

UPSC ENGINEERING SERVICES EXAMINATION

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

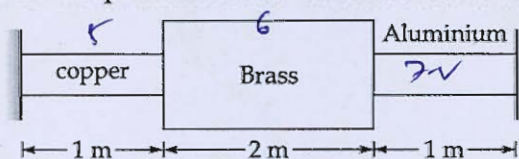
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Solid Mechanics

- Q.1 (a) A rod is made of three segments as shown in figure below. Calculate the stresses in each material due to rise in temperature of 40°C when the walls yield by 0.2 mm.



$$A_{\text{Copper}} = 200 \text{ mm}^2, A_{\text{Brass}} = 300 \text{ mm}^2, A_{\text{aluminium}} = 100 \text{ mm}^2, E_{\text{Copper}} = 1 \times 10^5 \text{ N/mm}^2, \\ E_{\text{brass}} = 0.8 \times 10^5 \text{ N/mm}^2, E_{\text{Aluminium}} = 0.5 \times 10^5 \text{ N/mm}^2, \alpha_{\text{Copper}} = 5 \times 10^{-6}/^\circ\text{C}, \\ \alpha_{\text{Brass}} = 6 \times 10^{-6}/^\circ\text{C}, \alpha_{\text{Aluminium}} = 7.5 \times 10^{-6}/^\circ\text{C}$$

[12 marks]

allum - P reaction per developed end to

$$\left(\frac{P_c l_c}{A_c E_c} + \frac{P_b l_b}{A_b E_b} + \frac{P_a l_a}{A_a E_a} \right) = (\sum L \alpha \Delta T) - 0.2$$

$$P_c = P_b = P_a = P$$

$$P = \frac{(\sum L \alpha \Delta T) - 0.2}{\sum \left(\frac{L}{AE} \right)}$$

$$= \frac{[(1 \times 5 + 2 \times 6 + 1 \times 7.5) \times 10^{-6} \times 40] \times 10000 - 0.2}{\dots}$$

$$\left(\frac{10000}{200 \times 10^5} + \frac{20000}{300 \times 0.8 \times 10^5} + \frac{10000}{0.5 \times 10^5} \right) \times \frac{1}{10^5}$$

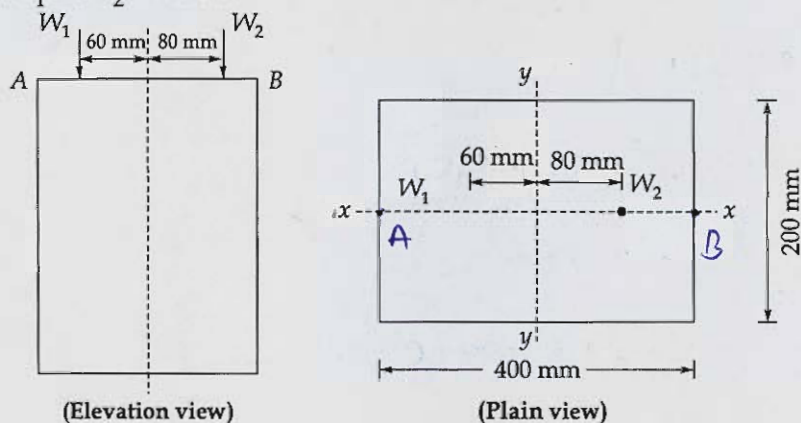
$$P = 2340 \text{ N}$$

$$G_c = \frac{2340}{200} = 11.7 \text{ MPa (C)}$$

$$G_b = \frac{2340}{300} = 7.8 \text{ MPa (C)}$$

$$G_a = \frac{2340}{100} = 23.4 \text{ MPa (C)}$$

- Q.1 (b) A short wooden pillar is rectangular in section $400 \text{ mm} \times 200 \text{ mm}$. It carries at the top, two point loads W_1 and W_2 in vertical plane as shown in figure below. If the stress is throughout compressive and extreme stress on the side in which W_1 acts i.e. at A is four times the extreme intensity on the other side i.e. at B, then compute the value of W_1 if $W_2 = 50 \text{ kN}$.



[12 marks]

$$G_A = 4 G_B$$

$$G_A = \left(\frac{W_1}{A} + \frac{W_2}{A} \right) + \frac{W_1 \times 60}{200 \times 400 \times \frac{6}{6}} - \frac{W_2 \times 80}{200 \times 400 \times \frac{6}{6}}$$

$$= \frac{W_1 \times 10^3}{200 \times 400} + \frac{50 \times 10^3}{200 \times 400} + \frac{W_1 \times 60}{200 \times 400 \times \frac{6}{6}} - \frac{50 \times 80 \times 10^3}{200 \times 400 \times \frac{6}{6}}$$

$$G_A = -0.125 + \frac{W_1 \times 10^3}{200 \times 400} + \frac{W_1 \times 60 \times 10^3}{200 \times 400 \times 6} \quad \text{--- (i)}$$

$$G_B = 1.378 + \frac{W_1 \times 10^3}{200 \times 400} - \frac{W_1 \times 10^3 \times 60 \times 6}{200 \times 400 \times 6} \quad \text{--- (ii)}$$

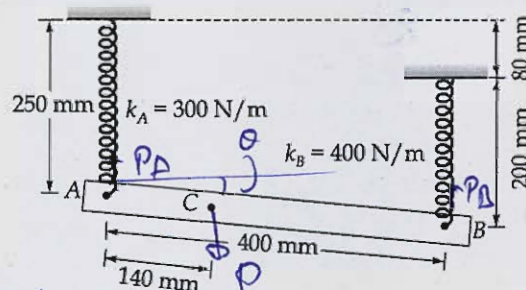
$$G_A = 4 G_B$$

on solving

$$\boxed{W_1 = 300 \text{ kN}}$$

12

- Q.1 (c) A uniform bar AB of weight 25 N is supported by two springs of natural length 250 mm and 200 mm respectively. Find the value of P that should be applied at C in order to bring the bar to a horizontal position.



at time of bar is horizontal

$$\Sigma M_C = 0$$

$$P_A \times 140 = P_B \times 260$$

$$P_A \times 140 = P_B \times 260$$

$$\frac{P_A}{P_B} = \frac{260}{140} \quad \text{--- (1)}$$

also:

$$0.250 + \frac{P_A}{k_A} = (0.200 + 0.080) + \frac{P_B}{k_B}$$

$$0.250 + \frac{260}{140} \times \frac{P_B}{300} = 0.280 + \frac{P_B}{400}$$

$$P_B = 0.129\text{ M}$$

[12 marks]

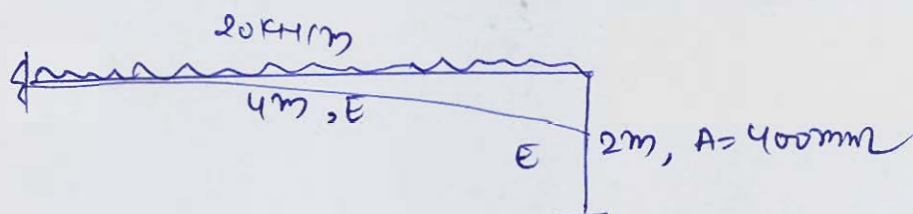
$$P_A = 15.096 \text{ M}$$

$$\text{total } P_{\text{req}} = 8.129 + 15.096$$

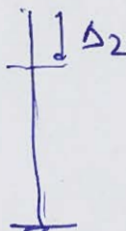
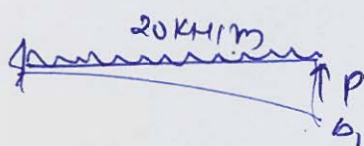
$$P = 23.225 \text{ M}$$

- Q.1(d) A cantilever beam of length 4 m is subjected to a uniformly distributed load of 20 kN/m throughout its length. It is supported by a strut of length 2 m and area of cross-section 400 mm² modulus of elasticity for beam and strut is same, then find the load taken by strut. Moment of inertia of beam given as $1.2 \times 10^6 \text{ mm}^4$.

