MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

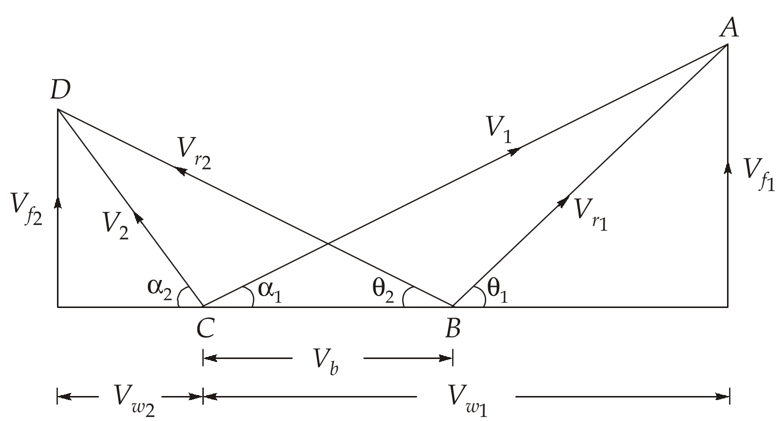
ESE-2025 Mains Test Series

Mechanical Engineering Test No : 3

Section A : Fluid Mechanics + Fluid Machinery + Power Plant

1. (a)

Given : $\alpha_1 = 25°$; $\theta_2 = \theta_1$; $k = 0.8$; $V_{r_2} = 0.8V_{r_1}$, $D_m = 0.6$ m, $V_1 = 780$ m/s, $P = 20 \times 10^3$ W



From maximum efficiency,

$$\rho = \frac{V_b}{V_1} = \frac{\cos\alpha_1}{2}$$

$$\therefore \quad V_b = \frac{V_1 \cos\alpha_1}{2} = \frac{780 \times \cos 25°}{2} = 353.46 \text{ m/s}$$

$$V_{w_1} = V_1\cos\alpha_1 = 780 \times \cos 25° = 706.92 \text{ m/s}$$

$$V_{f_1} = V_1 \sin\alpha_1 = 780 \times \sin 25° = 329.64 \text{ m/s}$$

$$\tan\theta_1 = \frac{V_{f_1}}{V_{w_1} - V_b} = \frac{329.64}{706.92 - 353.46} = 0.9326$$

$$\theta_1 = \tan^{-1}(0.9326)$$

$$\theta_1 = 43° = \theta_2$$

As, $$V_{f_1} = V_{r_1} \sin\theta_1$$

$\therefore$ $$V_{r_1} = \frac{V_{f_1}}{\sin\theta_1} = \frac{329.64}{\sin 43°} = 483.34 \text{ m/s}$$

As we know, $$V_{r_2} = 0.8 V_{r_1} = 0.8 \times 483.34$$

$$V_{r_2} = 386.67 \text{ m/s}$$

$$V_{w_1} + V_{w_2} = V_{r_1}\cos\theta_1 + V_{r_2}\cos\theta_2$$

$$= 483.34 \times \cos 43° + 386.67 \times \cos 43° \quad [\text{As } \theta_1 = \theta_2]$$

$$= 636.29 \text{ m/s}$$

$$\text{Power, } P = \dot{m}\left(V_{w_1} + V_{w_2}\right) \times V_b$$

$$20 \times 10^3 = \dot{m} \times 636.29 \times 353.46$$

$$\dot{m} = 0.0889 \text{ kg/s}$$

or $$\dot{m} = 320.04 \text{ kg/hr}$$

Blade or diagram efficiency,

$$\eta_d = \frac{\dot{m}\left(V_{w_1} + V_{w_2}\right)V_b}{\frac{1}{2}\dot{m}V_1^2} = \frac{20 \times 10^3}{\frac{1}{2} \times 0.0889 \times 780^2} = 0.7395$$

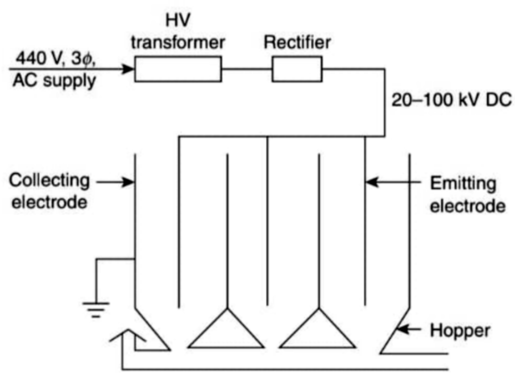
$$\eta_d = 73.96\%$$

1. (b)

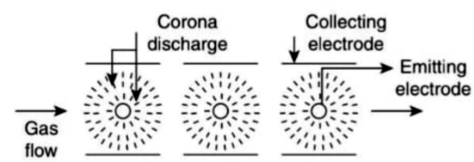
Electrostatic Precipitator : The electrostatic precipitator (ESP) is one of the most widely used fly ash collection devices. This is used in various industrial units such as:

1. Pulp and paper mills, non-ferrous metal industry and chemical industry.
2. Cement recovery furnace and steel plant for cleaning blast furnace gas.

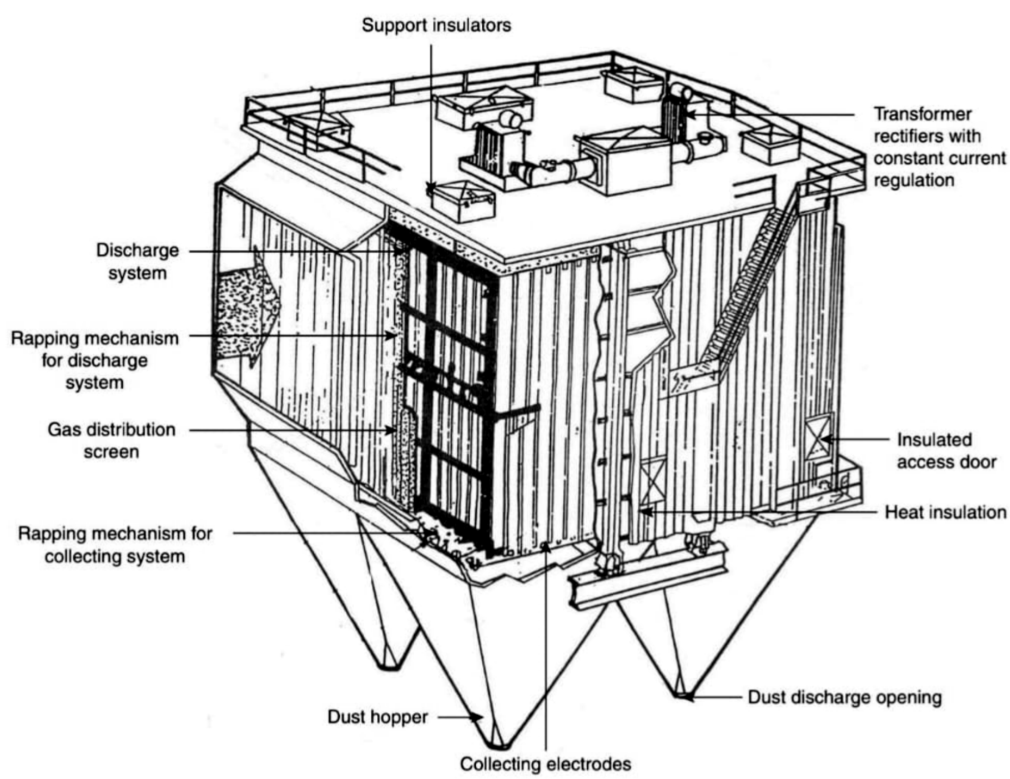
3. Removing tars from coke oven, sulphuric acid (Pyrite raw material) and phosphoric acid plant.
4. Petroleum industry and thermal power plant.



Elements of Electrostatic precipitator.



Production of corona.



View of an Electrostatic precipitator.

In this precipitator, the electrostatic forces are utilised to separate the dust particles from the gas to be cleaned. The principle upon which an ESP operates is that the individual particle of dust in the dust laden gases passing through the chamber is given an electric charge by absorption of free ions from a high-voltage DC ionising field.

Parts of Electrostatic Precipitator

ESP consists of the following parts:

1. Precipitator casing.
2. Hoppers.
3. Collecting electrode system.
4. Emitting electrode system.
5. Rapping mechanism.
6. High-voltage rectifier transformer with electronic controller.

The precipitator casing is a welded construction, consisting of prefabricated wall and roof panels. The casing rests on supports which allow for free thermal expansion of the casing during operation. There are two sets of electrodes in that equipment, one is emitting electrode and the other is collecting electrode. Emitting electrodes are generally wire shaped and connected with negative polarity. Each emitting electrode is located between two parallel plates. The plates are connected with the positive end. The emitting electrodes are connected with high-voltage current from a DC source. Because of the high voltage, a blue luminous glow known as corona is produced around these. When the gas passes over the emitting electrode, the particles present in the gas stream ionise negatively and are attracted by collecting electrode. In this way, ash particles are deposited on collecting plates. To remove the ash particles from the collecting plates, continuous hammering system is used, which hammers continuously on the flat plate and the particles drop to the bottom ash hopper by the mechanism of rapping. Intermittent hammering system is utilised to remove dust layers from emitting electrodes. Collected ash in the ash hopper is evacuated by creating high vacuum either in dry or in wet conditions by mixing water to ash pond. The compact view of an ESP.

Factors Responsible for Electrostatic Precipitator Performance:

The performance of an ESP depends on several factors, among which the prominent factors are as follows:

1. Voltage.
2. Plate size.
3. Characteristics of dust particles.
4. Dust loading.
5. Chemical composition of dust particles.
6. Electrical resistively of dust.
7. Adhesive/cohesive properties.

8. Particle size distribution.
9. Hammering system.
10. Characteristics of gases.
11. Temperature of gases.
12. Moisture content.
13. Quantity to be handled.

Advantages

Advantages of ESP includes the following:

1. High collection efficiency.
2. Capable to handle a large volume of high temperature gas.
3. Very less draught loss.
4. Easy cleaning.

Disadvantages

The disadvantages of ESP includes the following:

1. High establishment cost.
2. High running cost.
3. Requirement of more space.

1. (c)

The velocity field corresponding to the given stream function can be written as

$$u = \frac{\partial \psi}{\partial y} = 3x^2 + 8(2+t)y$$

and

$$v = -\frac{\partial \psi}{\partial x} = -6xy$$

where u and v are velocity components along the x and y directions respectively.

At time, $t = 2$

$$(u)_{t=2} = 3x^2 + 32y$$

$$(v)_{t=2} = -6xy$$

The volume flow rate across the face perpendicular to the x-direction and with edge OB as seen in the xy plane is found as

$$Q_{OB} = \int_{A_{OB}} (u)_{\substack{t=2 \\ x=0}} (dA)_{OB}$$

(direction of u and dA are same means Q entering the prism)

$$= \int_0^2 32y\,dy = 32\left[\frac{y^2}{2}\right]_0^2$$

$$Q_{OB} = 64 \text{ units} \qquad \text{(entered into the prism)}$$

Similarly, the flow rate across the face with OA (last seen in the xy plane) and perpendicular to the y-direction becomes

$$Q_{OA} = \int_{A_{OA}} (v)_{\substack{t=2\\y=0}} (dA)_{OA}$$

$$\Rightarrow \qquad Q_{OA} = 0$$

Volume flow rate across the inclined face with face with AB as the edge seen on the xy plane can be written as

$$Q_{AB} = \int \hat{n}\,dA_{AB} \cdot \vec{V}$$

where $\hat{n}$ is the unit vector along the normal to the element of surface dA_{AB}, taken positive when directed outwards as shown in figure. Hence we can write the above equation as

$$Q_{AB} = \int_{A_{AB}} \left[\hat{i}dy + \hat{j}dx\right]\left[\left(3x^2 + 32y\right)\hat{i} + \left(-6xy\right)\hat{j}\right]$$

$$Q_{AB} = \int_{A_{AB}} \left(3x^2 + 32y\right)dy - \int_{A_{AB}} (6xy)dx \qquad \text{...(i)}$$

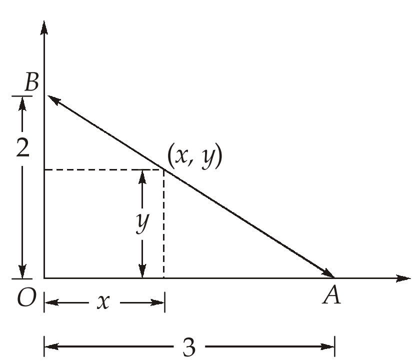
From OAB using geometry,

$$\frac{y}{3-x} = \frac{2}{3} \qquad \text{...(ii)}$$

$$\Rightarrow \qquad x = 3 - \frac{3}{2}y$$

and

$$y = 2 - \frac{2}{3}x$$



Using the relation in equation (i), we have

$$Q_{AB} = \int_0^2 \left[3\left(3-\frac{3}{2}y\right)^2 + 32y\right]dy - \int_0^3 6\left(2-\frac{2}{3}x\right)xdx$$

$$= 3\int_0^2 \left[9+\frac{9}{4}y^2 - 9y\right]dy + \int_0^2 32ydy - 6\int_0^3 \left(2x - \frac{2}{3}x^2\right)dx$$

$$= 3\left[9y + \frac{9}{4}\frac{y^3}{3} - \frac{9y^2}{2}\right]_0^2 + \left[\frac{32y^2}{2}\right]_0^2 - 6\left[\frac{2x^2}{2} - \frac{2}{3}\frac{x^3}{3}\right]_0^3$$

$$= 3\left[9\times 2 + \frac{3}{4}\times 8 - \frac{9}{2}\times 4\right] + 16(4) - 6[9-6]$$

$$= (3 \times 6) + (16 \times 4) - 6 \times 3$$

$$= 64 \text{ units (Fluid existing from } AB \text{ from prism)}$$

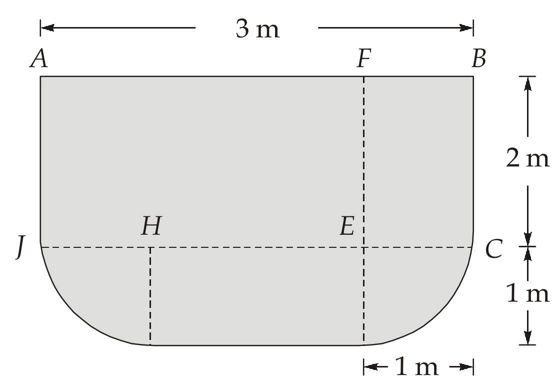
If stream function exist then divergence is zero means no fluid generation so the amount of fluid entered equals to fluid exited so both entry and exit are same which 64 units.

