MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

E & T Engineering Test No : 3

Section A : Analog Circuits + Electromagnetics

Q.1 (a) Solution:

For a two element antenna array,

$$E_T = E_1 + E_1 e^{j\psi}$$

where $\psi = \beta d \cos\theta + \delta$ is the phase difference between the waves radiated from two antennas

$$E_T = E_1 + E_1(\cos\psi + j\sin\psi) = E_1[(1+\cos\psi) + j\sin\psi]$$

$$|E_T| = E_1\sqrt{(1+\cos\psi)^2 + \sin^2\psi} = E_1\sqrt{2(1+\cos\psi)}$$

$$|E_T| = 2E_1\cos(\psi/2)$$

Since the antenna element is the Hertz dipole, the normalized primary element radiation pattern is given by $f(\theta) = \sin\theta$. Thus, radiation pattern of the array is,

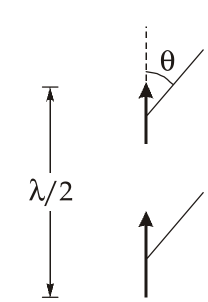
$$|E_T| = 2\cos\left(\frac{\psi}{2}\right)\cdot\sin\theta$$

Where, $$\psi = \beta d\cos\theta + \delta = \frac{2\pi}{\lambda}d\cos\theta + \delta$$

Elements are excited in phase, thus $\delta = 0$

The array phase ψ is,

$$\psi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{2}\right)\cos\theta + 0 = \pi\cos\theta$$



Array of collinear Hertz dipoles

The normalized field pattern is,

$$F(\theta, \phi) = \sin\theta \cdot \cos\left(\frac{\pi\cos\theta}{2}\right)$$

$\therefore$ Directivity, $D = \dfrac{4\pi}{\int\limits_{\phi=0}^{2\pi}\int\limits_{\theta=0}^{\pi} |F(\theta,\phi)|^2 \sin\theta\, d\theta\, d\phi}$

$$= \frac{4\pi}{\int\limits_{\phi=0}^{2\pi}\int\limits_{\theta=0}^{\pi} \sin^2\theta \cdot \cos^2\left(\frac{\pi\cos\theta}{2}\right)\cdot \sin\theta \cdot d\theta \cdot d\phi}$$

$\Rightarrow$ $D = \dfrac{4\pi}{\int\limits_{\phi=0}^{2\pi} d\phi \int\limits_{\theta=0}^{\pi} \sin^3\theta \cdot \cos^2\left(\frac{\pi\cos\theta}{2}\right) d\theta}$

$\Rightarrow$ $D = \dfrac{4\pi}{[\phi]_0^{2\pi} \int\limits_{\theta=0}^{\pi} \sin^3\theta \cdot \cos^2\left(\frac{\pi\cos\theta}{2}\right) d\theta}$ ...(i)

Let $\dfrac{\pi\cos\theta}{2} = t$

$\left\{ \begin{array}{l} \textbf{Limits:}\ \theta = 0 \Rightarrow t = \frac{\pi}{2} \\ \qquad\qquad \theta = \pi \Rightarrow t = -\frac{\pi}{2} \end{array} \right\}$

$\Rightarrow$ $-\sin\theta \times d\theta = dt\left(\dfrac{2}{\pi}\right)$

$$\sin\theta\, d\theta = -\frac{2}{\pi}dt$$

Also, $\left(\dfrac{\pi}{2}\right)^2 \cos^2\theta = t^2$

$\Rightarrow$ $\left(\dfrac{\pi}{2}\right)^2 \left[1 - \sin^2\theta\right] = t^2$

$\Rightarrow$ $1 - \sin^2\theta = \dfrac{4t^2}{\pi^2}$

$\Rightarrow$ $\sin^2\theta = 1 - \dfrac{4t^2}{\pi^2}$

Substituting in equation (i),

$$D = \frac{4\pi}{2\pi \int_{\pi/2}^{-\pi/2} \left(1 - \frac{4t^2}{\pi^2}\right) \cdot \cos^2 t \left(-\frac{2}{\pi} dt\right)}$$

$$\Rightarrow \quad D = \frac{2}{\frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \left(1 - \frac{4t^2}{\pi^2}\right) \cdot \cos^2 t \cdot dt}$$

$$\Rightarrow \quad D = \frac{\pi}{\int_{-\pi/2}^{\pi/2} \left(1 - \frac{4t^2}{\pi^2}\right)\left(\frac{1+\cos 2t}{2}\right) dt}$$

$$\Rightarrow \quad D = \frac{\pi}{\int_{-\pi/2}^{\pi/2} \left(\frac{1}{2} + \frac{\cos 2t}{2} - \frac{2t^2}{\pi^2} - \frac{2t^2}{\pi^2}\cos 2t\right) dt}$$

$$D = \frac{\pi}{\frac{1}{2}[t]_{-\pi/2}^{\pi/2} + \frac{1}{2}\left[\frac{\sin 2t}{2}\right]_{-\pi/2}^{\pi/2} - \frac{2}{\pi^2}\left[\frac{t^3}{3}\right]_{-\pi/2}^{\pi/2} - \frac{2}{\pi^2}\left[\frac{t^2}{2}\sin 2t + \frac{t}{2}\cos 2t - \frac{1}{4}\sin 2t\right]_{-\pi/2}^{\pi/2}}$$

$$D = \frac{\pi}{\frac{1}{2}[\pi] + \frac{1}{2}[0] - \frac{2}{\pi^2}\left[\frac{\pi^3}{12}\right] - \frac{2}{\pi^2}\left[\frac{\pi}{4}(-1) + \frac{\pi}{4}(-1)\right]}$$

$$\Rightarrow \quad D = \frac{\pi}{\left[\frac{\pi}{2} - \frac{\pi}{6} + \frac{1}{\pi}\right]} = 2.3$$

$\therefore$ Directivity, $D = 2.3$

Q.1 (b) Solution:

Here, we make use of the boundary conditions of the electric field i.e.

1. The tangential component of the electric field is continuous across the boundary i.e. $E_{1t} = E_{2t}$.
2. The normal component of the electric field density (D) is continuous across the boundary if there are no free charges at the interface i.e. $D_{1n} = D_{2n}$

(i) At the interface between ε_0 and $2\varepsilon_0$:

$$E_{1n} = E_0 \cos 30° = \frac{\sqrt{3}}{2} E_0$$

$$E_{1t} = E_0 \sin 30° = \frac{E_0}{2}$$

We have, $$E_{2t} = E_{1t} = \frac{E_0}{2}$$

and, $D_{2n} = D_{1n} \Rightarrow$ $$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{1n} = \frac{\varepsilon_0}{2\varepsilon_0}\left(\frac{\sqrt{3}}{2}E_0\right) = \frac{\sqrt{3}}{4}E_0$$

The angle that E makes with the z-axis in region A is,

$$\theta_1 = \tan^{-1}\left(\frac{E_{2t}}{E_{2n}}\right) = \tan^{-1}\left(\frac{\left(\frac{E_0}{2}\right)}{\frac{\sqrt{3}}{4}E_0}\right) = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right) = 49.11°$$

(ii) At the interface between $2\varepsilon_0$ and $3\varepsilon_0$:

$$E_{3t} = E_{2t} = \frac{E_0}{2}$$

and, $D_{3n} = D_{2n}$ $$\Rightarrow \varepsilon_3 E_{3n} = \varepsilon_2 E_{2n}$$

$$E_{3n} = \frac{\varepsilon_2}{\varepsilon_3} E_{2n} = \frac{2\varepsilon_0}{3\varepsilon_0}\left(\frac{\sqrt{3}}{4}E_0\right) = \frac{E_0}{2\sqrt{3}}$$

The angle that E makes with the z-axis in region B is,

$$\theta_2 = \tan^{-1}\left(\frac{E_{3t}}{E_{3n}}\right) = \tan^{-1}\left(\frac{\frac{E_0}{2}}{\frac{E_0}{2\sqrt{3}}}\right)$$

$$= \tan^{-1}\left(\sqrt{3}\right) = 60°$$

(iii) At the interface between $3\varepsilon_0$ and ε_0:

$$E_{4t} = E_{3t} = \frac{E_0}{2}$$

and, $D_{4n} = D_{3n} \Rightarrow$ $$\varepsilon_4 E_{4n} = \varepsilon_3 E_{3n}$$

$$E_{4n} = \frac{\varepsilon_3}{\varepsilon_4} E_{3n} = \frac{3\varepsilon_0}{\varepsilon_0} E_{3n} = 3\left(\frac{E_0}{2\sqrt{3}}\right) = \frac{\sqrt{3}E_0}{2}$$

The angle that E makes with the z-axis in region C is,

$$\theta_3 = \tan^{-1}\left(\frac{E_{4t}}{E_{4n}}\right) = \tan^{-1}\left(\frac{\frac{E_0}{2}}{\frac{\sqrt{3}}{2}E_0}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Q.1 (c) Solution:

Given,

$$Z_0 = 300\ \Omega$$

$$Y_L = 0.01 + j\,0.02\ ℧$$

The characteristic admittance is,

$$Y_0 = \frac{1}{Z_0} = \frac{1}{300} = 3.333 \times 10^{-3}\ ℧$$

(i) The reflection coefficient at the load-end is,

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{(3.333\times10^{-3}) - (0.01 + j0.02)}{(3.333\times10^{-3}) + (0.01 + j0.02)}$$

$$\Rightarrow \quad \Gamma_L = \frac{(-6.667\times10^{-3}) - j\,0.02}{(0.013333 + j\,0.02)}$$

$$\Rightarrow \quad \Gamma_L = -0.8462 - j0.23075$$

(ii) Reflection coefficient at a distance of 0.2λ from the load-end:

$$\Gamma(l) = \Gamma_L e^{-j2\beta l}$$

$$\Gamma(0.2\lambda) = (-0.8462 - j\,0.23075)\ e^{-j2\times\frac{2\pi}{\lambda}\times0.2\lambda}$$

$$= (-0.8462 - j\,0.23075) \times (-0.80902 - j0.5878)$$

$$\Rightarrow \quad \Gamma(0.2\lambda) = 0.549 + j0.6841$$

(iii) Impedance at a distance of $l = 0.2\,\lambda$ from the load-end :

$$Z(l) = Z_0\left[\frac{1+\Gamma(l)}{1-\Gamma(l)}\right]$$

$$\Rightarrow \quad Z(0.2\lambda) = 300\left[\frac{1+0.549+j0.6841}{1-0.549-j0.6841}\right]$$

$$\Rightarrow \quad Z(0.2\lambda) = 103.042 + j611.36\ \Omega$$

Q.1 (d) Solution:

Using the concept of virtual short, $V_- = V_+ = 0$

Apply KCL at inverting node of op-amp:

$$\frac{V_i}{R_1} + \frac{V_0}{1/sC} + \frac{V_0}{R_F} = 0$$

$$\frac{V_0}{V_i} = A_V = \frac{-R_F / R_1}{(1+sCR_F)}$$

Substituting $s = j\omega$,

$$\left|\frac{V_0}{V_i}\right| = |A_V| = \frac{R_F / R_1}{\sqrt{(\omega CR_F)^2 + 1}}$$

At $\omega = 0$;

$$|A_V|_{\omega = 0} = \frac{R_F}{R_1}$$

$$10 = \frac{R_F}{R_1}$$

$$R_F = 10R_1 \quad \text{...(i)}$$

From the characteristics, corner frequency (ω_c) = 10 k rad/s

Since; at corner frequency, gain is reduced by 3 dB from its peak value.

From the given characteristics, $|A_V|_{\omega=0}$ = 20 dB = 10. Thus,

Therefore, $$\omega_{3\text{ dB}} = \frac{1}{R_F C}$$

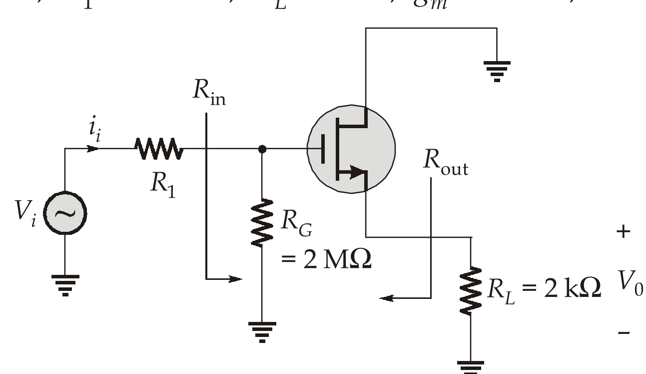
$$10 \times 10^3 = \frac{1}{R_F \cdot (10^{-8})}$$

$$R_F = 10 \text{ k}\Omega$$

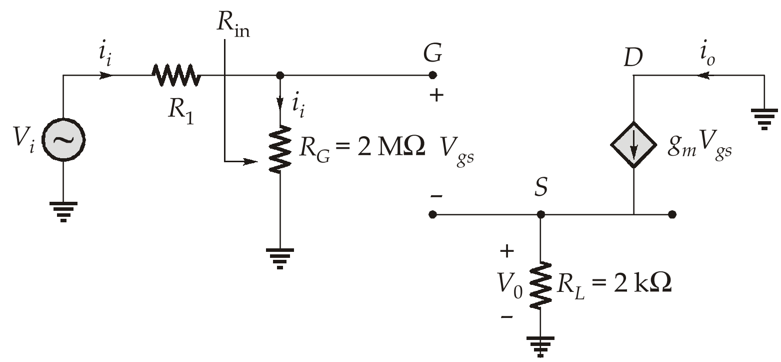
From equation (i), we get $R_1 = 1 \text{ k}\Omega$

Q.1 (e) Solution:

(i) Given, $R_G = 2$ MΩ; $R_1 = 100$ kΩ; $R_L = 2$ kΩ; $g_m = 10$ mS; $\lambda = 0$; $r_0 = \infty$



Small signal equivalent model:



$$R_{in} = R_G = 2\text{ M}\Omega$$

KVL in input loop:

$$-V_i + (R_1 + R_G)i_i = 0$$

$$\Rightarrow \quad V_i = [(100 \times 10^3) + (2 \times 10^6)]i_i$$

$$\Rightarrow \quad V_i = (2.1 \times 10^6)i_i \quad ...(1)$$

We have,

$$i_0 = g_m V_{gs} \quad ...(2)$$

$$V_0 = g_m V_{gs} R_L \quad ...(3)$$

Gate voltage, $V_g = V_i\left[\frac{R_G}{R_1 + R_G}\right] = V_i\left[\frac{(2\times10^6)}{(100\times10^3)+(2\times10^6)}\right]$

$$\Rightarrow \quad V_g = 0.9524V_i$$

Source voltage, $V_s = V_0 = i_0 R_L$

$$\therefore \quad V_{gs} = V_g - V_s = 0.9524\ V_i - V_0$$

Using equation (3),

$$V_0 = (10 \times 10^{-3}) \times [0.9524V_i - V_0] \times (2000)$$

$$V_0 = 20[0.9524V_i - V_0]$$

$$V_0 = 19.048V_i - 20V_0$$

$$\Rightarrow \quad A_v = \frac{V_0}{V_i} = \frac{19.048}{21} = 0.90705$$

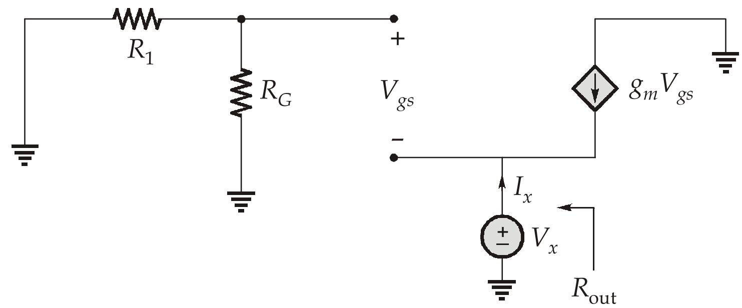
Using equation (2),

$$i_0 = g_m V_{gs} = (10 \times 10^{-3}) \times [(2 \times 10^6)i_i - (2 \times 10^3)i_0]$$

$$\Rightarrow \quad i_0 = 20000i_i - 20i_0$$

$$\Rightarrow \quad A_i = \frac{i_0}{i_i} = \frac{20000}{21} = 952.38$$

To find R_{out}: Disable 'V_i', disconnect R_L and connect a voltage source V_x across the load terminals.



