try to avoid calculation mistake

• Highlights final answers.

• use Headings

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ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electric Circuits + Engineering Mathematics

Name : ....

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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	34
Q.2	41
Q.3	
Q.4	50
Section-B	
Q.5	43
Q.6	39
Q.7	
Q.8	
Total Marks Obtained	207

Signature of Evaluator: Sourabh kumar

Cross Checked by ....

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CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

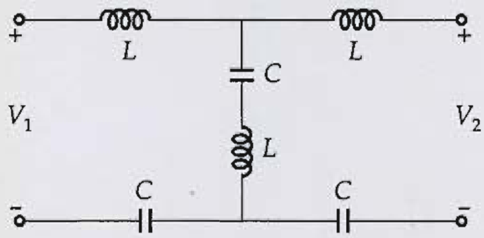
DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Do not write in this margin

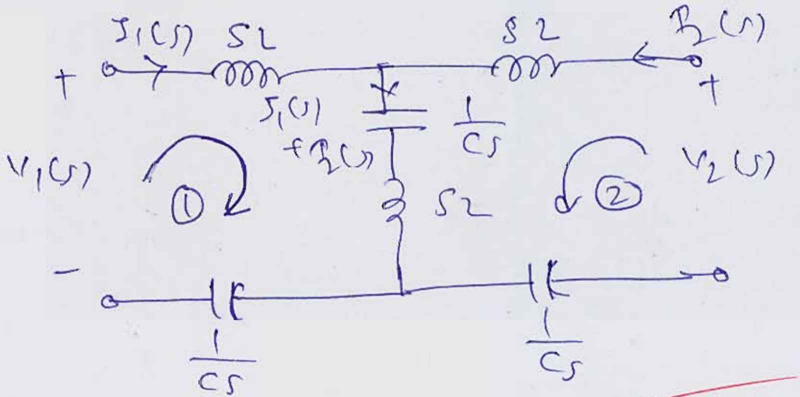
Section A : Electric Circuits

(a) Determine Z-parameters for network shown



[12 marks]

In s- domain -



KVL in loop 1 —

$$V_1(s) = sL\, I_1(s) + \left[\frac{1}{Cs} + sL\right]\left[I_1(s) + I_2(s)\right] + \frac{1}{Cs} I_1(s)$$

$$V_1(s) = \left[sL + \frac{1}{Cs} + sL + \frac{1}{Cs}\right] I_1(s) + \left[\frac{1}{Cs} + sL\right] I_2(s)$$

$$V_1(s) = \left[2sL + \frac{2}{Cs}\right] I_1(s) + \left[sL + \frac{1}{Cs}\right] I_2(s) \quad — (1)$$

KVL in loop 2 -

$$V_2(s) = sL\, I_2(s) + \left[\frac{1}{Cs} + sL\right]\left(I_1(s) + I_2(s)\right) + \frac{1}{Cs} I_2(s)$$

$$V_2(s) = \left[sL + \frac{1}{Cs}\right] I_1(s) + \left[sL + \frac{1}{Cs} + sL + \frac{1}{Cs}\right] I_2(s)$$

$$V_2(s) = \left[sL + \frac{1}{Cs}\right] I_1(s) + \left[2sL + \frac{2}{Cs}\right] I_2(s) \quad —(2)$$

from eq - (1) and (2) —

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} 2sL + \frac{2}{Cs} & sL + \frac{1}{Cs} \\ sL + \frac{1}{Cs} & 2sL + \frac{2}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

∴ Z-parameter for the given network is –

$$Z = \begin{bmatrix} 2\left(sL + \frac{1}{Cs}\right) & \left(sL + \frac{1}{Cs}\right) \\ \left(sL + \frac{1}{Cs}\right) & 2\left(sL + \frac{1}{Cs}\right) \end{bmatrix}$$

Good Approach

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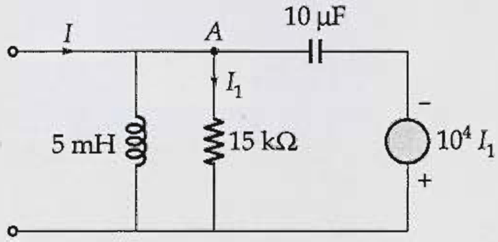
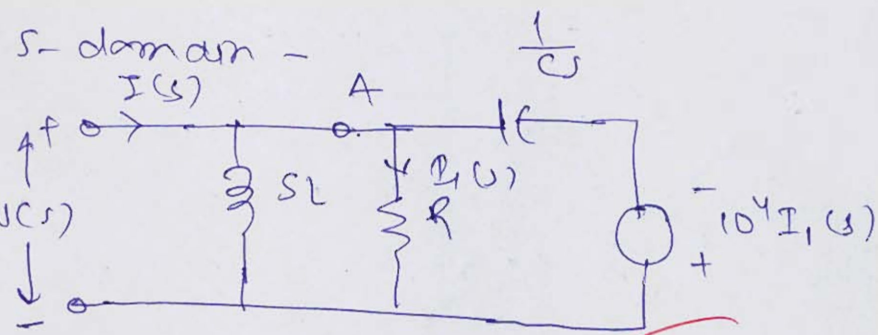
(b) Determine the resonant frequency, quality factor Q_0 and bandwidth of the given circuit.