1. (d)

For the vertical face, $F = wAy_c$

$$= w\times(BC \times \text{unit length}) \times \frac{BC}{2}$$

$$= \left(9.81 \times 0.7 \times 10^3\right) \times (2\times 1) \times 1 = 13.734 \text{ kN}$$



This force acts horizontally towards and its point of application is given by,

$$y_p = y_c + \frac{I}{Ay_c} = 1 + \frac{1\times 2^3/12}{(1\times 2)\times 1} = 1.333 \text{ m}$$

For the curved surface *CD*.

The horizontal component of hydrostatic pressure force on the curved corner CD equals the pressure force on its projected area on a vertical plane.

F_h = Specific weight × (Vertical projected area) × (Depth of centre of vertical projection)

$$= w \times (DE \times \text{unit length}) \times \left(FE + \frac{ED}{2}\right)$$

$$= (9.81 \times 0.7) \times (1 \times 1) \times \left(2 + \frac{1}{2}\right) = 17.1675 \text{ kN}$$

The vertical component of hydrostatic pressure force on the curved corner equals the weight of liquid supported by curve *CD* upto the free surface of liquid *FB*.

$$F_C = \text{Weight of liquid in portion } BCDEF$$

$$= 9.81 \times 0.7\left[(2 \times 1 \times 1) + \frac{\pi}{4} \times (1)^2 \times 1\right] = 19.1273 \text{ kN}$$

The angle made by the resultant with horizontal is given by

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{19.1275}{17.1675}\right)$$

$$\theta = 48.08°$$

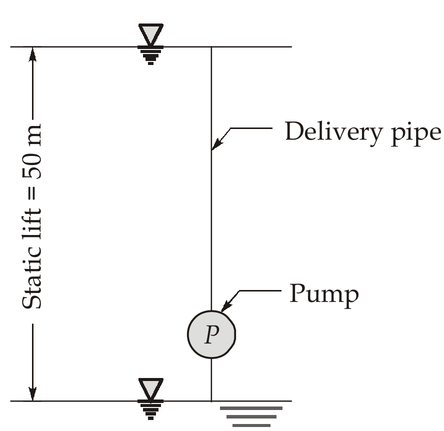
$$\text{Force on DG} = \text{Weight above DG}$$

$$= 0.7 \times 1000 \times 9.81 \times 3 \times (3 - 1 - 1)$$

$$= 20.601 \text{ kN vertically downwards}$$

1. (e)

Given : D_2 = 0.6 m; N = 1250 rpm; η_m = 85%; f = 20°; b_2 = 0.033 m; H_d = 50 – 4 = 46 m; H_{Ld} = 7 m; D_{pipe} = 37 cm



$$\text{Net heat, } H = \text{Static lift + Friction loss}$$
$$= 50 + 2.5 + 7 = 59.5 \text{ m}$$

Peripheral velocity at outlet,

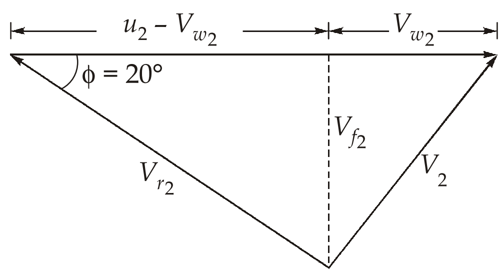
$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi(60)(1250)}{60}$$
$$u_2 = 39.27 \text{ m/s}$$

Since the flow is radial at the inlet

So, $V_{w_1} = 0$

$$\text{Manometric efficiency, } \eta_m = \frac{gH}{u_2 V_{w_2}}$$
$$0.85 = \frac{9.81 \times 59.5}{39.27 \times V_{w_2}}$$
$$V_{w_2} = 17.49 \text{ m/s}$$
$$\text{Blade angle, } \phi = 20°$$

From outlet velocity triangle,



$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$
$$\tan 20° = \frac{V_{f_2}}{39.27 - 17.49}$$
$$V_{f_2} = 7.93 \text{ m/s}$$
$$\text{Discharge, } Q = \pi D_2 b_2 V_{f_2}$$
$$= \pi \times 0.6 \times 0.033 \times 7.93$$
$$= 0.4933 \text{ m}^3\text{/s}$$

$$\text{Velocity in delivery pipe } (V_d) = \text{Velocity in suction pipe } (V_s) = \frac{Q}{A}$$

$$V_d = V_s = \frac{0.4933}{\frac{\pi}{4}\times(0.37)^2} = 4.59 \text{ m/s}$$

Let the pressure at the delivery side = P_d

$$\frac{P_d}{\rho g} + \frac{V_d^2}{2g} = H_d + H_{Ld}$$

$$\frac{P_d}{\rho g} + \frac{(4.59)^2}{2\times 9.81} = 46 + 7$$

$$P_d = \frac{51.93\times 9.81\times 1000}{1000}$$

$$= 509.34 \text{ kPa (gauge)}$$

Let the pressure on suction side be P_s and the atmospheric pressure be P_{atm}.

then,
$$\frac{P_{atm}}{\rho g} = H_s + H_L + \frac{P_s}{\rho g} + \frac{V_s^2}{2g}$$

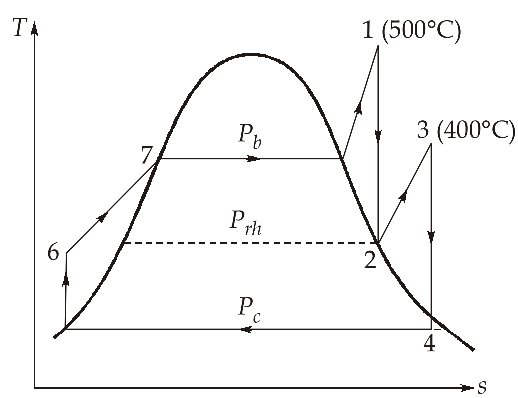
$$\frac{101.325\times 1000}{1000\times 9.81} = 4 + 2.5 + \frac{P_s}{1000\times 9.81} + \frac{(4.59)^2}{2\times 9.81}$$

$$P_s = 27.026 \text{ kPa}$$

2. (a)

Given : $T_1 = 500°C$; $T_3 = 400°C$; $T_4 = 50°C$

At 50°C, from steam table



$$P_c = 0.1235 \text{ bar}$$

$$h_5 = (h_f)_{50°C} = 209.34 \text{ kJ/kg}$$

$$v_5 = (v_f)_{50°C} = 0.00101215 \text{ m}^3\text{/kg}$$

$$s_5 = (s_f)_{50°C} = 0.70381 \text{ kJ/kgK}$$

$$s_3 = s_4 = (s_f)_{50°C} + 0.82 \times (s_{fg})_{50°C}$$
$$= 0.0.70381 + 0.82 \times 7.3710$$
$$s_3 = s_4 = 6.74803 \text{ kJ/kgK}$$
$$h_4 = (h_f)_{50°C} + 0.82 \times (h_f)_{50°C}$$
$$= 209.34 + 0.82 \times 2381.9 = 2162.498 \text{ kJ/kg}$$

(i) At 400°C, superheated temperature and entropy 6.74803 kJ/kgK

Corresponding pressure is $\dfrac{P_{rh} - 40}{(6.740803 - 6.7714)} = \dfrac{(45-40)}{(6.707 - 6.7714)}$

∴ The reheat pressure, P_{rh} = 41.814 bar

At reheat pressure, corresponding enthalpy.

$$h_3 = \left(\frac{41.814 - 40}{45 - 40}\right) \times (3205.6 - 3214.5) + 3214.5$$
$$h_3 = 3211.27 \text{ kJ/kg and } s_g = 6.050918 \text{ kJ/kgK}$$

(ii) At 500°C, superheated temperature and entropy 6.050918 kJ/kgK.

Corresponding pressure is $\dfrac{P_b - 220}{(6.050918 - 6.0705)} = \dfrac{(250-220)}{(5.9642 - 6.0705)}$

∴ The boiler pressure is, P = 225.526 bar ≃ 255.5 bar

At boiler pressure, corresponding enthalpy

$$h_1 = \left(\frac{225.5 - 220}{250 - 220}\right) \times (3165.9 - 3211.8) + 3211.8$$
$$h_1 = 3203.4 \text{ kJ/kg}$$

(iii) Work done by the pump is

$$w_p = V_5 \times (P_b - P_c)$$
$$= \frac{0.00101215 \times (225.5 - 0.1235) \times 10^5}{10^3}$$
$$= 22.81 \text{ kJ/kg}$$

Also, $w_p = h_6 - h_5$

∴ $h_6 = w_p + h_5$

$$= 22.81 + 209.34$$
$$= 233.15 \text{ kJ/kg}$$

Work done by turbine is, $w_T = (h_1 - h_2) + (h_3 - h_4)$

$$w_T = (3203.4 - (h_g)_{41.814\text{ bar}}) + (3211.26 - 2162.49)$$

$$w_T = (3203.4 - 2799.9) + (3211.27 - 2162.49)$$

$$w_T = 1452.27 \text{ kJ/kg}$$

∴ Net specific output is $(w_{net}) = w_T - w_P$

$$= 1452.27 - 22.81$$

$$= 1429.46 \text{ kJ/kg}$$

(iv) Cycle efficiency, $\eta_c = \dfrac{w_T - w_P}{Q_1} = \dfrac{1452.27 - 22.81}{(h_1 - h_6)} = \dfrac{1429.46}{3203.4 - 233.15}$

$$\eta_c = 0.4813 \text{ or } 48.13\%$$

(v) Steam rate $= \dfrac{3600}{w_{net}} = \dfrac{3600}{1429.46} = 2.52$ kg/kWh

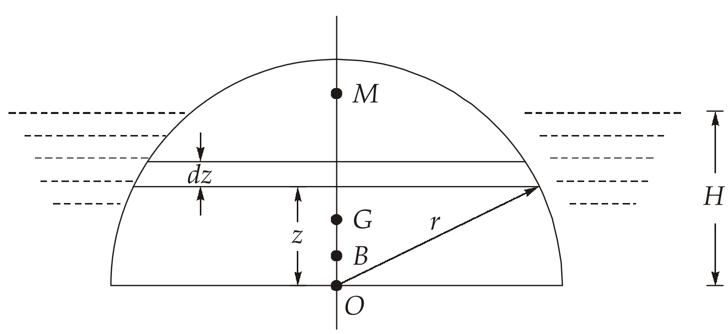
2. (b)

The hemisphere in its floating condition is shown in figure. Let V be the submerged volume. Then from equilibrium under floating condition.

Mass of solid hemisphere = Mass of fluid displaced

$$\frac{2}{3}\pi r^3 \times \rho = V \times \rho_1$$

$$V = \frac{2}{3}\pi r^3 \times \frac{\rho}{\rho_l}$$



A solid hemisphere floating in a liquid

The centre of gravity G will lie on the axis of symmetry of the hemisphere. The distance of G along this line from the base of the hemisphere can be found by taking moments of elemental circular strips as shown in figure about the base as

$$OG = \frac{\int_0^r \pi\left(r^2 - z^2\right) z dz}{\frac{2}{3}\pi r^3}$$

$$= \frac{3}{2\pi r^3} \times \left[\frac{\pi r^2 z^2}{2} - \frac{\pi z^4}{4}\right]_0^r = \frac{3}{2\pi r^3} \times \left[\frac{\pi r^4}{4}\right] = \frac{3}{8} r$$

In a similar way, the location of centre of buoyancy which is the centre of immersed volume V is found as

$$OB = \frac{\int_0^H \pi\left(r^2 - z^2\right) z dz}{\frac{2}{3}\pi r^3 \frac{\rho}{\rho_l}} = \frac{\left[\frac{\pi r^2 z^2}{2} - \frac{\pi z^4}{4}\right]_0^H}{\frac{2}{3}\pi r^3 \frac{\rho}{\rho_l}}$$

$$= \frac{\frac{\pi r^2 H^2}{2} - \frac{\pi H^4}{4}}{\frac{2}{3}\pi r^3 \frac{\rho}{\rho_l}} = \frac{3}{8}\frac{\rho_l}{\rho} r \frac{H^2}{r^2}\left(2 - \frac{H^2}{r^2}\right) \quad \text{...(i)}$$

where H is the depth of immersed volume as shown in figure.

If r_w is the radius of cross-section of the hemisphere at water line, then we can write

$$H^2 = r^2 - r_w^2$$

Substituting the value of H in equation (i), we have

$$OB = \frac{3}{8}\frac{\rho_l}{\rho} r \left(1 - \frac{r_w^4}{r^4}\right)$$

The height of the metacentre M above the centre of buoyancy B is given by,

$$BM = \frac{I}{V_{fd}} = \frac{\pi r_w^4}{4\left\{\left(\frac{2}{3}\pi r^3\right)\frac{\rho}{\rho_l}\right\}} = \frac{3}{8}\frac{\rho_l}{\rho} r \frac{r_w^4}{r^4}$$

Therefore, the metacentric height MG becomes,

$$MG = MB - BG = MB - (OG - OB)$$

$$= \frac{3}{8}\frac{\rho_l}{\rho} r \frac{r_w^4}{r^4} - \frac{3}{8} r + \left[\frac{3}{8}\frac{\rho_l}{\rho} r \left(1 - \frac{r_w^4}{r^4}\right)\right]$$

$$= \frac{3}{8}\frac{\rho_l}{\rho} r \frac{r_w^4}{r^4} - \frac{3}{8} r + \frac{3}{8} r \frac{\rho_l}{\rho} - \frac{3}{8}\frac{\rho_l}{\rho} r \frac{r_w^4}{r^4}$$

$$= -\frac{3}{8}r + \frac{3r}{8}\frac{\rho_l}{\rho} = \frac{3}{8}r\left(\frac{\rho_l}{\rho} - 1\right)$$

Since $\rho_l > \rho$, $MG > 0$, and hence, the equilibrium is stable.

2. (c)

Rate of air consumption, $\dot{m}_a = 50$ kg/s

$\Delta h = 300$ kJ/kg

Velocity coefficient, $z = 0.96$

Air fuel rate = 70 : 1

$\eta_{combustion} = 95\%$

Calorific value = 42000 kJ/kg

Aircraft velocity, $C_a = \frac{1800 \times 1000}{3600} = 500$ m/s

(i) Exit velocity of jet $C_j = z\sqrt{2 \times \Delta h \times 1000}$

$$= 0.96\sqrt{2 \times 300 \times 1000} = 743.61 \text{ m/s}$$

(ii) Fuel flow rate:

Rate of fuel consumption, $\dot{m}_f = \frac{\text{Rate of air consumption}}{\text{Air-fuel ratio}} = \frac{50}{70}$

$\dot{m}_f = 0.71428$ kg/s

(iii) Thrust is force produced due to change of momentum.

