ESE Main Examination

Civil Engineering : Paper-I

(Previous Years Solved Paper 1999)

Contents

MADE EASY PUBLICATIONS

1 Building Materials

1. Cement

1.1 **Describe the hydration of portland cement and outline the ways in which the Vicat apparatus and the Le-Chatelier apparatus can be used to assess the properties of fresh and hardened pastes.**
[15 marks : 1999]

Solution:

The chemical reactions that takes place between cement and water is known as **hydration of cement**. On account of hydration certain products are formed. These products are important because they have cementing or adhesive value. The quality, quantity, continuity, stability and the rate of formation of the hydration products are important.

Anhydrous cement compounds when mixed with water, react with each other to form hydrated compounds of very low solubility. The hydration of cement can be visualized in two ways. The first is "through solution" mechanism. In this the cement compounds dissolve to produce a super saturated solution from which different hydrated products get precipitated. The second possibility is that water attacks cement compounds in the "solid state" converting the compounds into hydrated products starting from the surface and proceeding to the interior of the compounds with time. It is probable that both "through solution" and "solid state" types of mechanism may occur during the course of reactions between cement and water. The former mechanisms may predominate in the early stages of hydration in view of large quantities of water being available and the latter mechanism may operate during the later stages of hydration.

The reaction of cement with water is exothermic. The reaction liberates a considerable quantity of heat. This liberation of heat is called heat of hydration. The hydration process is not an instantaneous one. The reaction is faster in the early period and continues indefinitely at a decreasing rate. Complete hydration can not be obtained under a period of one year or more unless the cement is very finely ground and reground with excess of water to expose fresh surfaces at intervals. During the course of reaction of C_3S and C_2S with water, calcium silicate hydrate (C–S–H) and calcium hydroxide $Ca(OH)_2$ are formed. Calcium silicate hydrate is the essence that determines the properties of concrete. It makes up 50-60 per cent of the volume of solids in a completely hydrated cement paste. On the other hand, calcium hydroxide is a compound which is responsible for the lack of durability. The calcium hydroxide also reacts with sulphates presents in soils or water to form calcium sulphate which reacts further with C_3A and cause deterioration of concrete which is known as sulphate attack. The only advantage of $Ca(OH)_2$ is that, being alkaline in nature, it maintain pH value around 13 in the concrete which resist the corrosion of reinforcements.

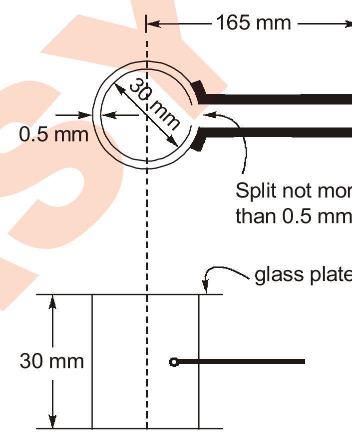
The hydration of aluminates (C_3A) results in a calcium aluminate system $CaO\text{-}Al_2O_3\text{-}H_2O$. This compound do not contribute anything to the strength of concrete. On the other hand their presence is harmful to the durability of the concrete particularly where the concrete is likely to be attacked by sulphates. As it hydrates fast it may contribute a little to early strength.

On hydration, C_4AF is believed to form a system of the form $CaO\text{-}Fe_2O_3\text{-}H_2O$. The hydrates of C_4AF also do not contribute anything to the strength but they show a comparatively higher resistance to the attack of sulphates than the hydrates of calcium aluminate.

Vicat apparatus is used for determining the **normal consistency and setting time for cement**. A known weight of cement is taken and a paste is prepared with a weighed quantity of water (24% by weight of cement) for the first trial. The Paste is then filled in a mould and the plunger of the apparatus is brought down to touch the surface of the paste in test block and quickly released allowing it to sink into the paste by its own weight. Similarly, trials are conducted with higher and higher water cement ratios till such time the plunger penetrates for a depth of 33-35 mm from the top. That particular percentage of water is known as the percentage of water required to produce a cement paste of standard consistency.

For setting times the plunger is replaced by a needle (for initial setting time) or a circular attachment (for final setting time).

Le Chatelier apparatus can be used to determine **soundness in cement**. Unsoundness in cement is due to excess of lime, magnesia or sulphates. Cement is gauged with 0.78 times the water required for standard consistency in a standard manner and filled into the mould kept on a glass plate. The mould is covered on the top with another glass plate. The whole assembly is immersed in water at a temperature of 27°C - 32°C and kept there for 24 hours. Now the distance is measured between the indicator points. The mould is again submerged in water and water is heated to brought to boiling point in about 25-30 minutes and it is kept boiling for three hours. The mould is now removed from water and allowed to cool. The distance between indicator points is measured again. The difference between these two measurements represent the expansion of cement. This must not exceed 10 mm. The Le Chatelier test detects the unsoundness due to free lime only.



1.2 **Explain how sulphate resisting cement and rapid hardening portland cement differ from OPC and specific circumstance in which these cements would be used.**

[15 marks : 1999]

Solution:

Sulphate Resisting Cement: Ordinary portland cement is vulnerable to sulphate attack. Sulphate attack is greatly accelerated if accompanied by alternate wetting and drying which normally takes place in marine structures in the zone of tidal variations.

To prevent the sulphate attack, the use of cement with low C_3A content is found to be effective. Such a cement with low C_3A content and comparatively low C_4AF content is known as sulphate resisting cement. In other words, this cement has a higher silicate content than OPC.

It is not often possible (feasible) to reduce the Al_2O_3 content of the raw materials. So Fe_2O_3 may be added to the mix so that C_4AF content increase at the expense of C_3A. Many of the physical properties of sulphate resisting cement are similar to ordinary portland cement.

The use of sulphate resistant cement is recommended under the following conditions:

(i) Concrete to be used in marine conditions.

(ii) Concrete to be used in foundation and basement, where soil is infested with sulphates.

(iii) Concrete used for fabrication of pipes which are likely to be marshy region or sulphate bearing soils.

(iv) Concrete to be used in the construction of sewage treatment works.

Rapid Hardening Cement: This cement is similar to OPC but with higher C_3S content and finer grinding. It gains strength more quickly than OPC, though the final strength is only slightly higher. The one day strength of this cement is equal to three day strength of OPC with the same water-cement ratio. This cement is used where a rapid strength development is required. The rapid gain of strength is accompanied by a higher rate of heat development during the hydration of cement. It is about 10 per cent costlier than

OPC.

The use of rapid hardening cement is recommended in the following situations:

(i) In prefabricated concrete construction.

(ii) Where formwork is required to be removed early for reuse elsewhere.

(iii) Road repair works.

(iv) In cold weather concrete where the rapid rate of development of strength reduces the vulnerability of concrete to the frost damage.

1.3 Name the principal compounds in portland cement, their relative rates of reaction with water and their approximate proportions.

