MADE EASY

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Leading Institute for ESE, GATE & PSUs

ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-1

Section A : Solid Mechanics [All topics]

Section B : Structural Analysis [All topics]

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Kolkata ☐	Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	60
Q.2	60
Q.3	39+2=41+2=43
Q.4	
Section-B	
Q.5	18
Q.6	
Q.7	35
Q.8	
Total Marks Obtained	212+2=214+2=216

Signature of Evaluator .... Cross Checked by ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

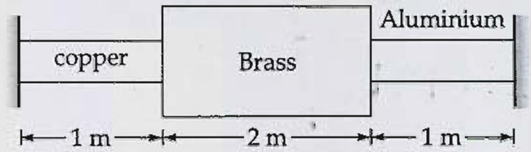
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Solid Mechanics

Q.1 (a) A rod is made of three segments as shown in figure below. Calculate the stresses in each material due to rise in temperature of 40°C when the walls yield by 0.2 mm.



$A_{Copper} = 200\ \text{mm}^2$, $A_{Brass} = 300\ \text{mm}^2$, $A_{aluminium} = 100\ \text{mm}^2$, $E_{Copper} = 1 \times 10^5\ \text{N/mm}^2$, $E_{brass} = 0.8 \times 10^5\ \text{N/mm}^2$, $E_{Aluminium} = 0.5 \times 10^5\ \text{N/mm}^2$, $\alpha_{Copper} = 5 \times 10^{-6}/°\text{C}$, $\alpha_{Brass} = 6 \times 10^{-6}/°\text{C}$, $\alpha_{Aluminium} = 7.5 \times 10^{-6}/°\text{C}$

[12 marks]

$\Delta T = 40°C$, yielding $= 0.2\,mm$

let the compressive stress be $\sigma_c, \sigma_b, \sigma_{Al}$

change in length = yielding of walls

$\sigma_{cu} \rightarrow$ Cu $\leftarrow \sigma_{cu}$ $\quad \sigma_{br} \rightarrow$ Br $\leftarrow \sigma_{br}$ $\quad \sigma_{Al} \rightarrow$ Al $\leftarrow \sigma_{Al}$

$$\left[L_c \alpha_c \Delta T - \frac{(\sigma_c) L_c}{E_c}\right] + \left[L_b \alpha_b \Delta T - \frac{\sigma_b L_b}{E_b}\right] + \left[L_{Al} \alpha_{Al} \Delta T - \frac{\sigma_{Al} L_{Al}}{E_{Al}}\right] = \text{yielding} \quad (1)$$

By force eq^n

$\sigma_{cu} A_{cu} = \sigma_b A_b \qquad \sigma_b \times A_b = \sigma_{Al} A_{Al}$

$\Rightarrow (\sigma_{cu}) 200 = \sigma_b \times (300) = \sigma_{Al} \times (100)$

$\boxed{\sigma_{Al} = 2(\sigma_{cu}) = 3(\sigma_b)}$

$\boxed{\sigma_{Al} = 3\sigma_b}$

$\boxed{\sigma_{cu} = 1.5\,\sigma_b}$

$$\left[1000 \times 5 \times 10^{-6} \times 40 - \frac{(1.5\sigma_b) \times 1000}{10^5}\right] + \left[2000 \times 6 \times 10^{-6} \times 40 - \frac{(\sigma_b) \times 2000}{0.8 \times 10^5}\right]$$

$$+ \left[1000 \times 7.5 \times 10^{-6} \times 40 - \frac{(3\sigma_b) \times 1000}{0.5 \times 10^5}\right] = 0.2$$

$$0.98 - 0.1\,\sigma_b = 0.2$$

$$0.78 = 0.1\,\sigma_b$$

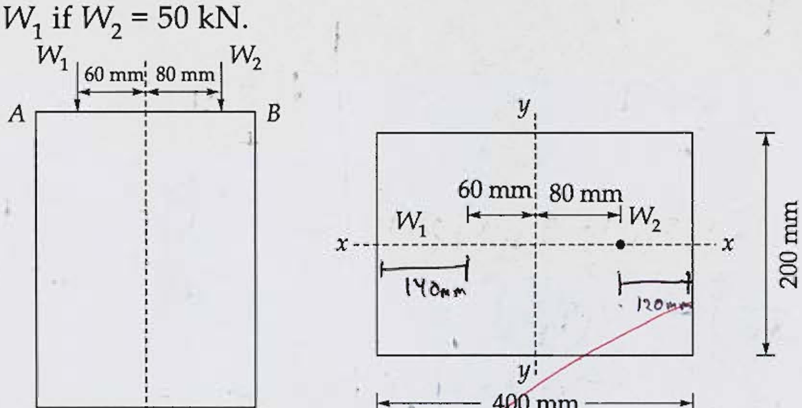
$$\boxed{\sigma_b = 7.8\,MPa}$$

$$\boxed{\sigma_{cu} = 11.7\,MPa}$$

$$\boxed{\sigma_{Ac} = 23.4\,MPa}$$

12

Q.1 (b) A short wooden pillar is rectangular in section 400 mm × 200 mm. It carries at the top, two point loads W_1 and W_2 in vertical plane as shown in figure below. If the stress is throughout compressive and extreme stress on the side in which W_1 acts i.e. at A is four times the extreme intensity on the other side i.e. at B, then compute the value of W_1 if W_2 = 50 kN.



(Elevation view) (Plain view)

[12 marks]

$$\boxed{\sigma_A = 4 \times \sigma_b}$$

$W_1 = ?$

$W_2 = 50\ kN$

bending stress due to W_1 on extreme side

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M}{I} \times y$$

$$(\sigma_{W_1})_b = \frac{W_1 \times 60 \times 10^3 \times 12 \times 200}{400^3 \times 200} = \boxed{0.01125\ W_1}$$

compressive stress due to W_1

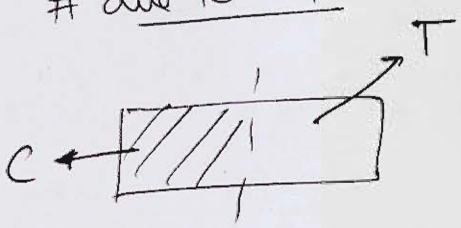
$$(\sigma_{W_1})_c = \frac{W_1 \times 10^3}{400 \times 200} = \boxed{0.0125\ W_1}$$

compressive stress due to W_2

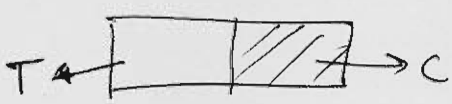
$$(\sigma_{W_2})_c = 0.625\ MPa$$

$$(\sigma_{W_2})_b = \frac{50 \times 10^3 \times \cancel{8}^{12} \times 80 \times 200}{400^3 \times 200} = \boxed{0.75\ MPa}$$

due to W_1



due to W_2



→ let compressive stress be +ve

total stress at A $= \left[(0.01125 W_1) + (0.0125 W_1) + 0.625 - 0.75\right]$

total stress at B $= \left[-0.01125 W_1 + 0.0125 W_1 + 0.625 + 0.75\right]$

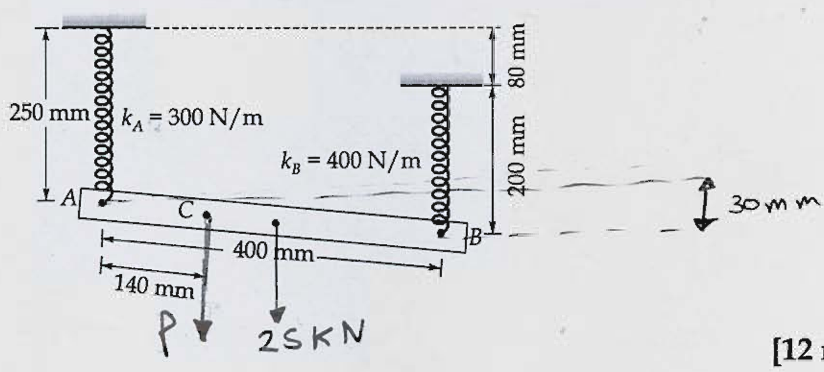
$$(\sigma_A)_T = 4 \times (\sigma_D)$$

$$[0.02375 W_1 - 0.125] = 4 \times [1.25 \times 10^{-3} W_1 + 1.375]$$

$$\boxed{W_1 = 300 \text{ KN}}$$

12

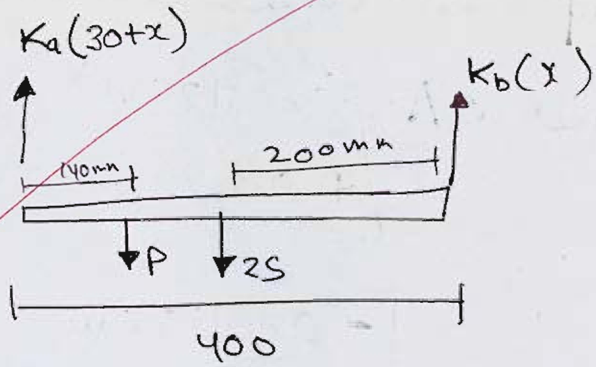
Q.1 (c) A uniform bar *AB* of weight 25 N is supported by two springs of natural length 250 mm and 200 mm respectively. Find the value of *P* that should be applied at *C* in order to bring the bar to a horizontal position.