[12 marks]



assume reaction P gets generated in strut and
compatibility eq



$$\Delta_1 = \Delta_2$$

$$\frac{20 \times 4^4}{8EI} - \frac{P \times 4^3}{3EI} = \frac{P \times 2}{AE}$$

$$\frac{20 \times (4000)^4}{8I} - \frac{P \times (4000)^3}{3I} = \frac{P \times 2000}{400}$$

$$P = 29991.56 \text{ N}$$

$$P = 30 \text{ kN}$$

12

- Q.1 (e) A solid shaft of mild steel 30 cm in diameter is to be replaced by a hollow shaft of 30 cm diameter of alloy steel, for which the allowable shear stress is 40% greater. If the power to be transmitted is 35% greater to that transmitted by solid shaft and speed of rotation of hollow shaft is 3% greater than that of solid shaft, determine the maximum internal diameter of hollow shaft. Take external diameter of shaft is 30 cm.

[12 marks]

Solid shaft - $D = 300 \text{ mm}$

$$\tau_{\max} = \frac{16T}{\pi(300)^3}$$

$$P = T\omega$$

$$= \left[\tau_{\max} \times \frac{\pi(300)^3}{16} \right] \omega \quad \text{--- (i)}$$

Hollow shaft - $D_o = 300 \text{ mm}$

$$D_i = ?$$

$$T' = \left\{ 1.40 \cdot \tau_{\max} \times \frac{\pi}{16} \left[\frac{(300)^4 - D_i^4}{300} \right] \right\}$$

$$P' = T' \omega'$$

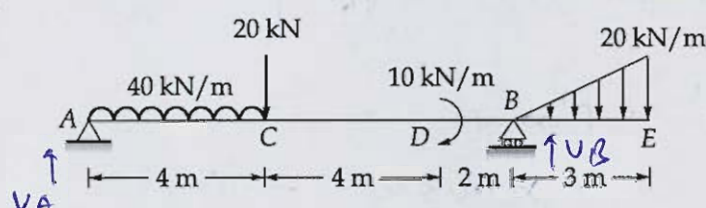
$$= T' \times 1.03 \omega \quad \text{--- (ii)}$$

$$\frac{P'}{P} = 1.35$$

$$\frac{\left[1.40 \times \frac{(300)^4 - D_i^4}{300} \times 1.03 \right]}{(300)^3} = 1.35$$

$$D_i = 150.77 \text{ mm} = \underline{\underline{15.08 \text{ cm}}}$$

Q.2 (a) Draw the shear force diagram and bending moment diagram of the beam shown below.



Also, find the maximum positive and negative bending moment.

[20 marks]

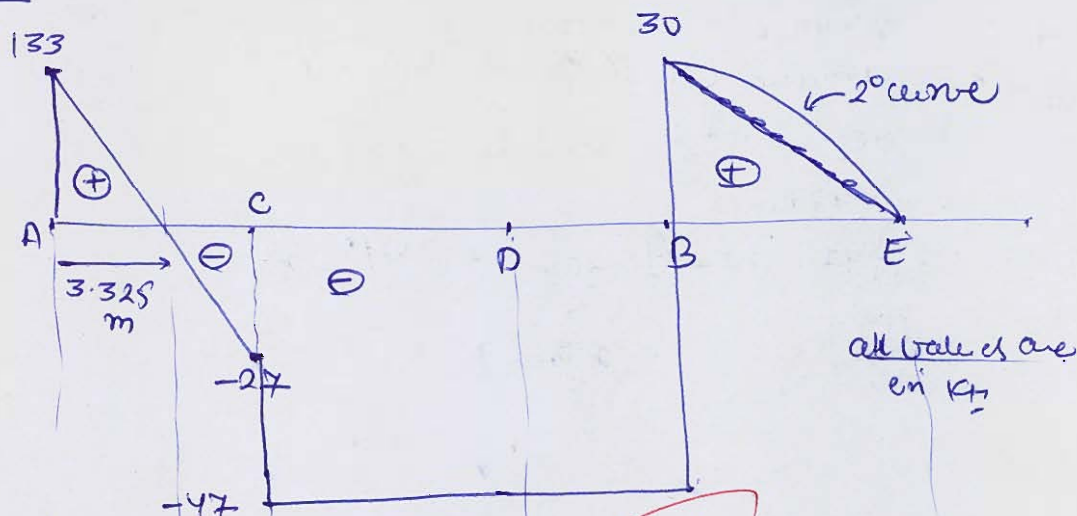
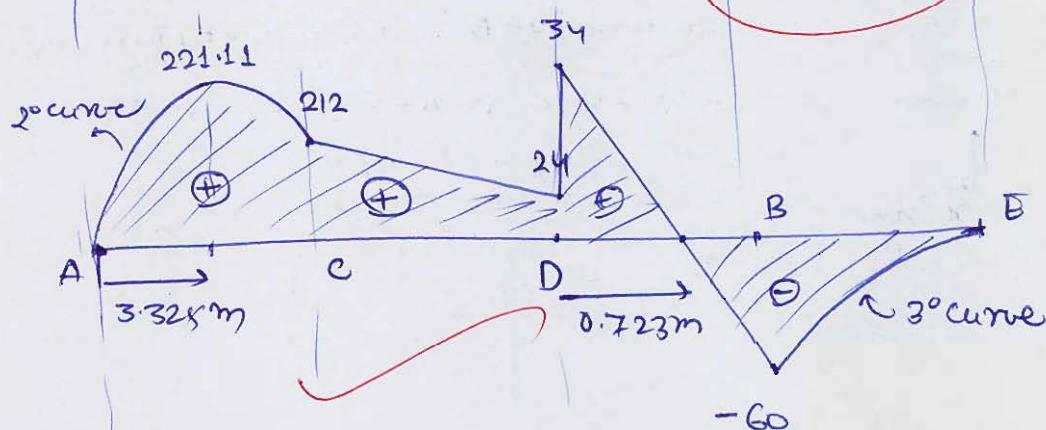
$$V_A + V_B = 40 \times 4 + 20 + \frac{1}{2} \times 20 \times 3$$

$$V_A + V_B = 210 \quad (1)$$

$$\Sigma M_A = 0 \Rightarrow V_B \times 10 = 40 \times 4 \times 2 + 20 \times 4 + 10 + \left(\frac{1}{2} \times 20 \times 3 \right) \times \left(10 + \frac{2}{3} \times 3 \right)$$

$$V_B = 77 \text{ kN} \quad (1)$$

$$V_A = 133 \text{ kN} \quad (1)$$

SFDBMD

$$\begin{aligned} \text{Max BM} &= 221.11 \text{ kN-m (+)} \\ &= -60 \text{ kN-m (-)} \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Max BM} &= 221.11 \text{ kN-m (+)} \\ &= -60 \text{ kN-m (-)} \end{aligned}} \right\} \text{ } \quad \text{✓}$$

SFD ⇒

Location of zero shear Max BM -

$$\frac{133}{x} = \frac{27}{4-x}$$

$$\boxed{x = 3.325 \text{ m}}$$

Location of zero BM

$$\frac{34}{x} = \frac{60}{2-x}$$

$$\boxed{x = 0.723 \text{ m}}$$

SFD →

$$AC \rightarrow V = V_A - 40x = 133 - 40x$$

due to sudden load at C, SF jumps down by 20 kN

$$CD \rightarrow V = C_{Vg} = 133 - 40 \times 4 - 20 = -47 \text{ kN}$$

$$CB \rightarrow V = -47 \text{ kN}$$

$$BC \rightarrow V = -47 + 77 + \left(\frac{1}{2} \times \frac{20}{3} \times x^2 \right) \times \frac{2}{3}$$