[10 marks : 1999]

Solution:

The principal compounds in portland cement are known as Bogue's compounds. They are as follows:

(i) Tricalcium silicate (C_3S) or Alite

(ii) Dicalcium silicate (C_2S) or Belite

(iii) Tricalcium aluminate (C_3A) or Celite

(iv) Tetracalcium aluminoferrite (C_4AF) or Felite

The reaction of water with C_3A is very fast and in the process **flash setting** i.e. stiffening without strength development can occur because the C-A-H phase prevents the hydration of C_3S and C_2S. To prevent this flash set, gypsum is added at the time of grinding the cement clinker. The hydrated aluminates do not contribute anything to the strength of concrete. On the other hand, their presence is harmful to the durability of concrete particularly, where the concrete is likely to be attacked by sulphates. As it hydrates fast, it may contribute a little to the early strength.

$$C_3A + H_2O \rightarrow C_3AH_6$$

On hydration, C_4AF is believed to form a system of the form C-F-H. A hydrated calcium ferrite of this form is comparatively more stable. This hydrated product also does not contribute anything to the strength. However, the hydrates of C_4AF show a comparatively higher resistance to the attack of sulphates than the hydrates of calcium aluminate.

$$C_3AF + H_2O \rightarrow C_3AH_6 + CFH$$

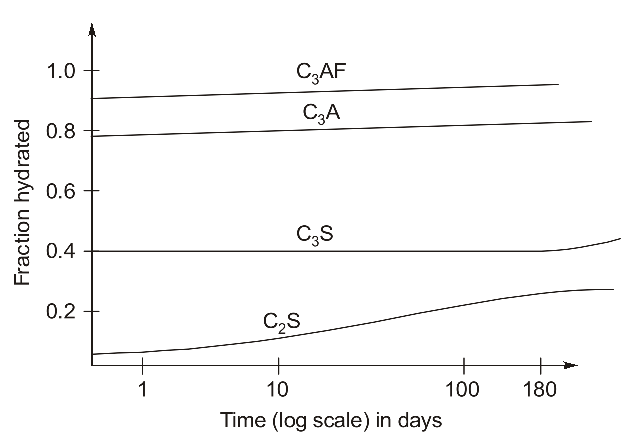
When C_3S and C_2S reacts with water, calcium silicate hydrate (C-S-H) and calcium hydroxide are formed. Calcium silicate hydrates are the most important products. It is the essence that determines the good properties of concrete.

$$C_3S + H_2O \rightarrow C_3S_2H_3 + 3Ca(OH)_2$$

$$C_2S + H_2O \rightarrow C_3S_2H_3 + Ca(OH)_2$$

It can be seen that C_3S produces a comparatively lesser quantity of calcium silicate hydrates and more quantity of $Ca(OH)_2$ than that formed in the hydration of C_2S. C_2S hydrates rather slowly than C_3S.

The relative rates of hydration of Bogue's compounds can be shown with the help of a graph as shown.



Rate of hydration of Bogue's Compounds

The approximate proportions of Bogue's compound are:

Compound	Percentage
C_3S	30-50
C_2S	20-45
C_3A	8-12
C_4AF	6-10

3. Concrete

3.1 What is non-destructive testing of concrete? What are its relative merits? Name methods of non-destructive testing and explain briefly anyone method.

[10 marks : 1998]

Solution:

Non-destructive testing of concrete is an indirect method in which specimen are not loaded to failure and as such the strength inferred or estimated cannot be expected to yield absolute values of strength. These methods therefore attempt to measure some other properties of concrete from which an estimate of its strength, durability and elastic parameters are obtained. Some such properties of concrete are hardness, resistance to penetration of projectiles, rebound number, resonant frequency and ability to allow ultrasonic pulse velocity to propagate through it.

Though non-destructive testing methods are relatively simple to perform, the analysis and interpretation of test results are not so easy. Therefore special knowledge is required to analyse the properties of hardened concrete.

Some of the non-destructive test methods are:

(i) Surface hardness tests
(ii) Rebound tests
(iii) Penetration and pull out techniques
(iv) Dynamic or vibration tests
(v) Combined methods
(vi) Radioactive and nuclear methods
(vii) Magnetic and electrical methods
(viii) Acoustics emission techniques

Dynamic or Vibration Method: This is an important method used in testing concrete strength and other properties. The fundamental principle on which the dynamic or vibration methods are based is velocity of sound through a material. A mathematical relationship could be established between the velocity of sound through specimen and its resonant frequency and the relationship of these two to the modulus of elasticity of the same material. The relationships which are derived for solid mediums considered to be homogeneous, isotropic and perfectly elastic, but they may be applied to heterogeneous material like concrete.

Fiber Reinforced Concrete: Plain concrete possesses a very low tensile strength, limited ductility and little resistance to cracking. Internal microcracks are inherently present in the concrete and its poor tensile strength is due to the propagation of such microcracks, eventually leading to brittle fracture of the concrete.

It has been recognised that the admission of small, closely spaced and uniformly dispersed fibres to concrete would act as crack arrester and would substantially improve its static and dynamic properties. This type of concrete is known as fiber reinforced concrete.

Fiber reinforced concrete can be defined as a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete, uniformly dispersed suitable fibres. Continuous meshes, woven fabrics and long wires or rods are not considered to be discrete fibres.

The fibers can be imagined as an aggregate with an extreme deviation in shape from the rounded smooth aggregate. The fibers interlock and entangle around aggregate particles and considerably reduce the workability, while the mix becomes more cohesive and less prone to segregation. The fibers suitable for reinforcing the concrete have been produced from steel, glass and organic polymers.

The major factors affecting characteristics of fiber reinforced concrete are water-cement ratio, Percentage (volume fraction) of fibers, diameter and length of fibers. Fiber reinforced concrete has special application in hydraulic structures, air field and highway pavements, bridge decks, heavy duty floors and tunnel linings.

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2 Solid Mechanics

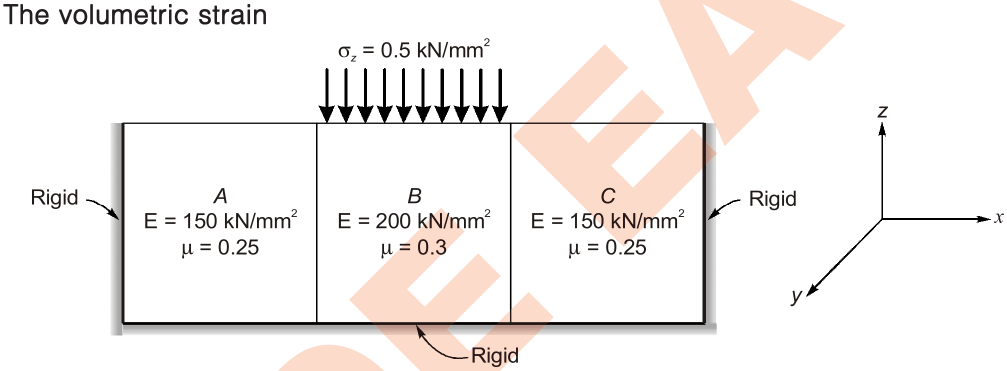
1. Simple Stress-Strain & Elastic Constants

1.1 (A) he figure below shows three metal cubes *A*, *B*, *C* of side 100 mm in direct contact, resting on a rigid base and confined in x-coordinate direction between two rigid end plate. If the upper face of the center cube *B* is subjected to uniform compressive stress of 0.5 kN/mm^2, compute for cube *B*, the following:

(*i*) The direct stress in x-direction (σ_x)

(*ii*) The direct strains in the three coordinates directions x, y and z

(*iii*) The volumetric strain



(B) State all assumptions made.

[30 + 10 marks : 1999]

Solution:

(A) (i) Since the cubes are confined in x-direction, hence

$$\Delta_{xA} + \Delta_{xB} + \Delta_{xC} = 0$$

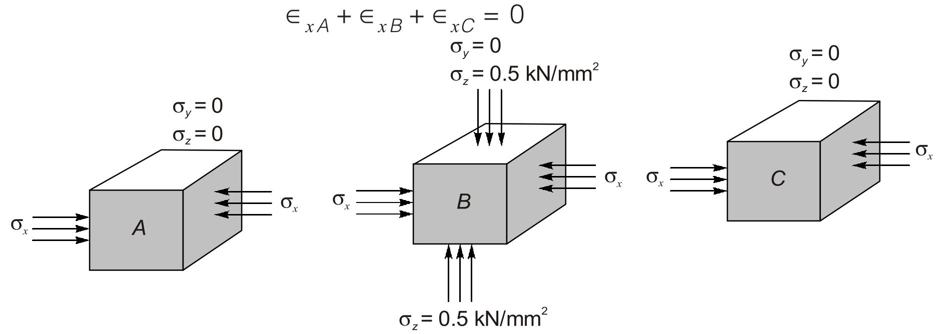
where, Δ_{xA} is change in length of cube *A* in x-direction