[12 marks]

$K_A = 0.3 \text{ N/mm}$

$K_B = 0.4 \text{ N/mm}$



if $K_B \rightarrow$ moves by x

$K_A \rightarrow$ moves by $(30+x)$

$$[P+2S]\times 10^3 = K_A \times (30+x) + K_B(x)$$

$$\boxed{(P+2S)\times 10^3 = 0.3\times(30+x) + 0.4(x)}$$

$$\Sigma M_A = 0$$

$$\boxed{[10^3 \times P]\times 140 + 2S\times 10^3 \times 200 = 0.4(x)\times 400}$$

$$\boxed{140P + 5000 = 0.16x}$$

$$\boxed{(P+2S)\times 10^3 = 9 + 0.7x}$$

Note $\rightarrow$ assuming K_A & $K_B \xrightarrow{in}$ kN/m

$$\boxed{P+2S = 9+0.7x}$$

$$\boxed{140P + 5000 = 160x}$$

$$140\times(9+0.7x-2S) + 5000 = 160x$$

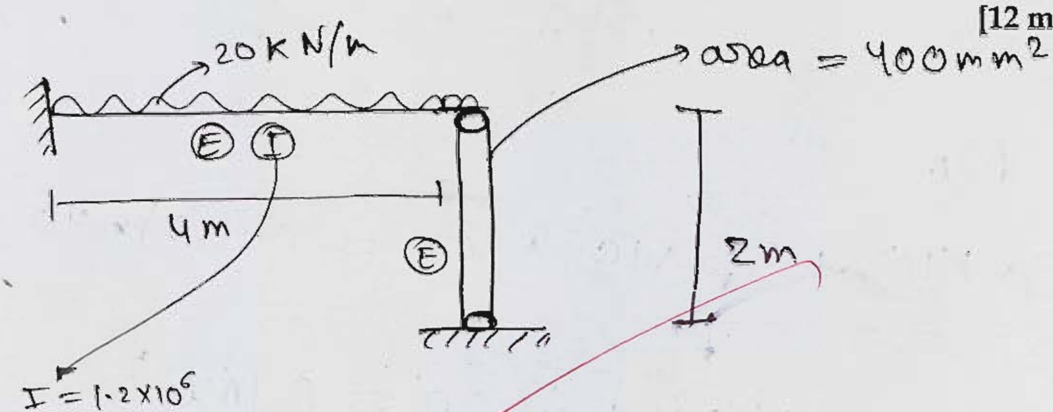
$$\boxed{x = 44.516\text{mm}}$$

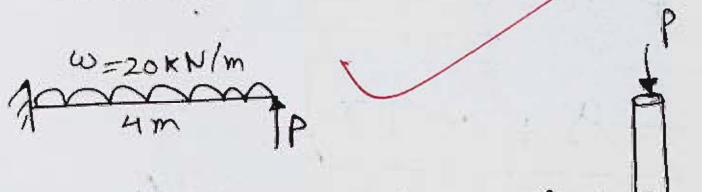
$$\Rightarrow \boxed{P = 15.1612 \text{ kN}}$$

12

Q.1 (d) A cantilever beam of length 4 m is subjected to a uniformly distributed load of 20 kN/m throughout its length. It is supported by a strut of length 2 m and area of cross-section 400 mm^2 modulus of elastcity for beam and strut is same, then find the load taken by strut. Moment of inertia of beam given as 1.2×10^6 mm^4.

[12 marks]





deflec^n of beam = axial deform^n of strut

$$\frac{wl^4}{8\cancel{E}I} - \frac{Pl^3}{3\cancel{E}I} = \frac{Pl_2}{A\cancel{E}}$$

$$\frac{(20)\times 4000^4}{8\times 1.2\times 10^6} - \frac{P\times 4000^3}{3\times 1.2\times 10^6} = \frac{P\times 2000}{400}$$

$$533.33\times 10^6 = (17782.78)\ P$$

$$P = 29.991\ kN$$

$$\boxed{P \simeq 30\ kN}$$

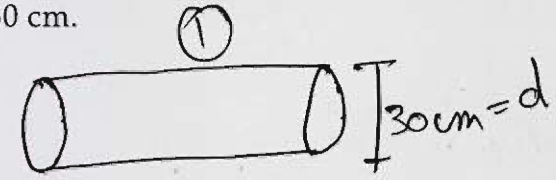
load taken by strut

12

Q.1 (e) A solid shaft of mild steel 30 cm in diameter is to be replaced by a hollow shaft of 30 cm diameter of alloy steel, for which the allowable shear stress is 40% greater. If the power to be transmitted is 35% greater to that transmitted by solid shaft and speed of rotation of hollow shaft is 3% greater than that of solid shaft, determine the maximum internal diameter of hollow shaft. Take external diameter of shaft is 30 cm.

[12 marks]

(1)



(2)

(τ_{max}) (P) (N)

($1.4\,\tau_{max}$) ($1.35\,P$) ($1.03\,N$)

By shear stress considerⁿ

$$\tau_{max} = \frac{T}{J} \times R = \frac{16\,T_1}{\pi d^3} \qquad (1.4)\tau_{max} = \frac{TR}{J} = \frac{16\,T_2}{\pi d_2^3\left[1-\left(\frac{d_1}{d_2}\right)^4\right]}$$

$$\cancel{\left[\frac{16\,T}{\pi \times (30)^3}\right] \times 1.4 = \frac{16\,T}{\pi \times 30^3 \times \left[1-\left(\frac{d_1}{30}\right)^4\right]}}$$

$$\cancel{1-\left(\frac{d_1}{30}\right)^4 = \frac{1}{1.4}}$$

$$\cancel{\Rightarrow \boxed{d_1 = 21.93\,mm}}$$

by Power considerⁿ

$$\boxed{\text{Power} = (\text{Torque}) \times (\omega)}$$

$$P = T_1\left(\frac{2\pi N}{60}\right) \qquad (1.35)\,P = T_2 \times \left(\frac{2\pi \times 1.03 N}{60}\right)$$

$$\Rightarrow 1.35 \times T_1 \times \left(\frac{2\pi N}{60}\right) = T_2 \times \left(\frac{2\pi \times 1.03\,N}{60}\right)$$

$$\boxed{T_2 = (1.31)\,T_1}$$

$$\left(\frac{16\,T_1}{\pi d^3}\right) \times 1.4 = \frac{16\,T_2}{\pi d_2^3\left[1-\left(\frac{d_1}{d_2}\right)^4\right]}$$

$$\left(\frac{\cancel{16}\,\cancel{T_1}}{\cancel{\pi}\times(30)^3}\right)\times 1.4 = \frac{\cancel{16}\times(1.31\,\cancel{T_1})}{\cancel{\pi}\times 30^3\times\left[1-\left(\frac{d_1}{30}\right)^4\right]}$$

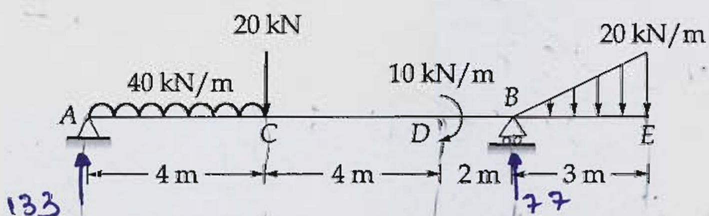
$$1-\left(\frac{d_1}{30}\right)^4 = \frac{1.31}{1.4}$$

$$\boxed{d_1 = 15.106\,mm}$$

maxm internal dia

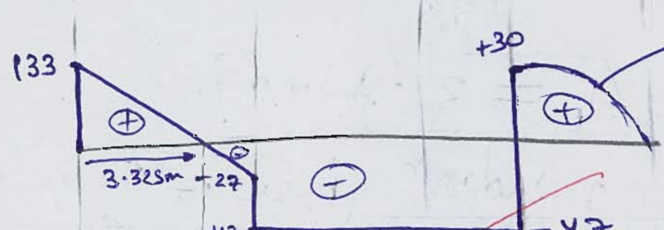
12

Q.2 (a) Draw the shear force diagram and bending moment diagram of the beam shown below.

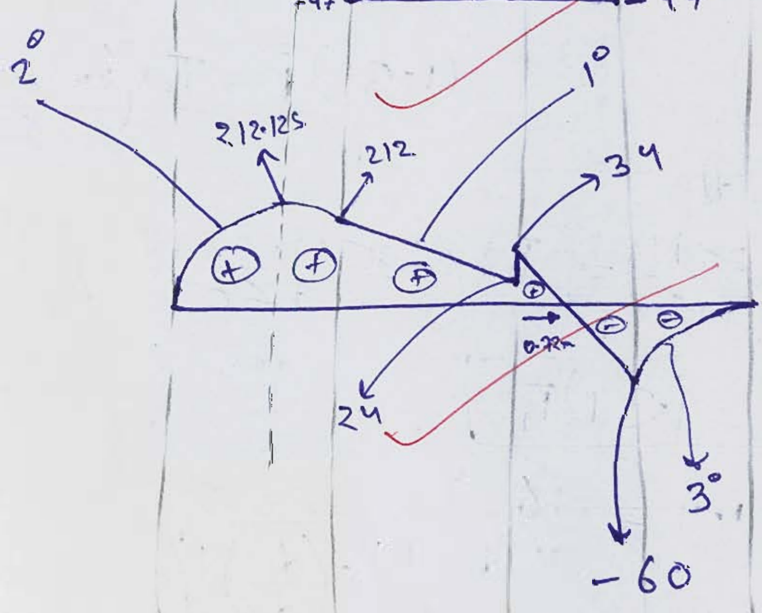


Also, find the maximum positive and negative bending moment.