Thrust produced $= \dot{m}_a(C_j - C_a)$

$$= 50(743.61 - 500) = 12180.5 \text{ N}$$

Thrust specific fuel consumption

$$= \frac{\text{Fuel consumption}}{\text{Thrust}}$$

$$= \frac{0.71428}{12180.5} = 5.864 \times 10^{-5} \text{ kg/N of thrust/s}$$

(iv) Thermal efficiency, $\eta_{thermal}$

$$\eta_{th} = \frac{\text{Work output}}{\text{Heat supplied}}$$

$$= \frac{\text{Gain in kinetic energy per kg of air}}{\text{Heat supplied by fuel per kg of air}}$$

$$= \frac{\left(C_j^2 - C_a^2\right)}{2\left(\frac{m_f}{m_a}\right) \times C.V \times \eta_{\text{combustion}} \times 1000}$$

$$= \frac{\left(743.61^2 - 500^2\right)}{2\left(\frac{1}{70}\right) \times 42000 \times 1000 \times 0.95} = 26.57\%$$

(v) Propulsive power

$$\text{Propulsive power} = \dot{m}_a \times \frac{\left(C_j^2 - C_a^2\right)}{2}$$

$$= \frac{50}{1000} \times \frac{\left(743.61^2 - 500^2\right)}{2} \text{kW}$$

$$= 7573.89 \text{ kW}$$

(vi) Propulsive efficiency pump,

$$\eta_{\text{prop}} = \frac{\text{Thrust power}}{\text{Propulsive power}} = \frac{2C_a}{C_j + C_a}$$

$$= \frac{2 \times 500}{743.61 + 500} = 0.8041 = 80.411\%$$

(vii) Overall efficiency,

$$\eta_0 = \frac{\left(C_j - C_a\right)C_a}{\left(\frac{m_f}{m_a}\right) \times CV \times \eta_{combustion}}$$

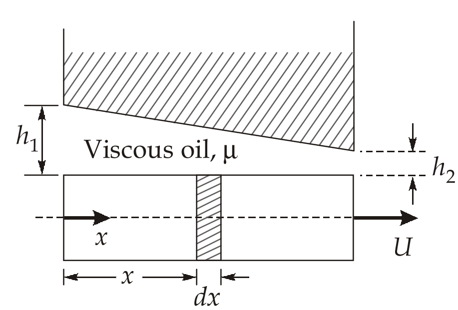
$$= \frac{(743.61 - 500) \times 500}{\left(\frac{1}{70}\right) \times 42000 \times 0.95 \times 1000} = 0.2137 = 21.37\%$$

3. (a) (i)

$$h_x = h_1 - (h_1 - h_2)\frac{x}{L}$$

$$dF = \tau dA = \mu \frac{U}{h_x} \times \pi D dx$$

$$= \frac{\mu U \pi D}{\left[h_1 - (h_1 - h_2)\frac{x}{L}\right]} dx$$



$$U = \frac{2\pi n}{60} \times \frac{D}{2}$$

$$dT = dF \times \frac{D}{2} = \frac{\mu\left(\frac{2\pi n}{60} \times \frac{D}{2}\right) \times \pi D \times \frac{D}{2}}{\left[h_1 - (h_1 - h_2)\frac{x}{L}\right]} dx$$

$$= \frac{\mu n \pi^2 D^3}{120} \frac{dx}{\left[h_1 - (h_1 - h_2)\frac{x}{L}\right]}$$

Integrating, $T = \dfrac{\mu n \pi^2 D^3}{120} \displaystyle\int_{x=0}^{x=L} \frac{dx}{\left[h_1 - (h_1 - h_2)\frac{x}{L}\right]}$

$$T = -\frac{\mu n \pi^2 D^3}{120} \left. \frac{\ln\left(h_1 - (h_1 - h_2)\frac{x}{L}\right)}{\frac{(h_1 - h_2)}{L}} \right]_{x=0}^{x=L}$$

$$= \frac{1}{120} \frac{\mu n \pi^2 D^3 L}{h_1 - h_2} \ln \frac{h_1}{h_2}$$

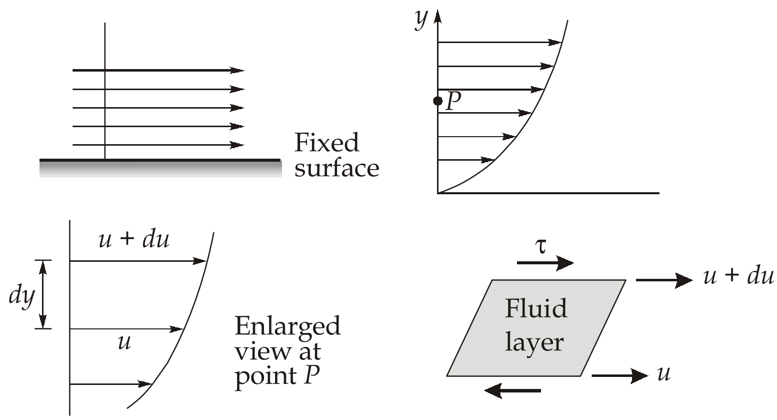
$$T = \frac{1}{120} \times \frac{(0.1) \times (1450)\pi^2 \times \left(80 \times 10^{-3}\right)^3 \left(400 \times 10^{-3}\right)}{(1.2 - 0.4) \times 10^{-3}} \ln\left(\frac{1.2}{0.4}\right)$$

$$= 3.35 \text{ Nm}$$

3. (a) (ii)

Viscosity is a property of the fluid by which it offers resistance to shear or angular defor-mation.

The resistance to flow because of internal friction is called viscous resistance, and the property which enables the fluid to offer resistance to relative motion between adjacent layers is called the viscosity of fluid. Viscosity is thus a measure of resistance to relative translational motion of adjacent layers of fluid. Molasses, tar and glycerine are examples of highly viscous liquids; the intermolecular force of attraction between their molecules is very large and consequently they cannot be easily poured or stirred.



Velocity profile and viscosity concept

Newton's law of viscosity

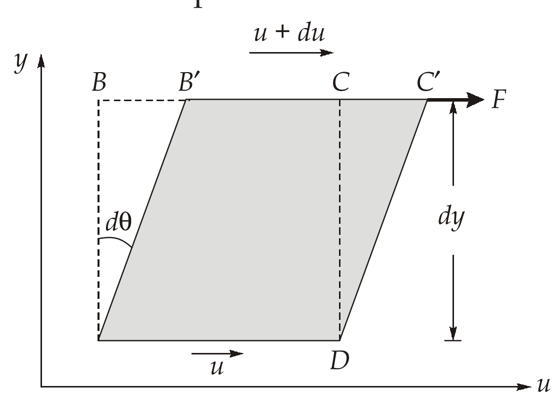
Consider two adjacent layers at an infinitesimal distance dy apart and moving with velocity u and $(u + du)$, respectively. The upper layer moving with velocity $(u + du)$ drags the lower layer along with it by exerting a force F.

$$\tau \propto \frac{du}{dy};\ \tau = \mu \frac{du}{dy}$$

Deformation of fluid elements can be prescribed in terms of the angle of shear strain $d\theta$. Figure indicates a thin sheet of fluid element ABCD placed between two plates distance dy apart.

The length and the width of the plates are much larger than the thickness dy so that the edge effects can be neglected.

When force F is applied to the upper plate, it causes it to move at a small speed du relative to the bottom plate. Velocity gradient sets up a shear stress $\tau = F/A$ which makes the fluid element distort to position AB′C′D after a short time interval dt.



$$\text{Distance BB}' = \text{CC}'$$
$$= \text{Speed} \times \text{Time} = du \times dt$$

For small angular displacement $d\theta$,

$$BB' = dy \times d\theta$$

$$\therefore \quad du \times dt = dy \times d\theta$$

$$\frac{du}{dy} = \frac{d\theta}{dt}$$

Invoke Newton's law of viscosity, i.e., express the shear stress in terms of velocity gradient.

$$\tau \propto \frac{d\theta}{dt}$$

Apparently the shear stress in fluids is dependent on the rate of fluid deformation = $\frac{d\theta}{dt}$.

3. (b)(i)

Radial flow reaction turbine : Radial flow turbines are those turbines in which the water flows in the radial direction. The water may flow radially from outwards to inwards (i.e., towards the axis of rotation) or from inwards to outwards. If the water flows from outwards to inwards through the runner, the turbine is known as inward radial flow turbine. And if the water flows from inwards to outwards, the turbine is known as outward radial flow turbine.

Reaction turbine means that the water at the inlet of the turbine possesses kinetic energy as well as pressure energy. As the water flows through the runner, a part of pressure energy goes on changing into kinetic energy. Thus the water through the runner is under pressure. The runner is completely enclosed in an air-tight casing and casing and the runner is always full of water.

Main Parts of a Radial Flow Reaction Turbine: The main parts of a radial flow reaction turbine are:

1. **Casing:** In case of reaction turbine, casing and runner are always full of water. The water from the penstocks enters the casing which is of spiral shape in which area of cross-section of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. The casing as shown in figure is made of spiral shape, so that the water may enter the runner at constant velocity throughout the circumference of the runner. The casing is made of concrete, cast steel or plate steel.
2. **Guide Mechanism:** It consists of a stationary circular wheel all round the runner of the turbine. The stationary guide vanes are fixed on the guide mechanism. The

guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet. Also by a suitable arrangement, the width between two adjacent vanes of guide mechanism can be altered so that the amount of water striking the runner can be varied.

3. **Runner:** It is a circular wheel on which a series of radial curved vanes are fixed. The surface of the vanes are made very smooth. The radial curved vanes are so shaped that the water enters and leaves the runner without shock. The runners are made of cast steel, cast iron or stainless steel. They are keyed to the shaft.

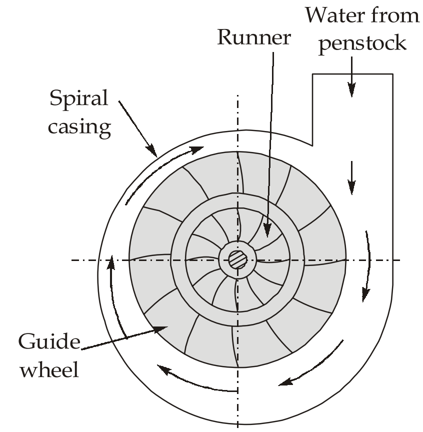


Fig. Main parts of a radial reaction turbines

4. **Draft-tube:** The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water at exit cannot be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging water from the exit of the turbine to the tail race. This tube of increasing area is called draft tube.

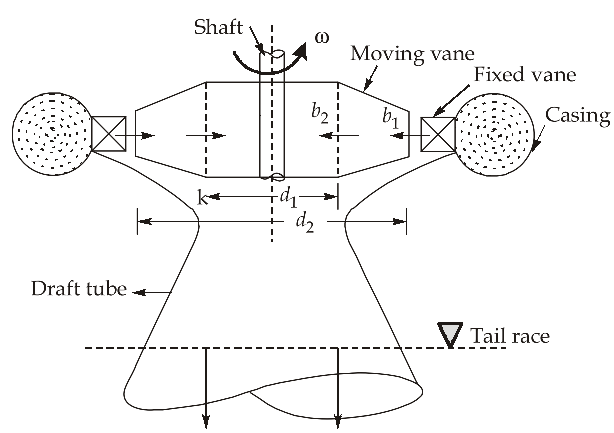


Fig. Cross-sectional view of radial reaction turbine

3. **(b)(ii)**

Given : η_0 = 0.75, S.P. = 150 kW, H = 7.5 m, N = 160 rpm, hydraulic losses = 20% of available energy

$$\text{Potential velocity, } u_1 = 0.25\sqrt{2\times 9.81\times 7.5} = 3.033 \text{ m/s}$$

$$\text{Velocity of flow at inlet, } V_{f_1} = 0.95\sqrt{2\times 9.81\times 7.5} = 11.524 \text{ m/s}$$

Since the discharge at the outlet is radial,

$$V_{w_2} = 0 \text{ and } V_{f2} = V_2$$

Hydraulic efficiency is given as

$$\eta_h = \frac{\text{Total head at inlet – Hydraulic loss}}{\text{Head at inlet}} = \frac{H - 0.2H}{H}$$

$$\eta_h = 0.8$$

But

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

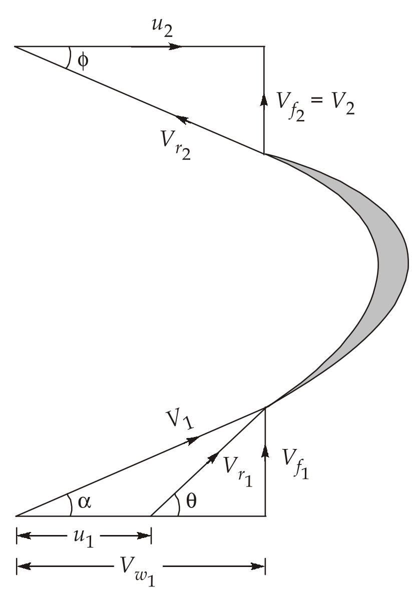
$$0.8 = \frac{V_{w_1}(3.033)}{9.81 \times 7.5}$$

$$V_{w_1} = 19.4 \text{ m/s}$$

(i) The guide blade angle i.e., α

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.524}{19.4}$$

$$\alpha = 30.711°$$



(ii) The wheel vane angle at inlet i.e. θ

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.524}{19.4 - 3.033}$$

$$\theta = 35.15°$$

(iii) Diameter of wheel at inlet (D_1)

We know $$u_1 = \frac{\pi D_1 N}{60}$$

$$3.033 = \frac{\pi \times D_1 \times (160)}{60}$$

$$D_1 = 0.362 \text{ m} = 36.2 \text{ cm}$$

(iv) Width of the wheel at inlet (B_1)

$$\eta_0 = \frac{S.P.}{W.P.} = \frac{150 \times 1000}{\rho g Q H}$$

$$0.75 = \frac{150 \times 1000}{1000 \times 9.81 \times Q \times 7.5}$$

$$Q = 2.718 \text{ m}^3/\text{s}$$

and also, $$Q = \pi D_1 B_1 V_{f_1}$$

$$2.718 = \pi \times 0.362 \times B_1 \times 11.524$$

$$B_1 = 0.207 \text{ m}$$

3. (c)

(i) Mass of dry flue gas produced per kg fuel

$$= \frac{C_{abs} \times (44CO_2 + 28CO + 32O_2 + 28N_2)}{12(CO_2 + CO)}$$

$$= \frac{0.63 \times (44 \times 0.125 + 28 \times 0.017 + 32 \times 0.08 + 28 \times 0.778)}{12(0.125 + 0.017)}$$

$$= 11.21 \text{ kg}$$

(ii) Energy loss due to dry exhaust gas is

$$m_{dfg} C_{pg}(t_g - t_a) = 11.21 \times 1.05 \times (180 - 35)$$

$$= 1706.72 \text{ kJ/kg}$$

$$= 1.71 \text{ MJ/kg fuel}$$

Energy loss due to incomplete combustion

$$= m_{dfg} \times \frac{28CO}{44CO_2 + 28CO + 32O_2 + 28N_2}\left(H_{R,CO} - H_{R,CO_2}\right) \times \frac{12}{28}$$

$$= \frac{11.21 \times 28 \times 0.017}{44 \times 0.125 + 28 \times 0.017 + 32 \times 0.08 + 28 \times 0.778}(33000 - 9500) \times \frac{12}{28}$$