$$V_g = 0 \Rightarrow V_{gs} = -V_s = -V_x$$

$$I_x = -g_m V_{gs} = -g_m(-V_x) = 10 \times 10^{-3} V_x$$

$$\therefore \quad R_{out} = \frac{V_x}{I_x} = \frac{V_x}{(10\times 10^{-3})V_x} = 100\ \Omega$$

Thus, for the given circuit,

$$A_v = (0.90705)\ \text{V/V}$$

$$R_{in} = 2\ \text{M}\Omega$$

$$R_{out} = 100\ \Omega$$

$$A_i = \frac{i_0}{i_i} = (952.38)\ \text{A/A}$$

Q.2 (a) Solution:

Given, $\sigma = 0$; $\mu = 2\mu_0$; $\varepsilon = 10\varepsilon_0$; $\vec{J_d} = 60\sin(10^9 t - \beta z)\hat{a}_x$ mA/m^2

(i) We know,

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t}$$

$$\Rightarrow \quad \vec{D} = \int \vec{J_d} \cdot dt$$

$$\Rightarrow \quad \vec{D} = \int 60\sin(10^9 t - \beta z)\hat{a}_x \times 10^{-3}\, dt$$

$$\Rightarrow \quad \vec{D} = \frac{-60\times 10^{-3}}{10^9}\cos(10^9 t - \beta z)\hat{a}_x$$

$$\vec{D} = -60\times 10^{-12}\cos(10^9 t - \beta z)\hat{a}_x$$

$$\therefore \quad \vec{D} = -60\times 10^{-12}\cos(10^9 t - \beta z)\hat{a}_x\ \text{C/m}^2$$

From the time-dependent Maxwell equations,

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

$$\Rightarrow \quad \vec{\nabla}\times\frac{\vec{D}}{\varepsilon} = -\mu\frac{\partial\vec{H}}{\partial t} \qquad \text{...(i)}$$

We can write,

$$\vec{\nabla}\times\frac{\vec{D}}{\varepsilon} = \frac{1}{\varepsilon}\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ D_x & D_y & D_z\end{vmatrix}$$

$$= \frac{1}{\varepsilon}\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -60\times10^{-12}\cos(10^9 t-\beta z) & 0 & 0\end{vmatrix}$$

$$\Rightarrow \quad \vec{\nabla}\times\frac{\vec{D}}{\varepsilon} = \frac{1}{\varepsilon}\left[\frac{\partial}{\partial z}(-60\times10^{-12}\cos(10^9 t-\beta z))\hat{a}_y\right]$$

$$\Rightarrow \quad \vec{\nabla}\times\frac{\vec{D}}{\varepsilon} = \frac{-60\times10^{-12}\beta}{\varepsilon}\sin(10^9 t-\beta z)\hat{a}_y$$

Using equation (i),

$$\vec{H} = -\frac{1}{\mu}\int\left(\nabla\times\frac{\vec{D}}{\varepsilon}\right)dt$$

$$\Rightarrow \quad \vec{H} = \frac{1}{\mu}\int\left[\frac{60\times10^{-12}\beta}{\varepsilon}\sin(10^9 t-\beta z)\hat{a}_y\right]dt$$

$$\Rightarrow \quad \vec{H} = \frac{-60\times10^{-12}\beta}{\mu\varepsilon\times10^9}\cos(10^9 t-\beta z)\hat{a}_y \text{ A/m}$$

$$\Rightarrow \quad \vec{H} = -60\times10^{-21}\frac{\beta}{\mu\varepsilon}\cos(10^9 t-\beta z)\hat{a}_y \text{ A/m}$$

(ii) We know,

$$\vec{\nabla}\times\vec{H} = \vec{J_c}+\vec{J_d}$$

As $\sigma = 0$, thus the conduction current density $\vec{J_c} = \sigma\vec{E} = 0.$ Thus, we get

$$\vec{\nabla}\times\vec{H} = \vec{J_d}$$

$$\therefore \quad \vec{J}_d = \vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\Rightarrow \quad \vec{J}_d = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -60\times10^{-21}\frac{\beta}{\mu\varepsilon}\cos(10^9t-\beta z) & 0 \end{vmatrix}$$

$$\vec{J}_d = -\frac{\partial}{\partial z}\left[-60\times10^{-21}\frac{\beta}{\mu\varepsilon}\cos(10^9t-\beta z)\right]\hat{a}_x$$

On equating with the given $\vec{J}_d$,

$$60\sin(10^9t-\beta z)\hat{a}_x\times10^{-3} = \frac{-\partial}{\partial z}\left[-60\times10^{-21}\frac{\beta}{\mu\varepsilon}\cos(10^9t-\beta z)\right]\hat{a}_x$$

$$60\times10^{-3}\sin(10^9t-\beta z)\hat{a}_x = 60\times10^{-21}\frac{\beta^2}{\mu\varepsilon}\sin(10^9t-\beta z)\hat{a}_x$$

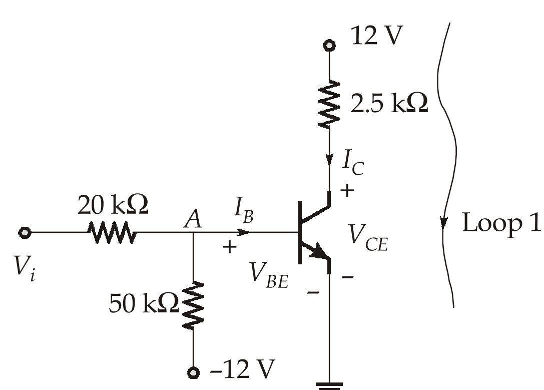
$$\Rightarrow \quad \frac{\beta^2}{\mu\varepsilon} = \frac{10^{-3}}{10^{-21}}$$

$$\Rightarrow \quad \beta = \sqrt{\mu\varepsilon\times10^{18}} = \sqrt{2\times4\pi\times10^{-7}\times10\times\frac{10^{-9}}{36\pi}\times10^{18}}$$

$$\Rightarrow \quad \beta = 14.907\ \text{rad/m}$$

Q.2 (b) Solution:

(i) The circuit is as given below,



On applying KVL in loop 1, we get

$$-12 + 2.5k\, I_C + V_{CE} = 0$$

$$I_C = \frac{12 - V_{CE}}{2.5k} \quad \text{...(i)}$$

On applying KCL at node A, we get

$$\frac{V_A - (-12)}{50k} + \frac{V_A - V_i}{20\text{ k}\Omega} = -I_B$$

Here, $V_A = V_B$ (Base voltage). Thus,

$$\frac{V_B + 12}{50k} + \frac{V_B - V_i}{20k} = -I_B$$

$$V_B\left[\frac{1}{50k} + \frac{1}{20k}\right] - \frac{V_i}{20k} + \frac{12}{50k} = -I_B \quad \text{...(ii)}$$

Now, assume the circuit is in saturation region i.e., $(V_{BE})_{\text{sat}} = 0.8$ V and $(V_{CE})_{\text{sat}} = 0.2$ V.

On substituting the above values in equation (i) and (ii), we get

$$(I_C)_{\text{sat}} = \frac{12 - 0.2}{2.5k}$$

$$(I_C)_{\text{sat}} = 4.72 \text{ mA} \quad \text{...(iii)}$$

and

$$-(I_B)_{\text{sat}} = 0.8\left[\frac{1}{50k} + \frac{1}{20k}\right] - \frac{V_i}{20k} + \frac{12}{50k}$$

$$-(I_B)_{\text{sat}} = (5.6 \times 10^{-5}) + (2.4 \times 10^{-4}) - \frac{V_i}{20k}$$

$$(I_B)_{\text{sat}} = -(2.96 \times 10^{-4}) + \frac{V_i}{20k} \quad \text{...(iv)}$$

Now, we know that the minimum base current required for transistor to be in saturation,

$$(I_B)_{\text{min}} = \frac{(I_C)_{sat}}{\beta} = \frac{4.72 \times 10^{-3}}{100}$$

$$(I_B)_{\text{min}} = 4.72 \times 10^{-5}$$

$$(I_B)_{\text{min}} = 47.2\ \mu\text{A} \quad \text{...(v)}$$

∴ For transistor to remain in saturation region,

$$(I_B)_{\text{sat}} > (I_B)_{\text{min}}$$

$$-\left[(2.96 \times 10^{-4}) - \frac{V_i}{20k}\right] > 47.2 \times 10^{-6}$$

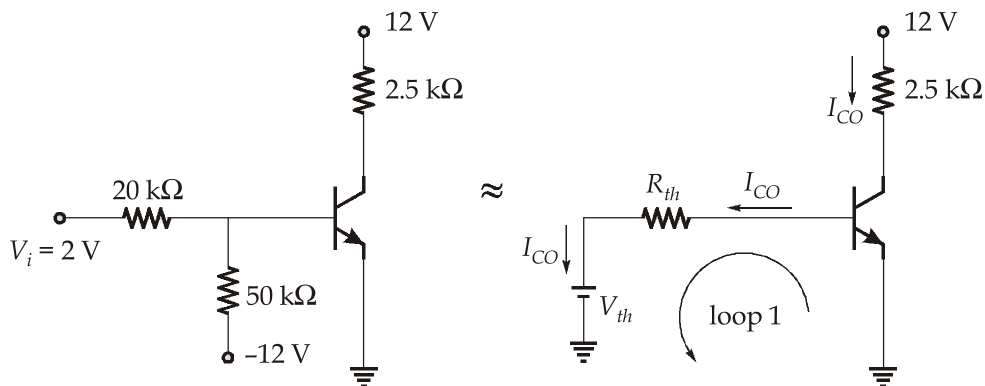
$$-2.96\times10^{-4}+\frac{V_i}{20k} > 47.2\times10^{-6}$$

$$\frac{V_i}{20k} > 47.2\times10^{-6}+2.96\times10^{-4}$$

$$V_i > 6.864\text{ V}$$

For given transistor to work in saturation region, input voltage must be greater than 6.864 Volt.

(ii) Drawing the Thevenin equivalent circuit,



where,
$$V_{\text{th}} = \frac{(-12\times20k)+50k\times2}{(20+50)k} = -2\text{ V}$$

$$R_{\text{th}} = (20\parallel50)k = 14.28\text{ k}\Omega$$

In the cut-off region, both the collector-base and base-emitter junctions are reverse biased. Assuming the base current to be zero in the cut-off region, we have

$$V_B = -2+I_{CO}R_{th}$$

The reverse saturation current I_{CO} increases with temperature and thus, V_B increases. For the transistor to be in the cut-off region, at 100°C, V_{BE} = 0.7 V. For T > 100°C, V_{BE} junction is forward biased.

$$-2+I_{CO}R_{th} = 0.7$$

$$I_{CO} = \frac{2.7}{14.28\times10^3} = 0.189\text{ mA}$$

We know that the reverse saturation current doubles for every 10°C rise in temperature. Thus,

$$(I_{CO})_{T2} = (I_{CO})_{T1}2^{\left(\frac{T_2-T_1}{10}\right)}$$

$$0.189 \text{ mA} = (I_{CO})_{T2} 2^{\left(\frac{100-37}{10}\right)}$$

$$(I_{CO})_{T2} = 2.4 \mu\text{A}$$

Thus at room temperature, reverse saturation current $I_{CO} = 2.4$ μA.

Q.2 (c) Solution:

(i) We know, s-parameters

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

where V_1^+ and V_2^+ represent the incident voltage at port 1 and port 2 respectively.

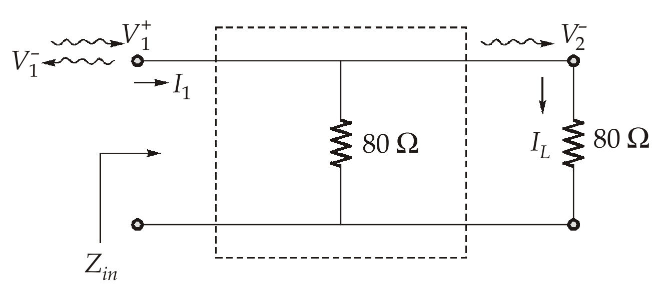
And, V_1^- and V_2^- represent the reflected voltage at port 1 and port 2 respectively

To find S_{11} and S_{21}:

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} \quad , \quad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

$V_2^+ = 0 \Rightarrow$ No wave is incident at port 2.

and the port 2 is matched to transmission line.



$$Z_{in} = [80 \,||\, 80] = 40\ \Omega$$

$$\therefore \quad S_{11} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{40-80}{40+80} = \frac{-1}{3}$$

Here, $$V_1 = V_1^+ + V_1^- = V_1^+(1+S_{11})$$

$$V_2^- = V_1 = V_1^+(1+S_{11})$$

$$S_{21} = \frac{V_2^-}{V_1^+} = 1 - S_{11} = 1 - \frac{1}{3} = \frac{2}{3}$$

As the network is symmetrical as well as reciprocal,

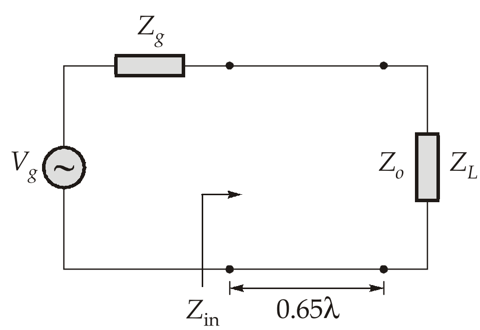
$$S_{11} = S_{22} = -\frac{1}{3}$$

$$S_{12} = S_{21} = \frac{2}{3}$$

$\therefore$ The s-parameters for the given two-port network is,

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

(ii) Given, $Z_o = 100\ \Omega$; $l = 0.65\lambda$; $V_g = 2$ V; $Z_g = 50\ \Omega$; $Z_L = 25 - j25\ \Omega$



1. The input impedance of the line:

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right]$$

$$= 100\left[\frac{25 - j25 + j100\tan\left(\frac{2\pi}{\lambda}\times 0.65\lambda\right)}{100 + j(25 - j25)\tan\left(\frac{2\pi}{\lambda}\times 0.65\lambda\right)}\right]$$

$$\Rightarrow \quad Z_{in} = 100\left[\frac{25 - j25 + j137.64}{100 + j(25 - j25)(1.3764)}\right]$$

$$\Rightarrow \quad Z_{in} = (37.59 + j74.18)\ \Omega = (83.161\ \angle 63.127°)\ \Omega$$

2. The input voltage:

$$V(x = 0) = \frac{Z_{in}}{Z_{in} + Z_g}V_g = \frac{(37.59 + j74.18)}{(37.59 + j74.18 + 50)}\times 2$$

$$\Rightarrow \quad V(0) = (1.3352 + j0.56305)\text{V} = 1.45\ \angle 22.87°\ \text{V}$$

The voltage and current on the transmisison line is given as

$$V(x) = V_0^+ e^{-\gamma x}(1+\Gamma(x))$$

$$I(x) = \frac{V_0^+}{Z_0} e^{-\gamma x}(1-\Gamma(x))$$

At the source ($x = 0$), we have

$$V(0) = V_g - Z_0 I(0)$$

$$I(x) = \frac{V_0^+}{Z_0} e^{-\gamma x}(1-\Gamma(x))$$

$$V(0) = V_g - Z_0 I(0)$$

$$V_0^+(1-\Gamma(x)) = V_g - V_0^+(1+\Gamma(x))$$

$$\Rightarrow \quad V_o^+ = \frac{V_g}{2} = \frac{1}{2}(V(0)+Z_o I_o) = \frac{1}{2}\left(V(0)+Z_o\left(\frac{V_g}{Z_{in}+Z_g}\right)\right)$$

$$= \frac{1}{2}\left[(1.45\angle 22.87°)+(100)\left(\frac{2}{37.59+j74.18+50}\right)\right]$$

$$= [1.333 - j0.2813] \text{ V}$$

$$= 1.3622 \angle -11.917° \text{ V}$$

Q.3 (a) Solution:

(i) Given, $\rho_v = \frac{\rho_o x}{a}$

Using Poisson's equation,

$$\nabla^2 V = \frac{-\rho_v}{\varepsilon}$$

Since the device is one-dimensional (x-direction), we can write

$$\frac{d^2V}{dx^2} = \frac{-\rho_o x}{\varepsilon a}$$

On integrating w.r.t. x, $\frac{dV}{dx} = \frac{-\rho_o x^2}{2\varepsilon a} + A$; where '$A$' is constant.