Δ_{xB} is change in length of cube *B* in x-direction

Δ_{xC} is change in length of cube *C* in x-direction

$$\frac{\Delta_{xA}}{L} + \frac{\Delta_{xB}}{L} + \frac{\Delta_{xC}}{L} = 0$$

$$\in_{xA} + \in_{xB} + \in_{xC} = 0 \quad \ldots(i)$$



From above FBD, we have,

$$\in_{xA} = -\frac{\sigma_x}{E_A}; \quad \in_{xB} = -\frac{\sigma_x}{E_B} + \mu_B \frac{\sigma_z}{E_B}; \quad \in_{xC} = -\frac{\sigma_x}{E_C}$$

Putting these values in (i), we get

$$\left[\frac{-\sigma_x}{E_A}\right] + \left[\frac{-\sigma_x}{E_B} + \mu_B \frac{\sigma_z}{E_B}\right] + \left[\frac{-\sigma_x}{E_C}\right] = 0 \qquad [\because E_A = E_C]$$

$$-\frac{\sigma_x}{E_A} - \frac{\sigma_x}{E_B} + \mu_B \frac{\sigma_z}{E_B} - \frac{\sigma_x}{E_c} = 0$$

$$-\frac{2\sigma_x}{E_A} - \frac{\sigma_x}{E_B} + \mu_B \frac{\sigma_z}{E_B} = 0$$

$$\sigma_x \left[\frac{2}{E_A} + \frac{1}{E_B}\right] = \frac{\mu_B \sigma_z}{E_B}$$

Direct stress in x-direction,

$$\sigma_x = \frac{\mu_B \sigma_z}{2 \cdot \frac{E_B}{E_A} + 1} = \frac{0.3 \times 0.5}{\left(2 \times \frac{200}{150}\right) + 1} = 0.041 \text{ kN/mm}^2$$

(ii) Direct strain in x,

$$\in_{xB} = -\frac{\sigma_x}{E_B} + \frac{\mu_B \sigma_z}{E_B} = \frac{-0.041}{E_B} + \frac{0.3 \times 0.5}{E_B} = 5.45 \times 10^{-4}$$

Direct strain in y,

$$\in_{yB} = \frac{+\mu_B \sigma_x}{E_B} + \frac{\mu_B \sigma_z}{E_B}$$

$$= \frac{0.3 \times 0.041}{E_B} + \frac{0.3 \times 0.5}{E_B} = 8.11 \times 10^{-4}$$

Direct strain in z,

$$\in_{zB} = -\frac{\sigma_z}{E_B} + \frac{\mu_B \sigma_x}{B} = \frac{-0.5}{E_B} + \frac{0.3 \times 0.041}{E_B} = -2.44 \times 10^{-3}$$

(iii) Volumetric strain,

$$\in_V = \in_{xB} + \in_{yB} + \in_{zB}$$

$$= 5.45 \times 10^{-4} + 8.11 \times 10^{-4} - 2.44 \times 10^{-3} = -1.084 \times 10^{-3}$$

(B) The assumptions made in the above analysis are:

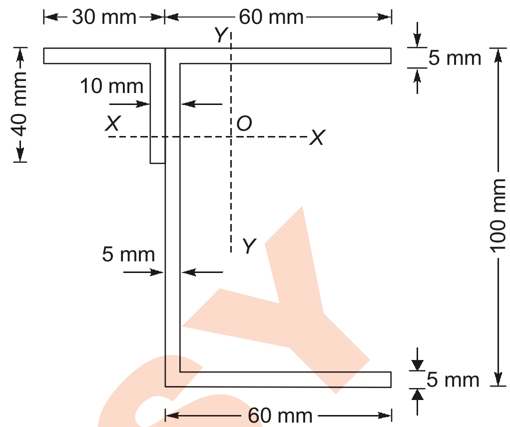
1. The material is linearly elastic which means it obeys Hooke's law.
2. The material is homogeneous i.e. properties of the material are same at any point of cross-section.
3. Material is isotropic i.e. properties are equal in all directions.
4. The cubes are placed on a perfectly rigid base i.e. base doesn't undergo any deformations.
5. The cubes do not have frictional force acting on their surfaces and also at the ends.

9. Shear Centre, Moment of Inertia & Principal Axes

9.1 The combined angle and channel section in figure below forms part of a runway beam. Calculate:

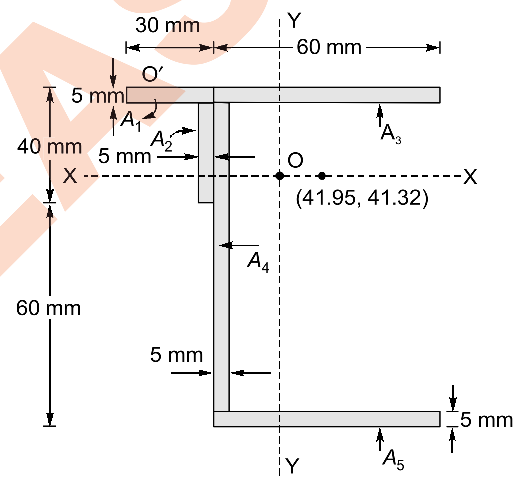
(a) Coordinate of the centroid
(b) Second moment of area about X-X
(c) Second moment of area about Y-Y
(d) Product of inertia about O

[10 × 4 = 40 marks : 1999]



Solution:

Dividing the whole arrangement into five parts of area A_1, A_2, A_3, A_4 and A_5 and taking O' as reference point (origin)



Area $A_1 = 30 \times 5 = 150\ \text{mm}^2$

$\bar{x}_1 = 15$ mm and $\bar{y}_1 = 2.5$ mm

Area $A_2 = 35 \times 5 = 175\ \text{mm}^2$

$\bar{x}_2 = 27.5$ mm and $\bar{y}_2 = 22.5$ mm

Area $A_3 = 60 \times 5 = 300\ \text{mm}^2$

$\bar{x}_3 = 60$ mm and $\bar{y}_3 = 2.5$ mm

Area $A_4 = 90 \times 5 = 450\ \text{mm}^2$

$\bar{x}_4 = 32.5$ mm and $\bar{y}_4 = 50$ mm

Area $A_5 = 60 \times 5 = 300\ \text{mm}^2$

$\bar{x}_5 = 60$ mm and $\bar{y}_5 = 97.5$ mm

(i) Coordinate of centroid

$$\bar{x} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 + A_4\bar{x}_4 + A_5\bar{x}_5}{A_1 + A_2 + A_3 + A_4 + A_5}$$

$$= \frac{(150\times15)+(175\times27.5)+(300\times60)+(450\times32.5)+(300\times60)}{150+175+300+450+300} = 41.95\ \text{mm}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 + A_5\bar{y}_5}{A_1 + A_2 + A_3 + A_4 + A_5}$$

$$= \frac{(150\times2.5)+(175\times22.5)+(300\times2.5)+(450\times50)+(300\times97.5)}{150+175+300+450+300} = 41.32\ \text{mm}$$

∴ Coordinate of the centroid of the area are 41.95 mm and 41.32 mm

(ii) Second moment of area about *x-x*

Coordinate of centroids of each area with respect to new origin *O*.

$$A_1 \rightarrow \begin{cases} \bar{x}_1 = -(41.95-15) = -26.95\ \text{mm} \\ \bar{y}_1 = +(41.32-2.5) = +38.82\ \text{mm} \end{cases}$$

$$A_2 \rightarrow \begin{cases} \bar{x}_2 = -(41.95-27.5) = -14.45\ \text{mm} \\ \bar{y}_2 = +(41.32-22.5) = +18.82\ \text{mm} \end{cases}$$

$$A_3 \rightarrow \begin{cases} \bar{x}_3 = +(60 - 41.95) = +18.05 \text{ mm} \\ \bar{y}_3 = +(41.32 - 2.5) = +38.82 \text{ mm} \end{cases}$$

$$A_4 \rightarrow \begin{cases} \bar{x}_4 = -(41.95 - 32.5) = -9.45 \text{ mm} \\ \bar{y}_4 = -(50 - 41.32) = -8.68 \text{ mm} \end{cases}$$

$$A_5 \rightarrow \begin{cases} \bar{x}_5 = +(60 - 41.95) = +18.05 \text{ mm} \\ \bar{y}_5 = +(97.5 - 41.32) = 56.18 \text{ mm} \end{cases}$$