= 1772.45 kJ/kg fuel = 1.77 MJ/kg fuel

(iii) Also, unaccounted loss = 2.5% of HHV

= 0.025 × 26 = 0.65 MJ/kg fuel

Total energy loss = (1.71 + 1.77 + 0.65)

= 4.13 MJ/kg fuel

$$\text{Hence, boiler efficiency} = \frac{HHV - \sum \text{Losses}}{HHV}$$

$$= \frac{26 - 4.13}{26} = 0.8412 \text{ or } 84.12\%$$

(iv) Energy balance around boiler gives

$$m_f \times \text{HHV} \times \eta_{\text{boiler}} = m_s(h_1 - h_f)$$

From steam table, at 120 bar and 500°C

$$h_1 = 3350 \text{ kJ/kg}$$

and at 150°C h_f = 632.18 kJ/kg

Steam generation rate = 175 T/h

= 175 × 1000 kg/h

$$\therefore \text{ Burning rate of the fuel, } m_f = \frac{175000 \times (3350 - 632.18)}{26000 \times 0.8412 \times 3600} = 6.041 \text{ kg/s}$$

(v) Theoretical air required per kg of fuel

$$(m_{\text{air}})_{\text{th}} = 11.5C + 34.5\left(H - \frac{O}{8}\right) + 4.3S$$

$$= 11.5 \times 0.63 + 34.5\left(0.018 - \frac{0.014}{8}\right) + 4.3 \times 0.009$$

= 7.844 kg

Actual air supplied per kg fuel,

$$m_{act} = \frac{3.04 N_2 C_{abs}}{CO_2 + CO} - \frac{1}{0.768} N$$

$$= \frac{3.04 \times 0.778 \times 0.63}{0.125 + 0.017} - \frac{1}{0.768} \times 0.017$$

= 10.47 kg

$$\text{Percentage of excess air used} = \frac{m_{act} - (m_{air})_{th}}{(m_{air})_{th}}$$

$$= \frac{10.47 - 7.84}{7.84} = 0.3349 \text{ or } 33.49\%$$

4. (a)

For velocity profile, $$\frac{U}{U_\infty} = \sin\left(\frac{\pi}{2}\cdot\frac{y}{\delta}\right)$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\int_0^\delta \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) dy$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\left[1 - \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] dy\right]$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\frac{-\cos\frac{\pi y}{2\delta}}{\frac{\pi}{2\delta}} - \left(\frac{1}{2}\cdot\frac{\frac{\pi y}{2\delta}}{\frac{\pi}{2\delta}} - \frac{\sin 2\left(\frac{\pi y}{2\delta}\right)}{4\times\frac{\pi}{2\delta}}\right)\right]_0^\delta$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\frac{-\cos\frac{\pi}{2}}{\frac{\pi}{2\delta}} + \frac{\sin\pi}{\frac{4\pi}{2\delta}} - \left(\frac{-1}{\frac{\pi}{2\delta}}\right) - \frac{\delta}{2}\right]$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\frac{-\delta}{2} + \frac{2\delta}{\pi}\right]$$

$$\Rightarrow \quad \frac{\tau}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left[\frac{2\delta}{\pi} - \frac{\delta}{2}\right]$$

$$\therefore \quad \frac{\tau}{\rho U_\infty^2} = \frac{4-\pi}{2\pi}\frac{\partial\delta}{\partial x} \qquad \text{...(i)}$$

As $$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$U = U_\infty \sin\left(\frac{\pi y}{2\delta}\right)$$

$$\frac{du}{dy} = U_\infty \cos\left(\frac{\pi y}{2\delta}\right)\times\frac{\pi}{2\delta}$$

$$\tau_0 = \mu \frac{U_\infty \pi}{2\delta} \quad ...(ii)$$

Equating τ_0 in (ii),

$$\rho U_\infty^2 \left(\frac{4-\pi}{2\pi}\right)\frac{\partial\delta}{\partial x} = \frac{\mu U_\infty \pi}{2\delta}$$

$$(\delta)d\delta = \frac{\mu U_\infty \pi}{2\delta U_\infty^2}\left(\frac{2\pi}{4-\pi}\right)dx$$

By integrating,

$$\frac{\delta^2}{2} = \frac{\mu}{2\rho U_\infty}\left(\frac{2\pi^2}{4-\pi}\right)x + C$$

$$\frac{\delta^2}{2} = \frac{\mu}{\rho U_\infty}\left(\frac{\pi^2}{4-\pi}\right)x + C$$

At $x = 0$, $\delta = 0$ so $C = 0$

$$\delta = \sqrt{\frac{2\mu}{\rho U_\infty}\left(\frac{\pi^2}{4-\pi}\right)x}$$

$$\delta = \sqrt{\frac{2\times\nu}{U_\infty}\left(\frac{\pi^2}{4-\pi}\right)x} = \sqrt{\frac{2\times 0.1\times 10^{-4}}{5}\times\left(\frac{\pi^2}{4-\pi}\right)\times 0.5}$$

$$= 4.795 \times 10^{-3} \text{ m} = 4.795 \text{ mm}$$

(ii) Shear stress at $x = 500$ mm

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$U = U_\infty \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

$$\left(\frac{du}{dy}\right) = U_\infty \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\cdot\frac{\pi}{2\delta}$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{U_\infty}{2\delta}$$

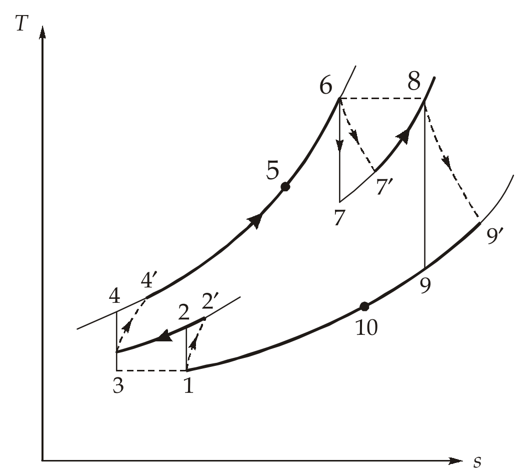
$$\tau_0 = \frac{\mu U_\infty \pi}{2\delta} = \frac{(\rho\times\nu)U_\infty \pi}{2\delta}$$

$$= \frac{1.2 \times 0.1 \times 10^{-4} \times 5 \times \pi}{2 \times 4.795 \times 10^{-3}} = 0.01965 \text{ N/mm}^2$$

4. (b)

Given : $T_1 = 25 + 273 = 298$ K; $T_6 = T_8 = 650 + 273 = 923$ K

$$(\eta_c)_{stage} = 0.82;\ (\eta_t)_{stage} = 0.85;\ \eta_{mech} = 0.92;\ \varepsilon = 0.8$$



Since, the pressure ratio and the isentropic efficiency of each compressor is the same then the work input required for each compressor is the same since both compressor have the same air inlet temperature i.e.,

$$T_1 = T_3 \text{ and } T_2' = T_4'$$

Also,
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ and } \frac{p_2}{p_1} = \sqrt{9} = 3$$

$\therefore$
$$T_2 = 298(3)^{\frac{1.4-1}{1.4}} = 407.88 \text{ K}$$

$$\eta_c(\text{L.P.}) = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.82 = \frac{407.88 - 298}{T_2' - 298}$$

$$T_2' = 432 \text{ K} = T_4' \qquad [\text{As } T_2' - T_4']$$

Work input per compressor stage

$$= c_{pa}(T_2' - T_1)$$

$$= 1.005(432 - 298)$$
$$= 134.67 \text{ kJ/kg}$$

The H.P. turbine is required to drive both compressors and to overcome mechanical friction.

$\therefore$ Work output of H.P. turbine = $\dfrac{2 \times 134.67}{0.92} = 292.76$ kJ/kg

Also, work output of H.P. turbine = $c_{pg}(T_6 - T_7')$

$$292.76 = 1.15 \times (923 - T_7')$$
$$T_7' = 668.43 \text{ K}$$
$$\eta_{turbine}\text{ (H.P.)} = \frac{T_6 - T_7'}{T_6 - T_7}$$
$$0.85 = \frac{923 - 668.43}{923 - T_7}$$
$$T_7 = 623.51 \text{ K}$$

As,
$$\frac{T_6}{T_7} = \left(\frac{P_6}{P_7}\right)^{\frac{\gamma-1}{\gamma}}$$
$$\left(\frac{923}{623.51}\right) = \left(\frac{P_6}{P_7}\right)^{\frac{1.333-1}{1.333}}$$
$$\frac{P_6}{P_7} = 4.81$$

Then,
$$\frac{P_8}{P_9} = \frac{9}{4.81} = 1.87$$

Again,
$$\frac{T_8}{T_9} = \left(\frac{P_8}{P_9}\right)^{\frac{\gamma-1}{\gamma}} = (1.87)^{\frac{1.333-1}{1.333}} = 1.17$$
$$T_9 = \frac{T_8}{1.17} = \frac{923}{1.17} = 788.89 \text{ K}$$

Also,
$$\eta_{turbine\ (L.P.)} = \frac{T_8 - T_9'}{T_8 - T_9}$$
$$0.85 = \frac{923 - T_9'}{923 - 788.89}$$

$$T_9' = 809.01 \text{ K}$$

Work output of low pressure turbine $= c_{pg}(T_8 - T_9') \times 0.92$

$$= 1.15(923 - 809.01) \times 0.92$$

$$= 120.60 \text{ kJ/kg}$$

Thermal ratio or effectiveness of heat exchanger,

$$\varepsilon = \frac{T_5 - T_4'}{T_9' - T_4'}$$

$$0.8 = \frac{T_5 - 432}{809.01 - 432}$$

$$T_5 = 733.608 \text{ K}$$

$$\text{Heat supplied} = (c_{pg}T_6 - c_{pa}T_5) + c_{pg}(T_8 - T_7')$$

$$= (1.15 \times 923 - 1.005 \times 733.608) + 1.15(923 - 668.43)$$

$$= 616.94 \text{ kJ/kg}$$

$$\text{Thermal efficiency } (\eta_m) = \frac{\text{Net work output}}{\text{Heat supplied}}$$

$$= \frac{120.60}{616.94} = 0.1955 = 19.55\%$$

(ii) Work ratio

$$\text{Gross work of the plant} = W_{\text{turbine(H.P.)}} + W_{\text{turbine(L.P.)}}$$

$$= 292.76 + 120.60$$

$$= 413.36 \text{ kJ/kg}$$

$$\therefore \quad \text{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{120.60}{413.36} = 0.292$$

(iii) As we know,

$$\dot{m} \times 120.60 = 4800$$

$$\therefore \quad \dot{m} = 39.80 \text{ kg/s}$$ **Ans.**

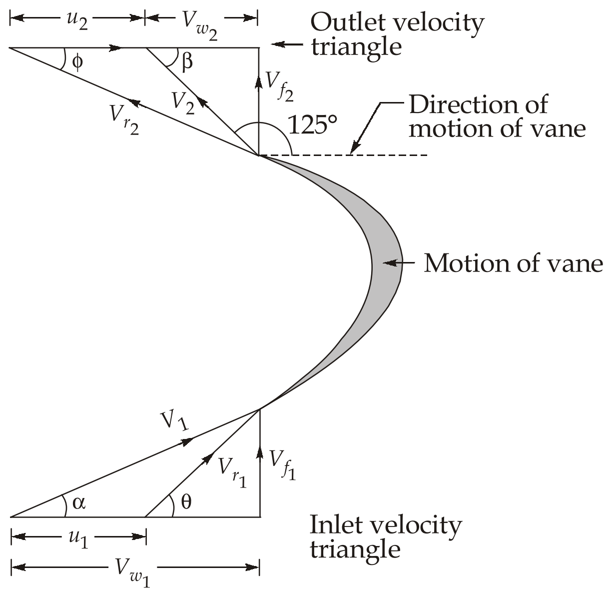
4. (c) (i)

Given : $V_1 = 30$ m/s, $u_1 = u_2 = 14$, $\alpha = 25°$

Angle made by the jet at the outlet with the direction of motion of vanes = 125°

$$\therefore \quad \text{Angle, } \beta = 180° - 125°$$

$$\beta = 55°$$



1. Angle of vanes tips, $V_{w_1} = V_1 \cos\alpha = 30\cos 25° = 27.19°$

$$V_{f_1} = V_1 \sin\alpha = 30\sin 25°$$

$$V_{f_1} = 12.68 \text{ m/s}$$

and, $$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{12.68}{27.19 - 14}$$

$$\theta = 43.87°$$

By sine rule, $$\frac{V_{r_1}}{\sin 90} = \frac{V_{f_1}}{\sin\theta}$$

$$V_{f_1} = \frac{12.68}{\sin(43.87°)}$$

Now, $$V_{r_1} = V_{r_2} = 18.296 \text{ m/s}$$

From outlet velocity triangle, by sine rule

$$\frac{V_{r_2}}{\sin 125} = \frac{u_2}{\sin(55 - \phi)}$$

$$\frac{18.296}{\sin 125} = \frac{14}{\sin(55 - \phi)}$$

$$55 - \phi = 38.814$$

$$\phi = 16.186°$$

2. Work done per unit weight of water entering = $\frac{1}{g}\left(V_{w_1} + V_{w_2}\right)u_1$

$$V_{w1} = 27.19 \text{ m/s and } u_1 = 14$$

The value of V_{w2} is obtained from outlet velocity triangle

$$V_{w2} = V_{r_2}\cos\phi - u_2$$
$$= 18.296\cos(16.186) - 14$$
$$V_{w2} = 3.57 \text{ m/s}$$

$$\therefore \quad \frac{\text{Work done}}{\text{Unit weight}} = \frac{1}{9.81}[27.19 + 3.57] \times 14 = 43.90 \text{ Nm/N}$$

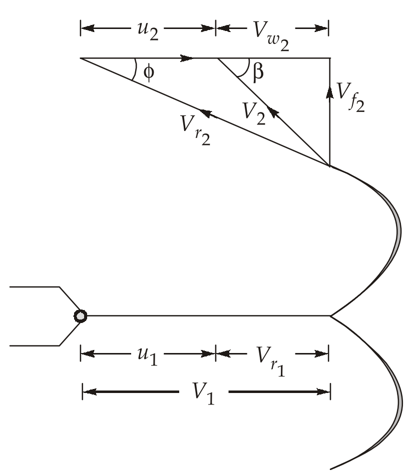
3. $$\text{Efficiency} = \frac{\text{Work done per kg}}{\text{energy supplied per kg}}$$

$$= \frac{43.90}{\frac{V_1^2}{2g}} = \frac{43.90 \times 2 \times 9.81}{30 \times 30}$$

$$\eta = 95.70\%$$

4. (c)(ii)

The figure shows a section through a bucket which is being acted on by a jet. The plane of the section is parallel to the axis of the wheel and contains the axis of the jet.