On integrating w.r.t. x, $V = \frac{-\rho_o x^3}{6\varepsilon a} + Ax + B$; where '$B$' is constant.

We know, $\vec{E} = -\vec{\nabla} V$

$\Rightarrow$ $\vec{E} = \frac{-dV}{dx}\hat{a}_x$

$$= -\left[\frac{-\rho_o x^2}{2\varepsilon a} + A\right]\hat{a}_x = \left(\frac{\rho_o x^2}{2\varepsilon a} + A\right)\hat{a}_x$$

Given $E = 0$ at $x = 0$, thus

$$A = 0$$

Therefore, $\vec{E} = \frac{\rho_0 x^2}{2\varepsilon A}\hat{a}_x$

$\Rightarrow$ $V = \frac{-\rho_o x^3}{6\varepsilon a} + 0 + B$

Given $V = 0$ at $x = a$, thus

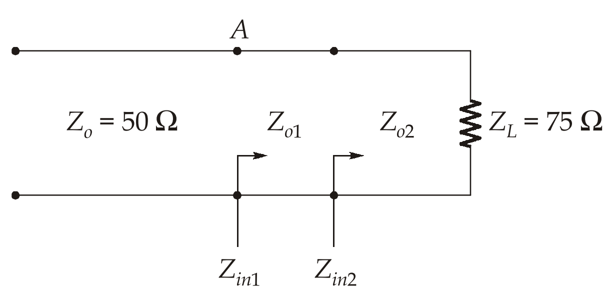
$$0 = \frac{-\rho_o a^3}{6\varepsilon a} + B$$

$\Rightarrow$ $B = \frac{\rho_o a^2}{6\varepsilon}$

Thus, $V = \frac{\rho_o}{6\varepsilon a}(a^3 - x^3)$

$$\vec{E} = \frac{\rho_o x^2}{2\varepsilon a}\hat{a}_x$$

(ii)



For $\frac{\lambda}{4}$ transformer the input impedance is given by, $Z_{in} = \frac{Z_o^2}{Z_L}$

$\Rightarrow$ $Z_{in2} = \frac{Z_{o2}^2}{Z_L} = \frac{(30)^2}{75} = 12\ \Omega$

Similarly, $$Z_{in1} = \frac{Z_{o1}^2}{Z_{in2}}$$

It is given that there is no reflected wave to the left of A, thus

$$Z_{\text{in1}} = Z_o = 50\ \Omega$$

$$\Rightarrow \quad 50 = \frac{(Z_{o1})^2}{12}$$

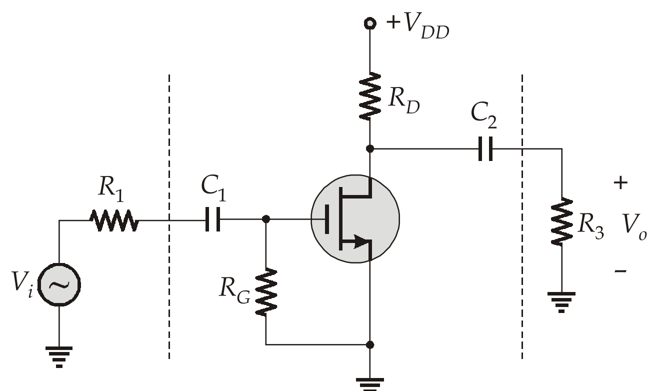
$$\Rightarrow \quad Z_{o1} = \sqrt{50 \times 12}$$

$$\Rightarrow \quad Z_{o1} = \sqrt{600}$$

$$\Rightarrow \quad Z_{o1} = 24.5\ \Omega$$

Q.3 (b) Solution:

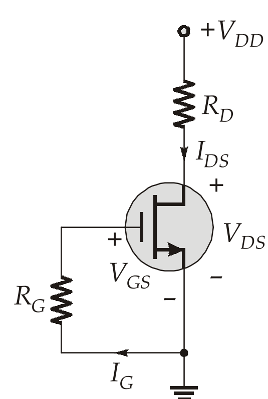
Given, amplifier circuit



(i) Given, V_{DD} = 15 V; K_n = 225 μA/V^2; V_{TN} = –3 V; R_G = 2.2 MΩ; R_D = 7.5 kΩ; R_1 = 10 kΩ; R_3 = 220 kΩ; λ = 0.015 V^{-1}

DC equivalent circuit:

Capacitors acts as open circuit for dc.



As the gate current is negligible, we assume $I_G = 0$. Thus we get $V_{GS} = 0$ V

MOSFET acts as a amplifier in saturation region. Thus,

$$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$$

Since $\lambda V_{DS} << 1$, $$I_{DS} \approx \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

$$\Rightarrow \quad I_{DS} = \frac{225\times10^{-6}}{2}[0-(-3)]^2 = 1.0125\times10^{-3}$$

$$\Rightarrow \quad I_{DS} = 1.0125 \text{ mA}$$

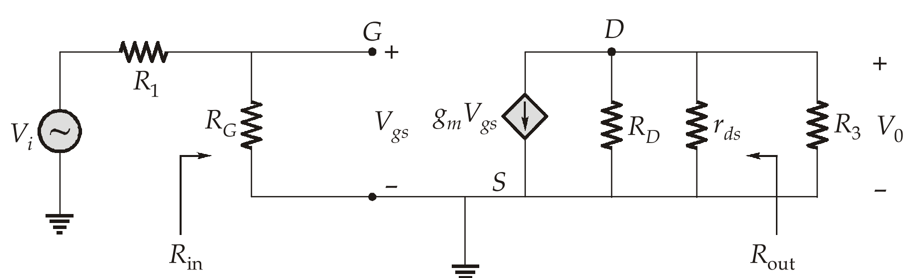
KVL: $-V_{DD} + I_{DS}R_D + V_{DS} = 0$

$$\Rightarrow \quad V_{DS} = V_{DD} - I_{DS}R_D$$
$$= 15 - (1.0125 \times 10^{-3} \times 7.5 \times 10^3)$$
$$= 7.40625 \text{ V}$$

As $V_{DS} > V_{GS} - V_{TN}$, MOSFET is in saturation region.

$$\text{The Q-point} = (V_{DS}, I_{DS})$$
$$= (7.40625 \text{ V}, 1.0125 \text{ mA})$$

(ii) **AC analysis:** As C_1 and C_2 are very large, the capacitors C_1 and C_2 are considered as short circuit in the ac analysis. The ac equivalent circuit can thus be drawn as below:

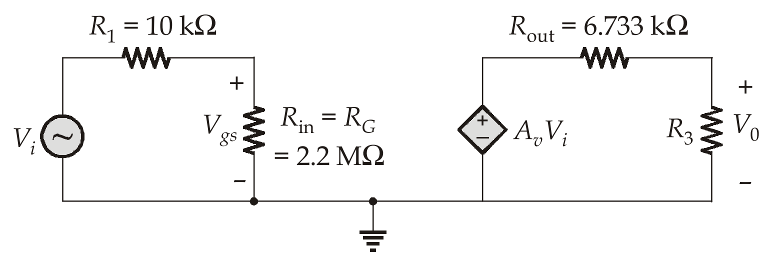


$$r_{ds} = \frac{1}{\lambda I_{DS}} = \frac{1}{(0.015\times1.0125\times10^{-3})} = 65.844 \text{ k}\Omega$$

$$\text{Output Resistance, } R_{out} = R_D \,||\, r_{ds} = (7.5k \,||\, 65.844k)$$
$$= \frac{7.5k\times65.844k}{(7.5k+65.844k)} = 6.733 \text{ k}\Omega$$

$$\text{Input Resistance, } R_{in} = R_G = 2.2 \text{ M}\Omega$$

The small-signal equivalent representation for the amplifier is as shown below:



Finding A_V:

$$V_{gs} = V_i \frac{R_G}{R_1 + R_G} = V_i\left[\frac{(2.2\times10^6)}{(10\times10^3)+(2.2\times10^6)}\right] = 0.9955\,V_i$$

$$V_0 = -g_m V_{gs}[R_D \,||\, r_{ds} \,||\, R_3] = -g_m V_{gs}[R_{out} \,||\, R_3]$$

where
$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = K_n(V_{GS} - V_{TN})(1+\lambda V_{DS})$$

$$= \frac{2I_{DS}}{(V_{GS}-V_T)} = \frac{2\times1.0125\times10^{-3}}{[0-(-3)]} = 6.75\times10^{-4}\,℧$$

$\Rightarrow$
$$V_0 = -6.75\times10^{-4}\times0.9955\,V_i\times[(6.733\times10^3)\,||\,(220\times10^3)]$$

$\Rightarrow$
$$A_v = \frac{V_0}{V_i} = -4.39$$

Q.3 (c) Solution:

(i) **1.** The direction of propagation is given by poynting vector. For poynting vector we need to find out magnetic field intensity.

Magnetic field intensity in terms of electric field intensity can be obtained from Maxwell's equation $\vec{\nabla}\times\vec{E} = -j\mu\omega\vec{H}$ as

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{a}_z = -j\omega\mu(H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z)$$

As electric field has only y component and it varies with z only, so

$$-\frac{\partial E_y}{\partial z}\hat{a}_x = -j\omega\mu H_x$$

$$\overline{E} = -E_0 e^{-j\beta z}\hat{a}_y$$

$$\overline{H} = \frac{\beta}{\omega\mu_0}E_0 e^{-j\beta z}\hat{a}_x = \frac{E_0}{\eta_0}e^{-j\beta z}\hat{a}_x$$

In time domain,

$$\overline{H} = \hat{a}_x \frac{1}{\eta_0} E_0 \cos(\omega t - \beta z)\ \text{A/m}$$

The pointing vector is,

$$\overline{P} = \overline{E} \times \overline{H} = \left[-\hat{a}_y E_0 \cos(\omega t - \beta z)\right]\left[\frac{1}{\eta_0} E_0 \cos(\omega t - \beta z)\hat{a}_x\right]$$

$$\overline{P} = \frac{E_0^2}{\eta_0}\cos^2(\omega t - \beta z)\,\hat{a}_z$$

The direction of power flow in z-direction. This is also the direction of propagation of the wave.

2. The instantaneous power density

$$\overline{P}(z,t) = \overline{E}(z,t) \times \overline{H}(z,t) = \frac{E_0^2}{\eta_0}\cos^2(\omega t - \beta z)\hat{a}_z$$

The time averaged power density is found by integrating the instantaneous power density over one cycle of the wave $\left(T = \frac{1}{f} = \frac{2\pi}{\omega}\right)$

$$\vec{P}_{av}(z) = \frac{1}{T}\int_0^T \frac{E_0^2}{\eta_0}\cos^2(\omega t - \beta z)\hat{a}_z\,dt$$

$$= \frac{1}{T}\frac{E_0^2}{\eta_0}\int_0^T \left[\frac{1}{2} + \frac{1}{2}\cos 2(\omega t - \beta z)\right]\hat{a}_z\,dt$$

$$= \frac{1}{T}\cdot\frac{E_0^2}{\eta_0}\int_0^T \frac{dt}{2}\hat{a}_z + \frac{1}{T}\frac{E_0^2}{\eta_0}\int_0^T \frac{1}{2}\cos 2(\omega t - \beta z)\hat{a}_z dt$$

$$= \frac{E_0^2}{\eta_0} \times \frac{1}{T} \times \frac{T}{2} + \frac{1}{T}\frac{E_0^2}{\eta_0} \times \frac{1}{2}\left.\frac{\sin 2(\omega t - \beta z)}{2\omega}\right|_0^T \hat{a}_z$$

$$= \frac{E_0^2}{2\eta_0}\hat{a}_z$$

$$\vec{P}_{av}(z) = \frac{E_0^2}{2\eta_0}\hat{a}_z = \frac{(1200)^2}{2 \times 377} = 1909.814\hat{a}_z\ \text{W/m}^2$$

3. The power density is uniform throughout space and doesn't depend on the location (except for phase, which varies in *z*-direction).

Thus, both the total instantaneous and time averaged power are infinite. This is true of any plane wave.