← x from B

BMD →

$$AC \rightarrow M(x) = V_A(x) - \frac{40x^2}{2}$$

$$= 133x - 20x^2$$

$$CD \rightarrow M(x) = 133x - 40 \times 4 \times (x-2) - 20(x-4)$$

due to sudden concentrated load at D, BM goes down by 10 kNm

$$BE \rightarrow M(x) = 133(x+10) - 40 \times 4 \times (x+8) - 20(6+x)$$

$$-10 - \left(\frac{1}{2} \times \frac{20}{3} \times \frac{x^2}{2} \right) \times \frac{2x}{3}$$

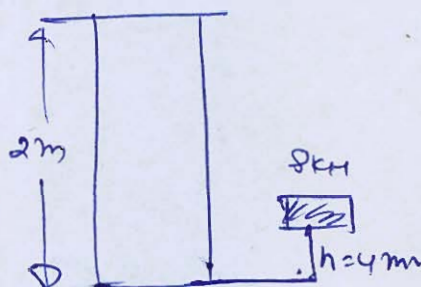
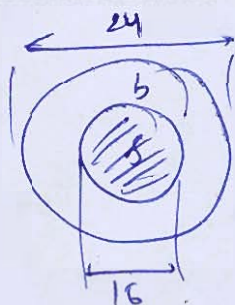
← x from B

Q.2 (b)

A vertical composite tie bar rigidly fixed at the upper end consists of a steel rod of 16 mm diameter enclosed in a brass tube of 16 mm internal diameter and 24 mm external diameter, each being 2 m long. Both are fixed together at the ends. The tie bar is suddenly loaded by a weight of 8 kN falling through a distance of 4 mm. Determine the maximum stresses in the steel rod and the brass tube

Take: Young's modulus of elasticity, $E_{\text{Steel}} = 205 \text{ GPa}$ and $E_{\text{Brass}} = 100 \text{ GPa}$.

[20 marks]



$$(\sigma_s/4) \times \pi \times (16)^2 + (\sigma_b) \times$$

$$\pi \times (24^2 - 16^2) =$$

both bars will carry equal share in lift. so stress

$$\sigma_s = \sigma_b$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\frac{\sigma_s}{\sigma_b} = \frac{E_s}{E_b} = \frac{205}{100} = 2.05 \quad \text{--- (1)}$$

also,

Loss of Energy of weight = gain of strain energy

$$mg(4+8) = \left[\frac{\sigma_s^2}{2E_s} \times \frac{\pi}{4} \times (16)^2 + \frac{\sigma_b^2}{2E_b} \times \frac{\pi}{4} \times (24^2 - 16^2) \right] \times 2000$$

$$8 \times 10^3 \times (4+8) = \left[\frac{(2.05\sigma_b)^2}{2E_s} \times \frac{\pi}{4} \times (16)^2 + \frac{\sigma_b^2}{2E_b} \times \frac{\pi}{4} \times (24^2 - 16^2) \right] \times 2000$$

$$8 \times 10^3 \times (4 + \frac{\sigma_b}{E_b} \times 2000) = 3.3175 \times 10^{-3} \times \sigma_b^2 \times 2000$$

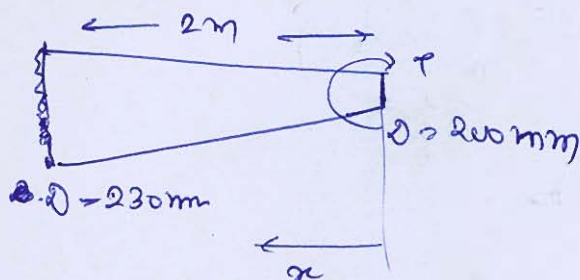
solving

$$\sigma_b = 82.54 \text{ MPa}$$

$$\sigma_s = 169.207 \text{ MPa}$$

- Q.2 (c) A solid circular shaft has a radius of 100 mm at one end and 115 mm at the other end the length of the shaft being 2 m. Calculate the percentage error in θ (angle of twist). If θ is calculated on the basis of the mean radius

[20 marks]



Assume Torque T is applied on the shaft then at any x distance

$$T(x) = T$$

$$d(x) = 0.200 + \frac{0.230 - 0.200}{2.000} \times (x)$$

$$= 0.200 + \frac{0.030}{2.000} \times x$$

0.03

$$d\theta = \frac{T dx}{GJ}$$

$$= \frac{T dx}{G \times \pi \left(0.200 + \frac{0.030}{2} x \right)^4}$$

32

0.015

$$d\theta = \frac{32T}{G\pi (0.200 + 0.015x)^4}$$

total angle of twist

$$\theta = \int_{x=0}^{2m} \frac{32T}{G\pi (0.200 + 0.015x)^4} dx$$

$$\theta = \frac{32T}{G\pi} \times 260 \text{ Radian}$$

with Mean radius $\bar{d}_m = \frac{200 + 230}{2} = 215 \text{ mm}$

$$\theta = \frac{32T}{G\pi (0.215)^4}$$

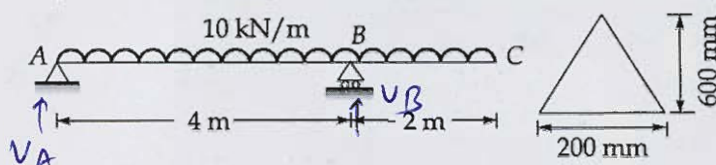
$$\% \text{ error} = \left[\left(\frac{327}{4\pi} \times 260 \right) - \frac{327}{4\pi (0.215)4} \right] \times 100$$

$$\frac{\left(\frac{327}{4\pi} \times 260 \right)}{\left(\frac{327}{4\pi} \times 260 \right)}$$

$$= \boxed{80\%} \text{ error}$$

Q.3 (a)

A cantilever beam of triangular cross-section is loaded as shown in figure below. Compute the maximum tensile, compressive stresses at the location of maximum negative and maximum positive bending moment and maximum shearing stress and its location.

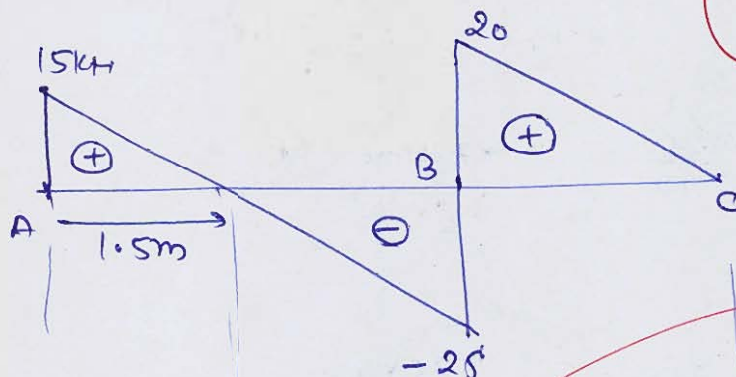
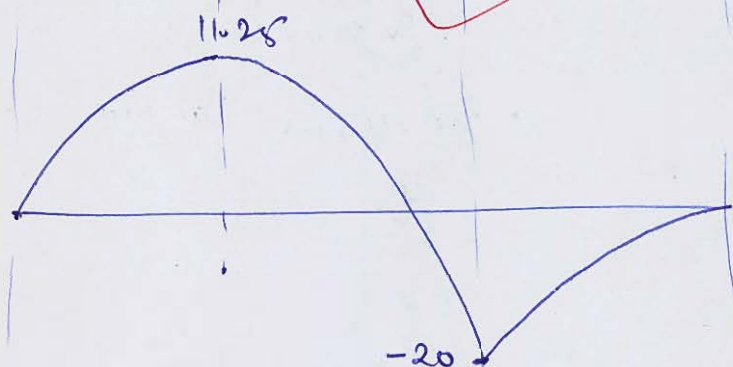