For Pelton turbine,

$$u_1 = u_2 = u = \frac{\pi DN}{60}$$

Force exerted by the jet of water on the bucket in the direction of motion of blade.

$$F_x = \rho A V_1 \left(V_{w_1} \pm V_{w_2}\right)$$

$$\text{Cross-section of jet, } A = \frac{\pi}{4} d^2; \quad d = \text{Diameter of jet}$$

$$\text{Work done, W.D.} = F_x u$$

$$= \rho A V_1 \left(V_{w_1} \pm V_{w_2}\right) u$$

and kinetic energy of jet per second $= \frac{1}{2} m V_1^2 = \frac{1}{2} \times (\rho A V_1) V_1^2$

$$\text{K.E.} = \frac{1}{2} \rho A V_1^3$$

$$\text{Hydraulic efficiency, } \eta_h = \frac{\rho A V_1 \left(V_{w_1} \pm V_{w_2}\right) u}{\frac{1}{2} \rho A V_1^3}$$

But for pelton wheel, $V_{w_1} = V_1, \quad V_{r_1} = V_1 - u = V_{r_2}$

Also from outlet velocity triangle,

$$V_{w_2} = V_{r_2} \cos\phi - u$$

$$= V_{r_1} \cos\phi - u$$

$$\eta_h = 2 \frac{\left[V_1 + (V_1 - u)\cos\phi - u\right] u}{V_1^2}$$

$$\eta_h = \frac{2\left[(V_1 - u)(1 + \cos\phi)\right] u}{V_1^2}$$

For maximum efficiency,

$$\frac{d}{du}\left[\frac{2(V_1 - u)(1 + \cos\phi)}{V_1^2} \times u\right] = 0$$

$$\frac{2(1 + \cos\phi)}{V_1^2} \frac{d}{du}\left(V_1 u - u^2\right) = 0$$

$$\therefore \quad u = \frac{V_1}{2}$$

Maximum hydraulic efficiency,

$$(\eta_h)_{max} = \left[\frac{2(V_1 - u)(1 + \cos\phi) \times u}{V_1^2}\right]_{u = \frac{V_1}{2}}$$

$$= \frac{1+\cos\phi}{2}$$

Section B : Fluid Mechanics + Fluid Machinery + Power Plant

5. (a) (i) Solution:

$$v_\theta = -\frac{c\sin\theta}{r^2}$$

1. Expression for radial velocity, v_r:

The continuity equation for a 2-D, steady incompressible flow is

$$\frac{1}{r}\frac{\partial}{\partial r}(r.v_r)+\frac{\partial(v_\theta)}{r\partial\theta} = 0$$

or, $$\frac{\partial}{\partial r}(rv_r)+\frac{\partial}{\partial\theta}(v_\theta) = 0 \quad \text{...(i)}$$

For the given velocity component:

$$\frac{\partial v_\theta}{\partial\theta} = \frac{\partial}{\partial\theta}\left(-\frac{c\sin\theta}{r^2}\right) = -\frac{c}{r^2}\cos\theta \quad \text{...(ii)}$$

for (i) and (ii), we have:

$$\frac{\partial}{\partial r}(rv_r) = \frac{c}{r^2}\cos\theta$$

$$rv_r = \int_0^r \frac{c}{r^2}\cos\theta\, dr \quad \text{(Integrating both side w.r.t. r)}$$

$$rv_r = -\frac{c\cos\theta}{r}$$

Radial component, $v_r = -\dfrac{c\cos\theta}{r^2}$ Answer

2. Resultant velocity:

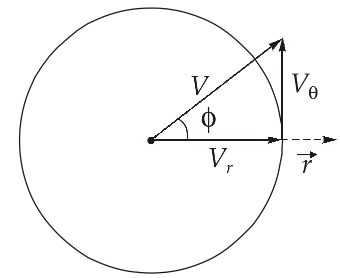
$$\text{Resultant velocity} = \sqrt{v_r^2+v_\theta^2}$$

$$= \left[\left(-\frac{c\cos\theta}{r^2}\right)^2+\left(-\frac{c\sin\theta}{r^2}\right)^2\right]^{1/2}$$

$$= \frac{c}{r^2}\left(\cos^2\theta+\sin^2\theta\right)^{1/2} = \frac{c}{r^2}$$ Answer

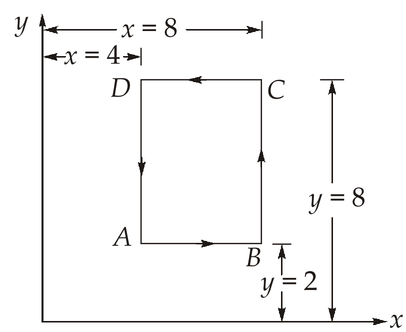
Direction of resultant velocity:

$$\tan\phi = \left(\frac{-c\dfrac{\sin\theta}{r^2}}{-c\dfrac{\cos\theta}{r^2}}\right)$$



$$\tan\phi = \tan\theta$$
$$\phi = \theta$$

5. (a) (ii) Solution:



$$u = 16y - 8x$$
$$v = 8y - 7x$$

$$\Gamma_{ABC} = \int_{AB}(udx+vdy)+\int_{BC}(udx+vdy)+\int_{CD}(udx+vdy)+\int_{DA}(udx+vdy)$$

$$= \int_4^8(16y-8x)dx+\int_2^2(8y-7x)dy+\int_8^8(16y-8x)dx+\int_2^8(8y-7x)dy+\int_8^4(16y-8x)dx+$$
$$\int_8^8(8y-7x)dy+\int_4^4(16y-8x)dx+\int_8^2(8y-7x)dy$$

$$\Gamma_{ABCD} = \int_4^8(16y-8x)dx+\int_2^8(8y-7x)dy+\int_8^4(16y-8x)dx+\int_8^2(8y-7x)dy$$

We can observe that for AB, y is invariant and for BC x is invariant and so on.

$$\Gamma_{ABCD} = \underset{\text{(i)}}{\left[16yx-4x^2\right]_4^8}+\underset{\text{(ii)}}{\left[4y^2-7xy\right]_2^8}+\underset{\text{(iii)}}{\left[16yx-4x^2\right]_8^4}+\underset{\text{(iv)}}{\left[4y^2-7xy\right]_8^2}$$

In integral (i): $y = 2$

In integral (ii): $x = 8$

In integral (iii): $y = 8$

In integral (iv): $x = 4$

Substituting these values, we have:

$$\Gamma_{ABCD} = [16 \times 2 \times 8 - 4 \times 8 \times 8 - 16 \times 2 \times 4 + 4^2 \times 4] + [4 \times 8^2 - 7 \times 8^2 - 4 \times 2^2 + 7 \times 2 \times 8] + [16 \times 8 \times 4 - 4 \times 4^2 - 16 \times 8 \times 8 + 4 \times 8^2] + [4 \times 2^2 - 7 \times 4 \times 2 - 4 \times 8^2 + 7 \times 4 \times 8]$$

$$\Gamma_{ABCD} = [256 - 256 - 128 + 64] + [256 - 448 - 16 + 112] + [512 - 64 - 1024 + 256] + [16 - 56 - 256 + 224]$$

$$= -64 - 96 - 320 - 72 = -552$$ Answer

Alternate solution,

Total circulation around a closed loop

$$\Gamma = \Gamma_{AB} + \Gamma_{BC} + \Gamma_{CD} + \Gamma_{DA}$$

$$\left(u - \frac{\partial u}{\partial y}\frac{\delta y}{2}\right)\delta x + \left(v + \frac{dv}{\partial x}\frac{\delta x}{2}\right)\delta y - \left(u + \frac{\partial u}{\partial y}\frac{\delta y}{2}\right)\delta x - \left(v - \frac{\partial v}{\partial x}\frac{\delta x}{2}\right)\delta y$$

$$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\delta x \delta y = \left(\frac{\partial(8y - 7x)}{\partial x} - \frac{\partial(16y - 8x)}{\partial y}\right)$$

$$= (-7 - 16) \times 24 = -552$$

5. (b)

Given : $A_1 = 1.8\ \text{m}^2$; $A_2 = 13.5\ \text{m}^2$; $V_1 = 12.5$ m/s

(i) Frictional loss in the draft tube,

$$H_L = 0.1 \times \frac{V^2}{2g} = 0.1 \times \frac{(12.5)^2}{2 \times 9.81} = 0.7964 \text{ m}$$

$$Q = A_1 V_1 = A_2 V_2$$

$$1.8 \times 12.5 = 13.5 V_2$$

$$V_2 = 1.667 \text{ m/s}$$

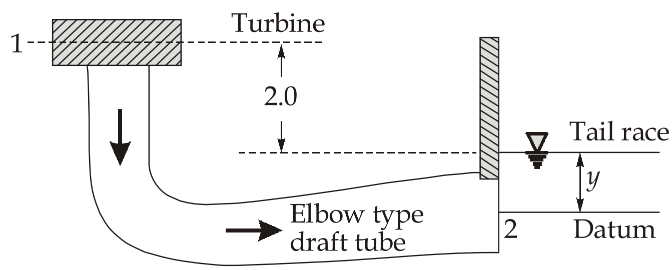


Figure: Draft tube set up

Applying Bernoulli's between (i) and (ii)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2+H_L$$

$$\frac{P_1}{\rho g}+\frac{(12.5)^2}{2\times 9.81}+(Z+y) = (0+y)+\frac{(1.667)^2}{2\times 9.81}+0+0.7964$$

$$\frac{P_1}{\rho g} = 0+\frac{(1.667)^2}{2\times 9.81}+0.7964-\frac{(12.5)^2}{2\times 9.81}-2$$

$$\frac{P_1}{\rho g} = -9.025 \text{ m (gauge)}$$ **Ans.**

(ii) Power wasted to the tail race,

$$P_L = \rho g Q\times\frac{V_2^2}{2g}$$

$$= 1000\times 9.81\times(1.8\times 12.5)\times\frac{(1.667)^2}{2\times 9.81}$$

$$= 31.26 \text{ kW}$$ **Ans.**

(iii) Efficiency of the draft tube

$$\eta_d = 1-\frac{H_L}{\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)}$$

$$= 1-\frac{0.7964}{\frac{(12.5)^2}{2\times 9.81}-\frac{(1.667)^2}{2\times 9.81}} = 0.8981 \text{ or } 89.81\%$$ **Ans.**

Q.5. (c) Solution:

Turbojet : A compressor pressurizes intake air before mixing it with fuel and igniting it, causing expansion through a turbine to power the compressor, and expel out to generate thrust.

Turbofan: A turbojet in which as little direct thrust is produced as possible, and instead as much energy as possible is extracted by the turbine and used to turn a large fan which bypasses the engine.

Turboprop: Just like a turbofan, but replace "fan" with propeller and add a transmission to adjust its speed.

Ramjet: Just like a turbojet, except instead of a mechanical compressor, a scoop is used to compress "ram air" being scooped up by the engine. Like all other engines so far, ramjets combust fuel-air mixture at subsonic speed, even if the aircraft is supersonic.

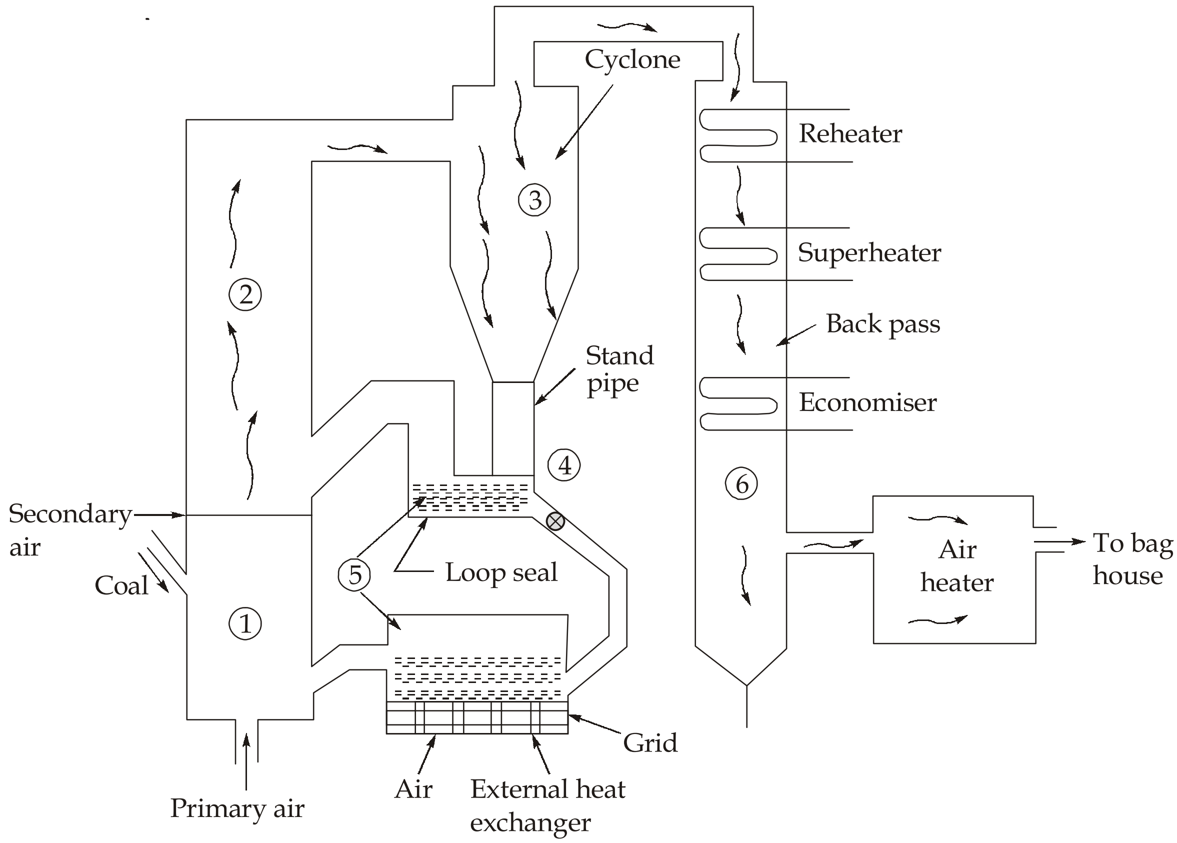
Scramjet: A ramjet that combusts the fuel-air mixture in a supersonic airflow.

