic natural drought cooling tower

Because of relatively small density difference, $\rho_0 - \rho_i$, H must be large as to result in the desired Δp_d, which must balance the air pressure losses in the tower. Natural draught cooling towers are, therefore, very tall. The tower body, above the water distribution system and the fill, is an empty shell of circular cross-section, but with a hyperbolic system profile. The hyperbolic profile offers superior strength and the greatest resistance to outside wind loading compared to other forms. Natural draught cooling towers are, therefore, often termed as hyperbolic towers. Made of reinforced concrete, they are an imposing sight and are conspicuous from a distance.

End of Solution

Q.5 **(d)** Distinguish among the following:
      (i)   Renewable energy
      (ii) Green energy
      (iii) Clean energy
      Also, mention the relative environmental effects of the above.

[12 marks : 2023]

Solution:

It is a common misconception that renewable energy, clean energy and green energy all mean the same thing. While there is some overlap, each term has a unique meaning. For instance, while green energy resources are all considered renewable, not all renewable energy resources can be considered green. Moreover, clean energy encompasses nuclear energy, which is neither green nor renewable.

Renewable energy : Renewable energy is any energy source that constantly replenishes itself over a relatively short time frame. In theory, renewable energy is considered virtually inexhaustible, meaning the method used to produce electricity never runs out. This makes it make viable long-term option compared to natural gas and coal, both of which are non-renewable resources that we have a finite supply of.

Green energy : Green energy is any form of energy that comes from natural sources, such as wind, sunlight or water. While green energy is renewable, not all renewable energy sources can be considered green. For example, the construction of a large-scale hydropower facility can have a heavy environment impact on nearby rivers and lakes. Even so, since green energy sources do not release significant amounts of green house gases into the atmosphere during the generation of electricity, they are considered clean energy resources.

Clean energy : Clean energy is electricity that does not create any greenhouse gases during its production-although it is not necessarily renewable. The "clean" means these sources have minimum or zero impact on the surrounding environment, which makes them ideal for the long-term survivability of the planet.

Relative environmental effects:

- Renewable energy can have an ecological impact, depending upon the process used to create the electricity.
- Green energy is considered the most environmentally friendly resource available to us today, with little to no ongoing environmental impact.
- Clean energy is power generation without creating adverse environment impacts like carbondioxide or GHGs. Most clean energy sources are renewable, including hydro energy, solar power and wind power.

MADE EASY

Q.5 **(e)** Describe the emission norms for Indian vehicles if they have to comply with Bharat Stage (BS) Emission standards-VI. Mention the devices and technology introduced to meet the BS-VI norms.

[12 marks : 2023]

Solution:

To regulate the pollution emitted by cars and two-wheelers, the government has put forth regulations known as Bharat stage emission standard (BSES). The central government has mandated that all vehicle manufacturers, both two-wheelers and four-wheelers, should manufacture, sell and register only BS6 (BSVI).

Emission norms for BS6

- BS6 emission norms allow a motorcycle to emit not more than 60 mg/km of NO_2 (Nitrogen oxides).

- The particulate matter (PM) for petrol vehicles has been restricted to 4.5 mg/km.

- The limit of NOx for diesel engines is 80 mg/km. The HC + NOx limit has been set to 170 mg/km and PM level limit has been set to 4.5 mg/km.

- The BS6 fuel has less sulphur and NOx. The content of sulphur in BS6 fuel is 10 ppm.

Devices and technology introduced to meet BS-VI norms.

- **Selective catalytic reduction technology :** It reduces oxides of nitrogen by injecting an aqueous urea solution into the system. Hence, NOx from diesel cars can be brought down by nearly 70%. In the petrol cars, they can be reduced by 25%.

- **Mandatory on-board diagnostics :** Which inform the vehicle owner or the repair technician about how efficient the systems in the vehicles are.

- **RDE (Real driving emission) :** is introduced for the first time that will measure the emission in real-world conditions and not just under test conditions.

End of Solution

Q.6 **(a)** A gasoline engine has a stroke volume of 0.002 m³ and a compression ratio of 6. At the end of the compression stroke, the pressure is 10 bar and the temperature is 400°C. Ignition is set so that the pressure rises along a straight line during combustion and attains its highest value of 30 bar after the piston has travelled

$\left(\dfrac{1}{40}\right)$ of the stroke. The charge consists of a gasoline-air mixture in proportion

of 1 : 18 by mass. Calculate the heat lost per kg of charge during combustion. Take R = 287 J/kg-K, calorific value of fuel = 45 MJ/kg, C_p = 1 kJ/kgK.

[20 marks : 2023]

Solution:
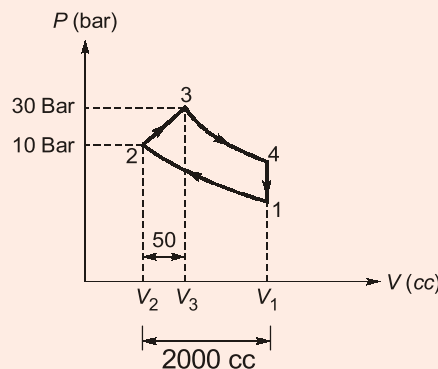
Given : Stroke volume V_s = 0.002 m³; Compression ratio, r = 6; P_2 = 10 bar; T_2 = 400°C = 623K; P_3 = 30 bar; F : A = 1 : 18; CV = 45 MJ/kg

Stroke volume, $V_s = V_1 - V_2 = 0.002$ m³

$V_1 - V_2 = 0.002 \times 10^6$ cc

$V_1 - V_2 = 2000$ cc   (i)

Compression ratio, $r = \dfrac{V_1}{V_2}$

$V_1 = rV_2 = 6V_2$ (ii)

From (i) and (ii), $6V_2 - V_2 = 2000$

$$V_2 = \frac{2000}{5} = 400 \text{ cc}$$

Since point 3 is attained when the piston has travelled $\left(\dfrac{1}{40}\right)$ of the stroke.