[20 marks]



SFD



B MD

$\Sigma M_A = 0 = (40 \times 4) \times 2 + (20) \times 4 + 10 + (10 \times 3) \times 12 - V_B \times 10$

$\boxed{V_B = 77}$ $\boxed{V_A = 133\,kN}$

* SFD [taking x from A]

AC →

$V_x = 133 - 40x \xrightarrow{x=0} \boxed{133}$, $\xrightarrow{x=4} \boxed{-27}$

$\hookrightarrow V_x = 0$ @ $\boxed{x = 3.325m}$

CB →

$V_x = 133 - 160 + 20 = \boxed{-47}$

at B → just left $= \boxed{-47}$

$\hookrightarrow$ just Right $= -47 + 77 = \boxed{30}$

In BE

$V_x = 30 - \left(\frac{20(x-10)}{3 \times 2}\right) \times (x-10) = 30 - \frac{10(x-10)^2}{3}$

$\xrightarrow{x=10} \boxed{30}$, $\xrightarrow{x=13} \boxed{0}$

* BMD [taking x from A]

AC →

$M_x = 133x - 20x^2 \xrightarrow{x=0} \boxed{0}$, $\xrightarrow{x=4} \boxed{212}$

$\xrightarrow{x=3.325m} \boxed{212.125\,kN\text{-}m}$

CD

$M_x = 133x - 160 \times (x-2) - 20(x-4)$

$M_x = -47x + 400 \xrightarrow{x=4} \boxed{212}$, $\xrightarrow{x=8} \boxed{24}$

$\xrightarrow{x=8^+} \boxed{34}$

at DE → $M_x = -47x + 400 + 10$

$\xrightarrow{x=10} \boxed{(-60)}$

BE ~~[taking x from B]~~

$$M_x = -47x + 410 + 77(x-10) - \frac{10}{3}(x-10)^2 \times \left[\frac{x-10}{3}\right] \times 2$$

$$M_x = 30x - 360 - \frac{20}{9}(x-10)^3$$

$\xrightarrow{x=10} \boxed{-60}$

$\xrightarrow{x=13} \boxed{0}$

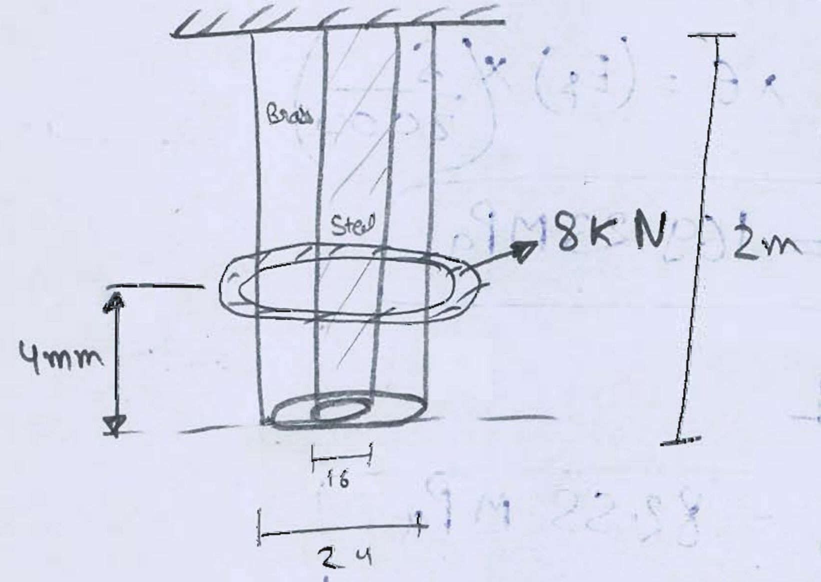
20

Do not write in this margin

Q.2 (b) A vertical composite tie bar rigidly fixed at the upper end consists of a steel rod of 16 mm diameter enclosed in a brass tube of 16 mm internal diameter and 24 mm external diameter, each being 2 m long. Both are fixed together at the ends. The tie bar is suddenly loaded by a weight of 8 kN falling through a distance of 4 mm. Determine the maximum stresses in the steel rod and the brass tube

Take: Young's modulus of elasticity, $E_{\text{Steel}} = 205$ GPa and $E_{\text{Brass}} = 100$ GPa.

$E_s = 2.05 \times 10^5$ MPa $\quad E_B = 10^5$ MPa **[20 marks]**



loss in PE of weight = gain in strain energy of brass & steel rod

$$mg(h+\delta) = \left[\frac{1}{2}\sigma_s \, \epsilon \times Vol^m\right]_{steel} + \left[\frac{1}{2}\sigma_b \, \epsilon (Vol^m)\right]_{brass}$$

$$mg(h+\delta) = \left[\frac{E_s}{2} \times (\epsilon^2) \times Vol^m\right]_{steel} + \left[\frac{E}{2} (\epsilon)^2 \times Vol^m\right]_{brass}$$

$$8000 \times (4+\delta) = \frac{2.05 \times 10^5}{2} \times \left[\frac{\delta}{2000}\right]^2 \times \frac{\pi}{4} \times 16^2 \times 2000$$

$$+ \frac{10^5}{2} \times \left(\frac{\delta}{2000}\right)^2 \times \frac{\pi}{4} \times (24^2 - 16^2) \times 2000$$

$$8000 \times (4+\delta) = \delta^2 \times (10304.42 + 6283.18)$$

$$(16587.6)\,\delta^2 - 8000(\delta) - 32000 = 0$$

$$\boxed{\delta = 1.651\,mm}$$

$$(\sigma_{steel}) = (E_s) \times \epsilon = (E_s) \times \left(\frac{\delta}{2000}\right)$$

$$\boxed{\sigma_{steel} = 169.23\,MPa}$$

20

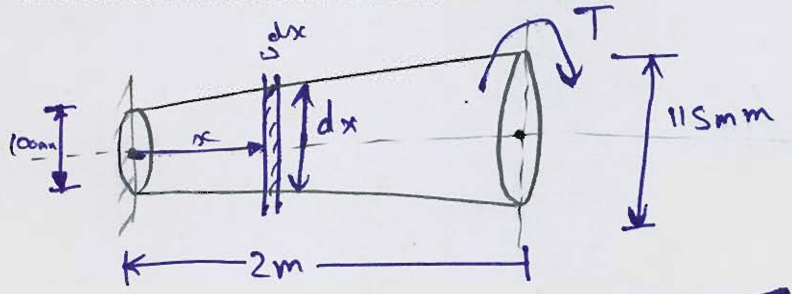
Similarly

$$\boxed{\sigma_{brass} = 82.55\,MPa}$$

Do not write in this margin

Q.2 (c) A solid circular shaft has a radius of 100 mm at one end and 115 mm at the other end the length of the shaft being 2 m. Calculate the percentage error in θ (angle of twist). If θ is calculated on the basis of the mean radius

[20 marks]



$$J = \frac{\pi d^4}{32}$$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \Rightarrow \left[\theta = \frac{TL}{GJ}\right]$$

$$\boxed{d_x = 100 + (7.5)x}$$

20

$$\int d\theta = \int_0^2 \frac{T\,dx \times 32}{G \times \pi \times [100 + 7.5x]^4}$$

actual value

$$\boxed{\theta = \left(\frac{32T}{\pi G}\right) \times 1.52215 \times 10^{-8}}$$

Based on mean radius $\Rightarrow d_{mean} = 107.5\,mm$

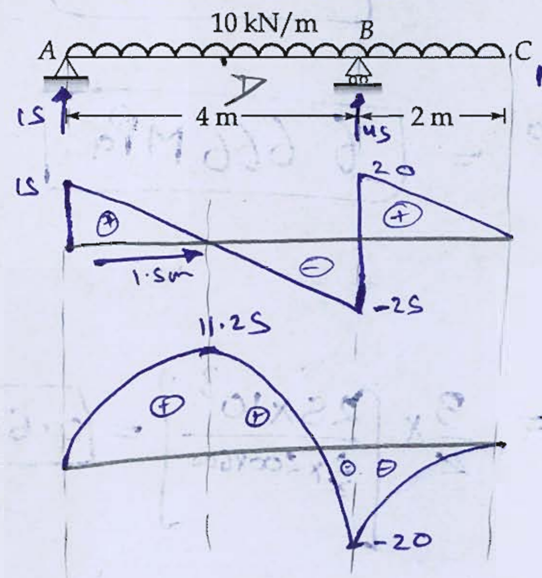
$$\boxed{\theta = \frac{32T \times 2}{\pi G \times 107.5^4} = \left(\frac{32T}{\pi G}\right) \times 1.4976 \times 10^{-8}}$$

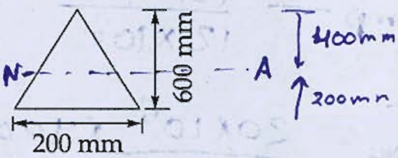
$$\%\text{ error} = \left[\frac{\text{actual} - \text{new value}}{\text{actual value}}\right] \times 100$$

$$\%\text{ error} = \frac{[1.52215 - 1.4976]}{(1.52215)} \times 100$$

$$\boxed{\%\text{ error} = 1.613\%}$$

Q.3 (a) A cantilever beam of triangular cross-section is loaded as shown in figure below. Compute the maximum tensile, compressive stresses at the location of maximum negative and maximum positive bending moment and maximum shearing stress and its location.