Pulsejet: A pulsejet engine (or pulse jet) is a type of jet engine in which combustion occurs in pulses. A pulsejet engine can be made with few or no moving parts, and is capable of running statically (i.e. it does not need to have air forced into its inlet typically by forward motion).

5. (d)

The CFB boiler is said to be the second generation fluidized bed boiler. It is divided into two sections. The first section consists of (a) furnace or fast fluidized bed, (b) gas-solid separator (cyclone), (c) solid recycle device (loop seal or L-valve), and (d) external heat exchanger (optional). These components form a solid circulation loop in which fuel is burned. The furnace enclosure of a CFB boiler is generally made of water tubes as in pulverized coal-fired (PC) boilers. A fraction of the generated heat is absorbed by these heat transferring tubes. The second section is the back-pass, where the remaining heat from the flue gas is absorbed by the reheater, superheater, economizer, and air preheater surfaces.

The lower part of the first section (furnace) is often tapered. Its walls are lined with refractory up to the level of secondary air entry. Beyond this the furnace walls are generally cooled by evaporative, superheater, or reheater surfaces. The gas-solid separator and the non-mechanical valve are also lined with refractory. In some designs, a part of the hot solids recycling between the cyclone and the furnace is diverted through an external heat exchanger, which is a bubbling fluidized bed with beat transfer surfaces immersed in it to remove heat from the hot solids. Coal is generally injected into the lower section of the furnace. It is sometimes fed into the loop-seal, from which it enters the furnace along with returned hot solids. Limestone is fed into the bed in a similar manner. Coal burns when mixed with hot bed solids.



1. *Furnace (below secondary air level) - Turbulent/bubbling fluidized bed.*
2. *Furnace (above secondary air level) - Fast fluidized bed.*
3. *Cyclone - Swirl flow.*
4. *Return leg (stand pipe) - moving packed bed.*
5. *Loop seal/external heat exchanger - bubbling fluidized bed.*
6. *Back pass - pneumatic transport.*

Figure : Flow regimes in a CFB boiler

The primary combustion air enters the furnace through an air distributor or grate at the furnace floor. The secondary air is injected at some height above the grate to complete the combustion. Bed solids are well mixed throughout the height of the furnace. Thus, the bed temperature is nearly uniform in the range 800-900°C, though heat is extracted along its height. Relatively coarse particles of sorbent (limestone) and unburned char, larger than the cyclone cut-off size, are captured in the cyclone and are recycled back near the base of the furnace. Finer solid residues (ash and spent sorbents) generated during combustion and desulphurization leave the furnace, escaping through the cyclones, but they are collected by a bag-house or electrostatic precipitator located further downstream.

5. (e)

Given : $\eta_V = 0.92$; $N_V = 80$ rpm; $l = 1.25d$; $T = 35°C$

From steam tables, corresponding to 35°C

$$P_s = 0.05629 \text{ bar}$$

The combined pressure of steam and air in the condenser,

$$P = 760 - 710 = 50 \text{ mm of Hg}$$

$$= \frac{50}{1000} \times 13600 \times 9.81 \times 10^{-5}$$

$$= 0.0667 \text{ bar}$$

$$\text{Partial pressure of air, } P_a = P - P_s$$

$$= 0.0667 - 0.05629$$

$$= 0.01041 \text{ bar}$$

Mass of air leakage in the condensor per minute,

$$m_a = \frac{14000 \times 2}{2800 \times 60} = \frac{1}{6} \text{kg/min}$$

Volume of air leakage in the condensor per minute

$$V_a = \frac{m_a R T_a}{P_a} = \frac{\frac{1}{6} \times 287 \times (35 + 273)}{(0.01041 \times 10^5)}$$

$$V_a = 14.1524 \text{ m}^3/\text{min}$$

Mass of steam condensed per minute = $\frac{14000}{60}$ kg

Volume of steam condensate per minute = $\frac{14000}{60 \times 1000} = 0.2333 \text{ m}^3/\text{min}$

[$\because$ Density of water = 1000 kg/m^3]

Volume of mixture (air + condensate) actually discharged/min

$$= 14.1524 + 0.2333$$

$$= 14.386 \text{ m}^3/\text{min}$$

$\therefore$ Discharging capacity of the air pump/min

$$= 14.386 \times \frac{100}{92} = 15.64 \text{ m}^3/\text{min}$$

$\therefore$ Discharging capacity of the air pump/stroke

$$= \frac{15.64}{80} \times 10^6 = 195500 \text{ cm}^3$$

$$\frac{\pi}{4} d^2 \times 1.25d = 195500$$

$$d = 58.396 \text{ cm} \qquad \textbf{Ans.}$$

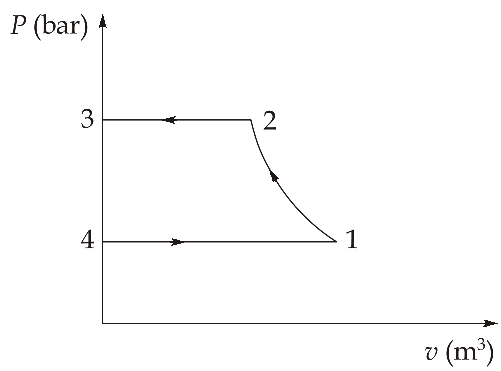
$$l = 1.25d = 1.25 \times 58.396$$

$$l = 72.995 \text{ cm} \qquad \textbf{Ans.}$$

6. (a)

Given : P_1 = 1 bar; T_1 = 25 + 273 = 298 K; P_2 = 8 bar; T_2 = 200 + 273 = 473 K;

N = 1250 rpm; P_{state} = 8.25 kW; $\dot{m}_a$ = 2 kg/min



(i) The actual volumetric efficiency, η_{vol}

Displacement volume (m^3/min)

$$V_d = \frac{\pi}{4} \times D^2 \times L \times N$$

$$= \frac{\pi}{4} \times \left(\frac{15}{100}\right)^2 \times \left(\frac{10}{100}\right) \times 1250 = 2.209 \text{ m}^3/\text{min}$$

$$FAD = \frac{mRT_1}{P_1}$$

$$= \frac{2 \times 0.287 \times 1000 \times 298}{1 \times 10^5} = 1.711 \text{ m}^3/\text{min}$$

$$\eta_{vol} = \frac{FAD}{v_d} \times 100 = \frac{1.711}{2.209} \times 100 = 77.46\%$$

(ii) The indicated power, (I.P.)

As we know, $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

Taken ln both sides,

We get $$\frac{n-1}{n} = \frac{\ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{P_2}{P_1}\right)}$$

or $$\frac{1}{n} = 1 - \frac{\ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{P_2}{P_1}\right)}$$

∴ $$\frac{1}{n} = 1 - \frac{\ln\left(\frac{473}{298}\right)}{\ln\left(\frac{8}{1}\right)}$$

$$n = 1.286$$

Hence, index of compression, $n = 1.286$

∴ Indicated power, I.P. $$= \left(\frac{n}{n-1}\right) mRT_1 \left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right\}$$

$$= \left(\frac{1.286}{1.286-1}\right)\frac{2}{60} \times 0.287 \times 298 \left\{(8)^{\frac{1.286-1}{1.286}} - 1\right\}$$

I.P. = 7.537 kJ/s or kW

I.P. = 7.537 kW

(iii) Thermal efficiency, η_{iso} :

Isothermal power $$= mRT_1 \ln\left(\frac{P_2}{P_1}\right)$$

$$= \frac{2}{60} \times 0.287 \times 298 \times \ln\frac{8}{1} = 5.928 \text{ kW}$$

$$\eta_{iso} = \frac{\text{Isothermal power}}{\text{Indicated power}} = \frac{5.928}{7.537} = 78.65\%$$

(iv) Mechanical efficiency, η_{mech}

$$\eta_{mech} = \frac{\text{Indicated power}}{\text{Shaft power}} \times 100$$

$$= \frac{7.537}{8.25} \times 100 = 91.36\%$$

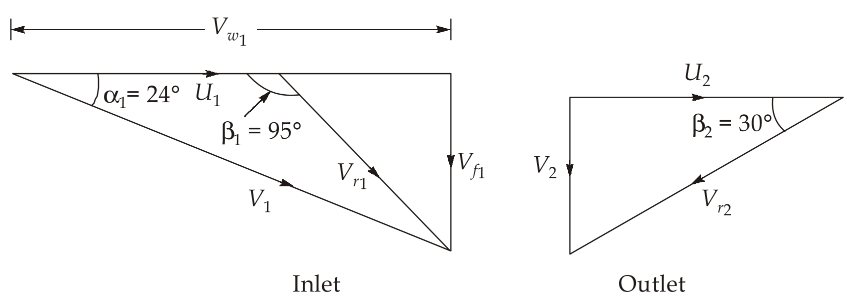
(v) The overall isothermal efficiency,

$$(\eta_{iso})_{overall} = \frac{\text{Isothermal power}}{\text{Shaft power}} \times 100$$

$$= \frac{5.928}{8.25} \times 100 = 71.85\%$$

6. (b)

The inlet and outlet velocity triangle are shown below.



Let W = work head given by the fluid to runner

h_{1r} = Head loss in runner inlet

h_{2r} = Head loss in runner outlet

Applying Bernoulli's equation between the inlet and the outlet of the runner,

we have: $$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + W + h_{1r} \quad ...(i)$$

Here subscript 1 represents the runner inlet while 2 represents the runner outlet.

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 55 \text{ mm} \quad \text{(given in the problem)}$$

$$h_{1r} = 0.18 \times 55 = 9.9 \text{ mm} \quad \text{(as per question)}$$

$$\text{Work output, } w = \frac{V_{w1}V_1}{g}$$

from inlet velocity triangle,

$$V_{w1} = V_1 \cos 24° = 0.9135\ V_1$$

from continuity equation,

$$K_1 V_{f1} D_1 B_1 = K_2 V_{f2} D_2 B_2$$

$$V_{f1} \times 450 \times 50 = V_{f2} \times 300 \times 75 \qquad (K_1 = K_2)$$

which gives $V_{f2} = V_{f1}$

Therefore, $V_{f2} = V_{f1} = V_1 \sin 24° = 0.4067\ V_1$

from the consideration of rotational speed,

$$u_1 = \frac{\pi D_1 N}{60},\ u_2 = \frac{\pi D_2 N}{60}$$

$$u_1 = \frac{D_1}{D_2} u_2,\ u_2 = \frac{\pi D_2 N}{60}$$

$$u_1 = \frac{D_1}{D_2} u_2 = \frac{450}{300} \times u_2 = 1.5\ u_2$$

Again, from the outlet velocity triangle,

$$u_2 = \frac{V_2}{\tan 30°} = \frac{0.4067 V_1}{\tan 30°} = 0.7044\ V_1$$

Hence, $u_1 = 1.5 \times 0.703\ V_1 = 1.057\ V_1$

Therefore, $W = \frac{V_{w1} u_1}{g} = \frac{0.913 \times 1.05}{g} V_1^2 = \frac{0.965 V_1^2}{g}$

Based on above calculated values, equation (i) can be written as

$$(55 - 9.9) \times 10^{-3} = -\frac{V_1^2}{2g} + \frac{(0.4067 V_1)^2}{2g} + \frac{0.965 V_1^2}{g}$$

$$45.1 \times 10^{-3} = \frac{V_1^2}{2g}\left[-1 + (0.4067)^2 + 2 \times 0.965\right]$$

$$45.1 \times 10^{-3} = 1.028 \frac{V_1^2}{2g}$$

Hence, $V_1 = \left[\frac{2 \times 9.81 \times 45.1 \times 10^{-3}}{1.028}\right]^{1/2} = 0.923 \text{ m/s}$

$$u_1 = 1.057 \times 0.923 = 0.98 \text{ m/s}$$

Therefore, $N = \frac{0.98 \times 60}{\pi \times 0.45} = 41.63 \text{ rpm}$

Rate of flow, $Q = 0.92\ \pi D_1 B_1 \times V_{f1}$

$$V_{f1} = 0.4067 \times 0.923 = 0.377 \text{ m/s}$$

Hence $Q = 0.92 \times \pi \times 0.45 \times 0.05 \times 0.377 = 0.0245 \text{ m}^3/\text{s}$

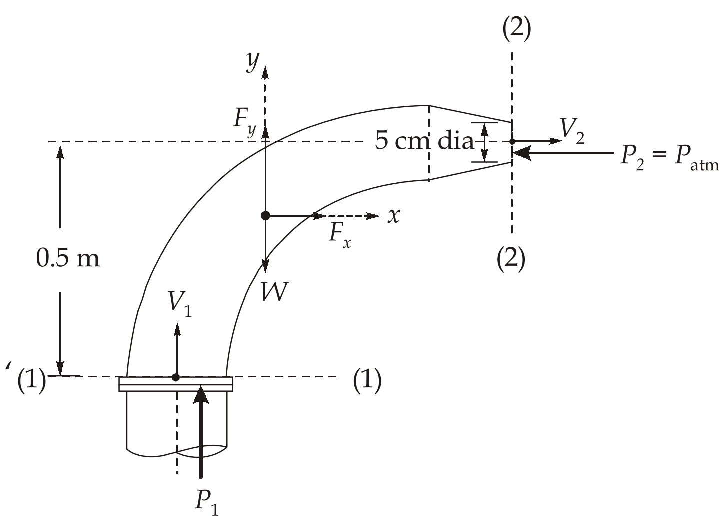
Therefore, power developed,

$$P = \rho Q V_{w1} U_1$$
$$= Q V_{w1} U_1 \text{ (kW)}$$
$$= 0.0245 \times 0.9135 \times 0.923 \times 0.98$$
$$= 0.02035 \text{ kW} = 20.35 \text{ W}$$

Ans.

6. (c)

Bend nozzle assembly is our system and F_x and F_y denotes hydrodynamic forces on system.



(a) Bend-nozzle

Discharge through bend-nozzle, $Q = \dfrac{2400 \times 10^{-3}}{60} = 0.04 \text{ m}^3/\text{s}$

Velocity at section 1, $V_1 = \dfrac{Q}{A_1} = \dfrac{0.04 \times 4}{\pi \times (0.1)^2} = 5.093 \text{ m/s}$

Velocity at nozzle exit, $V_2 = \dfrac{Q}{A_2} = \dfrac{4 \times 0.04}{\pi \times (0.05)^2} = 20.372 \text{ m/s}$

W = Weight of 18.2 litres of water

$= 18.2 \times 10^{-3} \times 9810 = 178.5 \text{ N}$

Applying Bernoulli's equation between sections (1) and (2) and taking the horizontal plane through the bottom of the bend as datum:

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+\text{Head loss}$$

$$\text{Head loss} = \text{Head loss in bend} + \text{Head loss in nozzle}$$

$$= 0.5\frac{V^2}{2g}+\frac{V^2}{g} = 2.5\frac{V^2}{2g}=2.5\frac{V_1^2}{2g}$$

(V_1 = velocity of water in pipe)

$$\frac{P_1}{\rho g}+\frac{(5.093)^2}{19.62}+0 = 0+\frac{(20.372)^2}{19.62}+0.5+\frac{2.5\times(5.093)^2}{19.62}$$

$$\frac{P_1}{\rho g} = 23.636 \text{ m}$$

$$P_1 = 9810 \times 23.636 = 231.868 \text{ kN/m}^2$$

Let F_x and F_y be the forces in positive x-direction and positive y-direction applied by fluid on bend-nozzle assembly.