$\Rightarrow$ $V_3 = \dfrac{2000}{40} + 400 = 450$ cc

Now, $\dfrac{P_2 V_2}{T_2} = \dfrac{P_3 V_3}{T_3}$

$$T_3 = T_2 \times \frac{P_3 V_3}{P_2 V_2} = 623 \times \frac{30 \times 450}{10 \times 400} = 2102.625 \text{ K}$$

Work done in process 2-3,

$W_{2\text{-}3}$ = Area under the line 2-3 on P-V diagram

$$W_{2\text{-}3} = \left[\left(\frac{30+10}{2}\right) \times 10^5\right] Pa \times \left[(50) \times 10^{-6}\right] m^3$$

$W_{2\text{-}3}$ = 100 J = 0.1 kJ

Total mass of mixture, $m = \dfrac{P_2 V_2}{RT_2} = \dfrac{10 \times 10^5 \times 400 \times 10^{-6}}{287 \times 623} = 2.237 \times 10^{-3}$ kg

Change in internal energy during process 2-3,

$$\Delta U = U_3 - U_2$$

$$\Delta U = 2.237 \times 10^{-3}(1 - 0.287)(2102.625 - 623)$$

$$\Delta U = 2.3599 \text{ kJ} \simeq 2.36 \text{ kJ}$$

Thus, heat given to the mixture in one cycle,

$$Q_{2\text{-}3} = W_{2\text{-}3} + \Delta U$$

$$Q_{2\text{-}3} = 0.1 + 2.36 = 2.46 \text{ kJ}$$

But actual heat given by fuel is

$$Q_{\text{act}} = m_f CV$$

$$= \frac{1}{18} \times 2.237 \times 10^{-3} \times 45000$$

$$= 5.5925 \text{ kJ}$$

Thus, heat lost, $Q = Q_{\text{act}} - Q_{2\text{-}3} = 5.5925 - 2.46$

$$= 3.1325 \text{ kJ}$$

On the basis of per kg of charge

$$Q = \frac{3.1325}{2.237 \times 10^{-3}} = 1400.31 \text{ kJ/kg}$$

Q.6 (b) A room is designed for air conditioning as per the following data:

Room sensible heat gain = 30 kW

Room latent heat gain = 10 kW

Inside design conditions are : 25°C DBT and 50% RH

Outside conditions are : 40°C DBT and 27°C WBT

Bypass factor of the cooling coil = 0.10

The return air from the space is mixed with the outside air before entering the cooling coil in the ratio of 4 : 1 by weight. Determine the following:

(i) Apparatus dew point

(ii) Condition of air leaving the cooling coil

(iii) Quantity of dehumidified air

(iv) Mass of ventilation air

(v) Volume flow rate of fresh air

(vi) Total refrigeration load

[Psychrometric chart is attached]

PSYCHROMETRIC CHART
BAROMETRIC PRESSURE 1·01325 bar
SEA LEVEL

Ref. Point for SHF is 25 °C, 50% RH
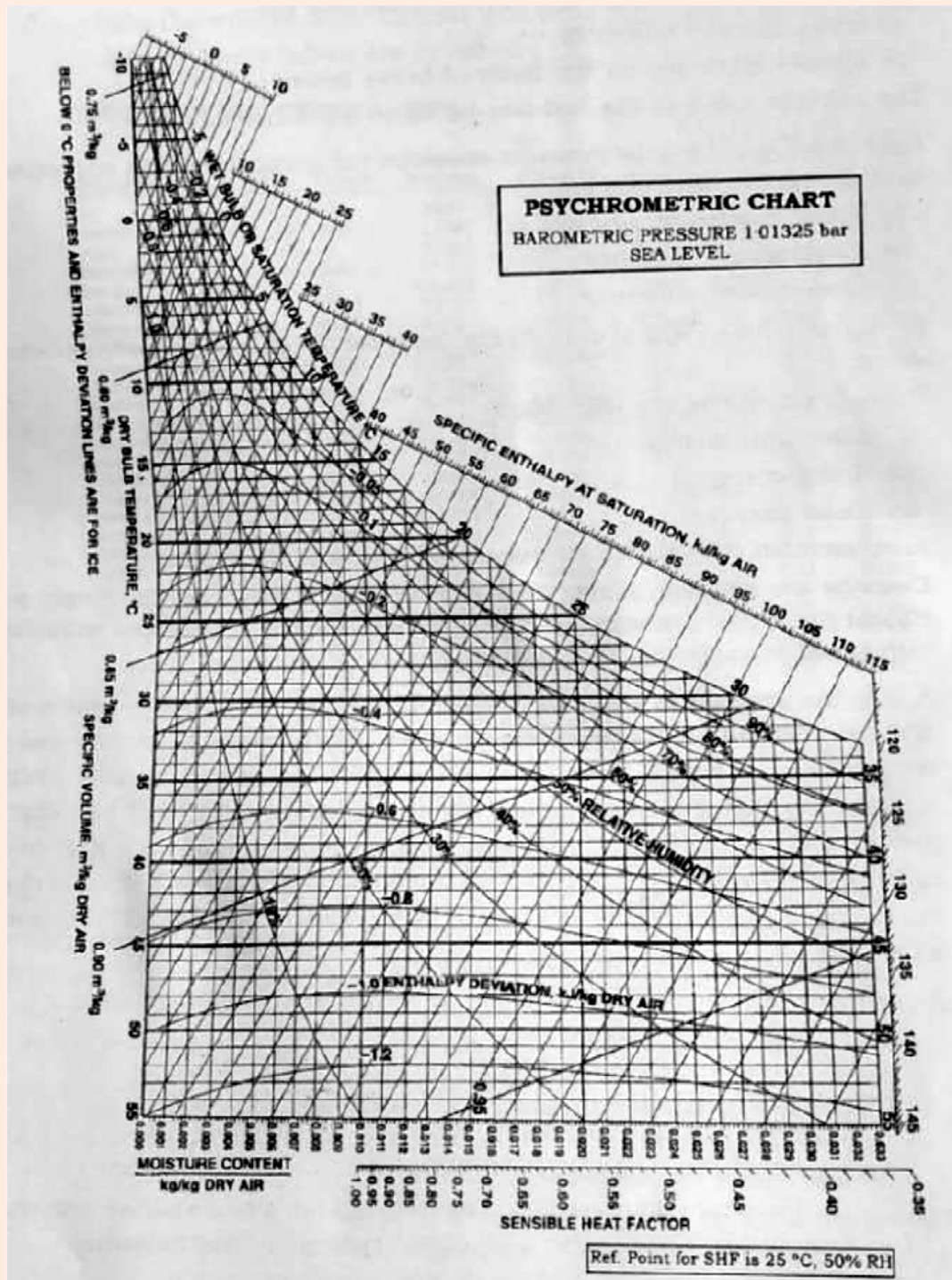
[20 marks : 2023]

Solution:

Given : RSH = 30 kW; RLH = 10 kW

Inside design conditions: 25°C DBT and 50% RH

Outside conditions : 40°C DBT and 27° WBT

By pass factor, X = 0.1
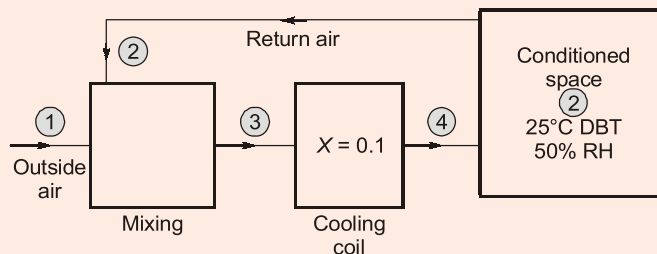
First of all mark the outside condition of air at 40°C DBT and 27°C WBT on the Psychrometric chart as point 1. Now mark the inside condition of air at 25°C DBT and 50% RH as point 2.
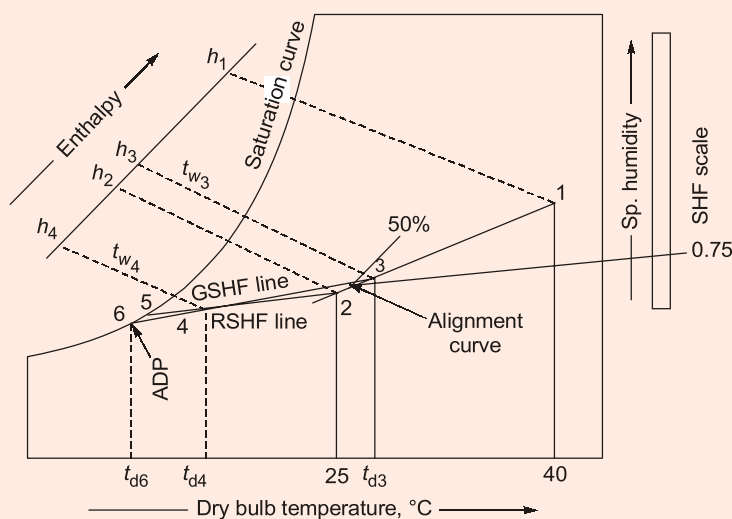
We know that RSHF = $\dfrac{\text{RSH}}{\text{RSH + RLH}} = \dfrac{30}{30+10} = \dfrac{30}{40} = 0.75$

Now mark this calculated value of RSHF on the sensible heat factor scale and join with the alignment circle (i.e; 26° DBT and 50% RH). From point 2, draw a line 2-5 parallel to this line. This line 2-5 is called RSHF line. Since the returned air from the conditioned space is mixed with outside air before entry to the cooling coil in the ratio 4 : 1 by weight therefore the condition of air entering the cooling coil after mixing process is marked on the line 1-2 by point 3 such that

$$\text{Length 2-3} = \dfrac{\text{Length 1-2}}{5}$$

Through point 3, draw a line 3-6 known as GSHF line intersecting the RSHF line at point 4 and the saturation curve at point 6 such that

$$\dfrac{\text{Length 4-6}}{\text{Length 3-6}} = \text{By pass factor} = 0.1$$

(i) The point 6 on the Psychrometric chart is the apparatus dew point. By reading the value of the chart, we find that

$$t_{ADP} = 11°C \qquad \text{Ans.}$$

(ii) Condition of air leaving the cooling coil : The point 3 and point 4 represent the entry and exit conditions of air for cooling coil as shown in the figure. From the Psychrometric chart, we find that

Dry bulb temperature of air leaving the cooling coil

$$T_4 = 14°C$$

From the Psychrometric chart, we find that enthalpy at point 2:

$$h_2 = 50 \text{ kJ/kg of dry air}$$

and enthalpy of air at point 4,

$$h_4 = 37 \text{ kJ/kg of dry air}$$

(iii) Quality of dehumidified air:

$$\dot{m}_a = \frac{\text{Room total heat}}{h_2 - h_4} = \frac{RSH + RLH}{h_2 - h_4}$$

$$= \frac{30 + 10}{50 - 37} = 3.0769 \text{ kg/s}$$

$$= 11076.84 \text{ kg/h} \qquad \text{Ans.}$$

(iv) Since mass of air contains return air and fresh air in the ratio of 4 : 1, therefore fresh air

$$\text{flow rate} = 11076.84 \times \frac{1}{5} = 2215.368 \text{ kg/h} \qquad \text{Ans.}$$

(v) The specific volume at condition 1 from the Psychrometric chart is,

$$v_1 = 0.91 \text{ m}^3/\text{kg}$$

∴ Volume flow rate of fresh air

$$V = \dot{m}_a \times v_1 = 2215.368 \times 0.91$$

$$= 2015.98 \text{ m}^3/\text{h} \qquad \text{Ans.}$$

(vi) Total refrigeration load:

The value of h_3 from the Psychrometric chart is 57 kJ/kg of dry air

∴   Refrigeration load, RC $= \dot{m}_a(h_3 - h_4) = 3.0769 \times (57 - 37)$

$$= 61.538 \text{ kW} = 17.58 \text{ TR} \qquad \text{Ans.}$$

End of Solution

Q.6 (c) The angles at inlet and discharge of the blading of a 50% reaction turbine are 35° and 20° respectively. The speed of rotation is 1500 rpm and at a particular stage, the mean ring diameter is 0.67 m and the steam condition is at 1.5 bar, 0.96 dry. Determine

(i)   the required height of blading to pass 3.6 kg/s of steam;
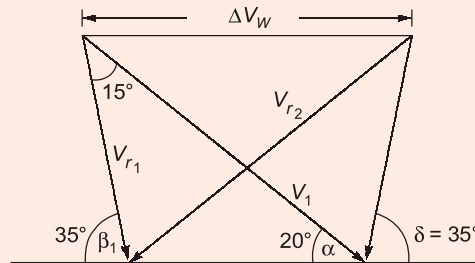
(ii) the power developed by the ring.

[Saturated steam table is attached at the end of booklet]

[20 marks : 2023]

Solution:

Given : $\beta_1 = 35°$; $\beta_2 = \alpha = 20°$; $D_m = 0.67$ m; $N = 1500$ rpm

$$V_b = \frac{\pi D_m N}{60} = \frac{\pi \times 0.67 \times 1500}{60} = 52.62 \text{ m/s}$$

By sine law:

$$\frac{V_1}{\sin 145°} = \frac{V_b}{\sin 15°} = \frac{V_{r_1}}{\sin 20°}$$

$$\Rightarrow \qquad V_1 = 52.62 \times \frac{\sin 145°}{\sin 15°} = 116.62 \text{ m/s} = V_{r_2}$$

$$\therefore \qquad V_{r_1} = 52.62 \times \frac{\sin 20°}{\sin 15°} = 69.54 \text{ m/s}$$

$$\Delta V_w = V_1 \cos\alpha + V_2 \cos\delta$$
$$= 116.62 \cos 20° + 69.54 \cos 35°$$
$$= 166.547 \text{ m/s}$$

At P = 1.5 bar using steam table,

$$v_f = 0.001053 \text{ m}^3/\text{kg}, \ v_g = 1.159 \text{ m}^3/\text{kg}$$

$$\therefore \qquad v_1 = 0.001053 + 0.96 \times (1.159 - 0.001053)$$
$$= 1.1127 \text{ m}^3/\text{kg}$$

$$\dot{m}_s = 3.6 \text{ kg/s}$$

$$\dot{m}_s = \frac{\pi D_m h_b V_1 \sin\alpha}{v_1}$$

$$3.6 = \frac{\pi \times 0.67 \times h_b \times 116.62 \sin 20°}{1.1127}$$

$$\Rightarrow \qquad h_b = 0.04771 \text{ m} = 47.71 \text{ mm}$$

(i) The required height of blading to pass 3.6 kg/s of steam is 47.71 mm.   **Ans.**