[20 marks]

SFD

$$\Sigma M_A = 0 = 10 \times 6 \times 3 - V_B \times 4 \Rightarrow \boxed{V_B = 45}$$

\# BMD [x from A]

\# AB

$$M_x = 15x - 5x^2 \quad x=0 \rightarrow M = 0, \quad x = 1.5 \rightarrow M = 11.25, \quad x = 4 \rightarrow M = -20$$

$$I_{NA} = \frac{bh^3}{36} = \boxed{12 \times 10^8 \ mm^4}$$

\# Max^m +ve B.M at D & Max^m -ve BM at B

& max^m shear stress at B

\# At D (Max^m +ve BM)

$$\sigma_{tensile} = \frac{M}{I} \times y_1 = \frac{11.25 \times 10^6 \times 200}{12 \times 10^8} = \boxed{1.875 \ MPa}$$

$$\sigma_{comp} = \frac{M}{I} \times y_2 = \frac{11.25 \times 10^6 \times 400}{12 \times 10^8} = \boxed{3.75 \ MPa}$$

B

AT C → (Max^{m} -ve B.M)

$$\sigma_{comp} = \frac{20\times10^6}{12\times10^8}\times200 = \boxed{3.333\,MPa}$$

$$\sigma_{Tens} = \frac{20\times10^6}{12\times10^8}\times400 = \boxed{6.666\,MPa}$$

B

at C

$V_{max} = 25\,kN$ (red correction: $95\,kN$)

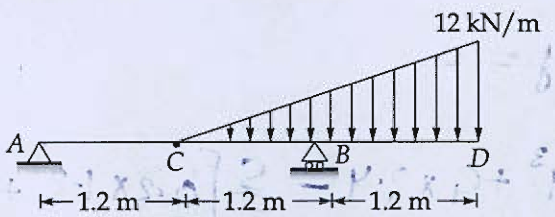
$$\tau_{max} = \left(\frac{3}{2}\right)(\tau_{avg}) = \frac{3}{2}\times\left[\frac{25\times10^3}{\frac{1}{2}\times200\times600}\right] = \boxed{0.625\,MPa}$$

↓

will occur at N-A

$18\frac{1}{2}$

Q.3 (b) An overhang beam *ABCD* is shown in figure. Determine the deflection at *C* and *D* and slope at *A* and *B* for the beam shown below.



Take flexural rigidity of beam as 300 kN-m^2.

$\boxed{EI = 300 \text{ kN-m}^2}$

[20 marks]

$\Sigma M_A = 0 = (6 \times 2.4) \times \left(1.2 + 2.4 \times \frac{2}{3}\right) - V_B \times 2.4$

$\boxed{V_B = 16.8 \text{ kN}}$

$\boxed{V_A = -24 \text{ kN}}$

using double integration

$EI \frac{d^2y}{dx^2} = M = [2.4 \times (x)] - \frac{5}{2}(x-1.2) \times \left[(x-1.2) \times \frac{2}{3}\right] \quad 16.8 \times (x-2.4) + R_B(x-2.4)$

$\delta_c = ?$ $\quad \delta_D = ?$ $\quad \theta_A = ?$ $\quad \theta_B = ?$

$$EI\frac{d^2y}{dx^2} = -2.4x \quad \Big| _{x=1.2} - \frac{5}{2}\times(x-1.2)\times\left[1.2+(x-1.2)\times\frac{2}{3}\right] \Big| _{x=2.4} + 16.8\times(x-2.4)$$

$$EI\frac{dy}{dx} = -1.2x^2 + C_1 \quad -\frac{5}{2}\left[0.6(x-1.2)^2 + \frac{2}{3}\frac{(x-1.2)^3}{3}\right] + 8.4(x-2.4)^2$$

$$EIy = -0.4x^3 + C_1x + C_2 \quad -\frac{5}{2}\left[0.2(x-1.2)^3 + \frac{2\times(x-1.2)^4}{36}\right] + 2.8(x-2.4)^3$$

boundary Conditions

$x = 0 \Rightarrow y = 0 \Rightarrow \boxed{C_2 = 0}$

$x = 2.4 \Rightarrow y = 0$

$$0 = -0.4\times 2.4^3 + C_1\times 2.4 - \frac{5}{2}\left[0.2\times 1.2^3 + \frac{2}{36}\times 1.2^4\right]$$

$$\Rightarrow \boxed{C_1 = 2.784}$$

\# Calcul^n of $\theta_A \Rightarrow x = 0$

$$EI\frac{dy}{dx} = 2.784$$

$$\frac{dy}{dx} = \theta_A = \frac{2.784}{300}$$

$$\boxed{\theta_A = 9.28\times 10^{-3}} \text{ radians}$$

\# Calculate $\theta_B \Rightarrow x = 2.4$

$$EI(\theta_B) = -1.2\times 2.4^2 + 2.784 - \frac{5}{2}\left[0.6\times 1.2^2 + \frac{2}{9}\times 1.2^3\right]$$

$$\theta_B = -\frac{5.328}{200} \qquad \boxed{\theta_B = -2.664 \times 10^{-2}}$$

3

\# Calculate δ_c $\Rightarrow x = 1.2$

$$EIy = -0.4 \times 1.2^2 + 2.784 \times 1.2$$

$$\boxed{\delta_c = 13.824 \text{ mm}} \uparrow$$

\# Calculate δ_D $\Rightarrow x = 3.6$

$$EI\,\delta_D = -0.4 \times 3.6^3 + 2.784 \times 3.6$$
$$- 2.5 \times \left[0.2 \times 2.4^3 + \frac{2}{36} \times (2.4)^4\right]$$
$$+ 2.8 \times (1.2)^3$$

$$\boxed{\delta_D = 76.608 \text{ mm}} \downarrow$$

Do not write in this margin

Q.3 (c) A hollow circular steel shaft is required to carry a torque of 40 kN-m and bending moment of 20 kN-m. If the internal diameter is 60% of external diameter, then determine size of shaft by

(i) Maximum principal stress theory

(ii) Maximum strain energy theory

Factor of safety = 2, $\mu = 0.3$ and $f_y = 250\ \text{N/mm}^2$

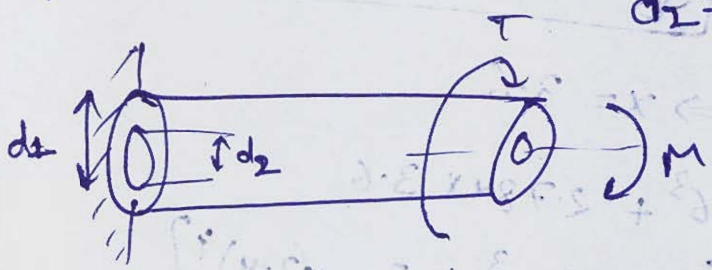
[8 + 12 = 20 marks]

$T = 40\ \text{KN-m}$

$M = 20\ \text{KNm}$

$d_1 = d$

$d_2 = 0.6d$



$$\sigma_b = \frac{M}{I} \times y = \frac{32M}{\pi d_1^3 \times \left[1 - \left(\frac{d_2}{d_1}\right)^4\right]} = \frac{32M}{\pi d_1^3 \times (0.8704)}$$

$$\tau = \frac{T}{J} \times R = \frac{16T}{\pi d_1^3 \times \left[1 - \left(\frac{d_2}{d_1}\right)^4\right]} = \frac{16T}{\pi d_1^3 \times (0.8704)}$$

$\frac{d_1}{d_2} = 0.6$

$$\sigma_{p_1/p_2} = \frac{\sigma_b}{2} \pm \frac{1}{2}\sqrt{(\sigma_b)^2 + 4(\tau)^2}$$

$$\sigma_{p_1/p_2} = \frac{16M}{\pi d^3 \times (0.8704)} \pm \frac{1}{\cancel{2}} \times \frac{16}{\pi d^3 \times 0.8704} \times \sqrt{\cancel{4}(M^2 + T^2)}$$

$$\sigma_{p_1/p_2} = \frac{16}{\pi d^3 \times (0.8704)} \times \left[M \pm \sqrt{M^2 + T^2}\right]$$

$M = 20 \times 10^6 \quad T = 40 \times 10^6$

$$\boxed{\sigma_{p_1} = \frac{378.703 \times 10^6}{d^3}} \qquad \boxed{\sigma_{p_2} = \frac{-144.65 \times 10^6}{d^3}}$$