[20 marks]

$$\sum M_A = 0 \Rightarrow V_B \times 4 = 10 \times 6 \times 3$$

$$V_B = 45 \text{ kN}$$

$$V_A = 15 \text{ kN}$$

SFDall values are
in
kNBMD

$$\text{Max +ve BM} = 15 \times 1.5 - \frac{10 \times 1.5^2}{2} = 11.25 \text{ kNm}$$

at $x = 1.5 \text{ m}$
from
A

$$\text{Max -ve BM} = 15 \times 4 - \frac{10 \times 4^2}{2} = -20 \text{ kNm}$$

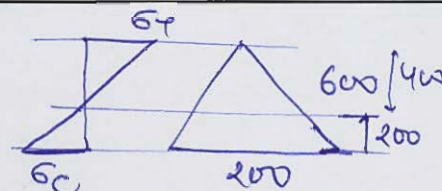
$$\text{Max shear} = 25 \text{ kN (at left of support B)}$$

17

at Max +ve BM.

$$\sigma_c = \frac{20 \times 10^6 \times 200}{12 \times 10^8}$$

$$= 3.33 \text{ MPa (at bottom fiber)}$$



$$I = \frac{200 \times 600^3}{36}$$

$$\sigma_t = 3.33 \times \frac{400}{200} = 6.66 \text{ MPa (tensile) } = 12 \times 10^8 \text{ mm}^4$$

at Max +ve BM:

$$\sigma_c = \frac{11.25 \times 10^6 \times 400}{12 \times 10^8} = 3.75 \text{ MPa (at top fiber)}$$

$$\sigma_t = 3.75 \times \frac{200}{400} = 1.875 \text{ MPa (at bottom fiber)}$$

Max shear-

$$\tau_{max} = \frac{3}{2} \times \tau_{avg} = \frac{3}{2} \times \frac{25 \times 10^3}{\frac{1}{2} \times 200 \times 600}$$

$$= 0.625 \text{ N/mm}^2 \text{ (at NA)}$$

$$\frac{bh^3}{12}$$

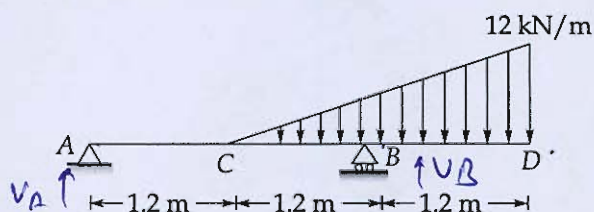
$$= \frac{62}{2}$$

$$\times \left(\frac{h}{2}\right)^2$$

$$= \frac{118}{12}$$

$$= \frac{8-2}{36}$$

- Q.3 (b) An overhang beam ABCD is shown in figure. Determine the deflection at C and D and slope at A and B for the beam shown below.



Take flexural rigidity of beam as 300 kN-m^2 .

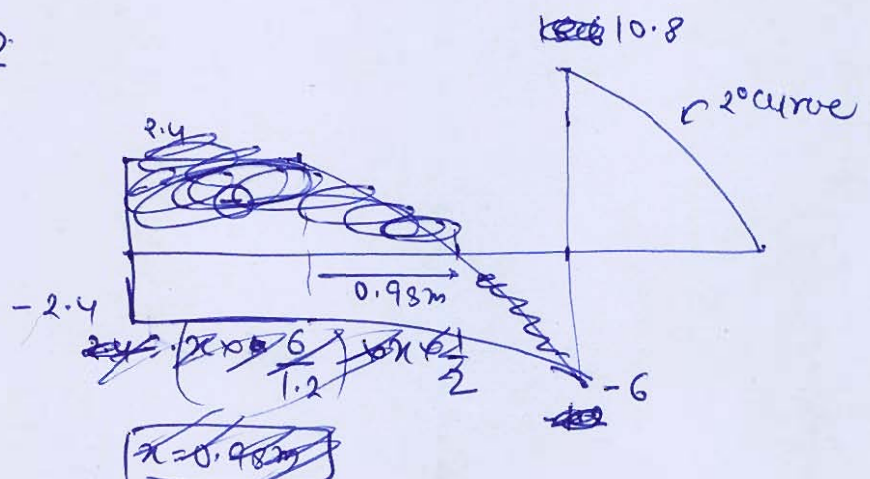
$$\sum M_A = 0 \Rightarrow V_B \times 2.4 = (12 \times 0.5 \times 2.4) \times \left(1.2 + \frac{2}{3} \times 2.4\right) \quad [20 \text{ marks}]$$

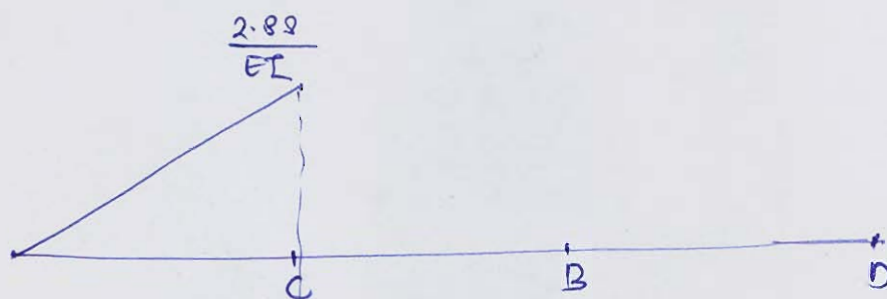
$$V_B = 16.8 \text{ kN} \quad (\uparrow)$$

$$V_A = -2.4 \text{ kN} \quad (\downarrow)$$

2

SFD





Q.3 (c) A hollow circular steel shaft is required to carry a torque of 40 kN-m and bending moment of 20 kN-m. If the internal diameter is 60% of external diameter, then determine size of shaft by

- Maximum principal stress theory
- Maximum strain energy theory

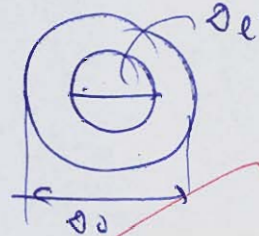
Factor of safety = 2, $\mu = 0.3$ and $f_y = 250 \text{ N/mm}^2$

[8 + 12 = 20 marks]

$$T = 40 \text{ kN-m}$$

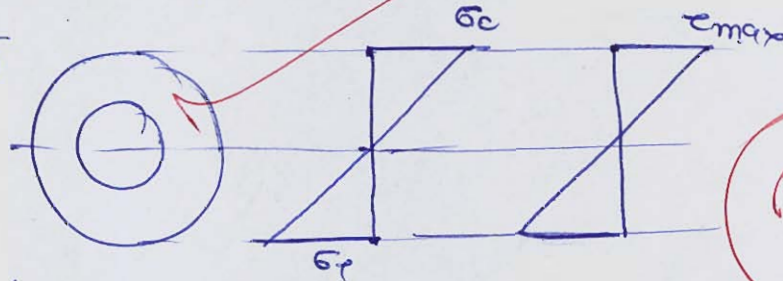
$$M = 20 \text{ kN-m}$$

$$d_i = 0.6 \times d_o$$



due to combined effect of torque & BM -

$$\sigma_x = \frac{16}{\pi d_o^3}$$



due to bending alone

$$\sigma = \frac{M}{Z} = \frac{M}{\left[\frac{\pi (d_o^4 - d_i^4)}{64} \right] \left(\frac{d_o}{2} \right)}$$

$$= \frac{32 M d_o}{\pi (d_o^4 - d_i^4)}$$

$$= \frac{32 M d_o}{\pi (d_o^4 - (0.6 d_o)^4)}$$

$$= \frac{11.7025 M}{d_o^3}$$

due to torque alone,

$$\sigma_{max} = \frac{T}{Z_p} = \frac{T}{\frac{\pi}{32} (d_o^4 - d_i^4) \left(\frac{d_o}{2} \right)} = \frac{5.951 T}{d_o^3}$$