(ii) The power developed by the ring:

$$P = \dot{m}_s \times \Delta V_w \times V_b$$

$$= \frac{3.6 \times 166.547 \times 52.62}{1000} = 31.55 \text{ kW} \qquad \textbf{Ans.}$$

Q.7 **(a)** The following data refer to a boiler unit consisting of an economizer, a boiler and a superheater:

Mass of water evaporated per hour = 5940 kg

Mass of coal burnt per hour = 675 kg

Lower calorific value of coal = 31600 kJ/kg

Pressure of steam at boiler stop valve = 14 bar

Temperature of feedwater entering economizer = 32°C

Temperature of feedwater leaving economizer = 115°C

Specific heat of superheated steam = 2.3 kJ/kg-K

Determine the following:

(i)   Percentage of heat in coal utilized in economizer, boiler and superheater.

(ii)   Overall efficiency of the boiler unit

Assume specific heat of water = 4.187 kJ/kg-K

[Saturated steam table is attached at the end of booklet]

[20 marks : 2023]

Solution:

Given: Mass of water evaporated = 5940 kg/h; Mass of coal burnt = 675 kg/hr; Lower calorific value of coal = 31600 kJ/kg; Pressure of steam at boiler stop valve, P_1 = 14 bar; Temperature of feedwater entering the economizer, t_{e_1} = 32°C; Temperature of feedwater leaving the economizer, t_{e_2} = 115°C; Dryness fraction of steam entering superheater, x = 0.96; Temperature of steam leaving the superheater, t_{sup} = 260°C; Specific heat of superheated steam, c_p = 2.3 kJ/kgK

Heat utilized by 1 kg of feed water in economiser,

$$h_{f_1} = 1 \times 4.18 \left(t_{e_2} - t_{e_1} \right) = 1 \times 4.18 (115 - 32)$$

$$h_{f_1} = 346.9 \text{ kJ/kg}$$

Heat utilized in boiler per kg of feed water

$$h_{boiler} = (h_f + x h_{fg}) - h_{f_1}$$

At P = 14 bar, from steam tables,

$$t_s = 195°C, \ h_f = 830.1 \text{ kJ/kg}, \ h_{fg} = 1957.7 \text{ kJ/kg}$$

$\therefore$
$$h_{boiler} = (830.1 + 0.96 \times 1957.7) - 346.9$$
$$= 2362.6 \text{ kJ/kg}$$

Heat utilized in superheater by 1 kg of feed water,

$$h_{superheater} = (1 - x) h_{fg} + c_p \left(T_{sup} - T_s \right)$$

$$= (1 - 0.96) \times 1957.7 + 2.3 (260 - 195)$$

$$= 78.3 + 149.5$$

$$= 227.8 \text{ kJ/kg}$$

Also, mass of water evaporated per hour per kg of coal burnt = $\dfrac{5940}{675}$ = 8.8 kg

(i)   Percentage of heat utilized in economiser = $\dfrac{346.9}{31600} \times 8.8 \times 100 = 9.66\%$

Percentage of heat utilized in boiler = $\dfrac{2362.6}{31600} \times 8.8 \times 100 = 65.7\%$

Percentage of heat utilized in superheater = $\dfrac{227.8}{31600} \times 8.8 \times 100 = 6.34\%$

(ii) Overall efficiency of boiler plant,

$$\eta_{overall} = \frac{\text{Total heat absorbed}}{31600}$$

$$= \frac{\left(h_{f_1} + h_{boiler} + h_{superheater}\right)8.8}{31600}$$

$$= \frac{(346.9 + 2362.6 + 227.8)8.8}{31600}$$

$$= 0.8179 \text{ or } 81.79\%$$

End of Solution

Q.7 (b) (i) Explain the various factors affecting anaerobic digestion process. Why do anaerobic microbes normally grow at a much lower rate than aerobic bacteria?

(ii) A family biogas plant is required to be designed to utilize the cow dung of five cows. The hydraulic retention time is 30 days. The temperature of the digester is to be maintained at 30°C. The dry matter consumption per day is 2 kg. The biogas yield is 0.25 m^3/kg. The efficiency of the burner is 60%. The heat of combustion of methane is 26 MJ/m^3. The methane proportion is 70%. The density of feedstock material may be taken as 50 kg/m^3. Find (1) the volume of biogas digester and (2) its thermal power.

[10 + 10 marks : 2023]

Solution:

(i)

Anaerobic digestion process involves 'microbial digestion' of biomass and is done in the 'absence of oxygen'. The process and end products depend upon the micro-organisms activated under culture conditions. The factors affecting anaerobic digestion process are:

- **Temperature :** The anaerobic fermentation process is temperature dependent. The process of the digestion and gasification proceeds at the highest rate when the temperature lies between 35°C - 38°C. The process becomes slow within temperature range of 45°C - 55°C and then rises to a peak between 55° - 60°C. Thus, the rate of gas production 'increases' with the increase in temperature but the percentage of methane 'decreases'.

- **Loading rate :** "Loading rating" is the weight of volatiles solids fed to a digester per day. It depends upon the plant capacity and also the retention period. Thus, for a given capacity of the digester, if the loading rate is increased, the 'retention period' is correspondingly decreased.

- **Solid concentration :** Normally, 7 to 9 parts of solid in 100 parts of the slurry is considered ideal.

- **Retention period :** The time period for which fermentable material resides inside the digester. This period ranges from 30 days to 50 days depending upon the climatic conditions.

- **Nutrients concentration :** The major nutrients required by the bacteria in the digester are C, H_2, O_2, N_2, P and *S*. To maintain proper balance of nutrients, an extra raw material, rich in P and N_2, should be added along with cattle dung to obtain maximum gas production.

- **Toxic substance :** The presence of ammonia, pesticides, detergents and heavy metals are considered as toxic substances to micro-organisms, since their presence reduces the fermentation rate.

Anaerobic microbes grow slower than aerobic bacteria because their energy yield from oxidizing organic molecules is smaller than that of aerobes. Aerobes obtain more energy from oxidizing their food because they use oxygen as their terminal electron acceptor. Anaerobes use other molecules as their terminal electron acceptor and these molecules often only release a tiny fraction of the energy that is liberated when when is used because they have a much lower electron affinity than oxygen.