(i) Max^m principle stress theory

$$(\sigma_p)_{max} \leq \frac{\sigma_y}{FOS}$$

$$\frac{378.703\times10^6}{d^3} \leq \frac{250}{2}$$

$$d \geq 144.698$$

$$\boxed{d = 145\,mm} \quad \boxed{d_i = 87\,mm}$$

(ii)
$$\frac{1}{2E}\left[\sigma_{p_1}^2 + \sigma_{p_2}^2 - 2\mu\,\sigma_{p_1}\sigma_{p_2}\right] \leq \frac{1}{2E}\left(\frac{\sigma_y}{FOS}\right)^2$$

$$\frac{10^{12}}{d^6}\times\left[197207.22\right] \leq \frac{250^2}{2\times2}$$

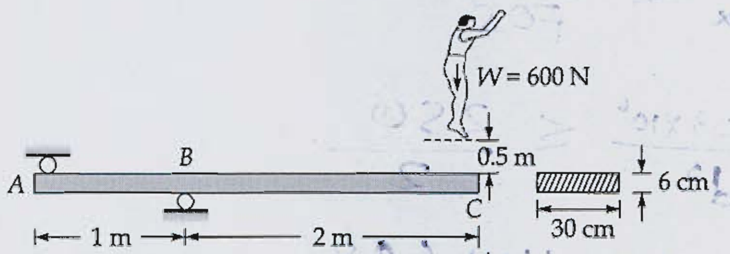
18+2

$$d \geq 152.58\,mm$$

$$\boxed{d = 155\,mm} \quad \boxed{d_i = 93\,mm}$$

$\rightarrow$ 152.5 $\qquad \rightarrow$ 91.8

Q.4 (a) A man weighing 600 N jumps from a height of 0.5 m on a diving board of dimensions 30 cm × 6 cm supported as shown in figure. Find the maximum stress produced in the board.



Take, $E = 10$ GPa.

[20 marks]

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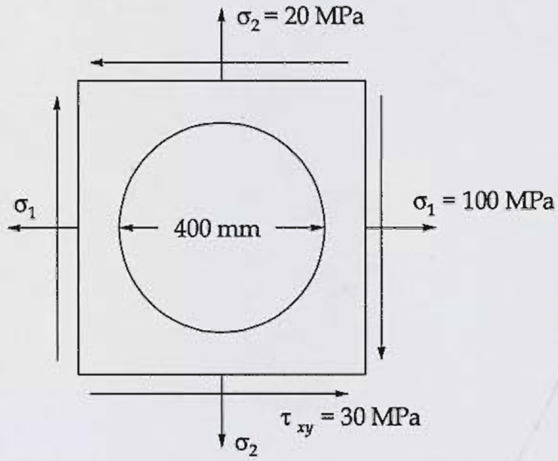
Q.4 (b) A hollow cast iron column with fixed ends supports an axial load of 800 kN. If the column is 3 m long and has an external diameter of 200 mm, find the thickness ofmetal required. Use Rankine's formula, taking a constant of $\frac{1}{1600}$ and assume a working stress of 90 N/mm^2.

[20 marks]

Q.4 (c) A circle of 400 mm diameter is scribed on a mild steel place before it is subjected to stresses as shown in figure. In stressing the circle deforms to an ellipse. Calculate the lengths of the major and minor axes of the ellipse and also find their directions.

Take $\mu = 0.286$ and $E = 205\ kN/mm^2$.



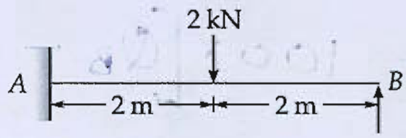
[20 marks]

Section B : Structural Analysis

Q.5 (a) For the propped cantilever beam shown in figure below using slope deflection equation, find the moments at support '*A*' when (EI = 2 × 10^6 kN-cm^2)

(i) The supports are at the same level.

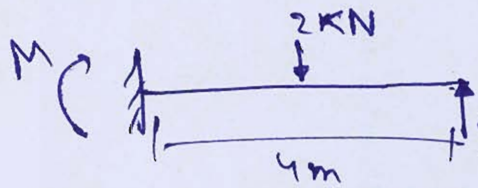
(ii) The support '*B*' sinks by 1 cm.



[6 + 6 = 12 marks]

$$\boxed{M_{AB} = (M_F)_{AB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)}$$

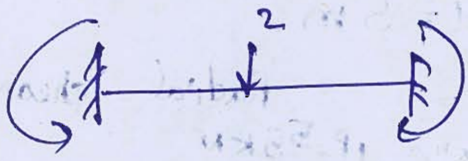
(i)



$\frac{5\times\cancel{2}}{\cancel{16}\,8} = \frac{5}{8}$

$(M)_{AB} = \frac{5}{8}\times 4 - 2\times 2 = (-1.5\ kN-m)$

~~MAB~~



1

$(M_F)_{AB} = -\frac{2\times 4}{8} = -1$

$$M_{AB} = (-1) + \frac{2EI}{4}\left[2\cancel{\theta_A}^{0} + \theta_B - \frac{3\cancel{\delta}^{0}}{l}\right]$$

$$\boxed{M_{AB} = -1 + \frac{EI\,\theta_B}{2}}$$

$$M_{AB} = -1 + \frac{\cancel{2}\times\cancel{10^6}^{100}\,\theta_B}{\cancel{2}\times\cancel{10^4}}$$

$$\boxed{M_{AB} = -1 + 100\,\theta_B}$$

(ii) If support B sinks by 1 mm

Similarly

1

$$M_{AB} = -1 + 100 \times \left[\theta_B - \frac{3 \times 1}{400} \right]$$

$$\boxed{M_{AB} = -1.75 + 100\,\theta_B}$$

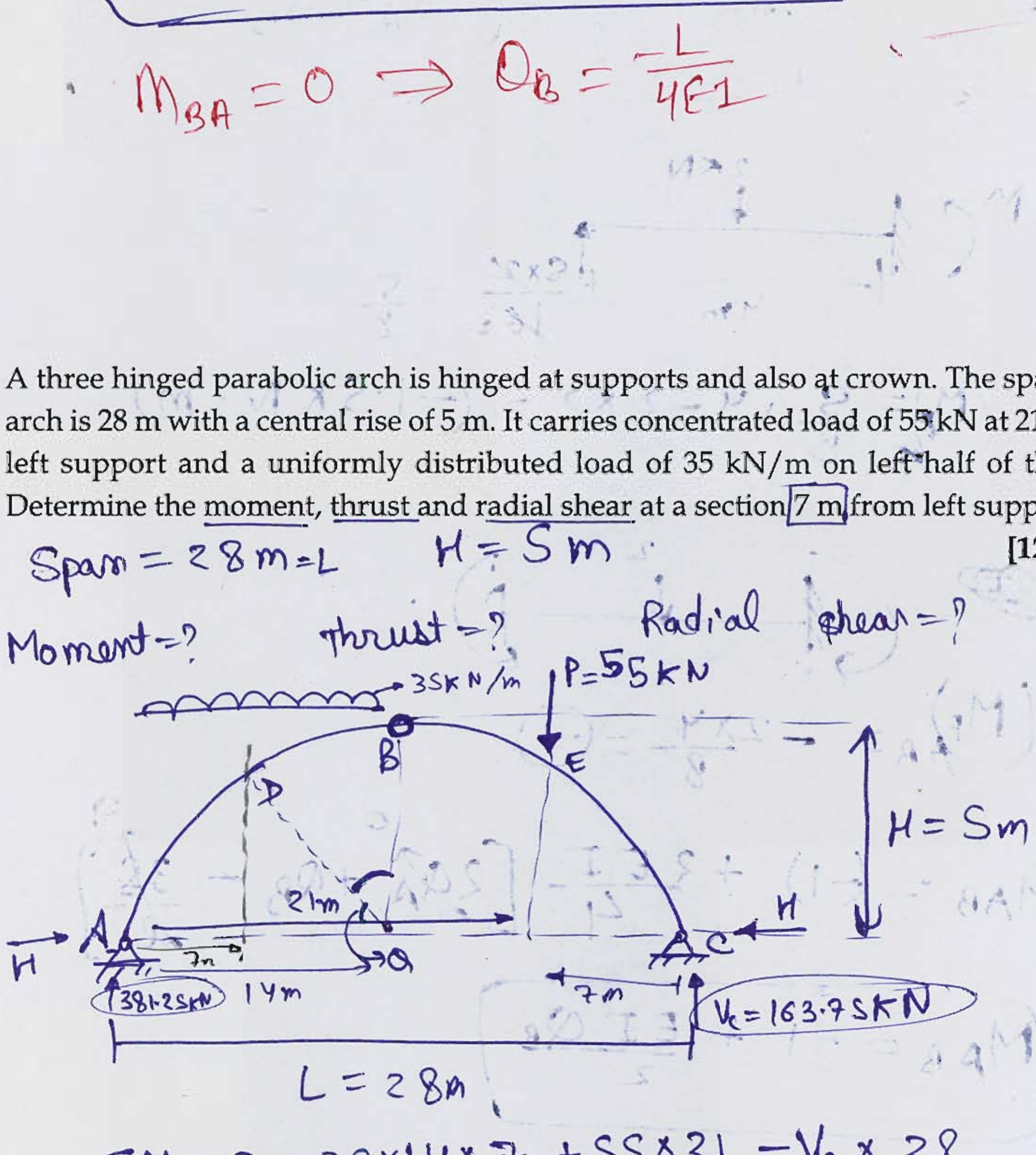
$$M_{BA} = 0 \Rightarrow \theta_B = \frac{-L}{4EI}$$

Q.5 (b) A three hinged parabolic arch is hinged at supports and also at crown. The span of the arch is 28 m with a central rise of 5 m. It carries concentrated load of 55 kN at 21 m from left support and a uniformly distributed load of 35 kN/m on left half of the span. Determine the moment, thrust and radial shear at a section 7 m from left support.