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{max}^2}$$

$$= \frac{11.7025 \frac{N}{mm^2}}{2} \pm \sqrt{\left(\frac{11.7025 \frac{N}{mm^2}}{2}\right)^2 + \left(\frac{5.851 \cdot \tau}{(200)^3}\right)^2}$$

$$\sigma_1 = \frac{1}{200^2} [378684674.7] \text{ N/mm}^2 \text{ (t)}_1$$

$$\sigma_2 = \frac{1}{200^2} [-144644674.7] \text{ N/mm}^2 \text{ (c)}$$

(i) Max principal stress then -

$$\sigma_1 \leq \left(\frac{f_u}{FOS} \right)$$

$$\sigma_1 \leq \frac{250}{2}$$

bulky bars fracture

$$\sigma_0 = 144.70 \text{ mm}$$

(ii) Maximum strain eqn.

$$\frac{1}{2E} [(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] \leq \frac{1}{2E} \left(\frac{f_u}{FOS} \right)^2$$

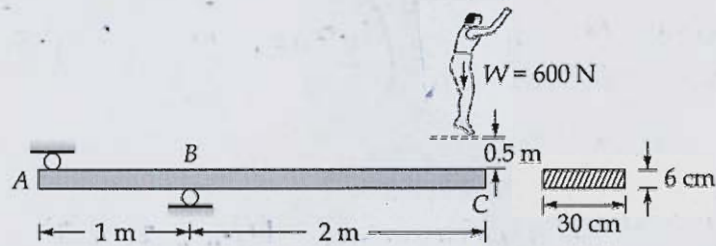
$$(\sigma_1^2 + \sigma_2^2 + 0) - 2 \times 0.3 \times [\sigma_1\sigma_2] \leq \left(\frac{250}{2} \right)^2$$

bulky bar

$$\sigma_0 = 28.55 \text{ mm}$$

$$\sigma_0 = 152.58 \text{ mm}$$

- Q.4 (a) A man weighing 600 N jumps from a height of 0.5 m on a diving board of dimensions $30 \text{ cm} \times 6 \text{ cm}$ supported as shown in figure. Find the maximum stress produced in the board.



Take, $E = 10 \text{ GPa}$.

[20 marks]

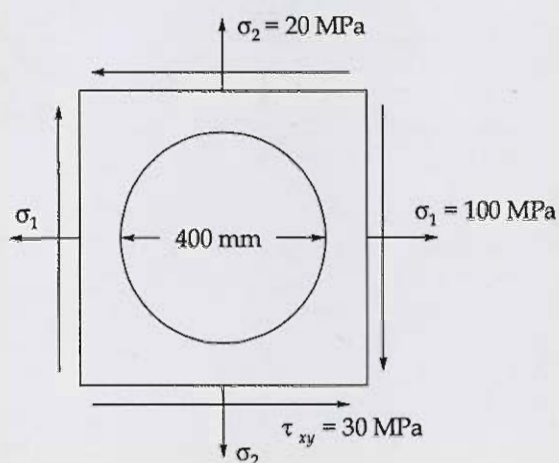
Q.4 (b)

A hollow cast iron column with fixed ends supports an axial load of 800 kN. If the column is 3 m long and has an external diameter of 200 mm, find the thickness of metal required. Use Rankine's formula, taking a constant of $\frac{1}{1600}$ and assume a working stress of 90 N/mm².

[20 marks]

- Q.4 (c) A circle of 400 mm diameter is scribed on a mild steel plate before it is subjected to stresses as shown in figure. In stressing the circle deforms to an ellipse. Calculate the lengths of the major and minor axes of the ellipse and also find their directions.

Take $\mu = 0.286$ and $E = 205 \text{ kN/mm}^2$.

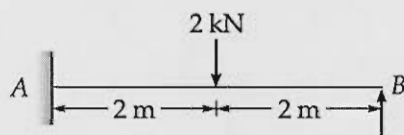


[20 marks]

Section B : Structural Analysis

Q.5 (a) For the propped cantilever beam shown in figure below using slope deflection equation, find the moments at support 'A' when ($EI = 2 \times 10^6 \text{ kN-cm}^2$)

- (i) The supports are at the same level.
- (ii) The support 'B' sinks by 1 cm.

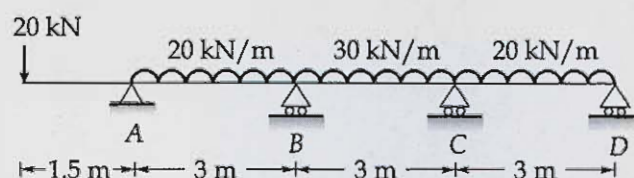


[6 + 6 = 12 marks]

- Q.5 (b)** A three hinged parabolic arch is hinged at supports and also at crown. The span of the arch is 28 m with a central rise of 5 m. It carries concentrated load of 55 kN at 21 m from left support and a uniformly distributed load of 35 kN/m on left half of the span. Determine the moment, thrust and radial shear at a section 7 m from left support.

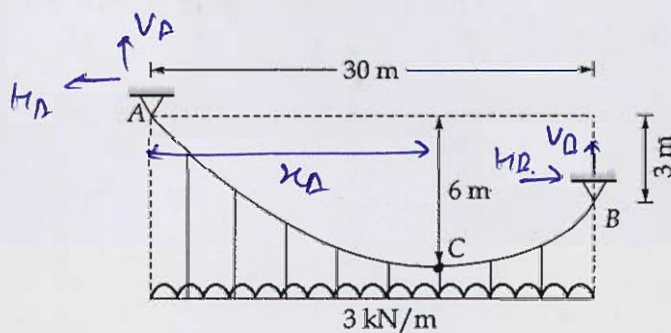
[12 marks]

- Q.5 (c) For the continuous girder shown in the figure. Find the support moment using theorem of three moments and draw the B.M. diagram with net bending moment at each span.



[12 marks]

- Q.5 (d) A cable is supported between two points 30 m horizontally apart. The left support is 3 m above the right support. The cable carries a load of 3 kN/m on the horizontal span. The lowest point of the cable is 6 m below the left support. Find the maximum and minimum tension in the cable.



$$V_A + V_B = 3 \times 30 = 90 \quad \text{--- (i)}$$

$$H_A = H_B \quad \text{--- (ii)}$$

$$M_A = 0$$

~~Under UDL~~
Under UDL cable takes shape of parabola \hookrightarrow

$$\frac{6}{(x_A)^2} = \frac{3}{(30 - x_A)^2}$$

$$\boxed{x_A = 17.57 \text{ m}} \quad \checkmark$$

4

[12 marks]

$$M_C = 0$$

$$V_A \times 17.57 - \frac{3 \times 17.57^2}{2} = H_A \times 6 \quad \text{--- (i)}$$

$$V_B \times 12.43 - \frac{3 \times 12.43^2}{2} = H_B \times 3 \quad \text{--- (ii)}$$