(ii)

Given that number of cows in family home = 5 cows

$$\text{Retention time} = 30 \text{ days}$$
$$\text{Dry matter consumed/day} = 2 \text{ kg/day/cow}$$
$$\text{Biogas produced} = 0.25 \text{ m}^3/\text{kg}$$
$$\eta_{\text{burner}} = 0.60$$
$$\text{Heat of combustion of methane} = 26 \text{ MJ/m}^3$$
$$\text{Methane proportion} = 0.70$$
$$\rho_{\text{feed stock}} = 50 \text{ kg/m}^3$$

Find V (digester) = ?   Thermal power = ?

Biogas (0.1 V)

Digester (V)

Slurry (0.9 V)
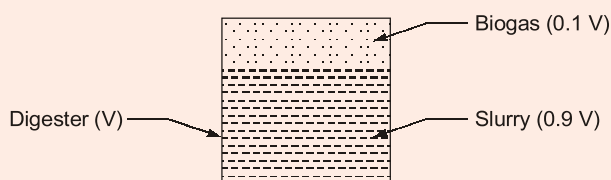
$$\text{Total dry matter consumed per day} = \frac{2\text{kg}}{\text{day cow}} \times 5 \text{ cow} = 10 \text{ kg/day}$$

Let, equal amount of water is added per day in the slurry

Mass of water added per day in slurry = 10 kg/day

Volume of slurry added per day = $V_{\text{feed stock}} + V_{\text{water}}$

$$= \frac{m_{\text{feed stock}}}{\rho_{feed\ stock}} + \frac{m_{\text{water}}}{\rho_{water}}$$

$$= \frac{10 \text{ kg/day}}{50 \text{ kg/m}^3} + \frac{10 \text{ kg/day}}{1000 \text{ kg/m}^3}$$

$$= 0.21 \text{ m}^3/\text{day}$$

$$\text{Retention time (RT)} = \frac{V_s}{V_d}$$

$$\Rightarrow \qquad 30 \text{ days} = \frac{V_s}{0.21 \text{ m}^3/\text{day}}$$

$$N_S = 6.3 \text{ m}^3$$

$$\text{Volume of digester } (V) = \frac{6.3 \text{ m}^3}{0.9} = 7 \text{ m}^3 \qquad\qquad \text{Ans.}$$

Thermal power available by biogas per day = Gas produced per day (m^3/day) $\times$ 0.70 $\times$ Heat combustion of methane

$$= 0.25 \text{ m}^3/\text{kg} \times 10 \text{ kg/day} \times 0.70 \times 26 \text{ MJ/m}^3$$

$$= 45.5 \text{ MJ/day}$$

$$\eta_{\text{burner}} = \frac{\text{Thermal power available from biogas at burner}}{\text{Total thermal power available by biogas per day}}$$

$$0.60 = \frac{\text{Thermal power available from biogas at burner}}{455 \text{ MJ/day}}$$

Thermal power available from biogas at burner = 27.3 MJ/day

$$\text{Thermal power} = 27.3 \times \frac{10^6 \text{J}}{24 \times 3600 \text{ J}}$$

$$= 315.972 \text{ Watt} \qquad\qquad \text{Ans.}$$

Q.7 (c) (i) A refrigeration system with R-22 as refrigerant operates with an evaporating temperature of –10°C and a condensing temperature of 35°C. If the vapour leaves the evaporator saturated and is compressed isentropically, what is the COP of the cycle (1) if saturated liquid enters the expansion device and (2) if the refrigerant entering the expansion device is with 10% vapour? [R-22 refrigerant chart is attached]
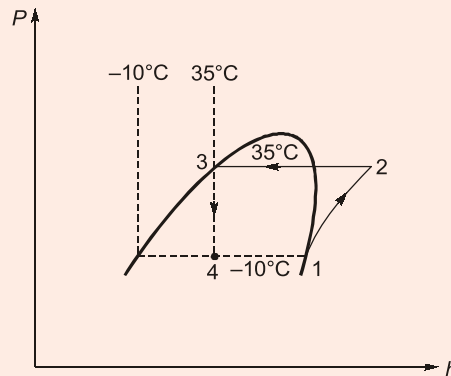
R-22 Refrigerant Chart
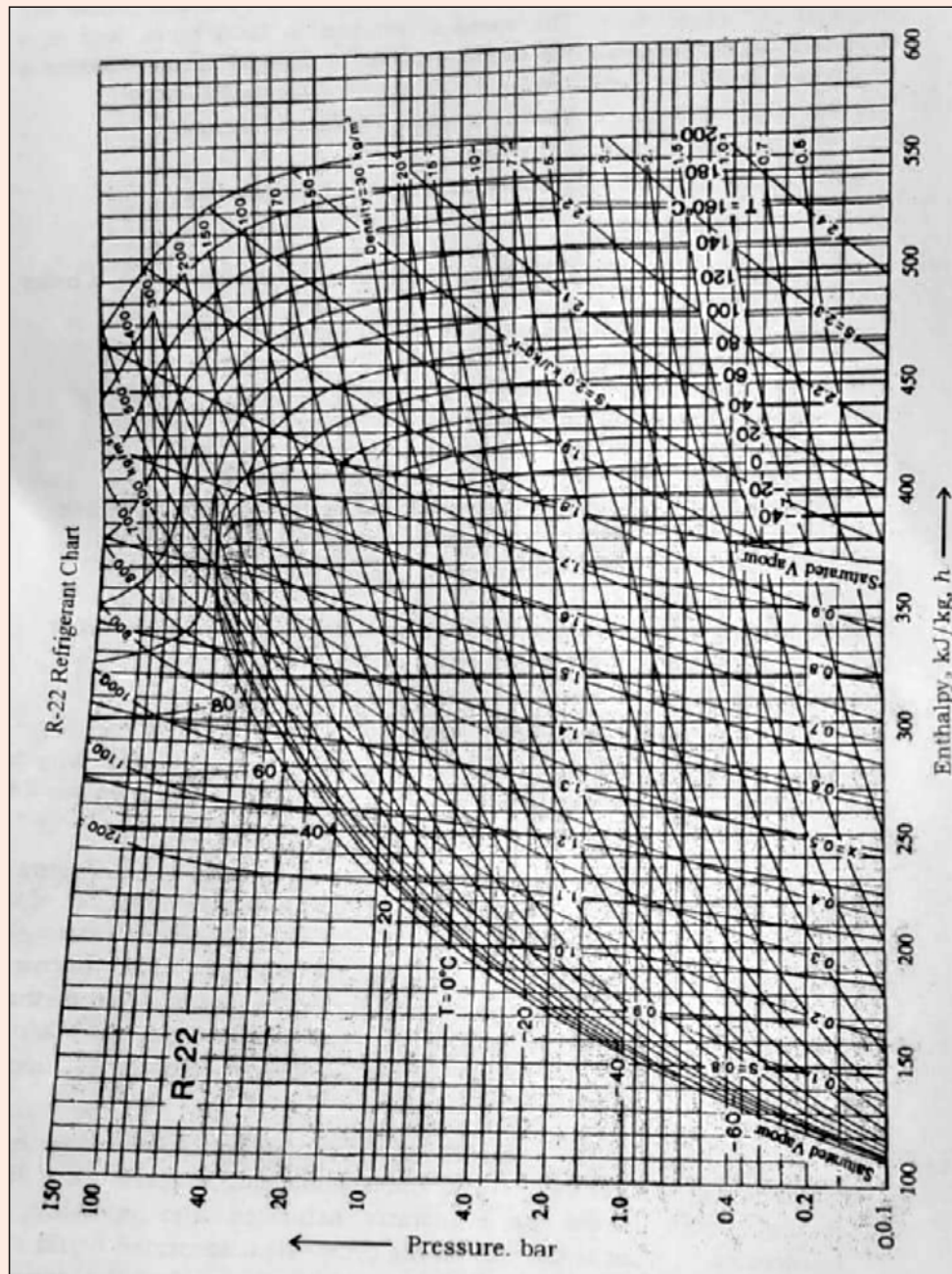
(ii) What is the liquid-to-suction heat exchanger in refrigeration and air conditioning? Illustrate the benefits of liquid-to-suction heat exchanger.