4. The amount of power received by antenna equals the power density multiplied by the surface area of the antenna (S).

For 2 m diameter dish, the instantaneous power received is

$$P_t = \left|\vec{P}(z,t)\right| \cdot \vec{S} = \frac{E_0^2 \pi d^2}{4\eta_0} \cos^2(\omega t - \beta z),$$

where $S = \pi d^2/4$ in a direction perpendicular to the surface i.e. in +z-direction.

$$P_t = \frac{(1200)^2 \times \pi (2)^2}{4 \times 377} \cos^2(8\pi \times 10^8 t - \beta z)$$

$$= 11999.717 \cos^2(8\pi \times 10^8 t \;\; \beta z)$$

$$P_t \cong 12000 \cos^2(8\pi \times 10^8 t - \beta z)$$

The time averaged power received is

$$P_{\text{avg}} = |P_{av}| S = \frac{E_0^2 \pi d^2}{8\eta_0} = \frac{(1200)^2 \times \pi \times (2)^2}{8 \times 377}$$

$$\cong 6000 \text{ W}$$

(ii) According to Maxwell's third equation or Faraday's law, we have

$$\overline{\nabla} \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

We have, $\overline{B} = \overline{\nabla} \times \overline{A}$ ($\overline{A} \rightarrow$ Magnetic vector potential)

$$\overline{\nabla} \times \overline{E} = -\frac{\partial}{\partial t}(\overline{\nabla} \times \overline{A}) \Rightarrow \overline{E} = \frac{-\partial \overline{A}}{\partial t}$$

The static electric field intensity,

$\overline{E_s} = -\overline{\nabla}\, V$ must also be added to this electric field intensity.

$$\overline{E} = \frac{-\partial \overline{A}}{\partial t} - \overline{\nabla}\, V$$

As per Maxwell's first equation

$$\overline{\nabla} \cdot \overline{D} = \rho$$

$$\overline{\nabla} \cdot \overline{D} = \in \overline{\nabla} \cdot \overline{E} = \in \overline{\nabla} \cdot \left(-\frac{\partial \overline{A}}{\partial t} - \overline{\nabla} V \right) = \rho$$

$$-\overline{\nabla} \cdot \frac{\partial \overline{A}}{\partial t} - \nabla^2 V = \frac{\rho}{\in}$$

$$\frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{A})+\nabla^2 V = -\frac{\rho}{\in} \quad \text{...(i)}$$

As per Lorentz condition,

$$\bar{\nabla}\cdot\bar{A} = -\mu\in\frac{\partial V}{\partial t}$$

Substituting this in (i),

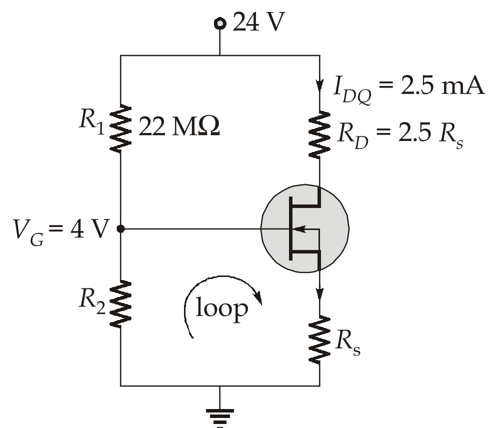
$$\nabla^2 V+\frac{\partial}{\partial t}\left(-\mu\in\frac{\partial V}{\partial t}\right) = -\frac{\rho}{\in}$$

$$\nabla^2 V-\mu\in\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\in}$$

$$\nabla^2 V-\mu\in\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\in}$$

Q.4 (a) Solution:

(i) The configuration of a voltage divider bias network is drawn below:



Using voltage divider rule, we get

$$V_G = \frac{V_{CC}\times R_2}{R_1+R_2}$$

$$4 = \frac{24\times R_2}{R_2+(22\times 10^6)}$$

$$88\times 10^6+4R_2 = 24R_2$$

$$20R_2 = 88\times 10^6$$

$$R_2 = 4.4\text{ M}\Omega$$

As we have,

$$I_{DQ} = I_{DSS}\left[1-\frac{V_{GS}}{V_P}\right]^2$$

$$2.5 \times 10^{-3} = 10\left[1-\frac{V_{GS}}{(-4)}\right]^2 \times 10^{-3}$$

$$\sqrt{0.25} = \left[1+\frac{V_{GS}}{4}\right]$$

$$0.5 \times 4 = 4 + V_{GS}$$

$$V_{GS} = -2 \text{ V}$$

Since, $V_{GS} = V_G - V_S = -2$

$$4 - V_s = -2$$

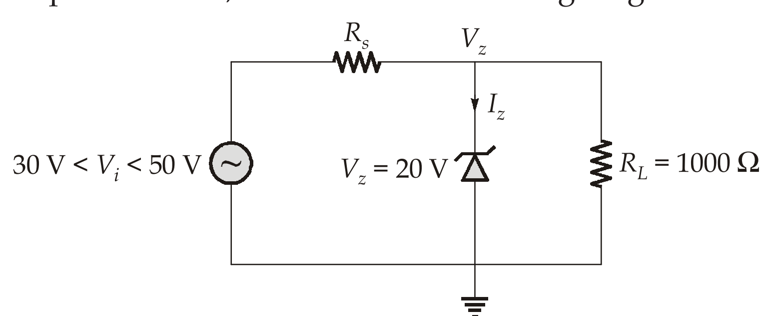
$$V_s = 6 = I_{DQ} \cdot R_s$$

$$R_s = \frac{6}{I_{DQ}} = \frac{6}{2.5\times10^{-3}} = 2.4 \text{ k}\Omega$$

and $R_D = 2.5\, R_s$

$$R_D = 2.5 \times 2.4 = 6 \text{ k}\Omega$$

(ii) For the given specifications, we can draw the voltage regulator as,



For zener diode to be in breakdown region, $V_z > 20$ V.

By voltage division rule, we get

$$V_z = \frac{V_i \times R_L}{R_L + R_S}$$

$$20 < \frac{V_i \times 1000}{1000 + R_s}$$

Consider $V_i = V_{i(\min)} = 30$ V; thus

$$20(1000 + R_s) < 1000 \times 30$$

$$1000 + R_s < 1500$$

$$R_s < 500\ \Omega$$

Hence, we can assume $R_s = 500\ \Omega$.

For $V_{in} = 30$ V, $I_s = I_{s\,\min}$

$$I_{s\,min} = \frac{30-20}{500} = \frac{10}{500} = 20 \text{ mA}$$

For $V_{in} = 50$ V; $I_s = I_{s\,max}$

$$I_{s\,max} = \frac{50-20}{500} = \frac{30}{500} = 60 \text{ mA}$$

Using KCL, $I_s = I_z + I_L$

where $I_L = \frac{V_z}{R_L} = \frac{20}{1000} = 20 \text{ mA}$

$$I_{zm} = I_{s\,max} - I_L$$

$$= 60 - 20 = 40 \text{ mA}$$

Q.4 (b) Solution:

(i) Given, $\sigma_c = 5.8 \times 10^7$ S/m

$\sigma_d = 10^{-4}$ S/m

$\varepsilon = 2.6\varepsilon_0$, $\mu = \mu_0$

$f = 12$ GHz

$a = 2$ cm

$b = 1$ cm

The propagation constant in a rectangular waveguide with dielectric loss is given by

$$\gamma = \alpha_d + j\beta_g = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\in_d}$$

where $\in_d$ is the complex permittivity of the medium in the waveguide. We have,

$$\in_d = \in - j\frac{\sigma_d}{\omega}$$

Thus, we get

$$\gamma = \alpha_d + j\beta_g = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\in + j\omega\mu\sigma_d}$$

For TE_{10} mode,

$$\alpha_d + j\beta_g = \sqrt{\frac{-\omega^2\varepsilon_r}{\frac{1}{\mu_0\varepsilon_0}} + \frac{\pi^2}{a^2} + j\omega\mu\sigma_d}$$

$$= \sqrt{-\left(\frac{2\pi\times12\times10^9}{3\times10^8}\right)^2(2.6)+\frac{\pi^2}{(2\times10^{-2})^2}+j2\pi\times12\times10^9\times4\pi\times10^{-7}\times10^{-4}}$$

$$= \sqrt{-164230.22+24674.011+j9.475}$$

$$= \sqrt{-139556.209+j9.475}$$

$$\simeq 373.572\angle 89.998°$$

$$\alpha_d + j\beta_g \simeq 0.013 + j373.572 \text{ m}^{-1}$$

$$\Rightarrow \quad \alpha_d = 0.013 \text{ Np/m}$$

(ii) Given,

$$Z_0 = 60\ \Omega$$

$$Z_L = (60 + j60)\Omega$$

$$Z_{in} = (120 - j60)\Omega$$

$$\text{Input impedance, } Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right],$$

where l is the length of the transmission line or the distance of load from the generator

$$(120 - j60) = 60\left[\frac{(60+j60)+j60\tan\left(\frac{2\pi}{\lambda}l\right)}{60+j(60+j60)\tan\left(\frac{2\pi}{\lambda}l\right)}\right]$$

$$(2 - j) = \frac{(1+j)+j\tan\left(\frac{2\pi}{\lambda}l\right)}{1+j(1+j)\tan\left(\frac{2\pi}{\lambda}l\right)}$$

$$\Rightarrow \quad (2 - j) = \left[\frac{1+j\left(1+\tan\left(\frac{2\pi}{\lambda}l\right)\right)}{1-\tan\left(\frac{2\pi}{\lambda}l\right)+j\tan\left(\frac{2\pi}{\lambda}l\right)}\right]$$

$$\Rightarrow \quad 2-2\tan\left(\frac{2\pi}{\lambda}l\right)+j2\tan\left(\frac{2\pi}{\lambda}l\right)-j\left(1-\tan\left(\frac{2\pi}{\lambda}l\right)\right)+\tan\left(\frac{2\pi}{\lambda}l\right)$$

$$-1-j\left(1+\tan\left(\frac{2\pi l}{\lambda}\right)\right) = 0$$

$$\Rightarrow \quad 1-\tan\left(\frac{2\pi}{\lambda}l\right)+j2\left[\tan\left(\frac{2\pi}{\lambda}l\right)-1\right] = 0$$

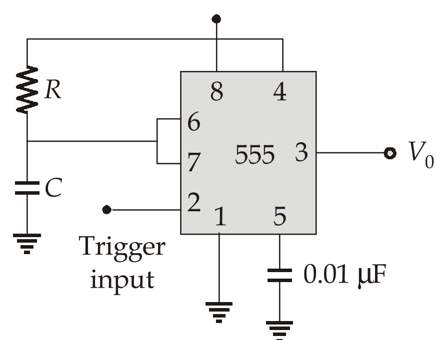
$$\Rightarrow \qquad 1 = \tan\left(\frac{2\pi}{\lambda}l\right)$$

$$\Rightarrow \qquad \frac{\pi}{4}+n\pi = \left(\frac{2\pi}{\lambda}l\right)$$

$$\Rightarrow \qquad l = \frac{\lambda}{8}(1+4n)\,;\ n = 0, 1, 2, 3....$$

Q.4 (c) Solution:

(i) The 555 IC can be configured to work as an monostable multivibrator as shown below:

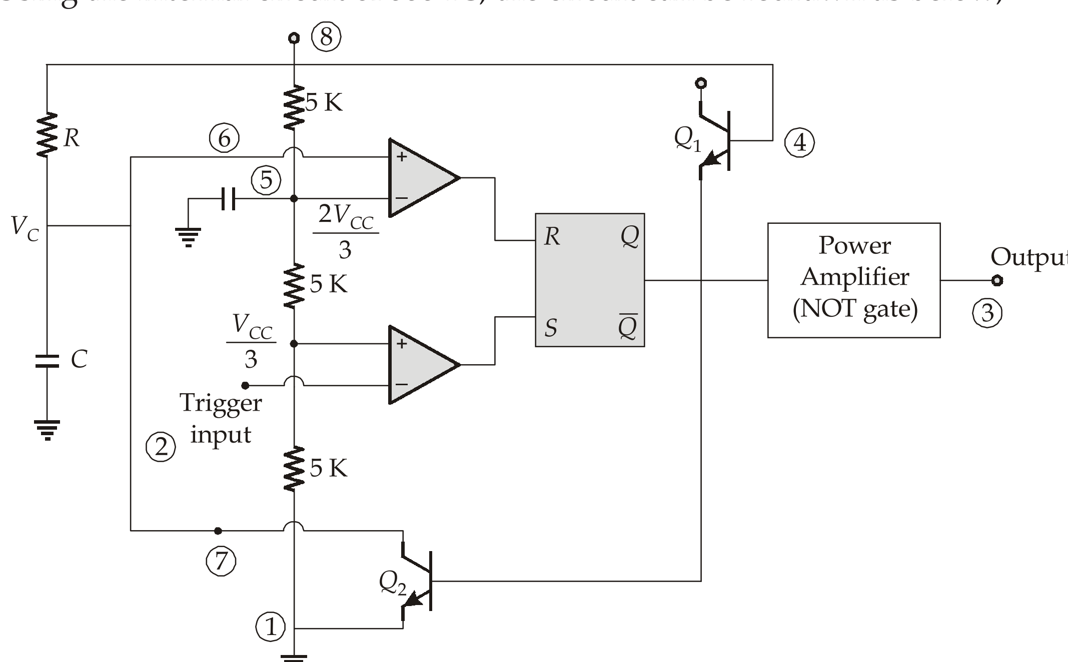


The output of the monostable multivibrator using 555 timer remains in its stable state until it gets a trigger. We have,

Stable state : $V_0 = 0$

Quasi stable state : $V_0 = \text{"1"}$ or V_{CC}

Using the internal circuit of 555 IC, the circuit can be redrawn as below,



Case-I (Trigger input is not applied):

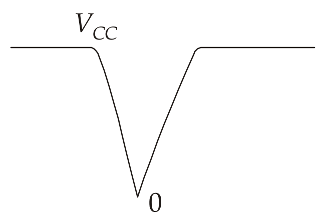
Output remain in stable state, hence

$$V_0 = 0$$

As $V_0 = 0$; $\bar{Q} = 1 \Rightarrow$ Transistor Q_2 is ON

Hence, capacitor discharge fully through Q_2 and voltage across capacitor, $V_C = 0$.

Case-II (Trigger input is applied):



If trigger input is a negative spike, voltage at trigger pin becomes zero when trigger is applied and lower comparator generates logic 1 output which causes SET input 'S' to flip flop as logic '1'.

If $S = 1$; $Q = 1$; $\bar{Q} = 0 \Rightarrow V_0 = 1$

Hence, output changes to Quasi stable state.

When $V_0 = 1 \Rightarrow \bar{Q} = 0 \Rightarrow$ Transistor Q_2 is OFF.

Since discharge path is disconnected, therefore capacitor can't discharge. Hence, capacitor starts charging through resistor R.

V_C increases exponentially upto $\frac{2V_{CC}}{3}$.

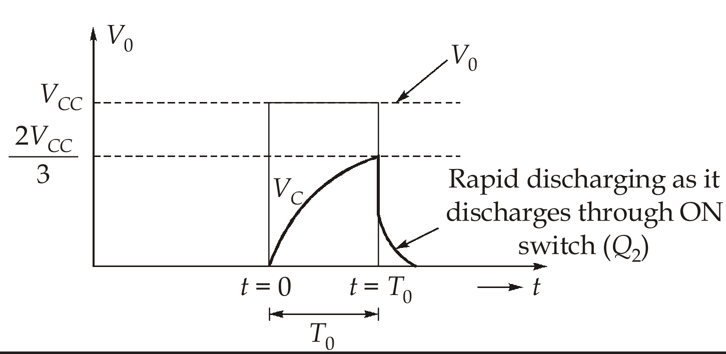
When V_C becomes $> \frac{2V_{CC}}{3}$, then upper comparator generates logic 1 output which acts as RESET input to flip flop.