Solving

$$V_A = 74.57 \text{ kN}$$

$$V_B = 15.43 \text{ kN}$$

$$H_A = H_B = 141.189 \text{ kN}$$

$$52.72$$

$$37.28$$

$$77.21$$

$$T_{\max} = \sqrt{74.57^2 + 141.189^2}$$

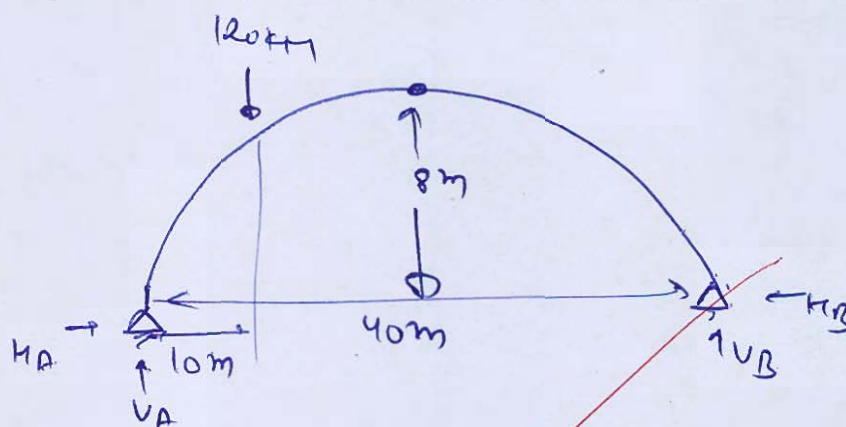
$$= \boxed{159.67 \text{ kN}}$$

$$\boxed{T_{\min} = 141.189 \text{ kN}}$$

$$V_A \times 17.57 - \frac{3 \times 17.57^2}{2} - H_A \times 6 = 0$$

- Q.5 (e) A three-hinged circular arch of span 40 m and rise 8 m carries a concentrated load of 120 kN at a horizontal distance of 10 m from the left end. Find the maximum positive and negative bending moments and draw the bending moment diagram.

[12 marks]

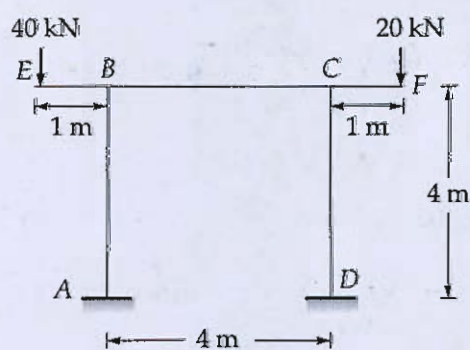


$$V_A + V_B = 120 \quad \text{--- (i)}$$

$$V_A \times 20 - 120 \times 10 = H_A \times 8 \quad \text{--- (ii)}$$

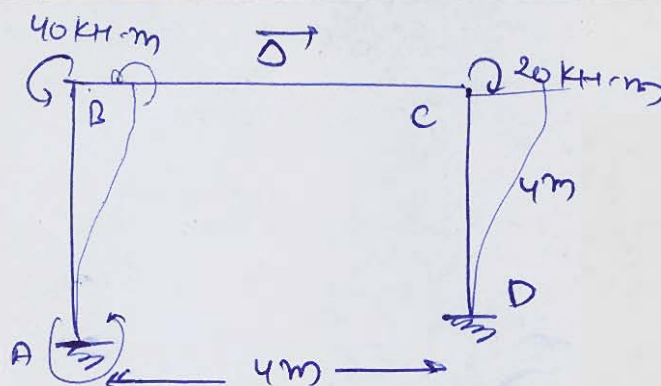


- Q.6 (a) Analyse the portal frame shown in figure by using slope-deflection method. Take EI as constant and draw the bending moment diagram.



[20 marks]

solution -

 $\theta_B, \theta_C \text{ \& } \Delta \rightarrow \text{KIP}$

$$M_{AB} = -10 + \frac{2EI}{4} \theta_B - \frac{6EI\Delta}{42}$$

$$M_{BA} = -20 + \frac{4EI}{4} \theta_B - \frac{6EI\Delta}{42}$$

$$M_{BC} = (-20 + 5) + \frac{4EI}{4} \theta_B + \frac{2EI}{4} \theta_C$$

$$M_{CB} = (-10 + 10) + \frac{4EI}{4} \theta_C + \frac{2EI}{4} \theta_B$$

$$M_{CD} = 10 + \frac{4EI}{4} \theta_C - \frac{6EI\Delta}{42}$$

$$M_{DC} = 5 + \frac{2EI}{4} \theta_C - \frac{6EI\Delta}{42}$$

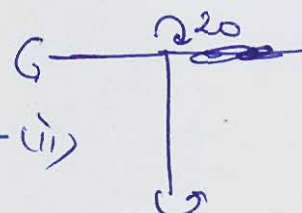
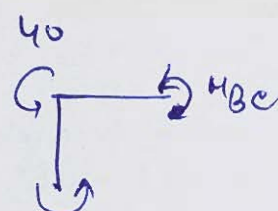
joint eqn \rightarrow

$$M_{BA} + M_{DC} + 40 = 0$$

$$2EI\theta_B + 0.5EI\theta_C - \frac{6EI\Delta}{42} = -75 \quad \text{--- (i)}$$

$$M_{CD} + M_{DC} = 20$$

$$2EI\theta_C + 0.5EI\theta_B = -\frac{6EI\Delta}{42} = +10 \quad \text{--- (ii)}$$



Shear eqn $H_A + H_D = 0$

$$\left(\frac{M_{AB} + M_{BA}}{4} \right) + \left(\frac{M_{DC} + M_{CD}}{4} \right) = 0$$

$$\frac{6}{4} EI \theta_B + \frac{6}{4} EI \theta_C - \frac{24 EI \Delta}{4L} = 15 \quad \text{--- (iii)}$$

solving

$$\left. \begin{aligned} \theta_B &= \frac{20}{EI} \text{ (C)} \\ \theta_C &= \frac{-10}{EI} \text{ (C)} \end{aligned} \right\} \begin{aligned} &\frac{-40}{7EI} \text{ (S)} \\ &\frac{30}{7EI} \text{ (C)} \end{aligned}$$