We know, $I_x = \Sigma I_{x\text{self}} + \Sigma A\bar{y}^2$

$$\Sigma I_{x\text{ self}} = \frac{30 \times 5^3}{12} + \frac{5 \times 35^3}{12} + \frac{60 \times 5^3}{12} + \frac{5 \times 90^3}{12} + \frac{60 \times 5^3}{12} = 0.323 \times 10^6 \text{ mm}^4$$

$$\Sigma A\bar{y}^2 = (150 \times 38.82^2) + (175 \times 18.82^2) + (300 \times 38.82^2) + [450 \times (-8.68)^2] + [300 \times (56.18)^2]$$

$$= 1.72 \times 10^6 \text{ mm}^4$$

Hence, $I_x = \Sigma I_{x\text{self}} + \Sigma A\bar{y}^2$

$$= 0.323 \times 10^6 + 1.72 \times 10^6 \text{ mm}^4 = 2.043 \times 10^6 \text{ mm}^4$$

(iii) Second moment of area about y-y

$$I_y = \Sigma I_{y\text{self}} + \Sigma A\bar{x}^2$$

$$\Sigma I_{y\text{ self}} = \frac{5 \times 30^3}{12} + \frac{35 \times 5^3}{12} + \frac{5 \times 60^3}{12} + \frac{90 \times 5^3}{12} + \frac{5 \times 60^3}{12} = 1.93 \times 10^5 \text{ mm}^4$$

$$\Sigma A\bar{x}^2 = [150 \times (-26.95)^2] + [175 \times (-14.45)^2] + (300 \times 18.05^2) + [450 \times (-9.45)^2] + (300 \times 18.05^2)$$

$$= 3.81 \times 10^5 \text{ mm}^4$$

Hence, $I_y = 1.93 \times 10^5 + 3.81 \times 10^5$

$$I_y = 5.74 \times 10^5 \text{ mm}^4$$

(Here $\bar{x}$ & $\bar{y}$ are distance of C.G. of individual areas from centroid of the whole section)

(iv) Product of inertia:

$$I_{xy} = \Sigma (I_{xy})_{\text{self}} + \Sigma A\bar{x}\bar{y}$$

Hence $(I_{xy})_{\text{self}}$ will be zero for each area because each area have symmetrical about their centroidal axes.

$$I_{xy} = \Sigma A\bar{x}\bar{y}$$

$$= [150 \times (-26.95) \times 38.82] + [175 \times (-14.45) \times 18.82)] + [300 \times 18.05 \times 38.82]$$
$$+ [450 \times (-9.45) \times (-8.68)] + [300 \times 18.05 \times (-56.18)]$$

$$I_{xy} = -26.16 \times 10^4 \text{ mm}^4$$

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3 Structural Analysis

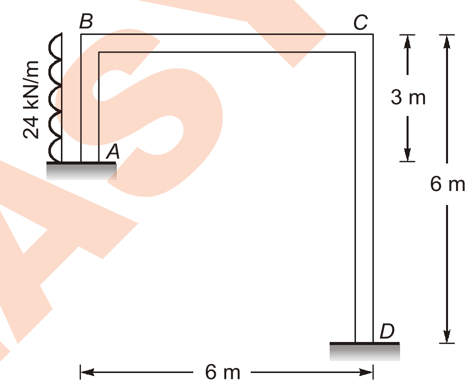
4. Methods of Structural Analysis

4.1 The rigid jointed portal frame shown in figure below is fully fixed at point *A* and *D* and is to be analysed by the moment distribution method, for the load system shown. It can be assumed that EI = 43500 kNm^2 for all members.

A. Draw the BMD throughout the structure.

B. Estimate the horizontal deflection at *B*.

[10 + 30 marks : 1999]



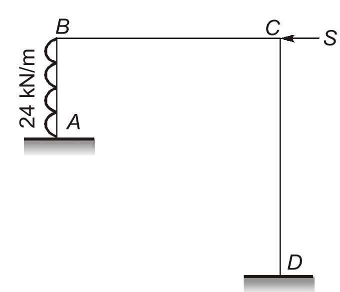
Solution:

A. Distribution Factors

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution Factors
B	*BA*	$\frac{I}{3}=\frac{2I}{6}$	$\frac{3I}{6}$	$\frac{2}{3}$
	BC	$\frac{I}{6}$		$\frac{1}{3}$
C	*CB*	$\frac{I}{6}$	$\frac{2I}{6}$	$\frac{1}{2}$
	CD	$\frac{1}{6}$		$\frac{1}{2}$

(i) Non sway analysis

Fixed end moments

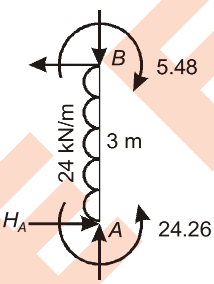


$$\bar{M}_{AB} = \frac{-24\times 3^2}{12} = -18 \text{ kN-m}$$

$$\bar{M}_{BA} = +\frac{24\times 3^2}{12} = +18 \text{ kN-m}$$

$$\bar{M}_{BC} = \bar{M}_{CB} = \bar{M}_{CD} = \bar{M}_{DC} = 0$$

A (AB)	B (BA)	B (BC)	C (CB)	C (CD)	D (DC)
	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	
–18	+18	0	0	0	0
	–12	–6	0	0	
–6		0	–3		0
	0	0	+1.5	+1.5	
0		+0.75	0		+0.75
	–0.50	–0.25	0	0	
–0.25		0	–0.125		0
	0	0	+0.06	+0.06	
0		+0.03	0		+0.03
	–0.02	–0.01	0	0	
–0.01			–0.005		0
–24.26	**+5.48**	**–5.48**	**–1.57**	**+1.57**	**+0.78**

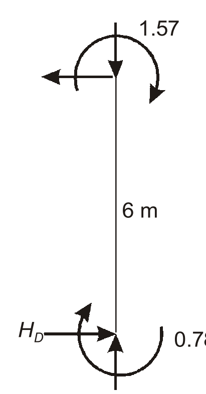


Taking moment about B,

$$3H_A + 24.26 - 5.48 + 24 \times 3 \times 1.5 = 0$$

$$\text{Horizontal reaction at } A = \frac{-24.26 + 5.48 - 24 \times 3 \times 1.5}{3}$$

$$= -42.26 \text{ kN } (\rightarrow) = 42.26 \text{ kN } (\leftarrow)$$



Taking moment about C,

$$6H_D - 1.57 - 0.78 = 0$$

$$\text{Horizontal reaction at } D = \frac{1.57 + 0.78}{6} = 0.39 \text{ kN } (\rightarrow)$$

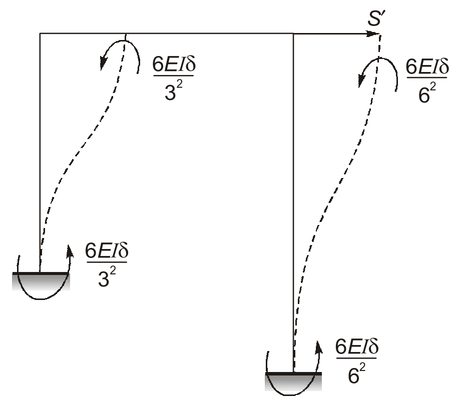
Force acting from left to right = Total UDL + Horizontal reaction at D

= (24 × 3) + 0.39 = 72. 39 kN

Force acting from right to left = 42.26 kN

∴ Sway force, S = 72.39 – 42.26 = 30.13 kN (→)

(ii) Sway analysis



Since sway of the frame is from left to right, the initial fixing moments due to deflection Δ will be in anticlockwise direction

$$\bar{M}_{AB} = \bar{M}_{BA} = \frac{-6EI\Delta}{L^2} = \frac{-6EI\Delta}{3^2} = \frac{-6EI\Delta}{9}$$