[12 marks]

Span = 28 m = L H = 5 m

Moment = ? thrust = ? Radial shear = ?



$$\Sigma M_A = 0 = 35 \times 14 \times 7 + 55 \times 21 - V_c \times 28$$

$$V_c = 163.75$$

$$V_A = 381.25\text{ kN}$$

$$(\Sigma M_B)_{Righ} = 0 = 55 \times 7 + H \times 5 - V_c \times 14$$

$\boxed{H = 381.5\ kN}$ → Horizontal thrust

4

Momont at 7m

~~M − 381.25 × 7 − 381.25~~

$$y = \frac{4h}{l^2}(l-x)(x) = \frac{4 \times 5}{28^2} \times (28-x)(x)$$

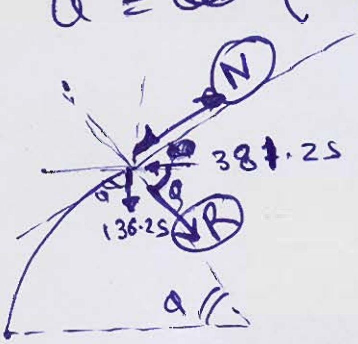
at $x = 7$

$$\boxed{y = 3.75\ m}$$

$$M = 381.25 \times 7 - 381.25 \times 3.75 - 35 \times 7 \times \frac{7}{2}$$

$$\boxed{M_{x=7} = 1239.0625\ kN\text{-}m}$$

$$\theta = \sin^{-1}\left(\frac{3.75}{14}\right) = \boxed{15.537^\circ}$$



Radial shear $= R = 136.25 \sin\theta - 381.25 \cos\theta$

$$\boxed{R = -330.82\ kN}$$

Thrust $= N = 136.25 \cos\theta + 381.25 \sin\theta$

$$\boxed{N = 233.4\ kN}$$

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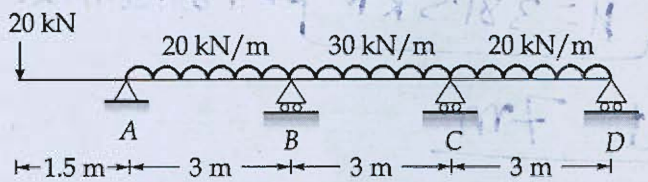
Q.5 (c) For the continuous girder shown in the figure. Find the support moment using theorem of three moments and draw the B.M. diagram with net bending moment at each span.

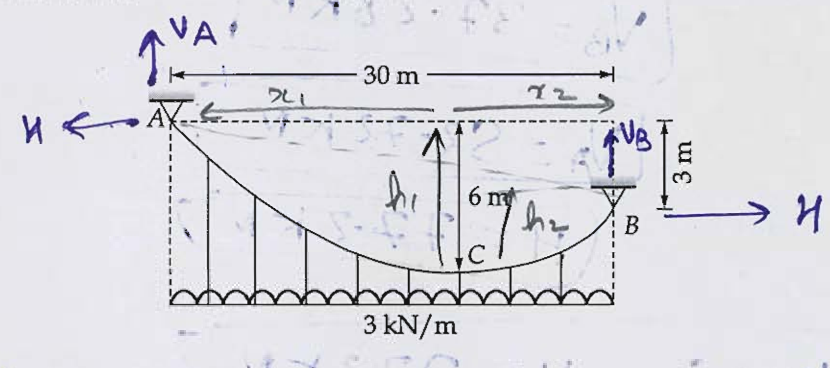


[12 marks]

$$M_1 \times \left(\frac{L_1}{I_1}\right) + M_2 \times \left(\frac{L_2}{I_2} + \frac{L_1}{I_1}\right) + M_3 \times \left(\frac{L_2}{I_2}\right)$$

$$+ \frac{6a_1\bar{x}_1}{A_1 I_1} + \frac{6a_2\bar{x}_2}{A_2 I_2} = 6E\left(\frac{\delta_3}{L_2}\right)$$

Q.5 (d) A cable is supported between two points 30 m horizontally apart. The left support is 3 m above the right support. The cable carries a load of 3 kN/m on the horizontal span. The lowest point of the cable is 6m below the left support. Find the maximum and minimum tension in the cable.



[12 marks]

minimum tension at C

max^m tension at A

$$\boxed{V_A + V_B = 90}$$

$$\Sigma M_A = 0 = 90 \times 15 - V_B \times 30 - H \times 3$$

$$\boxed{H + 10\,V_B = 450}$$

at C BM=0

12

$$\frac{x_1}{\sqrt{h_1}} = \frac{x_2}{\sqrt{h_2}}$$

$$\frac{x_1}{\sqrt{2}\cancel{\sqrt{6}}} = \frac{x_2}{\cancel{\sqrt{3}}}$$

$$\boxed{x_1 = \sqrt{2}\, x_2}$$

$$x_1 + x_2 = 30$$

$$\boxed{x_2 = 12.43 m}$$

$$\boxed{x_1 = 17.57 m}$$

$$3 \times \frac{(12.43)^2}{2} + H \times 3$$

$$\parallel$$

$$V_B \times 12.43$$

$$\boxed{12.43\ V_B = 3H + 231.76}$$

$$H + 10V_B = 450 \Rightarrow \boxed{H = 450 - 10V_B}$$

$$12.43\ V_B = 3 \times (450 - 10V_B) + 231.76$$

$$\boxed{V_B = 37.28 KN}$$

$$\boxed{V_A = 52.72 KN}$$

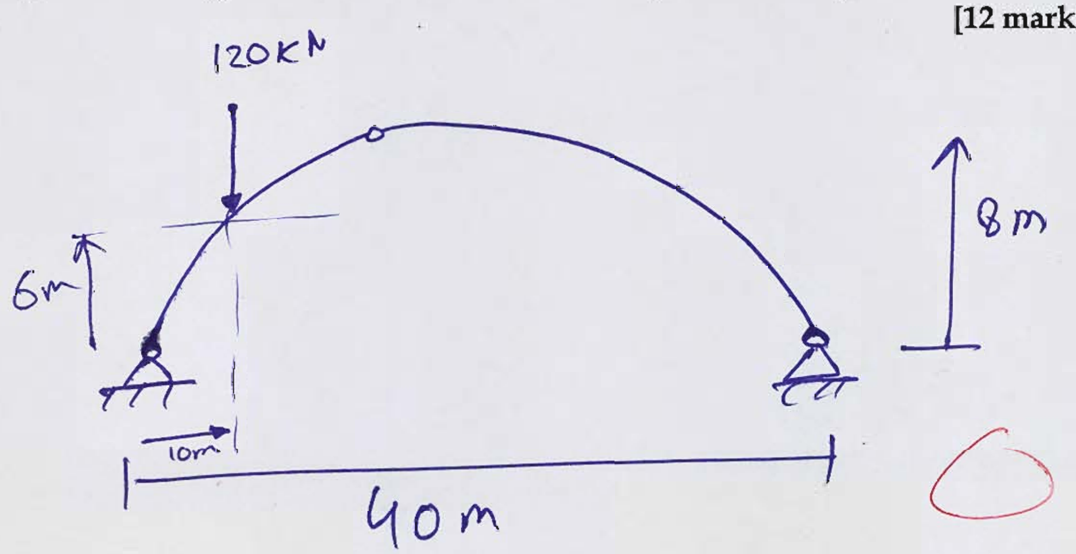
$$\boxed{H = 77.2 KN}$$

$$\boxed{\text{min}^m \text{ tension} = H = 77.2 KN}$$

$$\boxed{\text{max}^n \text{ tension} = \sqrt{V_A^2 + H^2} = 93.48 KN}$$

Q.5 (e) A three-hinged circular arch of span 40 m and rise 8 m carries a concentrated load of 120 kN at a horizontal distance of 10 m from the left end. Find the maximum positive and negative bending moments and draw the bending moment diagram.

[12 marks]



$$y = \frac{4h}{l^2}(l-x)\times x = \boxed{0.02(40-x)(x)}$$

at $x = 10$

$$\boxed{y = 6m}$$

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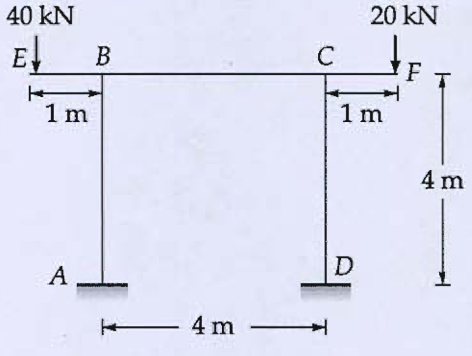
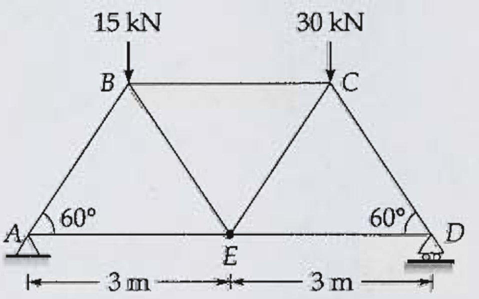
Q.6 (a) Analyse the portal frame shown in figure by using slope-deflection method. Take EI as constant and draw the bending moment diagram.