Applying momentum equation in x-direction:

$$-F_x+(-P_2A_2-0) = \rho Q(V_2-0)$$

$$F_x = 1000 \times 0.04 \times 20.372 = -814.9 \text{ N} \qquad (P_2 = P_{atm} = 0)$$

Applying momentum equation in y-direction:

$$(0+P_1A_1)+(-F_y)-W = \rho Q(0-v_1)$$

$$231868\times\frac{\pi}{4}\times0.1^2+(-F_y)-178.5 = 1000 \times 0.04 \times (-5.093)$$

$$1821.08+(-F_y)-178.5 = -203.72$$

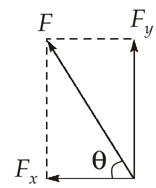
$$F_y = 1846.30 \text{ N}$$

Since the computed F_x is negative, its actual direction is reversed as shown in figure (b).

Resultant force on bend nozzle.

$$F_R = \sqrt{(-814.9)^2+(1846.30)^2} = 2018.14 \text{ N}$$

$$\text{Angle of resultant, } \theta = \tan^{-1}\left(\frac{1846.30}{814.9}\right) = 66.2° \text{ w.r.t. horizontal.}$$



(b) Force diagram

7. (a) (i)

Average height of surface protrusions,

$$k = 0.12 \text{ mm} = 0.12 \times 10^{-3} \text{ m}$$

Shear stress developed, $\tau_0 = 8.6 \text{ N/m}^2$

Density of water, $\rho = 1000 \text{ kg/m}^3$

kinematic viscosity, $\nu = 0.0093 \text{ stokes} = 0.0093 \times 10^{-4} \text{ m}^2/\text{s}$

$$\text{Shear velocity, } u_f = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{8.6}{1000}} = 0.09274 \text{ m/s}$$

$$\text{Roughness Reynolds number} = \frac{u_f k}{\nu} = \frac{0.09274 \times 0.12 \times 10^{-3}}{0.0093 \times 10^{-4}} = 11.966$$

Since $\frac{u_f k}{\nu}$ lies between 4 and 100, the pipe surface behaves as in 'Transition'.

Note: (i) For smooth boundary .... $\frac{u_f k}{\nu} < 4$

(ii) for rough boundary .... $\frac{u_f k}{\nu} > 100$ and

(iii) for boundary in transition stage .... $\frac{u_f k}{\nu}$ lies between 4 and 100.

7. (a) (ii)

The velocity distribution near the rough boundaries is given by:

$$\frac{u}{u_f} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5 \quad ...(i)$$

where, k = Average height of roughness elements, and

u_f = shear friction velocity

Equation (i) is known as Karman-Prandtl equation for the velocity distribution near hydrodynamically rough boundaries.

Let u = velocity at a distance (y) of 1 cm from the pipe wall

$1.2u$ = velocity at a distance (y) of 2 cm from the pipe wall

$$\frac{u}{u_f} = 5.75\log_{10}\left(\frac{1}{k}\right) + 8.5 \quad \text{...(ii)}$$

$$\frac{1.2u}{u_f} = 5.75\log_{10}\left(\frac{2}{k}\right) + 8.5 \quad \text{...(iii)}$$

Dividing (ii) by (iii), we get:

$$\frac{1}{1.2} = \frac{5.75\log_{10}\left(\frac{1}{k}\right) + 8.5}{5.75\log_{10}\left(\frac{2}{k}\right) + 8.5}$$

$$5.75\log_{10}\left(\frac{2}{k}\right) + 8.5 = 1.2 \times 5.75\log_{10}\left(\frac{1}{k}\right) + 1.2 \times 8.5$$

$$5.75\log_{10}\left(\frac{2}{k}\right) + 8.5 = 6.9\log_{10}\left(\frac{1}{k}\right) + 10.2$$

$$5.75 \log_{10}(2) - 5.75 \log_{10}(k) + 8.5 = 6.9 \log_{10}(1) - 6.9 \log_{10}(k) + 10.2$$

$$1.731 - 5.75 \log_{10}(k) + 8.5 = 0 - 6.9 \log_{10}(k) + 10.2$$

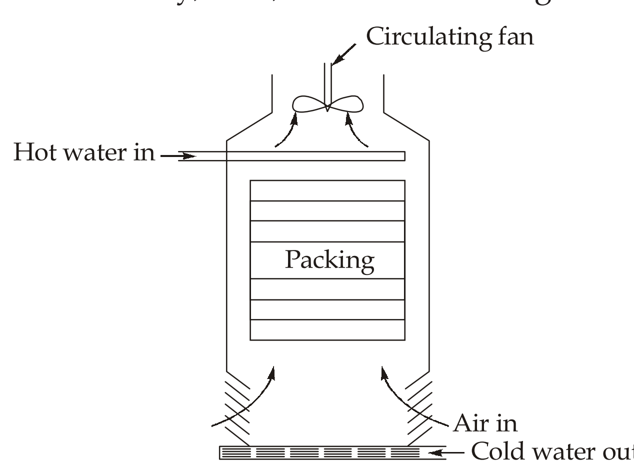
$$1.15 \log_{10}(k) = 10.2 - 8.5 - 1.73 = -0.031$$

$$\log_{10}(k) = -\frac{0.031}{1.15} = -0.02695 \quad \text{or} \quad k = 0.9398 \text{ cm}$$ Answer

7. (b) (i)

Cooling Towers : Cooling towers cool the warm water discharged from the condenser and feed the cooled water back to the condenser. They, thus, reduce the cooling water demand in the power plant.

Wet cooling towers have a hot water distribution system that showers or sprays water evenly over a lattice of horizontal slats or bars called fill or packing. The fill thoroughly mixes the falling water with air moving through the fill as the water splashes down from one fill level to another by gravity.



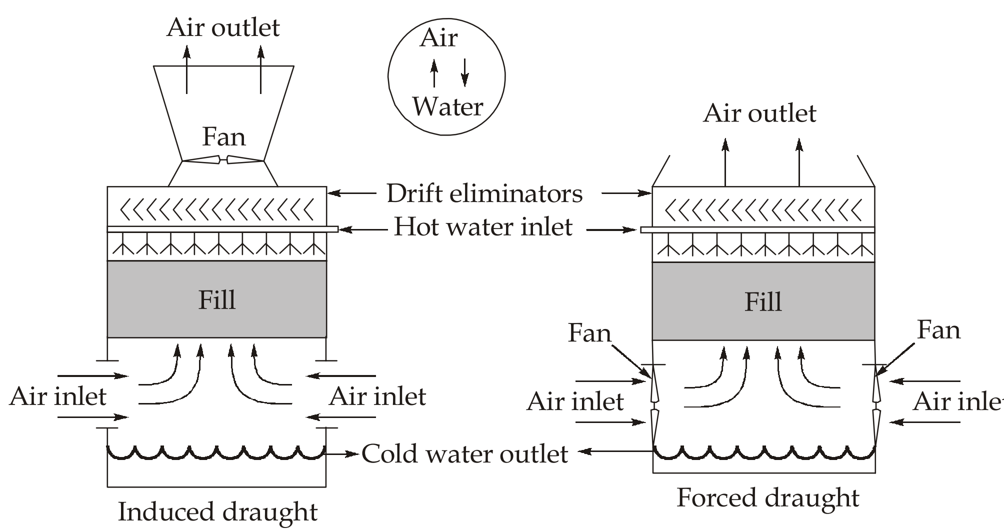
Packing or fill in a wet cooling tower

Outside air enters the tower through louvres on the side of the tower.

Intimate mixing of water and air enhances heat and mass transfer (evaporation), which cools the water. More the water evaporates, more will be the cooling since the latent heat of evaporation is taken from water itself (evaporative cooling). Cold water is collected in a concrete basin at the bottom of the tower, from where it is pumped back to the condenser. Hot and moist air leaves the tower from the top.

Air entering the tower is unsaturated and as it comes in contact with the water spray, water continues to evaporate till the air becomes saturated. So, the minimum temperature to which water can be cooled is the adiabatic saturation or wet bulb temperature of the ambient air. At this temperature (WBT), air is 100% saturated and cannot absorb any more water vapour. Hence, there will be no further evaporation and cooling. The humid air while moving up comes in contact with warm water spray and so the air temperature rises.

Wet cooling towers can be either mechanical draught or natural draught cooling towers. In mechanical draught cooling towers, air is moved through the fill by one or more fans driven by motors. As in steam generators, the fans could be of the forced draught (FD) type or induced draught (ID) type. The FD fan is mounted on the lower side of the tower. Since it operates on cooler air, it consumes less power. However, it has the disadvantages of (a) air distribution problems in the fill, often causing channelling of air flowing through paths of less flow resistance, (b) leakage and (c) recirculation of the hot and moist air back to the tower.



Induced draught counterflow cooling towers

Most of the mechanical draught cooling towers for utility applications are of the induced draught type. The ID fan is located at the top of the tower. Air enters the sides of the tower through large openings at low velocity and passes through the fill. Hot humid air is exhausted by the fan at the top to the atmosphere. It maintains the tower at a negative pressure thereby reducing leakage.

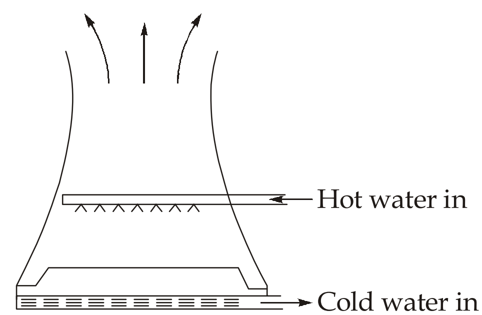
In natural draught cooling towers, the flow of air occurs due to the due to the natural pressure head caused by the difference in density between the cold outside air and the hot humid air inside.

Thus, the pressure head developed is

$$\Delta p_d = (\rho_o - \rho_i)gH$$

where, H = height of the tower above the fill, ρ_o = density of outside air, and ρ_i = density of inside air.

Because of relatively small density difference, $\rho_o - \rho_i$, H must be large so as to result in the desired Δp_d which must balance the air pressure losses in the tower.



Natural draught cooling tower

Natural draught cooling towers are, therefore, very tall. The tower body, above the water distribution system and the fill, is an empty shell of circular cross-section, but with a hyperbolic vertical profile. The hyperbolic profile offers superior strength and the greatest resistance to outside wind loading compared to other forms. Natural draught cooling towers are, therefore, often termed as hyperbolic towers. Made of reinforced concrete, they are an imposing sight and are conspicuous from a distance.

7. (b) (ii)

Given : t_{dbt_1} = 25°C; ϕ_1 = 50%; t_{dbt_2} = 30°C; ϕ_2 = 80%

Properties of air entering and leaving the tower are obtained from Psychrometric chart.

At t_{dbt} = 25°C and ϕ = 50%

$$h_1 = 50 \text{ kJ/kg dry air}$$

$$t_{wb_1} = 18°\text{C}$$

$$\omega_1 = 0.01 \text{ kg water vapour/kg dry air}$$

At t_{dbt} = 30°C and ϕ = 80%

$$h_2 = 81.5 \text{ kJ/kg dry air}$$

$$t_{wb_2} = 27.2°\text{C}$$

$$\omega_2 = 0.0218 \text{ kg water vapour/kg dry air}$$

From steam tables,

Enthalpies of water entering the tower and makeup water are

$$h_{w_3} = 146.72 \text{ kJ/kg} \qquad \text{(at 35°C and 1 bar)}$$

and $$h_w = 104.92 \text{ kJ/kg} \qquad \text{(at 25°C and 1 bar)}$$

From energy balance,

$$m_w\left(h_{w_3} - h_{w_4}\right) = m_a\left[(h_2 - h_1) - (\omega_2 - \omega_1)h_w\right]$$

$$1.2\left[h_{w_3} - h_{w_4}\right] = 1[(81.5 - 50) - (0.0218 - 0.01)104.92]$$

$$h_{w_3} - h_{w_4} = 25.218 \text{ kJ/kg}$$

$$t_{w_3} - t_{w_4} = \frac{25.218}{4.18} = 6.033$$

$$35 - t_{w_4} = 6.033$$

$$t_{w_4} = 28.97°\text{C}$$ **Ans.**

Fraction of water evaporated,

$$x = m_a(\omega_2 - \omega_1)$$

$$= 1(0.0218 - 0.01)$$

$$x = 0.0118$$ **Ans.**

$$\text{Approach} = t_{w_4} - t_{wb_1} = 28.97 - 18$$

$$= 10.97$$ **Ans.**

$$\text{Range} = t_{w_3} - t_{w_4} = 35 - 28.97$$

$$= 6.03$$ **Ans.**

7. (c)

Given data: Shaft power, $P = 22500$ kW $= 22.50 \times 10^6$ W; Head, $H = 20$ m; Speed, $N = 148$ rpm;

Hydraulic efficiency, $\eta_H = 95\% = 0.95$; Overall efficiency, $\eta_0 = 89\% = 0.89$;

Diameter of the runner, $D = 4.5$ m; Diameter of the hub, $d = 2$ m; Runner vane angle at outlet, $\beta_0 = 34°$

$$V_{fi} = V_{fo} = V_f$$

∴ Velocity of flow is constant.