If $R = 1$; $Q = 0$ i.e.

$$\bar{Q} = 1 \Rightarrow V_0 = 0$$

Hence, output returns back to stable state

The output voltage and the voltage across the capacitor when trigger is applied is as shown below:



Suppose at $t = 0$, the trigger input is applied and the capacitor starts charging through resistance R. When capacitor is charging,

$$V_C = V_{CC}(1 - e^{-t/RC})$$

At $t = T_0$;
$$V_C = \frac{2V_{CC}}{3}$$

$$\frac{2V_{CC}}{3} = V_{CC}(1 - e^{-T_0/RC})$$

$$e^{-T_0/RC} = 1 - \frac{2}{3} = \frac{1}{3}$$

$$T_0 = RC\ln(3) = 1.1\ RC$$

It is given that $T_0 = 1\ \mu s$ and $C = 325$ pF. Hence,

$$1.1\ \text{RC} = 1 \times 10^{-6}$$

$\therefore$
$$R = \frac{1 \times 10^{-6}}{325 \times 10^{-12}} = 3.076\ \text{k}\Omega$$

(ii) **1.** For FWR with inductor filter, ripple factor

$$r = \frac{R_L}{3\sqrt{2}\,\omega_o L} = \frac{650}{3\sqrt{2} \times 2\pi \times 60 \times 6} = 0.0677$$

2. DC Output voltage, $V_{DC} = \frac{2V_m}{\pi} - I_{DC}R = \frac{2V_m}{\pi} - \frac{V_{DC}}{R_L} \cdot R$

$$V_{DC}\left(1 + \frac{R}{R_L}\right) = \frac{2V_m}{\pi} \quad \text{...(i)}$$

where, $R = \frac{R_{sw}}{2} + R_f + R_{ind} = \frac{45}{2} + 20 + 30 = 72.5\ \Omega$

We have $\frac{V_m}{\sqrt{2}} = 50 \Rightarrow V_m = 50\sqrt{2} = 70.7106$ V

Putting above value in equation (i),

$$V_{DC}\left(1 + \frac{72.5}{650}\right) = \frac{2 \times 70.71}{\pi}$$

$$V_{DC}(1.1115) = 45.015$$

$$V_{DC} = \frac{45.015}{1.1115} = 40.5\ \text{V}$$

AC output voltage is V'_{rms} so

$$V'_{rms} = r\,V_{DC} = 0.0677 \times 40.5 = 2.74\ \text{V}$$

3.
$$\%\ \text{Regulation} = \frac{R}{R_L} \times 100\% = \frac{72.5}{650} \times 100 = 11.15\%$$

Section B : Analog Circuits + Electromagnetics

Q.5 (a) Solution:

Given,
$$f = 50 \text{ MHz}, r = 500 \text{ km}$$
$$E_{\theta S} = 10 \ \mu\text{V/m}$$
$$\theta = \frac{\pi}{2}$$

(i) The wavelength,
$$\lambda = \frac{c}{f} = \frac{3\times10^8}{50\times10^6} = 6 \text{ m}$$

Hence, the length of the half-wave dipole is
$$l = \frac{\lambda}{2} = \frac{6}{2} = 3 \text{ m}$$

(ii) The electric field from the half-wave dipole antenna is given by
$$E_{\theta s} = \frac{j\eta_0 I_0 e^{-jkr}\cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r\sin\theta}$$

Thus,
$$|E_{\theta s}| = \frac{\eta_0 I_0 \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r\sin\theta}$$

$\Rightarrow$
$$I_0 = \frac{|E_{\theta s}|2\pi r\sin\theta}{\eta_0 \cos\left(\frac{\pi}{2}\cos\theta\right)}$$
$$= \frac{10\times10^{-6}\times2\pi\times500\times10^3\sin\frac{\pi}{2}}{120\pi\times\cos\left(\frac{\pi}{2}\cos\frac{\pi}{2}\right)} = 83.33 \text{ mA}$$

(iii) For $\frac{\lambda}{4}$ monopole,
$$R_{rad} \simeq 36.56 \ \Omega$$

$\Rightarrow$
$$P_{rad} = \frac{1}{2}I_0^2R_{rad} = \frac{1}{2}(83.33)^2\times10^{-6}\times36.56$$
$$= 126.9 \text{ mW}$$

(iv) For $\frac{\lambda}{4}$ monopole, the total input impedance is $Z_{in} = 36.5 + j21.25 \ \Omega$.

Reflection coefficient,
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

($Z_L = Z_{in}$ in this case as antenna acts as the load)

$$\Gamma = \frac{36.5 + j21.25 - 75}{36.5 + j21.25 + 75} = -0.298141 + j\,0.2474$$

$\Rightarrow$ $\Gamma = 0.3874\angle 140.31°$

Standing wave ratio,

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.3874}{1-0.3874}$$

$\Rightarrow$ $S = 2.265$

Q.5 (b) Solution:

Given, $\vec{H} = y^2 z\hat{a}_x + 2(x+1)yz\hat{a}_y - (x+1)z^2\hat{a}_z$ A/m.

From Ampere's circuital law in differential form, the conduction current density is,

$$\vec{J} = \vec{\nabla} \times \vec{H}$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z & 2(x+1)yz & -(x+1)z^2 \end{vmatrix}$$

$$= \left\{\frac{\partial}{\partial y}\left[-(x+1)z^2\right] - \frac{\partial}{\partial z}\left[2(x+1)yz\right]\right\}\hat{a}_x - \left\{\frac{\partial}{\partial x}\left[-(x+1)z^2\right] - \frac{\partial}{\partial z}\left[y^2 z\right]\right\}\hat{a}_y$$

$$+ \left\{\frac{\partial}{\partial x}\left[2(x+1)yz\right] - \frac{\partial}{\partial y}\left[y^2 z\right]\right\}\hat{a}_z$$

$$= \{0 - 2(x+1)y\}\,\hat{a}_x - \left\{-z^2 - y^2\right\}\hat{a}_y + \left\{2yz - 2yz\right\}\hat{a}_z$$

We get,

$$\vec{J} = -2(x+1)y\hat{a}_x + \left(y^2 + z^2\right)\hat{a}_y \text{ A/m}^2$$

The conduction current density at point (2, 0, –1) is,

$$\vec{J}\Big|_{(2,0,-1)} = -2(x+1)y\hat{a}_x + \left(y^2 + z^2\right)\hat{a}_y$$

$$= -2(2+1)(0)\hat{a}_x + \left[0^2 + (-1)^2\right]\hat{a}_y$$

$$= \hat{a}_y$$

$$\Rightarrow \qquad \vec{J}\Big|_{(2,0,-1)} = 1.0\,\hat{a}_y\ \text{A/m}^2$$

The square loop is in *xz*-plane, i.e. perpendicular to the *y*-axis. The current enclosed by the square loop is,

$$I = \iint \vec{J}\cdot\vec{ds} = \iint\left(\vec{J}\right)\cdot\left(\hat{a}_y dx\,dz\right)$$

$$= \iint\left[-2(x+1)y\,\hat{a}_x + \left(y^2+z^2\right)\hat{a}_y\right]\cdot\left[\hat{a}_y dx\,dz\right] \text{ at } y = 1$$

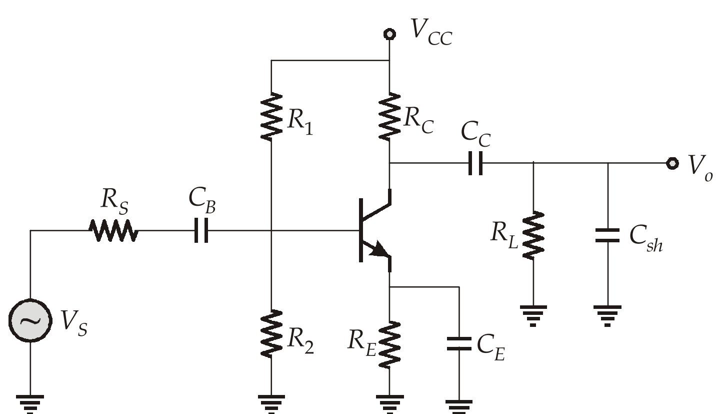
$$= \iint\left(1+z^2\right)dx\,dz = \int_{x=0}^{1} dx \int_{z=0}^{1}\left(1+z^2\right)dz$$

$$= [x]_0^1\cdot\left[z+\frac{z^3}{3}\right]_0^1 = (1)\cdot\left(1+\frac{1}{3}\right)$$

$$\Rightarrow \qquad I = \frac{4}{3}A$$

Q.5 (c) Solution:

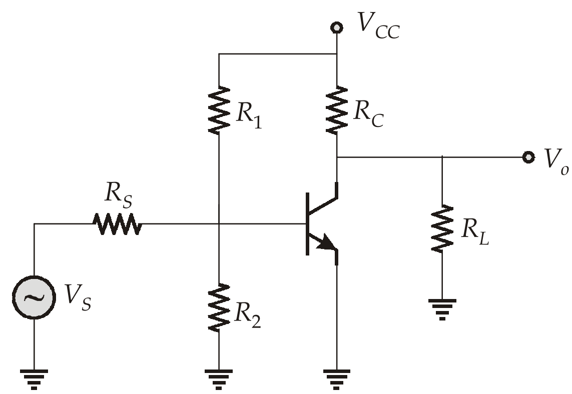
Let analyse the common emitter amplifier with bypass capacitor as shown in figure below:



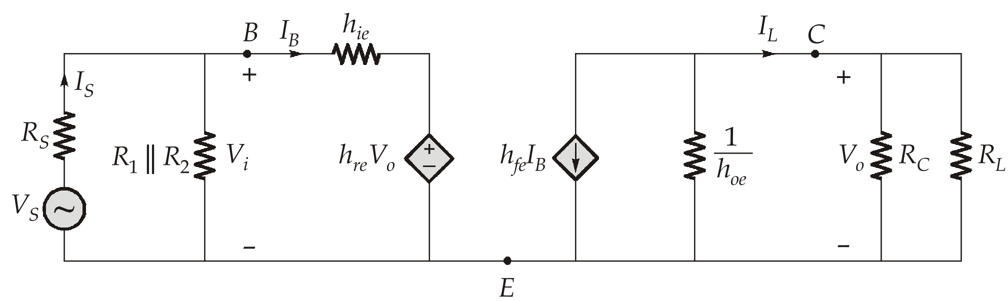
where, C_E = Emitter bypass capacitor

Now, Lets draw the AC equivalent circuit of the above amplifier.

In AC equivalent circuit, all the DC sources are disabled and large capacitances like Bypass capacitor, Blocking capacitor and Coupling capacitors acts as short circuit. Whereas, smaller capacitances like shunt capacitor C_{sh} is open circuited.



Using the h-parameter model of BJT, the circuit can be redrawn as below,



$$V_0 = -(h_{fe}I_B)\times R_L'' \quad \text{where } R_L'' = \left[R_L \| R_C \| \frac{1}{h_{oe}}\right] \qquad ...(i)$$

$$V_i = h_{ie}I_B + h_{re}V_0 \qquad ...(ii)$$

Using equation (i) in (ii),

$$V_i = h_{ie}I_B + h_{re}\left[-h_{fe}I_B R_L''\right]$$

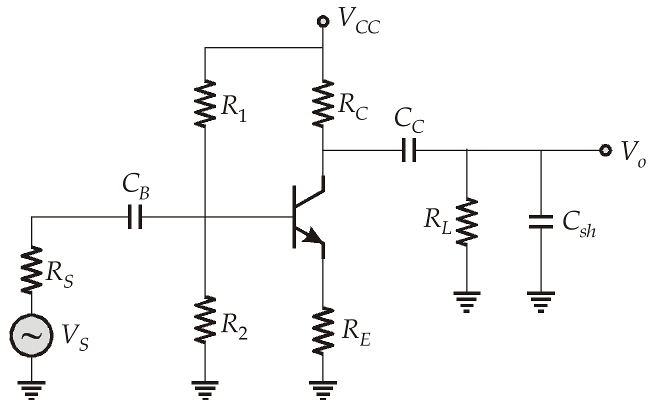
$$V_i = \left[h_{ie} - h_{re}h_{fe}R_L''\right]I_B \qquad ...(iii)$$

From equation (i) and (iii), we get,

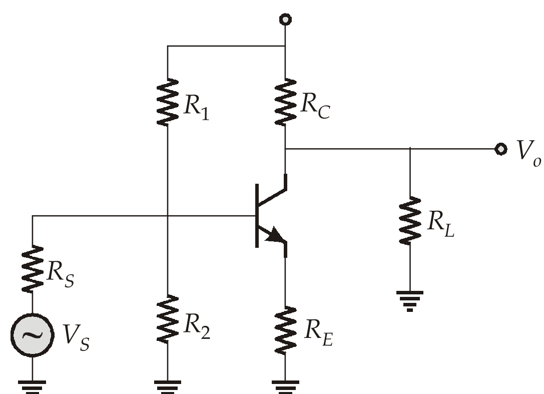
$$\frac{V_0}{V_i} = \frac{-h_{fe}I_B R_L''}{(h_{ie} - h_{re}R_L''h_{fe})I_B}$$

$$\frac{V_0}{V_i} = \frac{-h_{fe}R_L''}{h_{ie} - h_{re}R_L''h_{fe}} \qquad ...(iv)$$

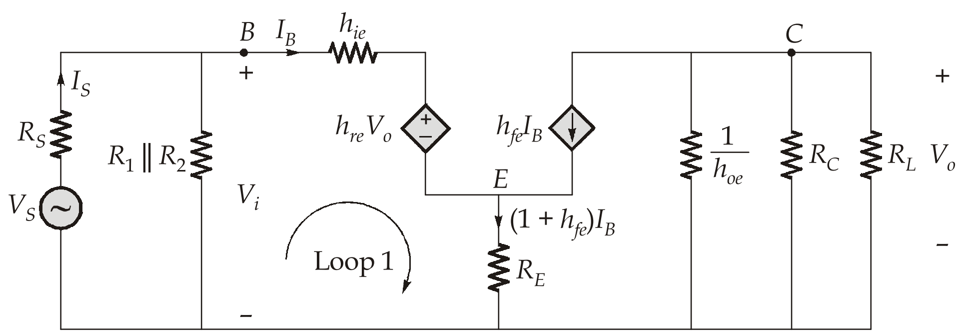
Now, analysing the circuit without bypass capacitor,



The ac equivalent circuit can be drawn as below,



Using the h-parameter model of BJT, the circuit can be redrawn as,



$$V_0 = -(h_{fe}I_B) \times R_L'' \quad \text{where } R_L'' = \frac{1}{h_{ie}} \| R_c \| R_L \qquad \text{...(v)}$$

On applying KVL in loop 1, we get

$$-V_i + I_B h_{ie} + h_{re}V_0 + R_E(1 + h_{fe})I_B = 0$$

$$V_i = I_B(h_{ie} + R_E(1 + h_{fe})) + h_{re}V_0$$

Using equation (v), we get

$$V_i = I_B(h_{ie} + R_E(1+h_{fe})) - h_{re}(h_{fe} I_B R_L'')$$

$$V_i = I_B[h_{ie} + R_E + R_E h_{fe} - h_{re} h_{fe} R_L''] \quad \text{...(vi)}$$

Using equation (v) and (vi), we get

$$\frac{V_0}{V_i} = \frac{-(h_{fe} R_L'')I_B}{(h_{ie} + R_E + R_E h_{fe} - h_{re} h_{fe} R_L'')I_B}$$

$$\frac{V_0}{V_i} = \frac{-h_{fe} R_L''}{h_{ie} + R_E(1+h_{fe}) - h_{re} h_{fe} R_L''} \quad \text{...(vii)}$$

We have,

$$\left.\frac{V_0}{V_i}\right|_{\text{with Bypass capacitor}} = \frac{-h_{fe} R_L''}{h_{ie} - h_{re} R_L' h_{fe}}$$

where negative sign represent 180° phase shift

$$\left.\frac{V_0}{V_i}\right|_{\text{with Bypass capacitor}} = \left[\frac{h_{fe} R_L''}{h_{ie} - h_{re} R_L'' h_{fe}}\right]$$

and $$\left.\frac{V_0}{V_i}\right|_{\text{without Bypass capacitor}} = \left[\frac{h_{fe} R_L''}{h_{ie} + R_E(1+h_{fe}) - h_{re} h_{fe} R_L''}\right]$$

Hence,

$$\left.\frac{V_0}{V_i}\right|_{\text{with Bypass capacitor}} > \left.\left|\frac{V_0}{V_i}\right|\right._{\text{without Bypass capacitor}}$$

The emitter resistance R_E provides bias stability but causes a decrease in the AC voltage gain at higher frequencies. Thus, to enhance the voltage gain of the amplifier, R_E is bypassed by capacitor so that it acts as a short circuit for AC.