[10 + 10 marks : 2023]

Solution:

(i)

Given : Evaporating temperature = –10°C; Condensing temperature = 35°C

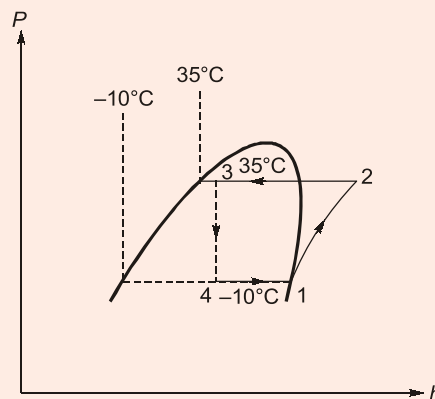(1) If saturated liquid enters the expansion device.

From P-h diagram given

$$h_1 = 430 \text{ kJ/kg}; \; h_2 = 480 \text{ kJ/kg}$$
$$h_3 = 240 \text{ kJ/kg} = h_4$$

$$\text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{430 - 240}{480 - 430} = 3.8 \qquad \text{Ans}$$

(2)   If the refrigerant entering the expansion device is with 10% vapour

From Psychrometric chart given

$$h_1 = 430 \text{ kJ/kg}; \; h_2 = 480 \text{ kJ/kg};$$

Here, $x_3 = 0.1$

So, point '3' will be intersection of $x = 0.1$ line and $T = 35°C$ line on *P-h* chart. Hence enthalpy corresponding to point '3'.

$$h_3 = h_4 = 258 \text{ kJ/kg}$$

$$\text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{430 - 258}{480 - 430} = 3.44 \qquad \text{Ans.}$$

(ii)

Required degree of subcooling and superheating may not be possible, if one were to rely only on heat transfer between the refrigerant and external heat source and sink. LVHE in vapour compression refrigeration systems is used to ensure subcooling of condenser liquid and superheating of saturated evaporated vapour at the same time.

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In a liquid-vapour regenerative heat exchanger, also called Liquid Suction Heat Exchanger (LSHX), the refrigerant vapour from the evaporator is superheated with subsequent subcooling of liquid from the condenser. The figures below represents the schematic and the corresponding T-s and p-h diagrams of VCRS cycle with LVHE. From energy balance,

$$h_1' - h_1 = h_3 - h_3'$$

Advantages of LVHE

- Subcooling of liquid ensures less flashing of liquid and less amount of vapour in expansion process.
- Superheating of vapour by LVHE ensures that no liquid enters the compressor which could have proved harmful for compressor.
- Increased refrigeration effect.
- System optimization and improved performance — Due to subcooling occurring outside condenser, condenser capacity can be more effectively utilized for heat rejection.

End of Solution

Q.8 **(a)** **(i)** Descrbibe the working principle of hydrogen fuel cell. Also, comment on the reversible energy conversion efficiency of fuel cells.

(ii) A flat plate solar collector measuring 2 m × 1.2 m has a loss resistance of 0.13 m^2 K/W and a plate transfer efficiency of 0.85. The glass cover has transmittance of 0.9 and the absorptance of the plate is also 0.9. Water enters at a temperature of 35°C. The ambient temperature is 20°C and the irradiance in the plane of the collector is 750 W/m^2. Calculate the flow rate needed to produce a temperature rise of 10°C. The density of water and its specific heat at mean film temperature may be taken as 1000 kg/m^3 and 4.2 J/g-°C respectively.
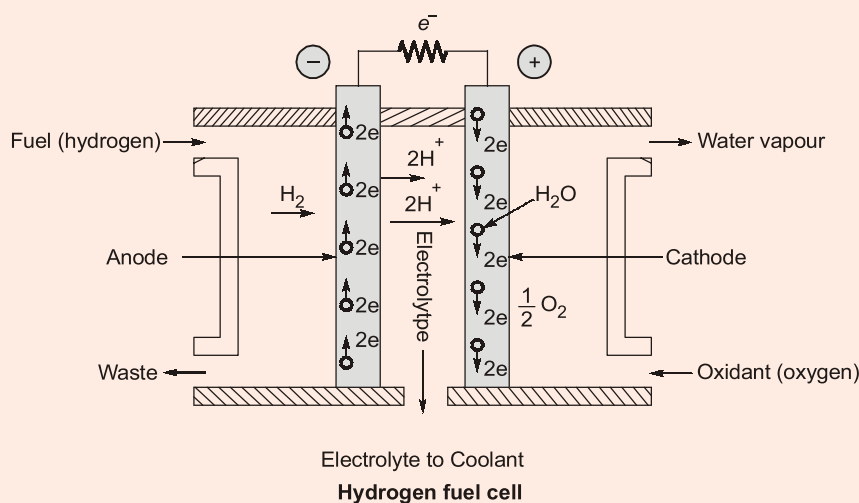
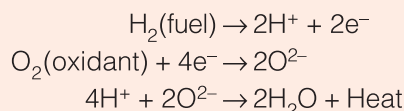[10 + 10 marks : 2023]

Solution:

(i)

A fuel cell is a device that generates electricity through an electrochemical reaction, not combustion. In a hydrogen fuel cell, hydrogen and oxygen are combined to generate electricity, heat and water.