Now, peripheral velocity of the runner,

$$u_i = u_0 = u = \frac{\pi DN}{60} = \frac{\pi \times 4.5 \times 148}{60} = 34.87 \text{ m/s}$$

Overall efficiency, $\eta_o = \frac{P}{\rho Q g H}$

$$0.89 = \frac{22.50 \times 10^6}{1000 \times Q \times 9.81 \times 20}$$

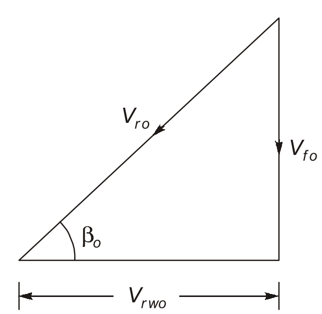
or Discharge, $Q = 128.85 \text{ m}^3/\text{s}$

also $Q = \frac{\pi}{4}[D^2 - d^2]V_{fi}$

$$128.85 = \frac{\pi}{4}\left[(4.5)^2 - (2)^2\right] \times V_{fi}$$

or $V_{fi} = 10.10 \text{ m/s} = V_{fo}$

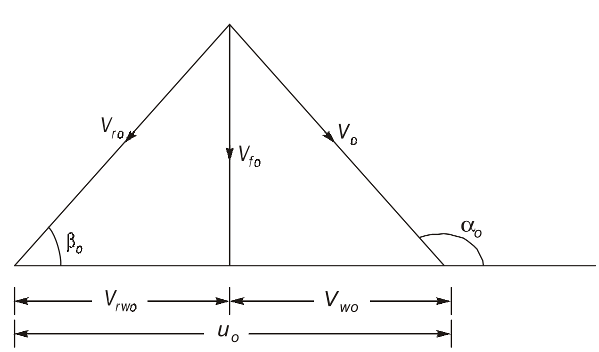
According to given data the partially velocity triangle at outlet as shown in figure.



$$\tan\beta_o = \frac{V_{fo}}{V_{rwo}}$$

or $V_{rwo} = \frac{V_{fo}}{\tan\beta_o} = \frac{10.10}{\tan 34°} = 14.97 \text{ m/s}$

Now, we know the suitable data to draw the complete velocity triangle at outlet:



$$u_o > V_{rwo}$$

$$V_{wo} = u_o - 14.97 = 19.90 \text{ m/s}$$

$$V_o = \sqrt{V_{fo}^2 + V_{wo}^2} = \sqrt{(10.10)^2 + (19.90)^2} = 22.32 \text{ m/s}$$

According to energy balance equation,

$$\rho QgH = \rho Q[V_{wi} - V_{wo}] + \rho Q\frac{V_o^2}{2}$$

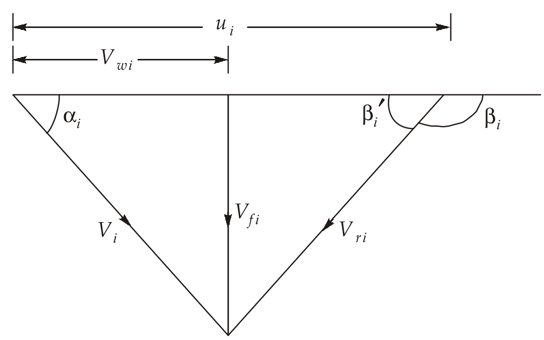
$$gH = [V_{wi} - V_{wo}]u + \frac{V_o^2}{2}$$

$$9.81 \times 20 = [V_{wi} - 19.88] \times 34.87 + \frac{(22.32)^2}{2}$$

$$196.2 = [V_{wi} - 19.90] \times 34.85 + 248.42$$

or $$V_{wi} = 18.38 \text{ m/s}$$

Now, we know the suitable data to draw the velocity triangle at inlet, $u_i > V_{wi}$



(i) Guide vane angle at inlet: α_1

$$\tan\alpha_i = \frac{V_{fi}}{V_{wi}} = \frac{10.10}{19.90} = 0.5075$$

$$\alpha_i = \tan^{-1}(0.5095) = \mathbf{26.91°}$$

(ii) Runner vane angle at inlet: β_i

$$\tan\beta_i' = \frac{V_{fi}}{u_i - V_{wi}} = \frac{10.10}{34.87 - 19.90} = 0.6746$$

$$\beta_i' = \tan^{-1}(0.6746) = 34°$$

and $$\beta_i = 180° - 34 = \mathbf{146°}$$

8. (a) (i)

Given : T_a = 290 K; T_g = 620 K; h = 22 mm; H = 35 mm

We know,
$$h = 353H\left[\frac{1}{290} - \left(\frac{m+1}{m}\cdot\frac{1}{T_g}\right)\right]$$

$$22 = 353\times35\left[\frac{1}{290} - \left(\frac{m+1}{m}\right)\left(\frac{1}{620}\right)\right]$$

$$m = 29.48 \text{ kg air/kg coal}$$ **Ans.**

Let H_g be the height of the hot gas column producing the draught, then

$$\Delta P = \rho_g g H_g = \rho_w \times g \times H_w$$

$$\rho_g H_g = 1000\times\frac{22}{1000}$$

$$\rho_g H_g = 22 \text{ kg/m}^2 \quad \text{...(i)}$$

and
$$\rho_g = 353\left(\frac{m+1}{m}\right)\frac{1}{T_g} = 353\left(\frac{29.48+1}{29.48}\right)\times\frac{1}{620}$$

$$\rho_g = 0.5886 \text{ kg/m}^3$$

Putting value of ρ_g in equation (i), we have

$$H_g = \frac{22}{0.5886}$$

$$H_g = 37.37 \text{ m}$$ **Ans.**

$$\text{Mass of flue gases} = \text{Mass of coal + Mass of air}$$

$$= \frac{2100(1+29.48)}{3600} = 17.78 \text{ kg/s}$$

$$\therefore \quad \text{Volume flow of flue gas} = \frac{17.78}{0.5886} = 30.207 \text{ m}^3/\text{s}$$

$$\text{Flue gas velocity, } V_g = \sqrt{2gH_g} = \sqrt{2\times9.81\times0.1\times37.37}$$

$$V_g = 8.56 \text{ m/s}$$

$$\text{Volume flow of fluid gases} = \frac{\pi}{4}d^2\times Vg$$

$$30.207 = \frac{\pi}{4}d^2\times8.56$$

$$d = 2.12 \text{ m}$$

8. (a) (ii)

Given : P = 12 bar; m_g = 45000 kg; x = 0.9; t_m = 30°C; m_c = 5000 kg; C.V. = 33500 kJ/kg

From steam table corresponding to 12 bar

$$h_f = 798.33 \text{ kJ/kg}; \quad h_{fg} = 1985.4 \text{ kJ/kg}$$

Now,
$$h = h_f + xh_{fg} = 798.33 + 0.9 \times 1985.4$$
$$= 2585.19 \text{ kJ/kg}$$

Heat of feed water, $h_{f_1} = 1 \times 4.18 \times (30 - 0) = 125.4$ kJ/kg

Total net heat given to produce 1 kg of steam = $h - h_f$

$$= 2585.19 - 125.4 = 2459.79 \text{ kJ/kg}$$

1. Factor of equivalents evaporation,

$$F_e = \frac{h - h_{f_1}}{2257} = \frac{2459.79}{2257} = 1.089$$

2. Equivalent evaporation from and at 100°C,

$$m_e = \frac{m_a\left(h - h_{f_1}\right)}{2257}$$

$$= \frac{\left(\frac{45000}{5000}\right)(2459.79)}{2257} = 9.801 \text{ kg of steam/kg of coal}$$

3. Efficiency of boiler, $\eta_{boiler} = \dfrac{m_a\left(h - h_{f_1}\right)}{CV}$

$$= \frac{9 \times (2459.79)}{33500} = 0.6608 \text{ or } 66.08\%$$ **Ans.**

8. (b) (i)

Let the diameter of orifice be d_0, and at any instant t, the height of the liquid level above the orifice be h. Then during an infinitesimal time dt, discharge through the orifice is

$$q = C_d \frac{\pi d_0^2}{4}\sqrt{2gh}\,dt = 0.60 \times \frac{1}{4}\pi d_0^2 \sqrt{2gh}\,dt$$

If the liquid level in the tank falls by an amount dh during this time, then from continuity.

$$-A_h dh = 0.60\frac{\pi d_0^2}{4}\sqrt{2gh}\,dt$$

where A_h is the area of the tank at height h. From the geometry of the tank,

$$\tan\alpha = \frac{(2.44 - 1.22)}{2 \times 3.05} = 0.2$$

Therefore the diameter of the tank at height h = 1.22 + 2 × 0.2h

Hence, $$A_h = \left(\frac{\pi}{4}\right)(1.22 + 0.4h)^2$$

Substituting the value of A_h in eq, we have

$$0.60\times\left(\frac{1}{4}\right)\pi d_0^2\sqrt{2\times 9.81h}\,dt = -\frac{\pi}{4}(1.22 + 0.4h)^2 dh$$

or $$d_0^2\int dt = -\frac{1}{0.60\times\sqrt{2\times 9.81}}\int_{3.05}^{0}(1.22+0.4h)^2 h^{-1/2}dh$$

Since, the time of emptying $\int dt = 360$ s

$$d_0^2 = -\frac{1}{0.60\times\sqrt{2\times 9.81}\times 360}\int_{3.05}^{0}(1.22+0.4h)^2 h^{-1/2}dh$$

Integrating and solving for d_0, we get

$$d_0^2 = 0.01043 \text{ m}^2$$

or $$d_0 = 0.1007 \text{ m} = 100.7 \text{ mm}$$

8 (b) (ii)

As per given information,

Gate width, b = 3 m

Height of gate, h = 8 m

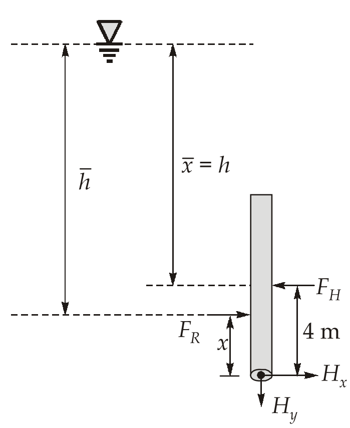
Area of gate = $A_c = b \times h$

$A_c = 3 \times 8 = 24$ m^2

$(F_H)_{max}$ = 3500 kN

Free body diagram,

1.



For gate hinged at bottom, $\Sigma M_H = 0$

So, $$4 \times F_H = F_R \times x \quad \text{... (i)}$$

$$F_R = \rho g A_c \bar{x} = 1000 \times 9.81 \times 24 \times h$$

$$\bar{h} = \bar{x} + \frac{I_x}{A_x \bar{x}} = h + \frac{\frac{3 \times 8^3}{12}}{3 \times 8 \times h} = \frac{5.33}{h} + h$$

$$x = h + 4 - \bar{h} = h + 4 - \left[\frac{5.33}{h} + h\right]$$

$$x = 4 - \frac{5.33}{h}$$

From equation (i), $$4 \times F_H = F_R \times x$$

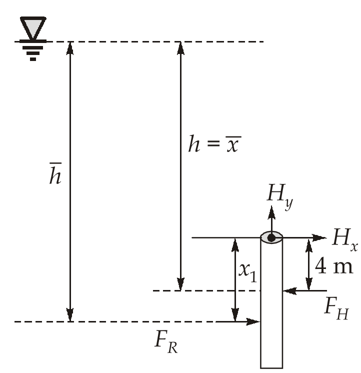
$$4 \times 3500 \times 10^3 = 235440 \times h \times \left(4 - \frac{5.33}{h}\right)$$

$$59.4631 = 4h - 5.33$$

$$h = 16.198 \approx 16.2 \text{ m}$$

2. When gate hinged at the top,

Free body diagram



For gate hinged at top.

$\Sigma M_H = 0$

So, $$4 \times F_H = F_R \times x_1 \quad \text{... (ii)}$$

where, $$x_1 = \bar{h} - h + 4 = \frac{5.33}{h} + h - h + 4$$

$$x_1 = \frac{5.33}{h} + 4$$

From equation (ii),

$$F_H \times 4 = F_R \times x_1$$

$$3500 \times 4 \times 10^3 = 235440 \times h \times \left(\frac{5.33}{h} + 4\right)$$

$$59.4631 = 5.33 + 4h$$

$$h = 13.533 \text{ m}$$

Therefore, maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom.

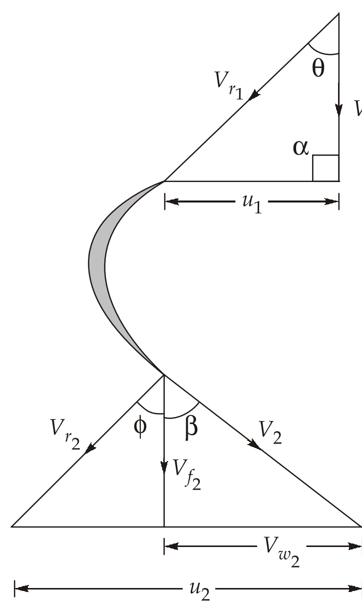
8. (c)

Given : P_1 = 1.01325 bar, d_{hub} = 55 cm = 0.55 m; T_1 = 300 K; d_{rotor} = 65 cm = 0.65 m; C_1 = 160 m/s; n = 100 rps

here $$u_1 = u_2 = u$$

$$u = \frac{\pi(d_{hub} + d_{rotor})}{2} \times N$$

$$= \frac{\pi(0.55 + 0.65)}{2} \times 100 = 188.5 \text{ m/s}$$



From velocity triangle, $$\tan\theta = \frac{u_1}{V_{f_1}} = \frac{188.5}{160}$$

$$\theta = \tan^{-1}\left(\frac{188.5}{160}\right) = 49.67°$$

As air is deflected by 28°, $$\phi = 49.67 - 28$$

$$= 21.67°$$

Also, $$\tan\phi = \frac{u_2 - V_{w_2}}{V_{f_2}}$$

$$u_2 - V_{w_2} = \tan\phi\, V_{f_2}$$

$$u_2 - V_{w_2} = 63.57 \qquad (\because\ V_{f_1} = V_{f_2})$$

and $$\tan\beta = \frac{188.5 - 63.57}{160}$$

$$\beta = 37.98°$$

Temperature, $$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 300 - \frac{(160)^2}{2\times 1005}$$

$$T_1 = 287.26 \text{ K}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 287.26(1.25)^{\frac{0.4}{1.4}}$$

$$T_2 = 306.17 \text{ K}$$

Given, $$\frac{P_2}{P_1} = 1.25$$

$$P_2 = 1.25 \times 1.01325 = 1.266 \text{ bar}$$

Density at exit, $$\rho_2 = \frac{P_2}{RT_2} = \frac{1.266\times 10^5}{287\times 306.17} = 1.44 \text{ kg/m}^3$$

Mass flow rate, $$\dot{m} = \frac{\pi}{4}\left(d_{rotor}^2 - d_{hub}^2\right)\times V_{f_2}\rho_2$$

$$= \frac{\pi}{4}\left(0.65^2 - 0.55^2\right)\times 160\times 1.44$$

$$= 21.714 \text{ kg/sec}$$

Power required, $$P = \phi_w u V_{f_1}\dot{m}(\tan\theta - \tan\phi)$$

$$= 1 \times 188.5 \times 160 \times (21.714)(\tan 49.67 - \tan 21.67)$$

$$= 511.2 \text{ kW}$$

Degree of reaction, $$R = \frac{V_f}{2u}(\tan\theta + \tan\phi)$$

$$= \frac{160}{2\times 188.5}(\tan 49.67 + \tan 21.67)$$

$$= 0.668$$

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