Q.5 (d) Solution:

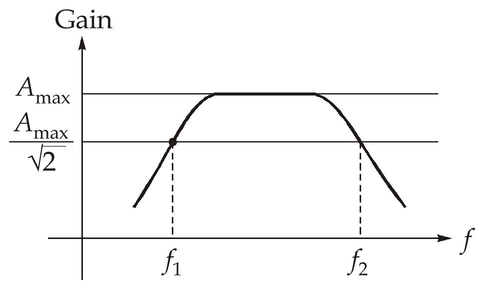
Given, roll-off rate = 40 dB/decade → 2nd order filter

$$f_1 = 2 \text{ kHz}, f_2 = 4 \text{ kHz}$$

$$R_1 = 1 \text{ k}\Omega$$

$$C = 10^{-8} \text{ F}$$

$$C_3 = 10^{-7} \text{ F}$$



In the bandpass filter, the low pass section defines the upper cut-off frequency $f_2 = 4$ kHz and high pass section defines the cut-off frequency $f_1 = 2$ kHz.

Given, circuit (a) is second order Sallen-key low pass filter. Thus,

$$f_0 = f_2 = \frac{1}{2\pi RC}$$

$$\Rightarrow \quad 4 \times 10^3 = \frac{1}{2\pi R \times 10^{-8}}$$

$$\Rightarrow \quad R = \frac{1}{2\pi \times 4 \times 10^3 \times 10^{-8}}$$

$$\Rightarrow \quad R = 3.98 \text{ k}\Omega$$

$$\text{Gain, } K = 1 + \frac{R_2}{R_1} = 1 + \frac{R_2}{1000} \quad \text{...(i)}$$

For the given circuit,

$$\frac{V_0}{V_i} = \frac{K}{1 - \omega^2 C^2 R^2 + j\omega RC(3-K)}$$

Comparing with the standard characteristic equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$, the Quality factor can be obtained as,

$$Q = \frac{1}{2\xi} = \frac{1}{3-K}$$

From equation (i),
$$\left(3 - \frac{1}{Q}\right) = 1 + \frac{R_2}{1000}$$

$$\Rightarrow \quad 3 - \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = 1 + \frac{R_2}{1000}$$

$$\Rightarrow \quad R_2 = 585.79 \ \Omega$$

Note: If $Q = \frac{1}{\sqrt{2}}$, then maximally flat frequency response is achieved, and such circuit is called 2nd order Butterworth LPF.

Given, circuit (b) is second order Sallen-key high pass filter. Thus,

$$f_0 = f_1 = \frac{1}{2\pi R_3 C_3} = 2\text{ kHz}$$

$$\Rightarrow \quad 2 \times 10^3 = \frac{1}{2\pi R_3 \times 10^{-7}}$$

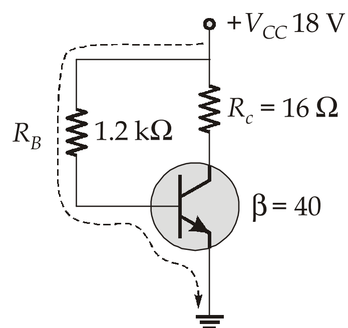
$$\Rightarrow \quad R_3 = \frac{1}{2\pi \times 2 \times 10^3 \times 10^{-7}}$$

$$\Rightarrow \quad R_3 = 795.775\ \Omega$$

Q.5 (e) Solution:

The given circuit is a series-fed class A amplifier.

For DC analysis



(i) On applying KVL in dotted loop, we get

$$I_B = \frac{V_{CC} - 0.7\text{ V}}{R_B} = \frac{18 - 0.7}{1.2} = 14.42\text{ mA}$$

We know that,

$$I_C = \beta I_B$$

$$I_C = 40 \times 14.42 \times 10^{-3}$$

$$I_C = 576.8\text{ mA}$$

Now,

$$V_{CE} = V_{CC} - 16\, I_C$$

$$= 18 - (16 \times 576.8 \times 10^{-3})$$

$$= 8.7712\text{ Volt}$$

Thus

Quiescent point $Q(I_{CQ}, V_{CEQ}) = Q(576.8\text{ mA}, 8.7712\text{ V})$

(ii) Input power, $P_{in} = V_{CC} \times I_{CQ}$

$$= 18 \times 576.8 \times 10^{-3}$$

$$= 10.38\text{ W}$$

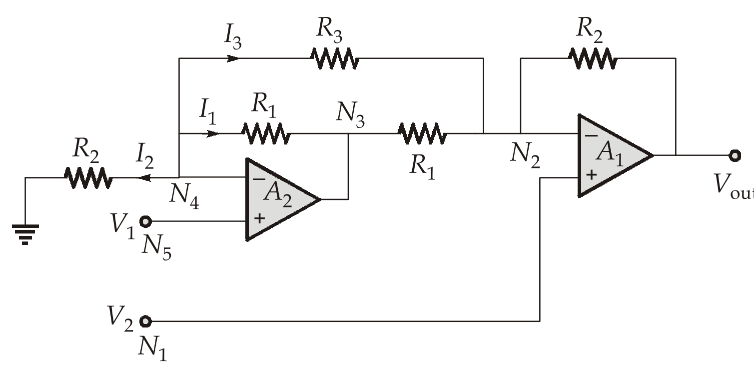
(iii) As small signal base current,

$$I_B = 5 \text{ mA rms}$$
$$I_{C\,rms} = \beta I_B = 40 \times 5 = 0.2 \text{ A}$$
$$\text{Output power, } P_{0(a.c)} = I_{c\,rms}^2 \times R_c = (0.2)^2 \times 16$$
$$= 0.64 \text{ W}$$

(iv) Power efficiency, $\%\eta = \dfrac{P_{0(a.c)}}{P_{i(d.c)}} \times 100\% = \dfrac{0.64}{10.38} \times 100 = 6.16\%$

Q.6 (a) Solution:

(i) We have,



For amplifier A_1, using the virtual short concept, node N_1 and N_2 have same potential.

Hence, $V_{N1} = V_{N2} = V_2$

Similarly, for amplifier A_2, the potential at node N_4 and N_5 is same, i.e.

$$V_{N5} = V_{N4} = V_1$$

On applying KCL at node N_2 and neglecting input current of op-amps, we get

$$\frac{V_{N2} - V_{N4}}{R_3} + \frac{V_{N2} - V_{N3}}{R_1} + \frac{V_{N2} - V_{out}}{R_2} = 0$$

Substituting $V_{N2} = V_2$ and $V_{N4} = V_1$, we get

$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - V_{N3}}{R_1} + \frac{V_2 - V_{out}}{R_2} = 0$$

$$V_1\left[\frac{-1}{R_3}\right] + V_2\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{V_{out}}{R_2} = \frac{V_{N3}}{R_1} \quad \text{...(i)}$$

Similarly, on applying KCL at node N_4,

$$\frac{V_{N4}-0}{R_2}+\frac{V_{N4}-V_{N3}}{R_1}+\frac{V_{N4}-V_{N2}}{R_3}=0$$

Substituting $V_{N4} = V_1$ and $V_{N2} = V_2$, we get

$$\frac{V_1}{R_2}+\frac{V_1-V_{N3}}{R_1}+\frac{V_1-V_2}{R_3} = 0$$

$$\frac{V_{N3}}{R_1} = V_1\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{V_2}{R_3} \quad ...(ii)$$

On equating equation (i) and (ii), we get

$$V_1\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{V_2}{R_3} = -\frac{V_1}{R_3}+V_2\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{V_{out}}{R_2}$$

On multiplying both sides by $R_1R_2R_3$, we get

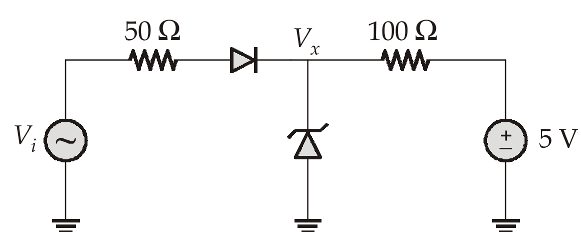
$$V_1[R_2R_3 + R_1R_3 + R_1R_2] - V_2R_1R_2 = -V_1R_1R_2 + V_2[R_2R_3 + R_1R_3 + R_1R_2] - R_1R_3V_{out}$$

$$R_1R_3V_{out} = (V_2 - V_1)(R_2R_3 + R_1R_3 + 2R_1R_2)$$

$$\frac{V_{out}}{V_2-V_1} = 1+\frac{R_2}{R_1}+\frac{2R_2}{R_3}$$

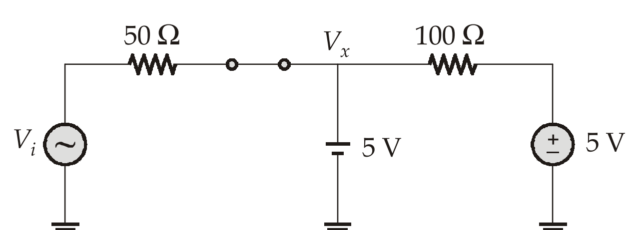
(ii) Let both diodes be OFF, then $V_x = 5$ V

If $V_i < 5$ V then both diodes are OFF.



Therefore, current through 100 Ω is zero for $V_i < 5$ V.

If $V_i > 5$ V, then normal diode becomes ON and zener diode undergoes breakdown.



Since zener diode is in breakdown region, hence $V_x = 0$ V. The current through 100 Ω resistor is given by,

$$I = \frac{5-5}{100} = 0\text{ A}$$

Since no current flows through the 100 Ω resistor for any value of V_i, hence power dissipated in 100 Ω resistor is zero.

Q.6 (b) Solution:

(i) Given, $a = 2$ cm; $b = 4$ cm; $P_{ave} = 2$ mW; $f = 10$ GHz

The dominant mode for $b > a$ is TE_{01} mode.

$$\text{Cut-off frequency}, f_{c01} = \frac{c}{2b} = \frac{3\times10^8}{2\times4\times10^{-2}} = 3.75\times10^9$$

$$\Rightarrow \quad f_{c01} = 3.75\text{ GHz}$$

We have,
$$\eta = \eta_{TE} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{3.75\times10^9}{10\times10^9}\right)^2}} = 406.68\ \Omega$$

For TE_{mn} mode, the electric field in rectangular waveguide is given by

$$E_{xs} = \frac{j\mu\omega}{h^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$E_{ys} = -\frac{j\mu\omega}{h^2}\left(\frac{m\pi}{a}\right)H_0\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

For TE_{01} mode, we have

$$E_{ys} = 0 \text{ and } |E_{xs}| = E_0\sin\left(\frac{n\pi y}{b}\right) \text{ where } E_0 = \frac{\mu\omega}{h^2}\left(\frac{\pi}{b}\right)H_0$$

Thus, the average power transmitted is,

$$P_{ave} = \int_{x=0}^{a}\int_{y=0}^{b}\frac{|E_{xs}|^2}{2\eta}dxdy$$

$$= \int_{x=0}^{a}\int_{y=0}^{b}E_0^2\sin^2\left(\frac{n\pi y}{b}\right)dxdy = \frac{E_0^2 ab}{4\eta}$$

$$\Rightarrow \quad E_0^2 = \frac{4\eta P_{ave}}{ab} = \frac{4\times406.68\times2\times10^{-3}}{2\times10^{-2}\times4\times10^{-2}} = 4066.8$$

$$\Rightarrow \quad E_0 = 63.77\text{ V/m}$$

∴ The peak value of the magnetic field in the waveguide is,

$$H_0 = \frac{E_0 h^2}{\mu\omega}\left(\frac{b}{\pi}\right)$$

where $h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \left(\frac{\pi}{b}\right)^2$ for TE_{01} mode

$$H_0 = \frac{\pi E_0}{\omega\mu b}$$

$$= \frac{\pi\times 63.77}{2\pi\times 10\times 10^9 \times 4\pi\times 10^{-7}\times 4\times 10^{-2}} = 63.43 \text{ mA/m}$$

(ii) Given, $\vec{H} = 0.2\cos(\omega t - \beta x)\hat{a}_z$ A/m

Let $f(x, y) = x + y - 1$

$$\vec{\nabla} f = \frac{\partial}{\partial x}(x+y-1)\hat{a}_x + \frac{\partial}{\partial y}(x+y-1)\hat{a}_y + \frac{\partial}{\partial z}(x+y-1)\hat{a}_z$$

$$\vec{\nabla} f = \hat{a}_x + \hat{a}_y$$

$$\left|\vec{\nabla} f\right| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

Thus, the unit vector normal to the plane $x + y = 1$ is given by

$$\hat{a}_n = \frac{\vec{\nabla} f}{\left|\vec{\nabla} f\right|} = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$$

The average power density is given by

$$\vec{P}_{ave} = \frac{1}{2}\eta H_0^2 \hat{a}_x$$

∴ Total power passing through a square plate of side 10 cm on plane $x + y = 1$ is,

$$P_t = \int \vec{P}_{ave}\cdot d\vec{s}$$

$$= \frac{1}{2}\eta H_0^2 \hat{a}_x \cdot \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right) \times (\text{Area of square plate})$$

$$= \frac{1}{2}\times 120\pi \times (0.2)^2 \times \left(\frac{1}{\sqrt{2}}\right) \times (10\times 10^{-2})^2$$

$$= 0.05331 = 53.31 \text{ mW}$$

Q.6 (c) Solution:

(i) Assuming transistor to be in saturation,

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)[V_{GS} - V_T]^2 \quad \text{...(i)}$$

Given: $V_G = 1.8\text{ V}, \quad \mu_n C_{ox}\left(\frac{W}{L}\right) = 2\text{ mA/V}^2$

$$V_S = 0.5\,I_D, \quad V_T = 1\text{ V}$$

Applying KVL in the input loop,

$$V_{GS} = 1.8 - 0.5I_D$$

$$\Rightarrow \quad I_D = \frac{1.8 - V_{GS}}{0.5} = 2[1.8 - V_{GS}]$$

$$I_D = 3.6 - 2V_{GS} \quad \text{...(ii)}$$

Now, from equation (i) and (ii)

$$3.6 - 2V_{GS} = \frac{1}{2}\times 2[V_{GS} - 1]^2$$

$$3.6 - 2V_{GS} = V_{GS}^2 + 1 - 2V_{GS}$$

$$V_{GS}^2 = 2.6$$

$$V_{GS} = 1.6\text{ Volt}$$

Hence, from equation (ii),

$$I_D = 0.4\text{ mA}$$

$$V_S = 0.5\,I_D = 0.2\text{ V}$$

Now,
$$V_D = 3.3 - I_D R_D$$
$$= 3.3 - 0.4 \times 10$$
$$V_D = -0.7\text{ V}$$

Hence, $V_{DS} = -0.7\text{ V} - 0.2\text{ V} = -0.9\text{ V} < V_{GS} - V_T$. Thus, the conditions for saturation are not satisfied.

Assuming transistor is in triode region, thus

$$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_D = 2\left[V_{GS} - V_T - \frac{V_{DS}}{2}\right]V_{DS} \quad \text{...(iii)}$$

Here, $V_D = 3.3 - 10I_D,\ V_S = 0.5I_D$

$$I_D = 2\left[V_G - V_S - V_T - \frac{V_D}{2} + \frac{V_S}{2}\right][V_{DS}]$$

$$= 2\left[1.8 - 0.5V_S - 1 - \frac{V_D}{2}\right][3.3 - 10.5I_D]$$

$$= 2[0.8 - 0.5V_S - 0.5V_D][3.3 - 10.5I_D]$$

$$= (1.6 - V_S - V_D)(3.3 - 10.5I_D)$$

$$= (1.6 - 0.5I_D - 3.3 + 10I_D)(3.3 - 10.5I_D)$$

$$= (9.5I_D - 1.7)(3.3 - 10.5I_D)$$

$$I_D = 31.35I_D - 99.75I_D^2 - 5.61 + 17.85I_D$$

$$99.75I_D^2 - 48.2I_D + 5.61 = 0$$

After solving we get,

$$I_D = 0.28778 \text{ mA}, 0.1954 \text{ mA}$$

Case (I):

$$I_D = 0.1954 \text{ mA}$$

$$V_S = 0.5 \times I_D = 0.0977 \text{ V}$$

$$V_D = 3.3 - 10I_D = 1.346 \text{ V}$$

$$V_{DS} = V_D - V_S = 1.2483 \text{ V}$$

$$(V_{GS} - V_T) = (1.8 - 0.0977 - 1) = 0.7023 \text{ V}$$

$V_{DS} > V_{GS} - V_T$ which is not valid, as transistor is assumed to be in triode region.