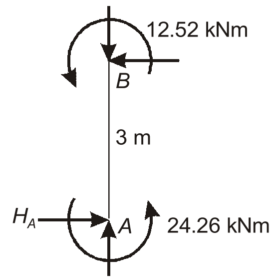
$$\bar{M}_{BC} = \bar{M}_{CB} = 0$$

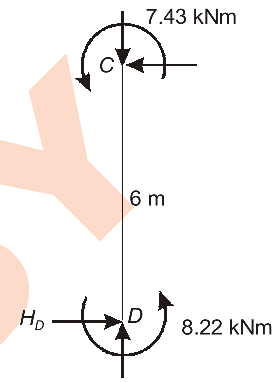
$$\bar{M}_{CD} = \bar{M}_{DC} = \frac{-6EI\Delta}{L^2} = \frac{-6EI\Delta}{6^2} = \frac{-6EI\Delta}{36}$$

∴	$\bar{M}_{AB}$	:	$\bar{M}_{BA}$	:	$\bar{M}_{BC}$	:	$\bar{M}_{CB}$	:	$\bar{M}_{CD}$	:	$\bar{M}_{DC}$
	$-\frac{1}{9}$	:	$-\frac{1}{9}$	:	0	:	0	:	$-\frac{1}{36}$	:	$-\frac{1}{36}$
	– 4	:	– 4	:	0	:	0	:	– 1	:	– 1
	– 36	:	– 36	:	0	:	0	:	– 9	:	– 9

	A	B		C		D
		2/3	1/3	1/2	1/2	
	–36	–36	0	0	–9	–9
		+24	+12	+4.5	+ 4.5	
	+12		+2.25	+6		+2.25
		–1.50	–0.75	–3	–3	
	–0.75		–1.50	–0.38		–1.50
		+1.00	+0.5	+0.19	+0.19	
	+0.5		+0.09	+0.25		+0.10
		–0.06	–0.03	–0.121	–0.13	
	–0.03		–0.06	–0.02		–0.07
		+0.04	+0.02	+0.01	+0.01	
	+0.02		0	+0.01		0
		0	0	0	0	
Col. (a)	**–24.26**	**–12.52**	**+12.52**	**+7.43**	**–7.43**	**–8.22**

Let the moments shown in col. (a) be due to a sway force *S′*.





Taking moment about B,

$$3H_A + 24.26 + 12.52 = 0$$

$$\text{Horizontal reaction at } A = \frac{-24.26 - 12.52}{3}$$

$$= -12.26 \text{ kN } (\rightarrow) = 12.26 \text{ kN } (\leftarrow)$$

Taking moment about C,

$$6H_D + 8.22 + 7.43 = 0$$

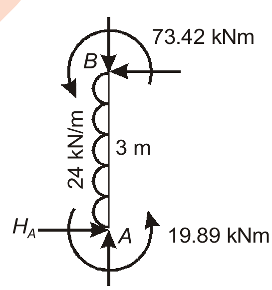
$$\text{Horizontal reaction at } D = \frac{-7.43 - 8.22}{6}$$

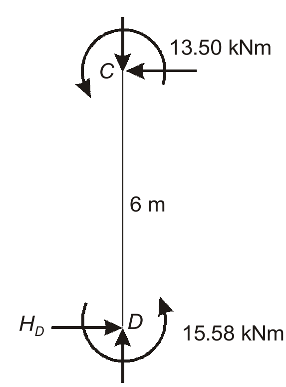
$$= -2.61 \text{ kN } (\rightarrow) = 2.61 \text{ kN } (\leftarrow)$$

∴ Sway force, $S' = 12.26 + 2.61 = 14.87 \text{ kN } (\rightarrow)$

Thus, the moments shown in col. (a) are due to a sway force of 14.87 kN. Hence, for the actual sway force of 30.13 kN the actual sway moments will be $\frac{30.13}{14.87} \times$ col. (a) moments.

	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
col. (a)	– 24.26	– 12.52	+ 12.52	+ 7.43	– 7.43	– 8.22
Actual Sway moments	– 49.16	– 25.37	+ 25.37	+ 15.06	– 15.06	– 16.66
Non-Sway moments	– 24.26	+ 5.48	– 5.48	– 1.57	+ 1.57	+ 0.78
Final moments	– 73.42	– 19.89	+ 19.89	+ 13.50	– 13.50	– 15.88





Taking moment about B,

$$3H_A + 19.89 + 73.42 + 24 \times 3 \times 1.5 = 0$$

$$\text{Actual horizontal reaction at } A = \frac{-73.42 - 19.89 - 24 \times 3 \times 1.5}{3}$$

$$= -67.10 \text{ kN} (\rightarrow) = 67.10 \text{ kN} (\leftarrow)$$

Taking moment about C,

$$6H_D + 15.58 + 13.50 = 0$$

$$\text{Actual horizontal reaction at } D = \frac{-13.50 - 15.88}{6} = -4.90 \text{ kN } (\rightarrow) = 4.91 \text{ kN } (\leftarrow)$$

$$\text{Vertical reaction at } A = -\left(\frac{19.89 + 13.50}{6}\right) = -5.57 \text{ kN}(\uparrow) = 5.57 \text{ kN} (\downarrow)$$

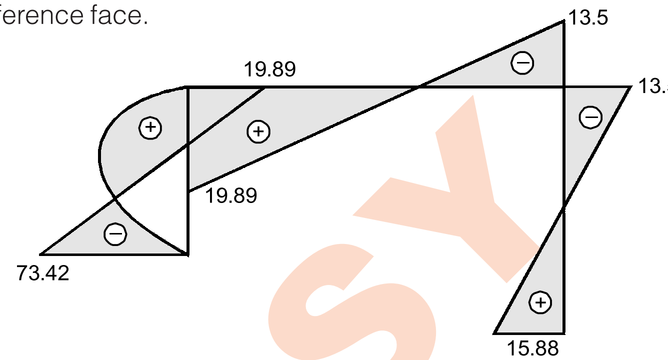
$\therefore$ Vertical reaction at D = 5.57 kN (↑)

Bending moment diagram

Taking outer face of the portal frame as reference face.

Simply supported moment for

$$AB = \frac{24 \times 3^2}{8} = 27 \text{ kN-m}$$



B. We have

$$M_{AB} = -73.42 \text{ kN-m}$$

and $$\bar{M}_{AB} = -18 \text{ kN-m}$$

$$M_{BA} = -19.89 \text{ kN-m}$$

and $$\bar{M}_{BA} = +18 \text{ kN-m}$$

The sway force is from left to right, hence, the deflection will be positive.

Writing slope deflection equations for M_{AB} and M_{BA}, we get

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$\Rightarrow$ $$-73.42 = -18 + \frac{2EI}{3}\left(0 + \theta_B - \frac{3\delta}{3}\right)$$

$\Rightarrow$ $$EI(\delta - \theta_B) = \frac{55.42 \times 3}{2}$$

$$\delta - \theta_B = \frac{83.13}{EI} \quad \text{... (i)}$$

and $$M_{BA} = \bar{M}_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$\Rightarrow$ $$-19.89 = 18 + \frac{2EI}{3}\left(2\theta_B + 0 - \frac{3\delta}{3}\right)$$

$\Rightarrow$ $$(\delta - 2\theta_B)\, EI = \frac{37.89 \times 3}{2}$$

$$\delta - 2\theta_B = \frac{56.835}{EI} \quad \text{...(ii)}$$

Solving (i) and (ii) we get

$$\theta_B = \frac{26.295}{EI} \text{ and } \delta = \frac{109.425}{EI}$$

But $$EI = 43500 \text{ kN-m}^2$$

$\therefore$ $$\delta = \frac{109.425}{43500}$$

$\Rightarrow$ $$\delta = 2.52 \text{ mm}$$

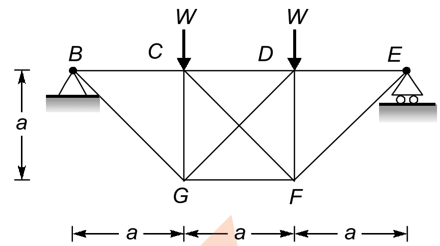
Thus the horizontal deflection at B is 2.52 mm from left to right.