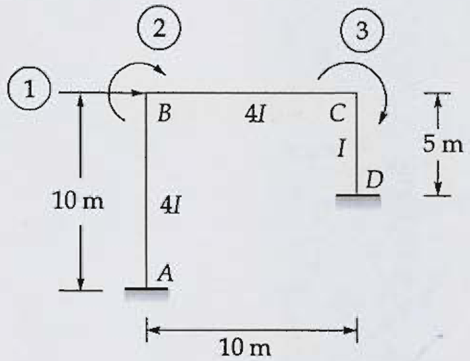
[20 marks]

Q.6 (b) Find the vertical deflection of the joint E of the truss shown in figure below. The sectional area of each member is 1200 mm^2. Take $E = 2 \times 10^5$ N/mm^2.



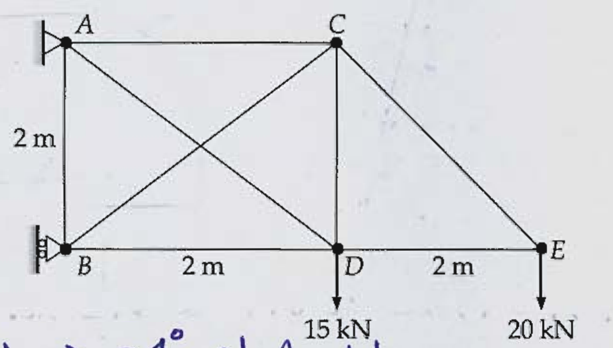
[20 marks]

Q.6 (c) Develop the stiffness matrix for portal frame ABCD with reference to the coordinates shown in figure.



[20 marks]

Q.7 (a) Find the forces in all members of the redundant plane truss shown in figure below. Cross-sectional area of each bar is 1000 mm² and $E = 2 \times 10^5$ N/mm².

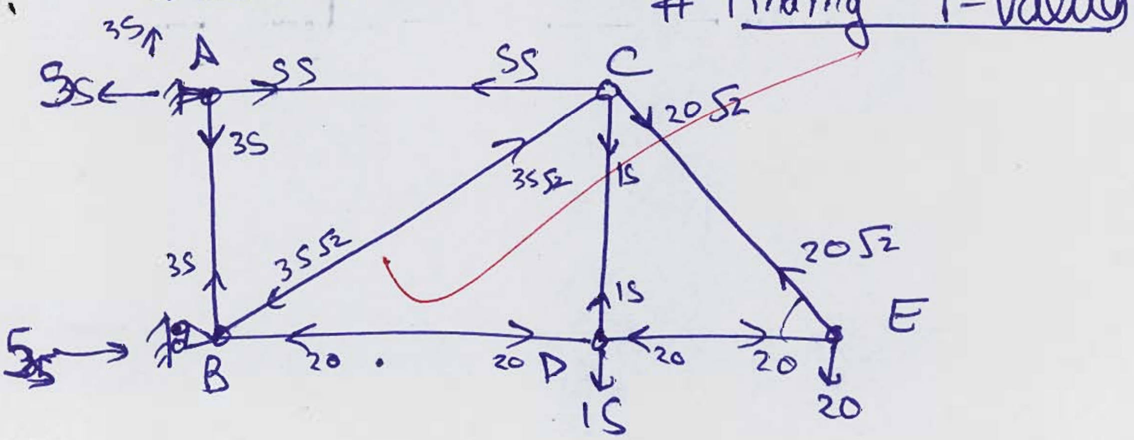


[20 marks]

$m-(2j-3)=1 \Rightarrow 1^\circ$ of Indeterminancy

Let us take AD as redundant member & remove it

Finding P-Value



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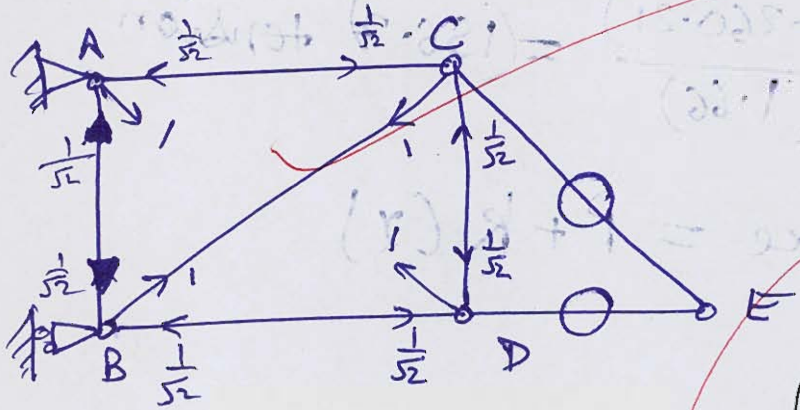
$\Sigma M_B = 0 = 15 \times 2 + 20 \times 4 - H_A \times 2$

$\boxed{H_A = 55}$

5

Finding K - Values



Positive

(Final Forces)

Member	P	R	l	PRl	$R^2 l$	$P + Rx$
AB	+35	$\frac{-1}{\sqrt{2}}$	2	−49.5	−1	−75.8
BD	−20	$-\frac{1}{\sqrt{2}}$	2	+28.28	−1	−130.8
CD	+15	$\frac{-1}{\sqrt{2}}$	2	−21.21	−1	−95.8
AC	+55	$-\frac{1}{\sqrt{2}}$	2	−77.78	−1	−55.8
AD	0	+1	$2\sqrt{2}$	0	2.83	156.7
BC	$-35\sqrt{2}$	+1	$2\sqrt{2}$	−140	2.83	107.2
CE	$+20\sqrt{2}$	0	$2\sqrt{2}$	0	0	$20\sqrt{2}$
DE	−20	0	2	0	0	−20
				$\Sigma = -260.21$	$\Sigma = 1.66$	

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$$x = F_{AD} = \frac{-\left(\Sigma \frac{PKl}{AE}\right)}{\left(\Sigma \frac{K^2 l}{AE}\right)} = \frac{-(\Sigma PKl)}{(\Sigma K^2 l)}$$

$$x = -\frac{(-260.21)}{(1.66)} = 156.7 \text{ tension}$$

Final Force $= P + K(x)$

Final Forces?

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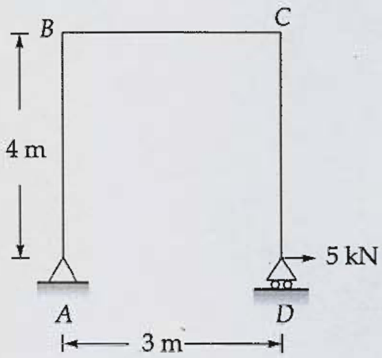
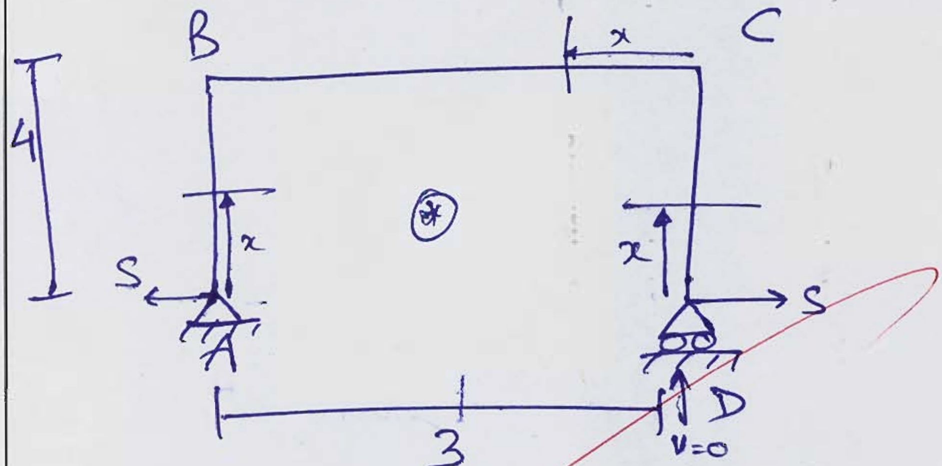
Q.7 (b) Determine the horizontal displacement of the roller support D of the portal frame shown in figure. Take EI as 8000 kNm^2 for all members.