Case (2):

$$I_D = 0.28778 \text{ mA}$$

$$V_S = 0.5 \times 0.28778 = 0.14389 \text{ V}$$

$$V_D = 3.3 - 10I_D = 0.4222 \text{ V}$$

$$V_{DS} = V_D - V_S = 0.27831 \text{ V}$$

$$(V_{GS} - V_T) = V_G - V_S - V_T = 1.8 - 0.14389 - 1 = 0.65611 \text{ V}$$

As $V_{DS} < (V_{GS} - V_T)$, therefore $I_D = 0.28778$ mA is valid value for which transistor is in triode region.

∴ Drain voltage, $V_D = 0.4222$ V

(ii) Transconductance: Transconductance is the ratio of change in drain current (∂I_D) to change in the gate to source voltage (∂V_{GS}) at a constant drain to source voltage (V_{DS} = constant)

$$g_m = \frac{\partial I_D}{\partial V_{GS}} \text{ at constant } V_{DS}$$

This value is maximum at $V_{GS} = 0$. This is denoted by g_{m0}.

We have, $$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_{GS(\text{off})}}\right)^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{-2I_{DSS}}{V_{GS(\text{off})}}\left[1-\frac{V_{GS}}{V_{GS(\text{off})}}\right]$$

where, $$g_{m0} = \frac{-2I_{DSS}}{V_{GS(\text{off})}}$$

$\therefore$ $$g_m = g_{m0}\left[1-\frac{V_{GS}}{V_{GS\,(\text{off})}}\right]$$

Dynamic Output Resistance : This is the ratio of change in drain to source voltage (∂V_{DS}) to the change in drain current (∂I_D) at a constant gate to source voltage (V_{GS} = constant). It is denoted as r_d.

$$r_d = \frac{\partial V_{DS}}{\partial I_D} \text{ at constant } V_{GS}$$

Amplification factor: It is defined as the ratio of change in drain voltage (∂V_{DS}) to change in gate voltage (∂V_{GS}) at a constant drain current (I_D = constant)

$$\mu = \frac{\partial V_{DS}}{\partial V_{GS}} \text{ at constant } I_D$$

There is a relation between transconductance (g_m), dynamic output resistance (r_d) and amplification factor (μ) given by

$$\mu = \frac{\partial V_{DS}}{\partial V_{GS}} = \frac{\partial V_{DS}}{\partial I_D}\times\frac{\partial I_D}{\partial V_{GS}}$$

$$\mu = r_d \times g_m$$

Q.7 (a) Solution:

(i) Given, a = 2 cm; b = 1.5 cm; f = 10 GHz; t_d = 50 n sec

The waveguide is operating in dominant mode (TE_{10}). The cut-off frequency of the TE_{mm} mode is,

$$\omega_c = \frac{1}{\sqrt{\mu\in}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

For TE_{10} mode, $$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\left[\frac{\pi}{a}\right]$$

$$\Rightarrow \qquad \omega_c = \frac{3\times10^8\pi}{2\times10^{-2}} = 1.5\,\pi \times 10^{10} \text{ rad/}s$$

The phase constant of the mode is,

$$\beta = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

The group velocity of the mode is,

$$v_g = \frac{\partial\omega}{\partial\beta} = \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

At 10 GHz, $\omega = 2\pi \times 10^{10}$ rad/sec

∴ The group velocity is,

$$v_g = \frac{3\times10^8}{\sqrt{1-\left(\frac{1.5\pi\times10^{10}}{2\pi\times10^{10}}\right)^2}}$$

$$\Rightarrow \qquad v_g = 4.536 \times 10^8 \text{ m/s}$$

For a delay of 50 nsec, the length of the wave guide is,

$$L = v_g t = 4.536 \times 10^8 \times 50 \times 10^{-9}$$

$$\Rightarrow \qquad L = 22.68 \text{ m}$$

∴ The length of the section to realize a delay of 50 *n*sec is 22.68 m.

(ii) The reflection coefficient at the load is given by,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta}$$

The voltage maxima occurs on the line where

$$2\beta x_{\max} = 2n\pi + \theta$$

$$2\left(\frac{2\pi}{\lambda}\right)x_{\max} = 2n\pi + \theta$$

$$x_{\max} = \frac{n\lambda}{2} + \frac{\theta\lambda}{4\pi}$$

Here, x is the distance from the load ($x = 0$) towards the generator. The distance between two consecutive maxima is $\lambda/2$. Further, consecutive minima and maxima are separated by $\lambda/4$.

1. For complex inductive load ($Z_L = R + jX$),

$$\Gamma = \frac{(R+jX)-Z_0}{(R+jX)+Z_0} = |\Gamma| e^{j\theta}$$

Thus, $\theta = \tan^{-1}\left(\frac{x}{R-Z_0}\right) - \tan^{-1}\left(\frac{x}{R+Z_0}\right) > 0.$

Thus, the first voltage maxima occurs for $n = 0$ in between $0 < x < \lambda/4$ followed by a minima in between $\lambda/4 < x < \lambda/2$.

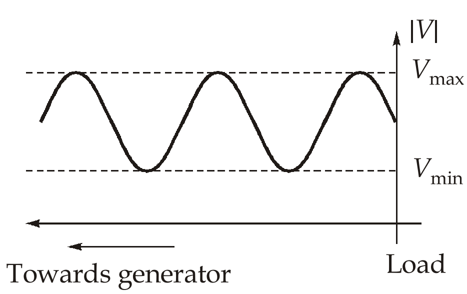
2. For complex capacitive load ($Z_L = R - jX$), $\theta < 0$. Hence, the first voltage maxima occurs for $n = 1$ in between $\lambda/4 < x < \lambda/2$. It means minima occurs first i.e. in between $0 < x < \lambda/4$.

3. For pure resistive load ($Z_L = R$), if $R > Z_0$, $\theta = 0$. In this case, the voltage maxima occurs first at $x = 0$ followed by a voltage minima at $x = \lambda/4$. If $R < Z_0$, $\theta = \pi$, hence the voltage maxima occurs at $x = \lambda/4$ and voltage minima occurs first at $x = 0$.

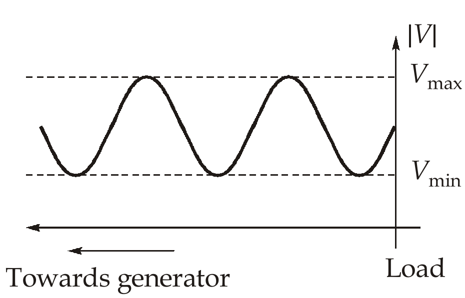
4. For pure inductive load ($Z_L = jX$), $\theta > 0$. Thus, the first voltage maxima occurs for $n = 0$ in between $0 < x < \lambda/4$ followed by a minima in between $\lambda/4 < x < \lambda/2$. Here, $|\Gamma| = 1$, thus, voltage minima, $V_{min} = V_0(1 - |\Gamma|) = 0$.

5. For pure capacitive load ($Z_L = -jX$), $\theta < 0$. Thus, the first voltage maxima occurs in between $\lambda/4 < x < \lambda/2$. It means minima occurs first i.e. in between $0 < x < \lambda/4$. Here, $|\Gamma| = 1$, thus, voltage minima, $V_{min} = V_0(1 - |\Gamma|) = 0$,

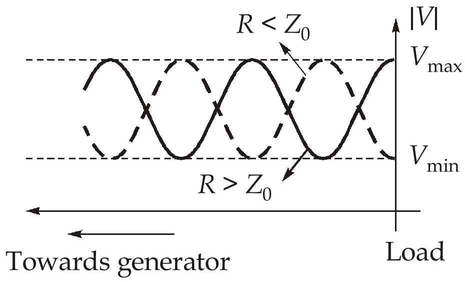
Using the above, the voltage standing wave patterns for the different types of load impedances can be drawn as below:

1. 

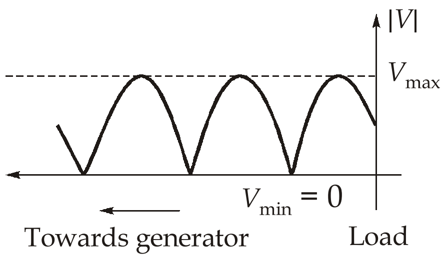
→ Complex Inductive Load ($R + jX$)

2. 

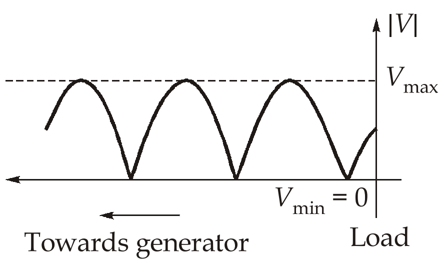
→ Complex capacitive load ($R - jX$)

3. 

→ Pure Resistive Load (R)

4. 

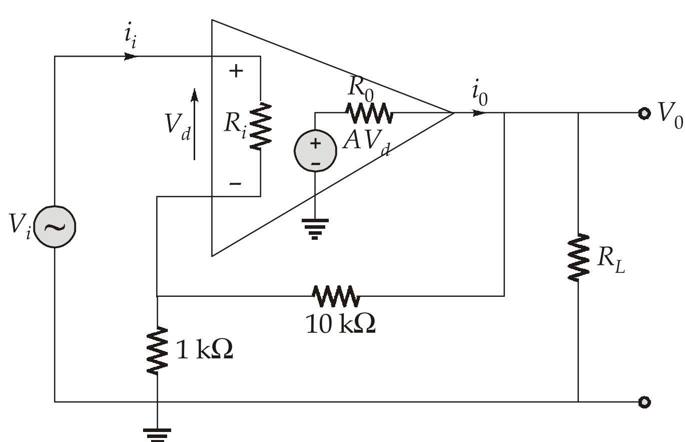
→ Pure inductive load ($+jX$)

5. 

→ Pure capacitive load ($-jX$)

Q.7 (b) Solution:

(i) Replacing the Op-Amp with the equivalent voltage controlled voltage source model, the given circuit can be redrawn as below,



$$V_d = V_i - V_0\left(\frac{1}{10+1}\right) \quad ...(1)$$

Assuming $R_L = \infty$ and neglecting the current into the terminals of op-amp, we can write

$$\Rightarrow \quad V_0 = AV_d\left(\frac{10+1}{10+1+R_0(\text{k}\,\Omega)}\right)$$

$$\Rightarrow \quad V_0 = AV_d\left(\frac{11}{11+0.075}\right)$$

$$\Rightarrow \quad V_0 = AV_d\left(\frac{11}{11.075}\right)$$

$$\Rightarrow \quad V_d = \frac{V_0}{A}\left(\frac{11.075}{11}\right) \quad \text{...(2)}$$

From equation (1) and (2),

$$\Rightarrow \quad \frac{V_0}{A}\left(\frac{11.075}{11}\right) = V_i - V_0\left(\frac{1}{11}\right)$$

$$\Rightarrow \quad \frac{V_0}{11}\left[\frac{11.075}{20000}+1\right] = V_i$$

$$V_i = 0.091\ V_0$$

$$V_0 = 11\ V_i \quad \text{...(3)}$$

$$\frac{V_0}{V_i} = A_f = 11$$

The given circuit is a voltage series feedback amplifier. Hence, the input impedance increases by a factor of (1 + $A\beta$) and the output impedance decreases by a factor of (1 + $A\beta$), where A is the voltage gain without feedback and β is the feedback factor.

Here, Feedback factor,

$$\beta = \frac{1}{10+1} = \frac{1}{11}$$

Thus, Input Resistance with feedback,

$$R_{if} = R_i(1+A\beta) = 2M\Omega \times \left(1+\frac{20000}{11}\right)$$

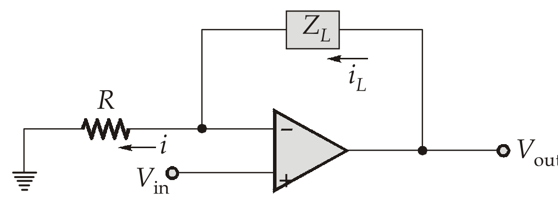
$$R_{if} = 3638.36\ \text{M}\Omega$$

Output Resistance with feedback,

$$R_{0f} = \frac{R_0}{1+A\beta} = \frac{75}{\left(1+\frac{20000}{11}\right)} = 0.0412\ \Omega$$

(ii) The voltage to current converter in which load resistor Z_L is floating (not connected to ground) is shown in below figure. For an ideal op-amp, the voltage at inverting and non-inverting terminal should be equal. Thus, voltage at inverting terminal, $V_- = V_{in}$.

Applying KCL at inverting terminal,



$$\frac{V_{in} - 0}{R} = i_L$$

$$\Rightarrow \quad i_L = \frac{V_{in}}{R}$$

Thus, the load current is independent of load impedance and depends only on the input voltage. Hence, the given circuit acts as a voltage to current converter.