Working principle of a hydrogen fuel cell

Hydrogen fuel cell

A fuel cell is composed of an anode, cathode, and an electrolyte membrane. A typical fuel cell works by passing hydrogen through the anode of a fuel cell and oxygen through the cathode. At the anode site, a catalyst splits the hydrogen molecules into electrons and protons. The protons pass through the porous electrolyte membrane, while the electrons are forced through a circuit, generating an electric current and excess heat. At the cathode, the protons, electrons and oxygen combine to produce water molecules. As these are no moving parts, fuel cells operate silently and with extremely high reliability. The resulting reaction is given by

$$H_2(\text{fuel}) \rightarrow 2H^+ + 2e^-$$
$$O_2(\text{oxidant}) + 4e^- \rightarrow 2O^{2-}$$
$$4H^+ + 2O^{2-} \rightarrow 2H_2O + \text{Heat}$$

Reversible energy conversion efficiency for fuel cells

For the fuel cell system the energy balance equation, can be written on a per unit mole of fuel basis, is

$$h_{in} - h_{out} + q - w = 0 \text{ or } h_{in} - h_{out} = -q + w \qquad \text{...(i)}$$

which indicates that the enthalpy change, $-\Delta h = h_{in} - h_{out}$, provides the energy available for conversion into the useful energy exhibited as work here, and it is the expense to be paid for the useful work output. At the same time, waste heat, q, is also generated, which would represent a degradation of energy. The amount of water heat generated can be determined from the second law expression, as

$$q = T\Delta s - T\phi \qquad \text{...(ii)}$$

and the useful energy output as work is,

$$w = -\Delta g - T\phi \qquad \text{...(iii)}$$

Therefore, the energy conversion efficiency for the fuel cell system,

$$\eta = \frac{w}{-\Delta h} = \frac{\Delta g + T\phi}{\Delta h} \qquad \text{...(iv)}$$

Note that both Δh and Δg are negative for power generation systems, including fuel cells. By the second law, the entropy generation per unit mole of fuel is

$$\phi \geq 0 \qquad \qquad \text{...(v)}$$

and the equality holds for all reversible processes, whereas entropy is always generated for irreversible processes. Therefore, the maximum possible efficiency allowed by the second law is, when the process is reversible (i.e. $\phi_s = 0$).

$$\eta = \frac{w_{max}}{-\Delta h} = \frac{\Delta g(T,P)}{\Delta h(T,P)} \qquad \qquad \text{...(vi)}$$

Since both the enthalpy and Gibbs function changes depend on the system temperature and pressure, the same holds for the energy conversion efficiency.

Thus, equation (vi) is valid for any power production system, be it electrochemical converter like fuel cells or conventional thermal energy converter like heat engines, as long as the process is reversible. Hence, it may be called the second law or reversible efficiency, since it is the maximum possible efficiency that is allowed by the second law of thermodynamics.

(ii)

$$T_a = 20°C; \ R_{th\ loss} = 0.13 \ m^2\text{-K/W}$$
$$\text{Area of plate (A)} = 2 \times 1.2 \ m^2$$
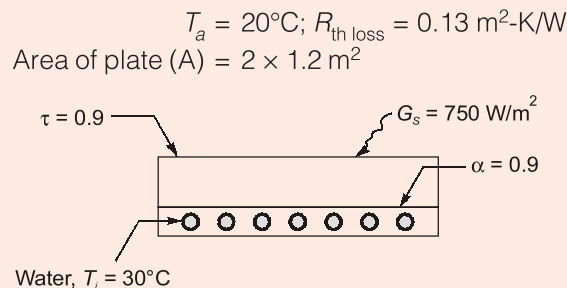
$$\Delta T \text{(Temperature rise of water)} = 10°C$$

Energy balance on water,

Rate of heat received by water = Rate of the increase of internal energy of water

$$q_u = \dot{m}_w c_p(\Delta T)$$

$$\Rightarrow \qquad q_{watt} = \dot{m}_w \times 4200 \times (10) \qquad \qquad \text{...(i)}$$

applying energy balance on absorber plate

$$q_v = \text{Rate of heat absorbed} \times \eta_{plate\ transfer}$$
$$q_v = G_s \times \alpha \times \tau \times \eta_{plate\ transfer}$$
$$q_v = 750 \ W/m^2 \times 0.9 \times 0.9 \times 0.85$$
$$q_v = 516.375 \ \text{Watt}$$

Q.8 (b) A two-pass surface condenser is required to handle the exhaust from a turbine developing 15 MW with specific steam consumption of 5 kg/kWh. The condenser vacuum is 660 mm of mercury when the barometer reads 760 mm of mercury. The mean velocity of water is 3 m/s and the water inlet temperature is 24°C. The condensate is saturated water and the outlet temperature of cooling water is 4°C less than the condensate temperature. The quality of exhaust steam is 0.9 dry. The overall heat transfer coefficient based on outer area of tubes is 4000 W/m²-°C. The water tubes are 38.8 mm in outer diameter and 29.6 mm in inner diameter. Calculate the following:

(i)   Mass of cooling water circulated in kg/min
(ii) Condenser surface area
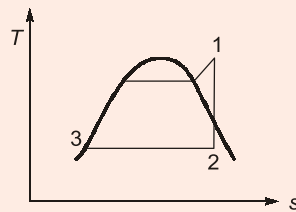(iii) Number of tubes required per pass
(iv) Tube length

Assume atmospheric pressure to be 760 mm of mercury or 1.01325 bar and specific heat of water = 4.187 kJ/kg-K

[Saturated steam table is attached at the end of booklet]

[20 marks : 2023]

Solution:

Given : $n_p = 2$; P = 15 MW; SSC = 5 kg/kWh; $P_{cond.}$ = 660 mm (vacuum); V = 3 m/s; T_{w1} = 24°C; $T_{w2} = T_{sat} - 4$°C ; x = 0.9; U = 4000 W/m²-°C; D_0 = 38.4 mm; D_i = 29.6 mm

Now,
$$SSC = \frac{\dot{m}_s}{\dot{p}}$$

∴
$$\dot{m}_s = \frac{15 \times 10^3 \times 5}{3600} = 20.83 \text{ kg/sec}$$

Condenser pressure (absolute)
$$P = 13600 \times 9.81 \times 0.1 = 0.13 \text{ bar}$$

From steam table, at 0.13 bar
$$T_{sat} = 51.062°C; h_f = 213.7 \text{ kJ/kg}; h_g = 2594 \text{ kJ/kg}$$

Now,
$$T_{w_2} = 51.062 - 4 = 47°C$$

Now,
$$h_2 = h_f + xh_{fg}$$
$$= 213.7 + 0.9 \times (2594 - 213.7)$$
$$= 2355.97 \text{ kJ/kg}$$
$$h_3 = h_f = 213.7 \text{ kJ/kg}$$