5. Trusses

5.1 Figure below shows a plane pinjointed truss *BCDEFG* of the given dimensions and carrying the loading shown. All member have the same axial rigidity.

(i) Calculate the axial force in member *FG*.

(ii) Evaluate the force in the same member if it is fabricatered 0.1% too short, and the structure carries no external load.



[30 + 10 = 40 marks : 1999]

Solution:

(i) The degree of determinacy may be given by

$$D_S = m + r_e - 2j$$

Here $m = 10,\ r_e = 3,\ j = 6$

$\therefore$ $D_S = 10 + 3 - 2 \times 6 = +1$

After removing *GF*, the static indeterminacy will be

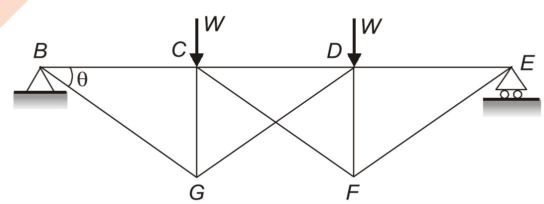
$$D_S = m + r_e - 2j$$

$\Rightarrow$ $D_S = (10 - 1) + 3 - 2 \times 6$

$\Rightarrow$ $D_S = 0$

P-system of forces

After removing *GF*, the resultant truss will be as in the given figure.



$$\Sigma F_y = 0$$

$\Rightarrow$ $V_B + V_E = 2W$

and $\Sigma M_B = 0$

$\Rightarrow$ $V_E \times 3a - W \times 2a - W \times a = 0$

$\Rightarrow$ $V_E = \dfrac{3Wa}{3a}$

$\Rightarrow$ $V_E = W$

$\therefore$ $V_B = W$

Also $\tan\theta = \dfrac{a}{a}$

$\Rightarrow$ $\tan\theta = 1,\quad \sin\theta = \dfrac{1}{\sqrt{2}}$ and $\cos\theta = \dfrac{1}{\sqrt{2}}$

Assuming tensile forces as positive and compressive forces as negative

Considering joint *B*

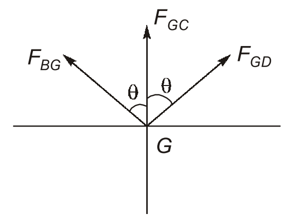
$\Sigma F_y = 0$ AND $\Sigma F_x = 0$

$\Rightarrow$ $W - F_{BG} \sin\theta = 0$ | $\Rightarrow$ $F_{BC} + F_{BG}\cos\theta = 0$

$\Rightarrow$ $F_{BG} = \dfrac{W}{\sin\theta}$ | $\Rightarrow$ $F_{BC} = -\sqrt{2}\, W \times \dfrac{1}{\sqrt{2}}$

$\Rightarrow$ $F_{BG} = \sqrt{2}\, W$ (Tension) | $\Rightarrow$ $F_{BC} = -W$ (Compression)

Considering joint G



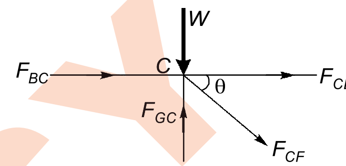
$\Sigma F_x = 0$ AND $\Sigma F_y = 0$

$\Rightarrow \quad F_{GD}\sin\theta - F_{BG}\sin\theta = 0 \qquad \Rightarrow \quad F_{GD}\cos\theta + F_{BG}\cos\theta + F_{GC} = 0$

$\Rightarrow \quad F_{GD} = F_{BG} \qquad \Rightarrow \quad F_{GC} = -\sqrt{2}\,W \times \frac{1}{\sqrt{2}} - \sqrt{2}\,W \times \frac{1}{\sqrt{2}}$

$\Rightarrow \quad F_{GD} = \sqrt{2}\,W$ (Tension) $\qquad F_{GC} = -2\,W$ (Compression)

Considering joint C



$\Sigma F_y = 0$ AND $\Sigma F_x = 0$

$\Rightarrow \quad F_{GC} - W - F_{CF}\sin\theta = 0 \qquad \Rightarrow \quad F_{CD} + F_{CF}\cos\theta + F_{BC} = 0$

$\Rightarrow \quad 2W - W = F_{CF}\sin\theta \qquad \Rightarrow \quad F_{CD} = -W\sqrt{2} \times \frac{1}{\sqrt{2}} - W$

$\Rightarrow \quad F_{CF} = W\sqrt{2}$ (Tension) $\qquad \Rightarrow \quad F_{CD} = -2W$ (Compression)

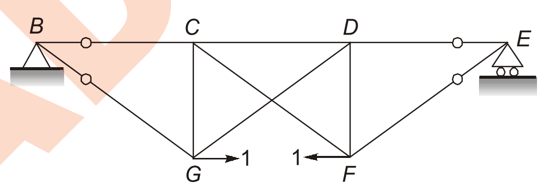
Now, as the truss is symmetrical, the forces in remaining members will be corresponding to the forces in the above members.

i.e.,
$$F_{BG} = F_{EF} = \sqrt{2}\,W \text{(Tension)}$$
$$F_{BC} = F_{ED} = -W \text{(Compression)}$$
$$F_{CG} = F_{FD} = -2W \text{(Compression)}$$

K- system of forces

The truss is now analysed by removing all external loads and applying unit tensile forces at G and F respectively.

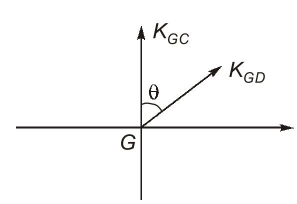
The only forces acting on the truss are unit tensile forces at G and F respectively which are opposite in nature. Therefore the vertical reaction will be zero at B and E respectively.



$\therefore \quad K_{BC} = 0$ and $K_{BG} = 0$

Also $\quad K_{ED} = 0$ and $K_{EF} = 0$ and $K_{GF} = +1$

Considering joint G



$\Sigma F_x = 0$ AND $\Sigma F_y = 0$

$\Rightarrow \quad K_{GD}\sin\theta + 1 = 0 \qquad K_{GC} + K_{GD}\cos\theta = 0$

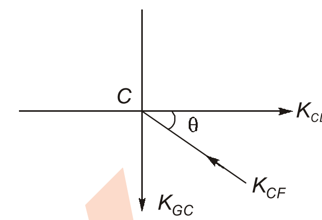
$\Rightarrow \quad K_{GD} = -\sqrt{2}$ (Compression) $\qquad \Rightarrow \quad K_{GC} = -(-\sqrt{2}) \times \frac{1}{\sqrt{2}}$

$\Rightarrow \quad K_{GC} = 1$ (Tension)

By symmetry

$$K_{CF} = -\sqrt{2} \text{ (Compression)}$$
$$K_{FD} = 1 \text{ (Tension)}$$

Considering joint C



$$\Sigma F_x = 0$$

$\Rightarrow$ $$K_{CD} - K_{CF}\cos\theta = 0$$

$\Rightarrow$ $$K_{CD} = \sqrt{2} \times \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$K_{CD} = 1$$

Member	***P***	***K***	***L***	***PKL***	***K²L***	***KX′***
BC	$-W$	0	a	0	0	0
BG	$\sqrt{2}W$	0	$a\sqrt{2}$	0	0	0
CD	$-2W$	1	a	$-2Wa$	a	X'
CF	$\sqrt{2}W$	$-\sqrt{2}$	$\sqrt{2}a$	$-2\sqrt{2}Wa$	$2\sqrt{2}a$	$-\sqrt{2}X'$
GC	$-2W$	1	a	$-2Wa$	a	X'
ED	$-W$	0	a	0	0	0
EF	$\sqrt{2}W$	0	$\sqrt{2}a$	0	0	0
FD	$-2W$	1	a	$-2Wa$	a	X'
GF	0	1	a	0	a	X'
GD	$\sqrt{2}W$	$-\sqrt{2}$	$\sqrt{2}a$	$-2\sqrt{2}Wa$	$2\sqrt{2}a$	$-\sqrt{2}X'$

$$\Sigma PKL = -(6Wa + 4\sqrt{2}\,Wa)$$
$$\Sigma K^2L = 4a + 4\sqrt{2}\,a$$

We know that,

$$X = -\frac{\Sigma \frac{PKL}{AE}}{\Sigma \frac{K^2L}{AE}}$$

Axial rigidity of members is constant i.e. AE is constant.