[20 marks]

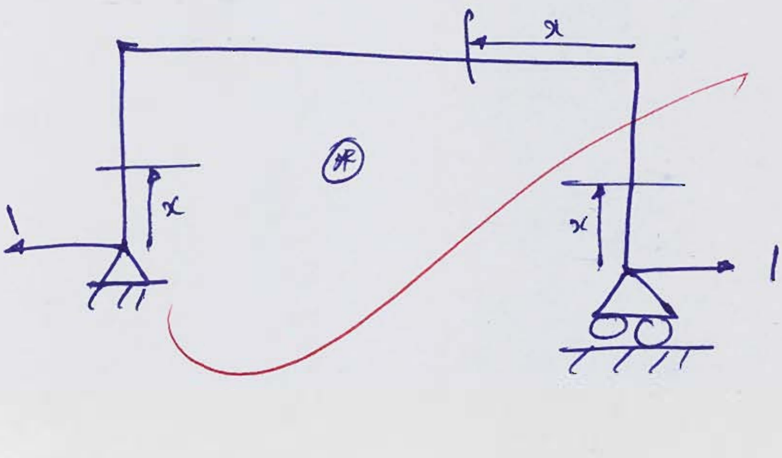
$$\boxed{\delta = \Sigma \int \frac{M m \, dx}{EI}}$$

Value of M



$\Sigma M_A = 0 = V \times 3 \Rightarrow V = 0$

Value of m



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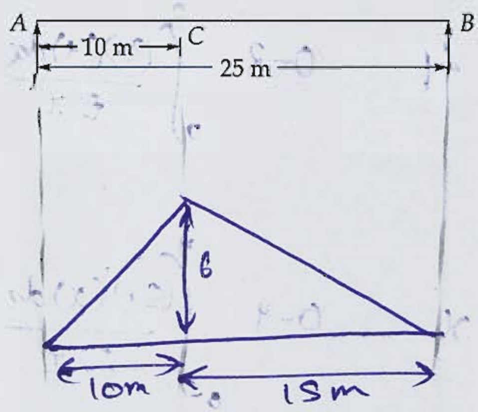
Member	M	m	limits	$\int \frac{M m dx}{EI}$
AB	$5x$	x	0–4	$\int_0^4 \frac{(x)(5x)dx}{EI} = \boxed{\frac{106.67}{EI}}$
BC	20	4	0–3	$\int_0^3 \frac{(20)(4)dx}{EI} = \boxed{\frac{240}{EI}}$
CD	$5x$	x	0–4	$\int_0^4 \frac{(5x)(x)dx}{EI} = \boxed{\frac{106.67}{EI}}$

20

$$(\delta_H)_D = \Sigma \left(\frac{Mmdx}{EI}\right) = \boxed{\frac{453.34}{EI}}$$

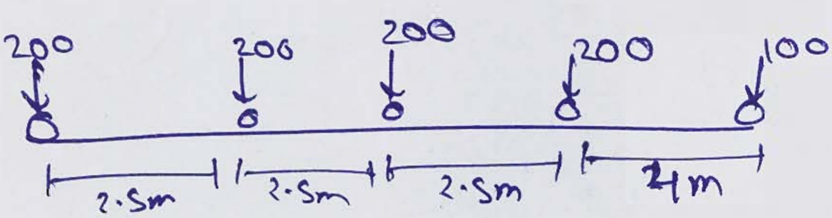
$$(\delta_H)_D = \frac{453.34}{8000} = \boxed{56.6675\,mm}$$

Q.7 (c) Draw the influence line diagram for bending moment at a point 10 m distant from the left-hand abutment on a bridge of span 25 m. Find the maximum bending moment at the point C due to a series of wheel loads 100 kN, 200 kN, 200 kN, 200 kN, 200 kN at centre to centre distance of 4 m, 2.5 m, 2.5 m, and 2.5 m. The loads can cross in either direction, 100 kN load leading in each case.



[20 marks]

BM for ILD $\Rightarrow$ Peak $= \dfrac{ab}{l} = \dfrac{10 \times 15}{25} = 6$



when first 100 kN crosses

avg load on left $= \dfrac{800}{10} = 80$

avg load on right $= \dfrac{100}{15} = 6.67$

when first 200 kN cross

avg ~~left~~ $\rightarrow \dfrac{600}{10} = 60$

avg right $\rightarrow \dfrac{300}{15} = 20$

when 2nd 200 kN crosses

avg left $\rightarrow \dfrac{400}{10} \rightarrow 40$

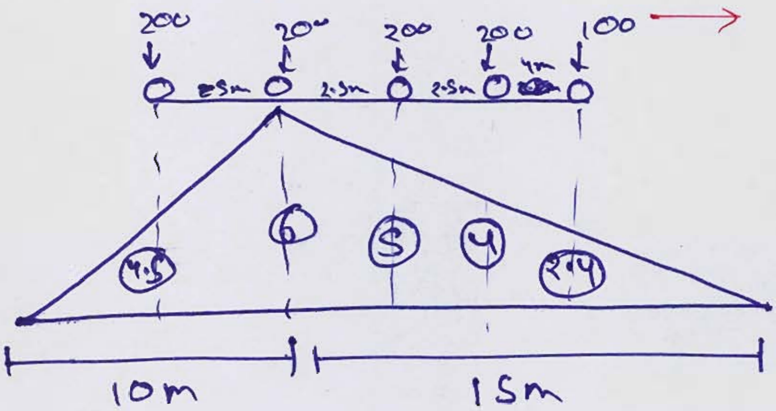
avg right $\rightarrow \dfrac{500}{15} \rightarrow 33.33$

when 3rd 200 kN at C

avg load on left $= \frac{200}{10} = 20$

avg load on right $= \frac{700}{15} = 46.67$

$\Rightarrow$ critical case

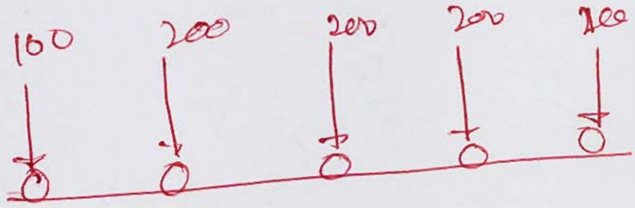


10

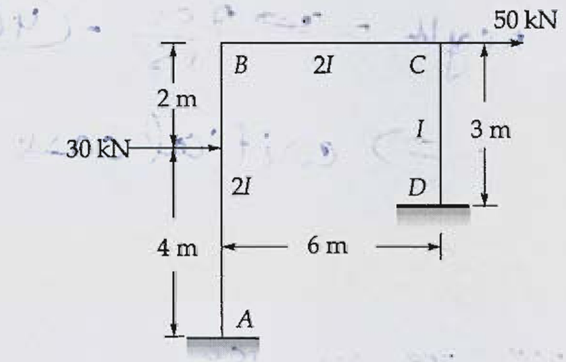
$$Max^m\ B.M = 200\times4.5 + 200\times6 + 200\times5 + 200\times4 + 100\times2.4$$

$$\boxed{(BM)_{max} = 4140\ kN\text{-}m}$$

2nd case ?



Q.8 (a) Analyse the frame shown in figure by moment distribution method and sketch bending moment diagram.



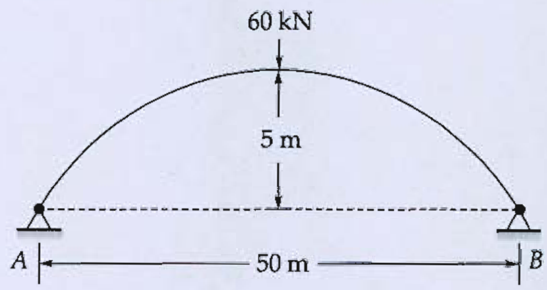
[20 marks]

Q.8 (b) A two-hinged parabolic arch of span 50 m and rise 5 m is subjected to a central concentrated load of 60 kN. It has an elastic support which yields by 0.0001 mm/kN.

Taking, $E = 200$ kN/mm^2, $I = 5 \times 10^9$ mm^4, Average area, $A_m = 10000$ mm^2, $\alpha = 12 \times 10^{-6}$/°C and assuming secant variation, calculate the horizontal thrust developed when the temperature rises by 20°C.

(i) Neglecting rib shortening.

(ii) Considering rib shortening.

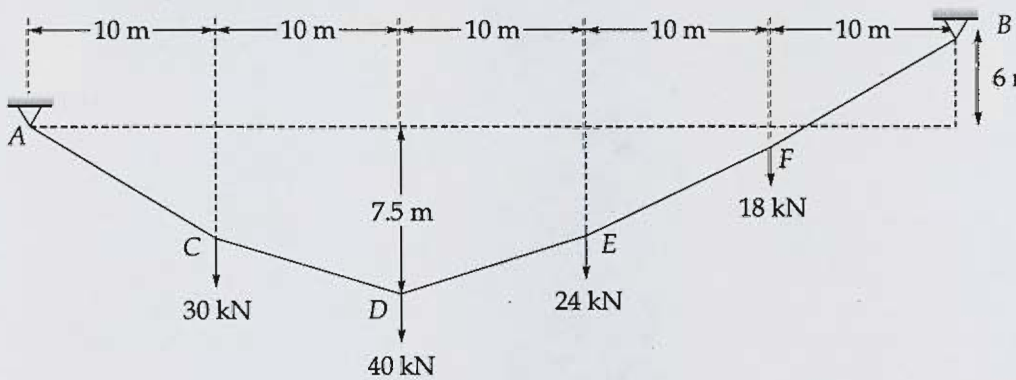


[10 +10 = 20 marks]

Q.8 (c) A cable ACDEFB supports a set of vertical hangers at four intermediate points (C, D, E, T). The span between the supports *A* and *B* is 50 m. The lowest point of the cable (D) is located 7.5 m below the left support A which in turn is located 6 m below the right support B. The vertical loads applied through the hangers at points C, D, E and F are 30 kN, 40 kN, 24 kN and 18 kN respectively, placed at equal intervals draw the funcular polygonal and find.

(i) The tension in each segment of cable and inclination of each segment of the cable from horizontal

(ii) The final length of the cable.



[20 marks]

■■■■

Space for Rough Work

Space for Rough Work