Energy balance, $\dot{m}_s(h_2 - h_3) = \dot{m}_w c_p (T_{w_2} - T_{w_1})$

$$20.83(2355.97 - 213.7) = \dot{m}_w \times 4.187(47 - 24)$$

∴
$$\dot{m}_w = 463.375 \text{ kg/sec} \qquad \text{Ans. (i)}$$

Now LMTD,

$$\theta_1 = 27.062°C$$
$$\theta_2 = 4.062°C$$
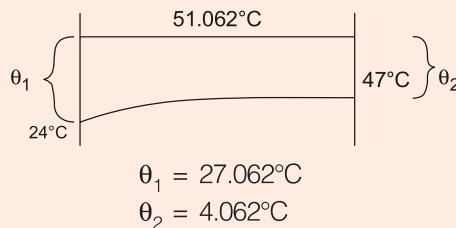
$$\therefore \qquad \theta_m = \frac{\theta_1 - \theta_2}{\ln\dfrac{\theta_1}{\theta_2}}$$

$$\theta_m = \frac{27.062 - 4.062}{\ln\left(\dfrac{27.062}{4.062}\right)} = 12.12°C$$

Heat transfer, $Q = \dot{m}_s(h_2 - h_3)$

$$= 20.83(2355.97 - 213.7)$$

or $\qquad UA\theta_m = 44623.48\ \text{kW}$

$$\therefore \qquad A = \frac{44623.48 \times 10^3}{4000 \times 12.12} = 920.45\ \text{m}^2 \qquad \text{Ans. (ii)}$$

Now, mass flow rate of water through single tube,

$$\dot{m}_{W_1} = \rho A_i v$$

$$= 10^3 \times \frac{\pi}{4} \times 0.0296^2 \times 3 = 2.0644\ \text{kg/s}$$

Number of tubes $= n = \dfrac{\dot{m}_w}{\dot{m}_{W_1}} = \dfrac{463.375}{2.0644}$

$$= 224.46 \simeq 225\ \text{tubes} \qquad \text{Ans. (iii)}$$

Tube length, $A = n\pi D_0 L_P \times p$

$$920.45 = 225 \times \pi \times 0.0384 \times L_P \times 2$$

$$L_P = 16.95\ \text{m} \qquad \text{Ans. (iv)}$$

End of Solution

Q.8 **(c)** The total pressure maintained in an Electrolux refrigerator is 15 bar. The temperature obtained in the evaporator is –15°C. The quantities of heat supplied to the generator are (i) 420 kJ to dissociate one kg of vapour and (ii) 1460 kJ/kg for increasing the total enthalpy of NH_3. The enthalpy of NH_3 entering the evaporator is 330 kJ/kg. Take the following properties of NH_3 at –15°C :

Pressure = 2.45 bar

Enthalpy of vapour = 1666 kJ/kg

Specific volume = 0.5 m³/kg

The hydrogen enters the evaporator at 25°C

Gas constant for H_2 = 4.218 kJ/kg-°C

C_P (for H_2) = 12.77 kJ/kg-°C

Find the COP of the system assuming NH_3 leaves the evaporator in saturated condition.

[20 marks : 2023]

Solution:

Given : P_T = 15 bar; T_E = –15°C = 258 K; Q_{G1} = 420 kJ/kg; Q_{G2} = 1460 kJ/kg

For NH_3:

h_1 = 330 kJ/kg; P_2 = 2.45 bar; h_2 = 1666 kJ/kg; v = 0.5 m³/kg

For H_2:

T_3 = 25°C = 298 K; R = 4.218 kJ/kg-°C; C_P = 12.77 kJ/kg-°C

The Schematic diagram for electrolux is shown below
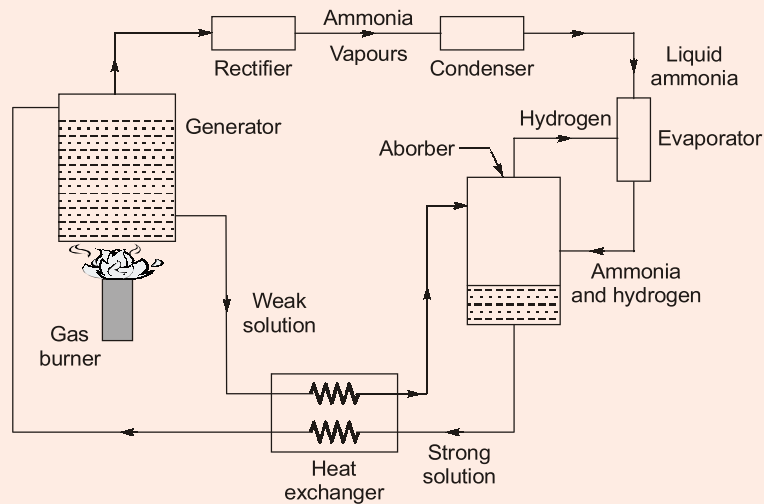
Consider the flow of 1 kg NH_3;

Heat given to the generator, $\qquad Q_G = Q_{G1} + Q_{G2}$

$\Rightarrow \qquad Q_G$ = 420 + 1460 = 1880 kJ/kg

Since, in the evaporator $\qquad P_T$ = 15 bar and $P_2 = P_{NH_3}$ = 2.45 bar.

So, pressure of H_2 :

$$P_{H_2} = P_T - P_{NH_3}$$
$$= 15 - 2.45 = 12.55 \text{ bar}$$
$$= 12.55 \times 10^5 \text{ N/m}^2$$
$$= 1255 \text{ kPa}$$
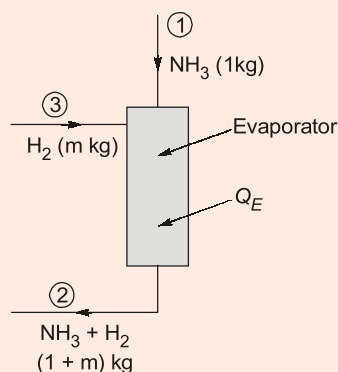
For NH_3 : h_1 = 330 kJ/kg; h_2 = 1666 kJ/kg

Let m kg of H_2 flow through the evaporator per kg of NH_3. The ammonia (NH_3) occupies

v = 0.5 m³/kg. The same is the volume of H₂ at –15°C i.e., 258 K.

$$m_{H_2} = \left(\frac{PV}{RT}\right)_{H_2} = \frac{1255 \times 0.5}{4.218 \times 258} = 0.5766 \text{ kg}$$

Considering the energy balance for the evaporator :

$$1 \times (h_2 - h_1) = \left[mc_p\left(T_3 - T_2\right)\right]_{H_2} + Q_E$$

$$1 \times (1666 - 330) = 0.5766 \times 12.77(298 - 258) + Q_E$$

$$\Rightarrow \qquad\qquad Q_E = 1041.472 \text{ kJ/kg of NH}_3$$

$$\therefore \qquad\qquad \text{COP} = \frac{Q_E}{Q_G} = \frac{1041.472}{1880} = 0.554 \qquad\qquad \text{Ans.}$$

End of Solution

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