$\therefore$ $$X = -\frac{\Sigma PKL}{\Sigma K^2L} - \frac{-\left(6Wa + 4\sqrt{2}\,Wa\right)}{4a + 4\sqrt{2}\,a}$$

$\Rightarrow$ $$X = \left(\frac{3 + 2\sqrt{2}}{2 + 2\sqrt{2}}\right) W = 1.207\ W$$

Force in $GF = P + KX = 0 + 1 \times 1.207\ W$

$$= 1.207\ W \text{ (Tension)}$$

(ii) GF is 0.1 % too short

$$\Delta = \frac{0.1}{100} \times a = \frac{a}{1000}$$

Let forces in all the members are expressed in terms of X'.

$\therefore$ $$S = P + KX'$$

But the structure carries no external load i.e. $P = 0$

$\therefore$ $$S = KX'$$

The total strain energy stored in the structure may be given by

$$U = U_{BG} + U_{BC} + U_{CD} + U_{CF} + U_{GC} + U_{ED} + U_{EF} + U_{FD} + U_{GF} + U_{GD}$$

$$\Rightarrow \quad U = 0 + 0 + \frac{(X')^2 a}{2AE} + \frac{\left(-\sqrt{2}X'\right)^2 \sqrt{2}a}{2AE} + \frac{(X')^2 a}{2AE} + 0 + 0 + \frac{(X')^2 a}{2AE}$$

$$+ \frac{(X')^2 a}{2AE} + \frac{\left(-\sqrt{2}X'\right)^2 \times \sqrt{2}a}{2AE}$$

$$\Rightarrow \quad U = \frac{4a(X')^2}{2AE} + \frac{4\sqrt{2}a(X')^2}{2AE}$$

$$\Rightarrow \quad U = \frac{4a(X')^2}{2AE}\left(\sqrt{2}+1\right)$$

$$\Rightarrow \quad \frac{\partial U}{\partial X'} = \frac{8a X'\left(\sqrt{2}+1\right)}{2AE} = \Delta$$

$$\therefore \quad \frac{4\left(\sqrt{2}+1\right)a X'}{AE} = \frac{a}{1000}$$

$$\Rightarrow \quad X' = \frac{AE}{4000\left(\sqrt{2}+1\right)}$$

Thus force in GF due to lack of fit = KX'

$$= 1 \times \frac{AE}{4000\left(\sqrt{2}+1\right)} = \frac{AE}{4000\left(\sqrt{2}+1\right)}$$

■■■■

5 Design of Concrete and Masonry Structures

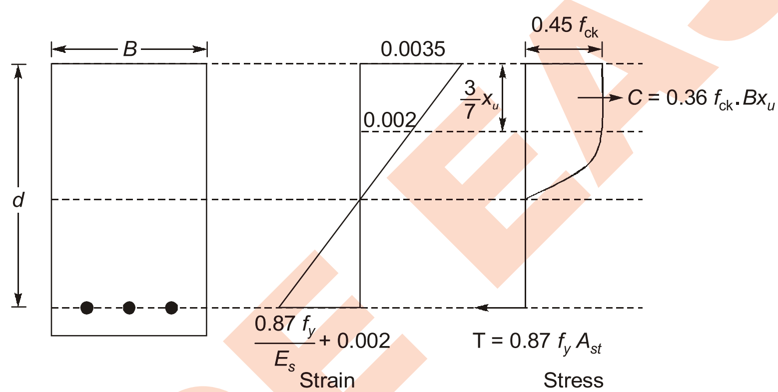
1. Fundamentals of RCC

1.1 Explain 'under-reinforced', 'balanced' and 'over-reinforced' sections in the ultimate load theory.

[10 marks : 1999]

Solution:

The stress block parameters are shown below:



Actual depth of neutral axis is given by

$$C = T$$

$$\Rightarrow \quad 0.36 f_{ck} Bx_u = 0.87 f_y A_{st}$$

$$\frac{x_u}{d} = \frac{0.87}{0.36} \cdot \frac{f_y}{f_{ck}} \cdot \frac{A_{st}}{Bd}$$

Also limiting value of neutral axis is given by.

$$\frac{x_{u,\max}}{d} = \frac{700}{1100 + 0.87 f_y}$$

Now

(i) when $\frac{x_u}{d}$ is less than the limiting value, it means that the reinforcement provided is less than its limiting value. Thus steel reinforcement reaches its yield stress before ultimate strain is reached in concrete. Such a section is called a **under reinforced section.**

(ii) When $\frac{x_u}{d}$ is equal to the limiting value, the case corresponds to balanced section design in which the steel reinforcement reaches its yield stress at the same instant when ultimate strain is reached in concrete. Such a section is called **balanced section.**

(iii) When $\frac{x_u}{d}$ is greater than the limiting value, the case corresponds to the beam in which percentage of steel is sufficient to ensure that steel yield does not take place. Failure occurs when the strain in extreme fibres in concrete reaches its ultimate value. Thus failure takes place due to crushing of concrete while strain in steel remains below the yield strain. Such a section is called **over reinforced section.** It should be noted that compression failure is sudden and therefore not desirable.

2. Beams and Slabs

2.1 (i) Calculate the ultimate moment of resistance of an RC rectangular beam with the following data:
Breadth of beam = 230 mm
Overall depth of beam = 550 mm
Tension steel consist of 4 numbers of 20 mm diameter bars of grade Fe 415
Clear cover = 30 mm, M20 concrete grade.

(ii) Hence determine the intensity of safe superimposed load (excluding self weight) this above beam can carry on a simply supported span of 5 m.

[25 + 5 = 30 marks : 1999]

Solution:

(i) $A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$

(a) Limiting depth of NA

$$x_{u,\lim} = 0.48\,d = 0.48 \times (550 - 30 - 20/2) = 0.48 \times 510 = 244.8 \text{ mm}$$

(b) Actual depth of NA

$$C = T$$

$$0.36\, f_{ck}\, B\, x_u = 0.87\, f_y\, A_{st}$$

$$\Rightarrow \quad x_u = \frac{0.87 \times 415 \times 1256.64}{0.36 \times 20 \times 230} = 273.98 \text{ mm};\ x_u > x_{u,\lim}$$

Hence the section is over reinforced and since over reinforced section is not allowed in LSM, the depth of actual NA is limited up to $x_{u,\lim} = 244.8$ mm

(c) Moment of Resistance,

$$MR = 0.36\, f_{ck}\, B x_{u,\lim}\,(d - 0.42\, x_{u,\lim})$$

$$= 0.36 \times 20 \times 230 \times 244.8\,(510 - 0.42 \times 244.8)$$

$$MR = 165.06 \text{ kN-m.}$$

(ii) If 'w' is the net super imposed load including self weight of the beam, then the bending moment for the beam will be

$$M_{max} = \frac{w_u l^2}{8} = MR$$

$$\Rightarrow \quad \frac{w_u \times 5^2}{8} = 165.06$$

$$\Rightarrow \quad w_u = \frac{165.06 \times 8}{25} = 52.82 \text{ kN/m}$$

But self weight of beam = $0.230 \times 0.550 \times 1 \times 25 = 3.1625$ kN/m

Hence **safe super imposed load** = $\left(\frac{52.82}{1.5}\right) - 3.1625 = 32.05$ kN/m

■■■■

6 Construction Practice, Planning and Management

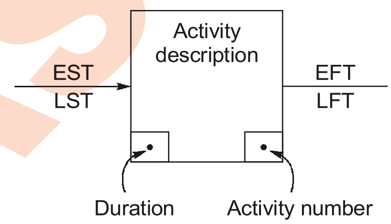
2. PERT and CPM

2.1 What are the main advantages of Activity on the Node networks over Activity on the Arrow networks? [15 marks : 1999]

Solution:

In AON system sometimes called precedence diagrams also, the nodes represent the activities and the arrow, their interdependence or precedence relationships.