$$\Delta = -\frac{80}{7EI} \text{ (←)}$$

then putting value back to slope deflection eqn

$$M_{AB} = -8.57$$

$$M_{BA} = -21.43$$

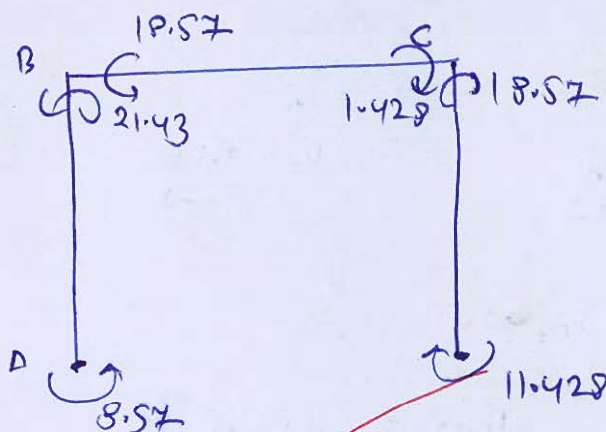
$$M_{BC} = -18.57$$

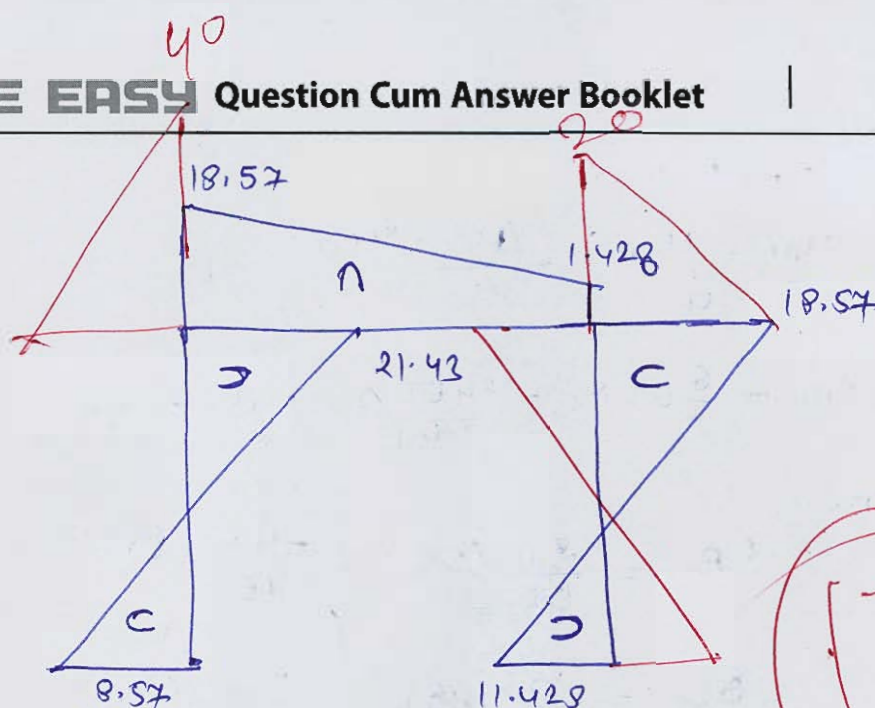
$$M_{CB} = 1.428$$

$$M_{CD} = 18.57$$

$$M_{DC} = 11.428$$

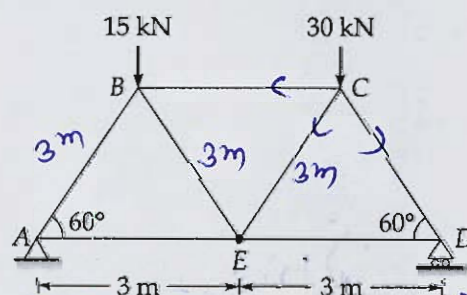
all values are in kN-m





BMD

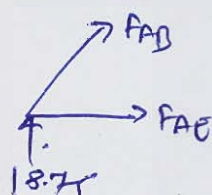
- Q.6 (b) Find the vertical deflection of the joint E of the truss shown in figure below. The sectional area of each member is 1200 mm^2 . Take $E = 2 \times 10^5 \text{ N/mm}^2$.



[20 marks]

due to external load -
 $V_A = 18.75 \text{ kN}$

$V_D = 26.25 \text{ kN}$

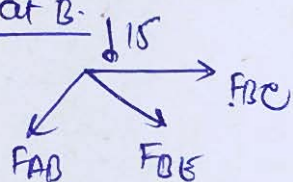


$$\frac{F_{AD}}{2} + F_{AE} = 20$$

$$F_{AD} \times \frac{\sqrt{3}}{2} = -18.75$$

$$F_{AD} = -21.65$$

$$F_{AE} = 10.825$$

Equivalent Load -at B:

$$F_{BC} + \frac{F_{BE}}{2} - \frac{F_{AB}}{2} = 0$$

$$F_{AB} \times \frac{\sqrt{3}}{2} + F_{BE} \times \frac{\sqrt{3}}{2} = -15$$

$$F_{BE} = 4.33 \text{ kN}$$

$$F_{BC} = -13 \text{ kN}$$

at C:

$$F_{CE} \times \frac{\sqrt{3}}{2} + F_{CD} \times \frac{\sqrt{3}}{2} = -30 \quad \text{--- (1)}$$

$$F_{BC} + \frac{F_{CE}}{2} = \frac{F_{CD}}{2}$$

$$F_{CE} - F_{CD} = 26 \quad \text{--- (2)}$$

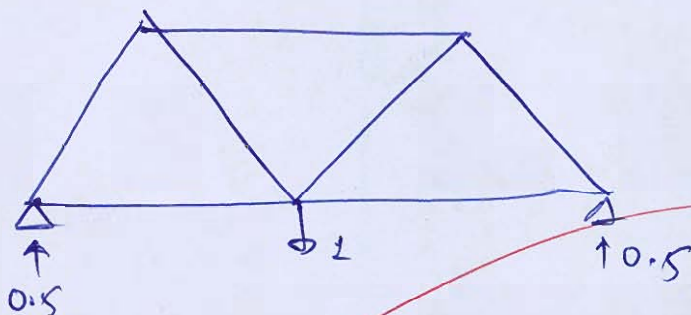
$$F_{CE} = -4.32$$

$$F_{CD} = -30.32$$

at D:

$$F_{DE} + \frac{F_{CD}}{2} = 0$$

$$F_{DE} = +\frac{30.32}{2} = 15.16$$

due to unit load at B -

$$u_{AB} = -0.577$$

$$u_{AE} = 0.289$$

$$u_{BE} = 0.577$$

$$u_{BC} = -0.577$$

$$u_{CE} = 0.577$$

$$u_{CD} = -0.577$$

$$u_{DE} = 0.2885$$

$$\Delta = \frac{\sum u_i P_i h}{AE}$$

$$= \frac{(\sum u_i P_i) \times h}{AE}$$

$$= \frac{44.995 \times 3000}{1200 \times 200}$$

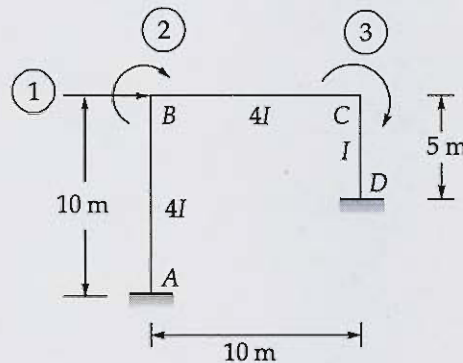
$$\Delta_e = 0.562 \text{ mm}$$

(d)

Σ

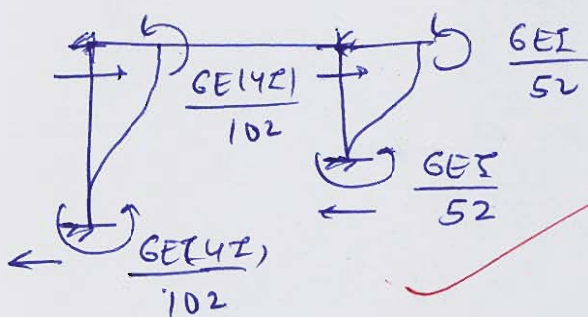
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- Q.6 (c) Develop the stiffness matrix for portal frame ABCD with reference to the coordinates shown in figure.