Q.7 (c) Solution:

Given,

$$\varepsilon_r = 10$$

$$\text{Loss tangent} = 10^{-2}$$

$$\omega = 10^{10} \text{ rad/sec}$$

$$P_{in} = 100 \text{ W/m}^2$$

Loss tangent,

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{\omega\varepsilon_0\varepsilon_r} = 10^{-2}$$

$$\Rightarrow \quad \sigma = 10^{-2} \times \omega\varepsilon_0\varepsilon_r$$

$$= 10^{-2} \times 10^{10} \times \frac{10^{-9}}{36\pi} \times 10 = \frac{1}{36\pi} (\text{S/m})$$

Power density of the incident wave is,

$$P_{in} = \frac{|E_i|^2}{2\eta_1} = \frac{|E_i|^2}{2\eta_0}$$

where E_i : Peak electric field value of incident wave.

$$\Rightarrow \quad 100 = \frac{|E_i|^2}{2 \times 120\pi}$$

$$\Rightarrow \quad |E_i| = \sqrt{100 \times 2 \times 120\pi} = 274.59 \text{ V/m}$$

The transmitted field at the surface is,

$$\Rightarrow \quad E_t = \tau E_i = \frac{2\eta_2}{\eta_1 + \eta_2} E_i \qquad \text{...(i)}$$

Here, $\eta_1 = \eta_0 = 120\,\pi$

$$\eta_2 = \sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\varepsilon_0\varepsilon_r}}$$

$\Rightarrow$
$$\eta_2 = \sqrt{\frac{j\times 10^{10}\times 4\pi\times 10^{-7}}{\frac{1}{36\pi} + j\times 10^{10}\times\frac{10^{-9}}{36\pi}\times 10}}$$

$$= \sqrt{\frac{j4000\pi^2}{\frac{1}{36} + j\frac{100}{36}}} = \sqrt{\frac{j4000\pi^2}{\frac{1}{36}(1 + j100)}}$$

$$\eta_2 = \sqrt{14211.52\angle 0.573^\circ} = 119.212\angle 0.2865^\circ$$

$\Rightarrow$ $\eta_2 = 119.21 + 0.6j\ \Omega$

Using equation (i),
$$|E_t| = \frac{2(119.212)}{\sqrt{(119.21 + 120\pi)^2 + (0.6)^2}}\times(274.59)$$

$\Rightarrow$ $|E_t| = 0.4805 \times 274.59 = 131.94$ V/m

Since, the dielectric slab is lossy, the field amplitude reduces exponentially ($e^{-\alpha z}$) as a function of distance where,

$$\alpha = \text{Re}\left\{\sqrt{j\omega\mu_0(\sigma + j\omega\varepsilon_0\varepsilon_r)}\right\}$$

$$= \text{Re}\left\{\sqrt{j\times 10^{10}\times 4\pi\times 10^{-7}\left(\frac{1}{36\pi} + j\times 10^{10}\times\frac{10^{-9}}{36\pi}\times 10\right)}\right\}$$

$$= \text{Re}\left\{\sqrt{j\frac{4000}{36} - \left(\frac{400000}{36}\right)}\right\}$$

$$= \text{Re}\left\{\sqrt{\left(-\frac{400000}{36} + j\frac{4000}{36}\right)}\right\}$$

$$= \text{Re}\left\{\sqrt{11111.67\angle 179.43^\circ}\right\}$$

$$= \text{Re}\{105.412\angle 89.715^\circ\}$$

$$= \text{Re}\{0.524337 + j\,105.411\}$$

$\Rightarrow$ $\alpha = 0.524$ nepers/m

The magnitude at 10 m distance from the surface is,

$$\left|E_t\right|\Big|_{z=10m} = \left|E_t\right|\Big|_{\text{surface}} \times e^{-\alpha z}$$

$$= 131.94 \times e^{-(0.524 \times 10)}$$

$$= 0.7 \text{ V/m}$$

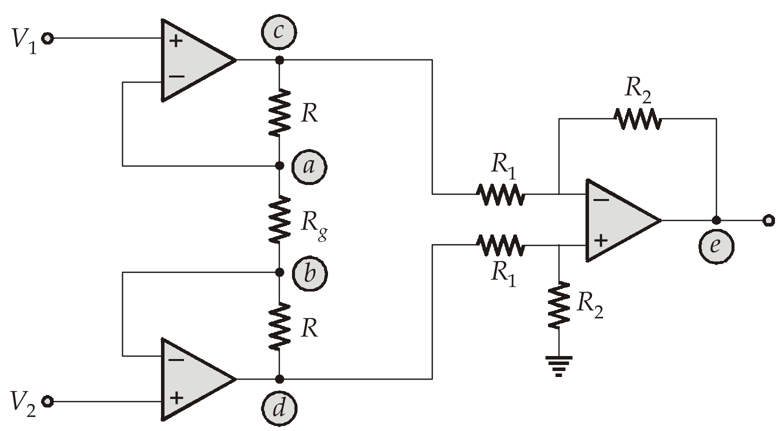
The power density of the wave is,

$$P = \frac{|E_t|^2}{2\times \text{Re}\{\eta_2^*\}} = \frac{(0.7)^2}{2\times \text{Re}\{(119.21+0.6j)^*\}}$$

$$= \frac{(0.7)^2}{2\times \text{Re}\{119.21-0.6j\}} = \frac{(0.7)^2}{2\times 119.21}$$

$$\Rightarrow \qquad P \approx 2 \text{ mW/m}^2$$

Q.8 (a) Solution:



(i) The circuit is known as instrumentation amplifier.

The circuit has two stages:

(a) Amplifier stage

(b) Difference amplifier stage

CMRR: Common Mode Rejection Ratio = $\frac{A_d}{A_{cm}}$

In instrumentation amplifier, differential mode signal (V_1 – V_2) is amplified by both stages (input stage and differential amplifier stage). The first stage amplify the difference signals (V_1 – V_2), however does not amplify the common mode signal. Second stage (differential amplifier stage) has common mode gain ≈ 0. Thus, overall common mode gain (A_{cm}) remain very low resulting in high CMRR for the circuit.
Thus, instrumentation amplifier provides high CMRR compared to op-amp.

(ii) Due to virtual short-circuit,

$$V_a = V_1$$
$$V_b = V_2$$

Current in ab branch,

$$i_{ab} = \frac{V_1 - V_2}{R_g}$$

Voltage at (c), $$V_c = V_a + i_{ab} R$$

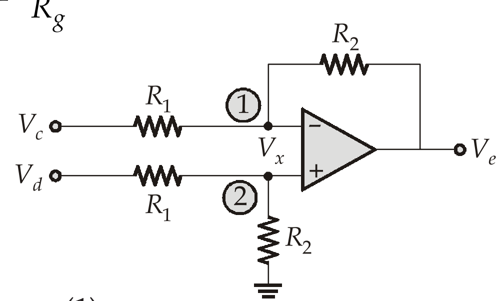
[∵ Current in the inverting terminals of op-amp is zero]

$$V_c = V_1 + (V_1 - V_2)\frac{R}{R_g} = V_1\left(1 + \frac{R}{R_g}\right) - (V_2)\frac{R}{R_g}$$

Similarly, voltage at (d), $$V_d = V_b - i_{ab}R = V_2 - \left(\frac{V_1 - V_2}{R_g}\right)R$$

$$V_d = V_2\left(1 + \frac{R}{R_g}\right) - V_1 \cdot \frac{R}{R_g}$$

Voltage at (e),



KCL at point (1),

$$\frac{V_x - V_c}{R_1} + \frac{V_x - V_e}{R_2} = 0$$

$$V_x\left(\frac{R_1 + R_2}{R_1 R_2}\right) = \frac{V_e}{R_2} + \frac{V_c}{R_1} \qquad ...(1)$$

KCL at point (2),

$$\frac{V_x - V_d}{R_1} + \frac{V_x}{R_2} = 0$$

$$\Rightarrow \qquad V_x\left(\frac{R_1 + R_2}{R_1 R_2}\right) = \frac{V_d}{R_1} \qquad ...(2)$$

From equation (1) and (2),

$$\frac{V_d}{R_1} = \frac{V_e}{R_2} + \frac{V_c}{R_1} \Rightarrow \frac{V_d - V_c}{R_1} = \frac{V_e}{R_2}$$

$$\Rightarrow \qquad V_e = \frac{R_2}{R_1}(V_d - V_c) \qquad ...(3)$$

Putting V_d and V_c in equation (3),

$$V_e = \frac{R_2}{R_1}\left(1+\frac{2R}{R_g}\right)(V_2 - V_1) \quad ...(4)$$

(iii) Given, $V_1 = 5\text{ V}, \quad V_2 = 5.05\text{ V}$

$$V_e = 5\text{ V}$$

Using equation (4), $$5 = \frac{R_2}{R_1}\left(1+\frac{2R}{R_g}\right)(5.05-5)$$

$\Rightarrow$ $$100 = \frac{R_2}{R_1}\left(1+\frac{2R}{R_g}\right) \quad ...(5)$$

We have, First stage gain $= \left(1+\dfrac{2R}{R_g}\right)$

Second stage gain $= \dfrac{R_2}{R_1}$

Given, $$\text{Ratio} = \frac{\left(1+\dfrac{2R}{R_g}\right)}{\left(\dfrac{R_2}{R_1}\right)} = 10$$

$$\left(\frac{R_2}{R_1}\right) = \frac{1}{10}\left(1+\frac{2R}{R_g}\right) \quad ...(6)$$

By equation (5) and (6),

$$\frac{1}{10}\left(1+\frac{2R}{R_g}\right)^2 = 100 \Rightarrow \left(1+\frac{2R}{R_g}\right)^2 = 1000$$

$\Rightarrow$ $$1+\frac{2R}{R_g} = \sqrt{1000}$$

$$\frac{2R}{R_g} = 31.62 - 1 \Rightarrow \frac{R}{R_g} = 15.31$$

and $$\frac{R_2}{R_1} = \frac{1}{10}\left(1+\frac{2R}{R_g}\right) = \frac{1}{10}(31.62) = 3.162$$

Q.8 (b) Solution:

(i) Given,

$$R_{rad} = 280\ \Omega$$
$$\eta = 0.8$$
$$I_0 = 5\text{ A}$$
$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}\text{A}$$

The average power radiated from the antenna will be,

$$P_{rad} = I_{rms}^2 \times R_{rad} = \left(\frac{5}{\sqrt{2}}\right)^2 \times 280$$
$$= 3500$$
$$\Rightarrow \quad P_{rad} = 3.5\text{ kW}$$

Efficiency factor, $$\eta = \frac{R_{rad}}{R_{rad} + R_{loss}}$$

$$\Rightarrow \quad 0.8 = \frac{280}{280 + R_{loss}}$$
$$\Rightarrow \quad 0.8\,(280 + R_{loss}) = 280$$
$$\Rightarrow \quad R_{loss} = 70\ \Omega$$

The ohmic loss is, $$P_{loss} = I_{rms}^2 \times R_{loss}$$
$$= \left(\frac{5}{\sqrt{2}}\right)^2 \times 70$$
$$\Rightarrow \quad P_{loss} = 875\text{ W}$$

(ii) Given,

$$P_{rad} = 100\text{ kW}$$
$$E = 12\text{ mV/m}$$
$$r = 20\text{ km}$$

1. Directive gain, $$G_d = \frac{U_{max}}{(P_{rad}/4\pi)} = \frac{4\pi r^2 P_{ave}}{P_{rad}} = \frac{4\pi r^2\left[\frac{1}{2}\frac{E^2}{\eta}\right]}{P_{rad}} = \frac{2\pi r^2 E^2}{\eta P_{rad}}$$

[$\because U(\theta, \phi) = r^2 P_{ave}$]

$$\Rightarrow \quad G_d = \frac{2\pi \times (20 \times 10^3)^2 \times (12 \times 10^{-3})^2}{120\pi \times 100 \times 10^3}$$ [$\eta = 120\pi$ for air]

$\Rightarrow \quad G_d = 9.6 \times 10^{-3} = 0.0096$

In decibels,

$$\text{Directivity, } G = 10 \log_{10} G_d = 10 \log_{10} (0.0096) = -20.18 \text{ dB}$$

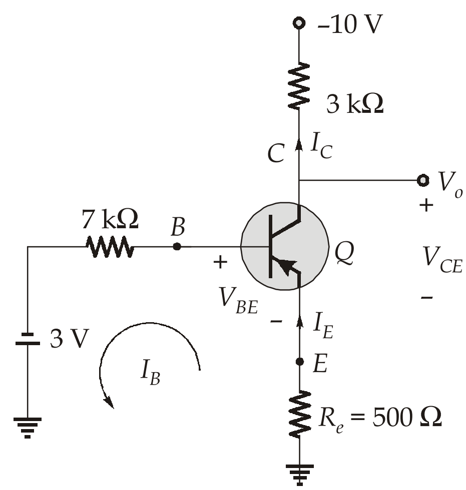
2. Maximum power gain,

$$G_P = \eta_r G_d = (0.98) \times (0.0096)$$

$$\Rightarrow \quad G_P = 9.408 \times 10^{-3}$$

Q.8 (c) Solution:

(i) Let the circuit is in saturation region i.e., $(V_{BE})_{sat} = -0.8$ V and $(V_{CE})_{sat} = -0.2$ V



Now, on applying KVL in input base loop, we get

$$(I_E \times 500) - V_{BE} + 7k(I_B)_{sat} - 3 = 0$$

where $\quad I_E = I_{Bsat} + I_{Csat}$

$$500I_{Bsat} + 500I_{Csat} + 7kI_{Bsat} = 3 + V_{BE}$$

$$500I_{Csat} + 7500I_{Bsat} = 3 - 0.8$$

$$500I_{Csat} + 7500I_{Bsat} = 2.2 \qquad ...(i)$$

Similarly on applying KVL in output collector loop, we get

$$+I_E 500 - V_{CE} + 3kI_{Csat} - 10 = 0$$

$$+500I_{Csat} + 500I_{Bsat} + 3000I_{Csat} = +10 + V_{CE}$$

$$3500I_{Csat} + 500I_{Bsat} = 10 - 0.2$$

$$3500I_{Csat} + 500I_{Bsat} = 9.8 \qquad ...(ii)$$

From equation (i) and (ii), we get

$$I_{Csat} = 2.78 \text{ mA}; I_{Bsat} = 0.1077 \text{ mA}$$

The minimum current required for the transistor to be in saturation region,

Calculate I_{Bmin}; $$I_{Bmin} = \frac{I_{Csat}}{\beta} = \frac{2.78 \text{ mA}}{100} = 27.8\ \mu\text{A}$$

We get, $$I_{Bmin} = 27.8\ \mu\text{A} = 0.0278 \text{ mA}$$

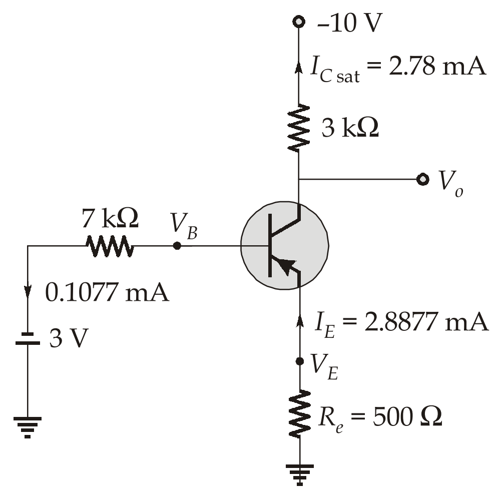
We have, $$I_{Bsat} = 0.1077 \text{ mA}$$

As, $$I_{Bsat} > I_{Bmin}$$

Thus, it implies the transistor is in saturation region.

Now, V_0

Using KCL at the collector terminal, we get



$$\frac{V_0 - (-10)}{3k} = 2.78 \times 10^{-3} \text{ A}$$

$$V_0 + 10 = (3 \times 2.78)$$

$$V_0 = (3 \times 2.78) - 10$$

$$V_0 = -1.66 \text{ Volt}$$

As $I_B = 0.1077$ mA then,

$$\frac{V_B - (-3)}{7k} = 0.1077 \text{ mA}$$

$$V_B + 3 = 0.1077 \times 7$$

$$V_B = -2.2461 \text{ Volt}$$

We know that in saturation region,

$$V_{BE} = -0.8 \text{ Volt}$$

i.e. $$V_B - V_E = -0.8$$

$$V_E = V_B + 0.8 = -2.2461 + 0.8 = -1.4461 \text{ Volt}$$

(ii) To prevent thermal runaway, the rate at which the heat is generated at the collector junction must not exceed the rate at which the heat can be dissipated i.e.

$$\frac{\partial P_C}{\partial T_j} < \frac{\partial P_D}{\partial T_j} \quad \text{...(i)}$$

where P_C is the heat generated at the collector junction.

In a steady state, in which the transistor is dissipating P_D watts, the temperature rise of the junction relative to the surrounding ambience can be expressed as follows:

$$T_j - T_A = \theta_{JA} \cdot P_D$$

Differentiating above equation w.r.t, we get

$$1 = \theta_{JA} \cdot \frac{\partial P_D}{\partial T_j}$$

$$\Rightarrow \qquad \frac{\partial P_D}{\partial T_j} = \frac{1}{\theta_{JA}}$$

From equation (i), to prevent themral runaway,

$$\frac{\partial P_c}{\partial T_j} < \frac{1}{\theta}$$

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