Nodes are usually represented by squares or rectangles but circles and other conventional geometrical shapes may also be used. One of the most common types of node representation is shown in figure:



Main advantages of *AON* system over *AOA* system are:

(i) *AON* system of the network completely eliminates the use of dummy activities.

(ii) This system can show activity which should be allowed to overlap each other or must be separated by a time delay.

(iii) This system is self sufficient as it contains all activity times (*EST*, *LST*, *EFT*, *LFT*) on the diagram itself. This facilitates efficient scheduling and control.

(iv) Revisions and modifications can be carried out easily without affecting most of the activities i.e. few activities only have to be altered to incorporate changes in duration, logic and activity sequence.

(v) Pre-operations and post-operations of the activity under consideration are distinctly visible.

(vi) This system adopts simple notations similar to the engineering flow charts and hence can be easily understood by non-specialists also.

(vii) It is more advantageous in projects involving highly repetitive nature of series of operations.

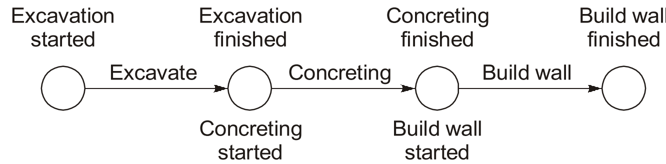
2.2 With the help of suitable diagrams compare and contrast Activity on the Arrow networks with Activity on the Node networks (Precedence Diagrams).

[10 marks : 1999]

Solution:

CPM has two systems, namely *AOA* system i.e. Activity on Arrow Network System and *AON* system i.e. Activity on Node Network System.

1. **AOA System:** In this system an activity is graphically represented by an arrow bounded by circle at each end. Activity name or symbol is printed on one side of the arrow and time duration of the activity on the other side of the arrow. A circle situated at the tail end of the arrow is called tail event and represents the instant of time when the activity will start and the circle situated at the head end or arrow end is called head event and represents the instant of time when the activity will finish as shown in figure.

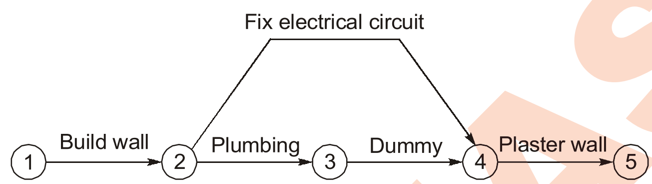


The AOA system uses dummies for the following purposes:

(i) To maintain a unique numbering system for different activity.

(ii) To keep the logical sequence of activity showing correct inter-relationship.

Consider the network shown below. Both activities namely fixing electrical circuit and plumbing start at one event and both ends at another event. So one dummy was introduced just to identify each activity by its own $i - j$ number.

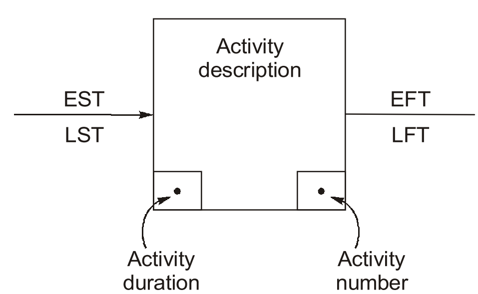


Fixing electrical circuit 2 – 4
Plumbing 2 – 3

Consider the network shown in figure. Let *E*, *F*, *G* represents the shuttering of three bays of a retaining wall and *L*, *M* and *N*, the corresponding concreting operation. In view of the nature of the activities, shuttering has to precede concreting.

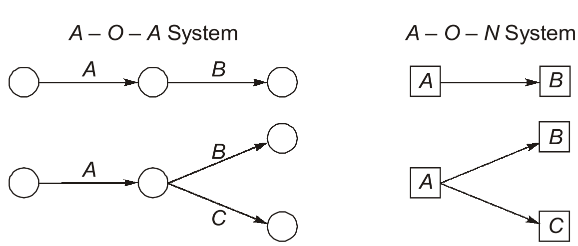
As regarding the labour, only one forward crew for shuttering and one forward crew for concreting are available. Here dummies have been used to represent the constraint and consequent interdependence. But number of dummies must be minimum for the efficient network.

2. **AON System:** In this system, the nodes represent the activities and the arrow, their interdependence or precedence relationships. Nodes are usually represented by squares or rectangles but circles and other conventional shapes may also be used. One of the most common types of node representation is shown in figure.



The AON system also called precedence diagram eliminates the use of dummies. Events have no place while activities have their places with durations. Start of an activity must be linked to the end of all activities which must be completed before that start may take place. The connecting lines runs in the left right direction.

Some examples are shown below:



2.3 What is the difference between 'Free Float' and 'Total Float'? Explain the significance of each term.

[15 marks : 1999]

Solution:

Total Float: It is the time span by which the starting (or finishing) of an activity can be delayed without delaying the completion of the project. In certain activities, it will be found that there is a difference between maximum time available and the actual time required to perform the activity. This difference is known as the total float.

Effectively, it is an inbuilt reserve of a resource (time) which is available for use in certain tail event and head event. Now, maximum time available to perform events can therefore be considered as setting the limits.

Consider an activity *i-j*. The time duration available for this activity is equal to the difference between its earliest start time $\left(T_E^i\right)$ and the latest finish time $\left(T_L^j\right)$.

$\therefore$ Maximum time available = $T_L^j - T_E^i$

activity time required = t^{ij}

$\therefore$ **Total float (F_T) = time available – time required**

$$F_T = \left(T_L^j - T_E^i\right) - t^{ij}$$

or $$F_T = T_L^j - \left(T_E^i + t^{ij}\right) = LFT - EFT$$

or $$F_T = \left(T_L^j - t^{ij}\right) - T_E^i = LST - EST$$

It should be clearly noted that the total float for each activity is a measure of its particular relationship to all other activities in the project, since the earliest start time ties in all preceding activities and the latest finish time ties in all succeeding activities.

Free Float: It is that portion of positive total float that can be used by an activity without delaying any succeeding activity (or without affecting the total float of the succeeding activity). The concept of free float is based on the possibility that all the events occur at their earliest times (i.e. all activities start as early as possible).

To get a clear concept of the free float, consider activity *i-j* and its successor activity *j-k*. Events *i* and *j* has earliest occurrence times as T_E^i and T_E^j. Earliest start time for activity *i - j* will be T_E^i, while EST for *j-k* will be T_E^j. However if t^{ij} is the activity time, activity *i-j* will be complete by $\left(T_E^i + t^{ij}\right)$ time, while activity *j-k* cannot start because its EST $\left(T_E^j\right)$ is greater than $\left(T_E^i + t^{ij}\right)$. The difference between the two is the free float for *i-j*.

$$F_F \text{ for } i\text{-}j = T_E^j - \left(T_E^i + t^{ij}\right)$$

But $\left(T_E^i + t^{ij}\right)$ is the earliest finish time (*EFT*) of the activity *i-j*, while T_E^j is the early start time for activity *j-k*.

$$F_F \text{ for } i\text{-}j = EST \text{ of successor activity} - EFT \text{ of present activity}$$

or $$\mathbf{F_F \text{ for } i\text{-}j = T_E^j - EFT}$$

Hence free float for an activity i-j is the difference between its earliest finish time and the earliest start time of its successor activity.

■■■■