[20 marks]

Make joint 1 free. - $\Delta_1 = 1$



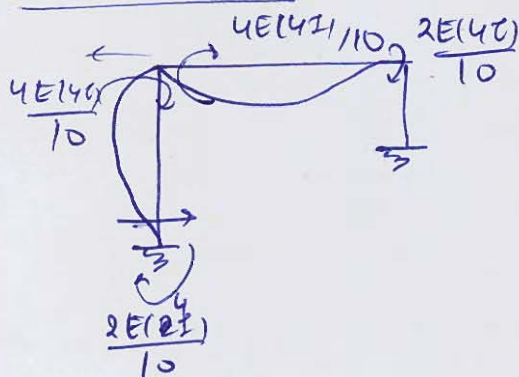
$$K_{11} = \frac{GE(4I)}{102 \times 10} + \frac{2 \times GEI}{52}$$

=

$$K_{21} = -\frac{GE(4I)}{102}$$

$$K_{31} = -\frac{GEI}{52}$$

again $\theta_2 = 1$

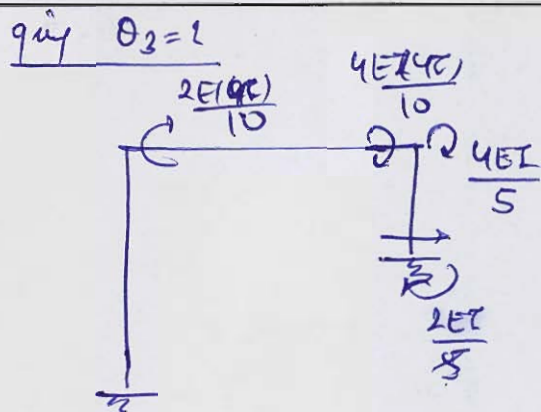


$$K_{12} = -\frac{GE(4I)}{102}$$

$$K_{22} = \frac{4EI(4I)}{10} + \frac{4EI(4I)}{10}$$

$$= \frac{32EI}{10}$$

$$K_{32} = \frac{2EI(4I)}{10} = \frac{8EI}{10}$$



$$K_{13} = -\frac{6EI}{52}$$

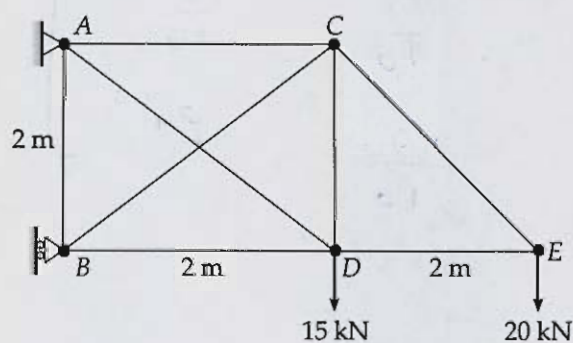
$$K_{23} = -\frac{8EI}{10}$$

$$K_{33} = \frac{4E(4I)}{10} + \frac{4EI}{5} = \frac{24EI}{10}$$

$$[K] = \begin{bmatrix} \frac{18}{125} & -\frac{24}{100} & -\frac{6}{25} \\ -\frac{24}{100} & \frac{32}{10} & \frac{8}{10} \\ -\frac{6}{25} & \frac{8}{10} & \frac{24}{10} \end{bmatrix} EI$$

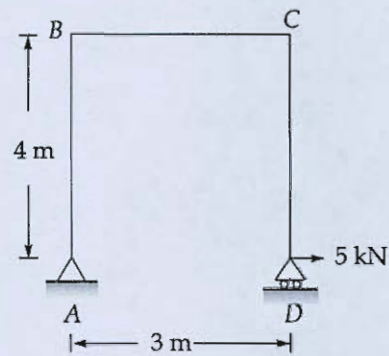
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- Q.7 (a) Find the forces in all members of the redundant plane truss shown in figure below. Cross-sectional area of each bar is 1000 mm^2 and $E = 2 \times 10^5 \text{ N/mm}^2$.



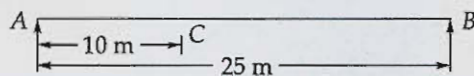
[20 marks]

- Q.7 (b) Determine the horizontal displacement of the roller support D of the portal frame shown in figure. Take EI as 8000 kNm^2 for all members.



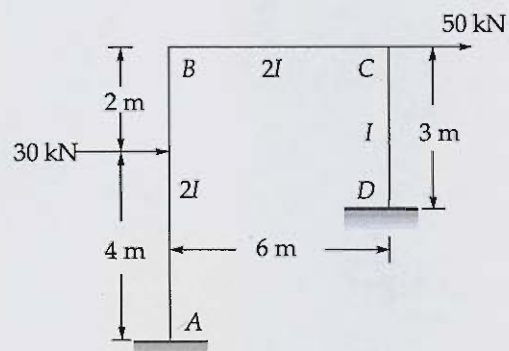
[20 marks]

- Q.7 (c) Draw the influence line diagram for bending moment at a point 10 m distant from the left-hand abutment on a bridge of span 25 m. Find the maximum bending moment at the point C due to a series of wheel loads 100 kN, 200 kN, 200 kN, 200 kN, 200 kN at centre to centre distance of 4 m, 2.5 m, 2.5 m, and 2.5 m. The loads can cross in either direction, 100 kN load leading in each case.



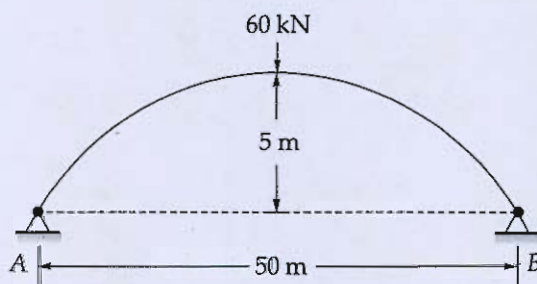
[20 marks]

- Q.8 (a) Analyse the frame shown in figure by moment distribution method and sketch bending moment diagram.



[20 marks]

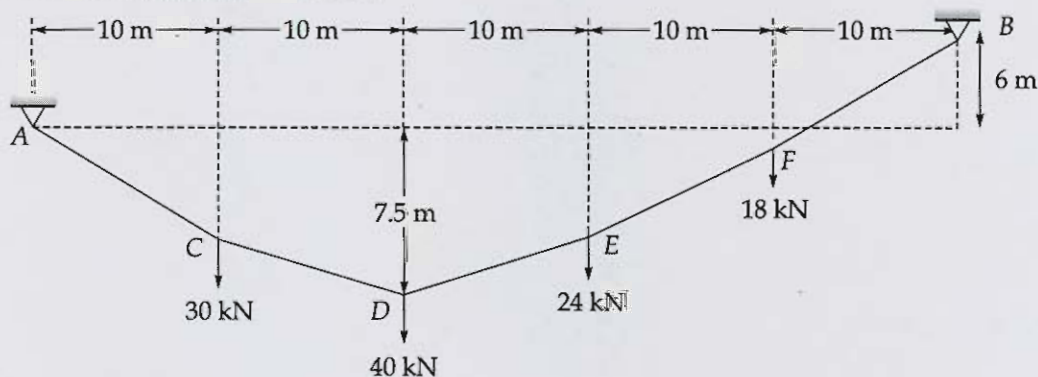
- Q.8 (b) A two-hinged parabolic arch of span 50 m and rise 5 m is subjected to a central concentrated load of 60 kN. It has an elastic support which yields by 0.0001 mm/kN. Taking, $E = 200 \text{ kN/mm}^2$, $I = 5 \times 10^9 \text{ mm}^4$, Average area, $A_m = 10000 \text{ mm}^2$, $\alpha = 12 \times 10^{-6}/^\circ\text{C}$ and assuming secant variation, calculate the horizontal thrust developed when the temperature rises by 20°C .
- (i) Neglecting rib shortening.
(ii) Considering rib shortening.



[10 + 10 = 20 marks]

- Q.8 (c) A cable ACDEFB supports a set of vertical hangers at four intermediate points (C, D, E, F). The span between the supports A and B is 50 m. The lowest point of the cable (D) is located 7.5 m below the left support A which in turn is located 6 m below the right support B. The vertical loads applied through the hangers at points C, D, E and F are 30 kN, 40 kN, 24 kN and 18 kN respectively, placed at equal intervals draw the funicular polygon and find.

- The tension in each segment of cable and inclination of each segment of the cable from horizontal
- The final length of the cable.



[20 marks]

Space for Rough Work