[12 marks]

In S-domain -



where, $R = 15\,k\Omega$, $L = 5\,mH$, $C = 10\,\mu F$

KCL at node A -

$$I(s) = \frac{V(s)}{sL} + \frac{V(s)}{R} + \frac{V(s) + 10^4 I_1(s)}{1/Cs}$$

where $I_1(s) = \frac{V(s)}{R}$

$$\therefore I(s) = \frac{V(s)}{sL} + \frac{V(s)}{R} + \frac{V(s) + \frac{10^4 V(s)}{R}}{\frac{1}{Cs}}$$

2

$$I(s) = V(s)\left[\frac{1}{sL} + \frac{1}{R} + Cs + \frac{10^4 Cs}{R}\right]$$

$$\therefore \frac{V(s)}{I(s)} = \frac{1}{\frac{1}{sL} + \frac{1}{R} + Cs + \frac{10^4 Cs}{R}}$$

$$= \frac{sLR}{R + sL + Cs^2RL + 10^4 Cs^2 RL}$$

$$= \frac{SLR}{s^2(10^4 CRL + CRL) + SL + R}$$

put value of ~~Res~~ the given value of R, L, C

$$= \frac{5 \times 5\times10^{-3} \times 15\times10^{3}}{s^2(10^4\times10\times10^{-6}\times15\times10^3\times5\times10^{-3} + 10\times10^{-6}\times15\times10^3\times5\times10^{-3}) + 5\times5\times10^{-3} + 15\times10^4}$$

$$= \frac{75 s}{s^2(7.5) + 5\times5\times10^{-3} + 15\times10^4}$$

$$\frac{V(s)}{I(s)} = \frac{10 s}{s^2 + 6.66\times10^{-4} s + 2\times10^4}$$

$\therefore$ CE $= s^2 + 2\xi\omega_n s + \omega_n^2$

comparing the CE we get -

resonant frequency, $\omega_0 = \sqrt{2\times10^4}$

$\omega_0 = 141.42$ r/s

Band width = coefficient of s

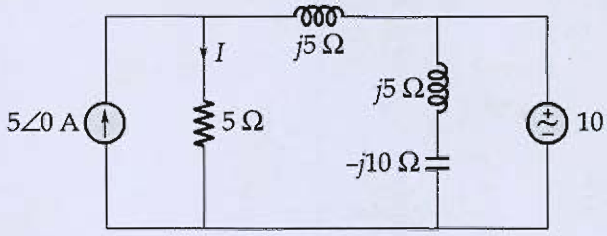
$BW = 6.66\times10^{-4}$ r/s

$\therefore$ Quality factor

$$Q = \frac{\omega_0}{Q} = \frac{141.42}{6.66\times10^{-4}}$$

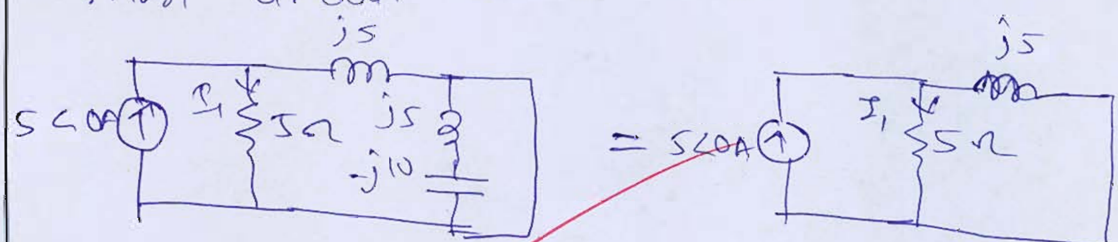
$Q = 2.12\times10^5$

(c) (i) Determine the current 'I' in the network shown using principle of superposition.



[6 marks]

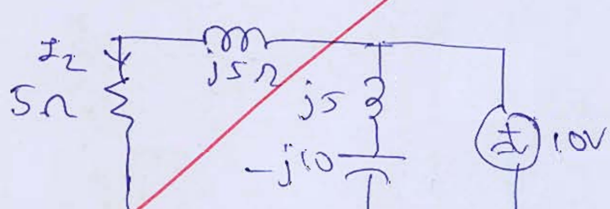
∴ only current source, voltage source short circuit —



By current divider rule —

$$I_1 = 5\angle 0^\circ \times \frac{j5}{5+j5} = 3.535\angle 45^\circ\ A$$

only voltage source, current source open circuit —



6

∴ current $I_2 = \dfrac{10\angle 0^\circ}{5+j5}$

$I_2 = 1.414\angle -45^\circ$ A

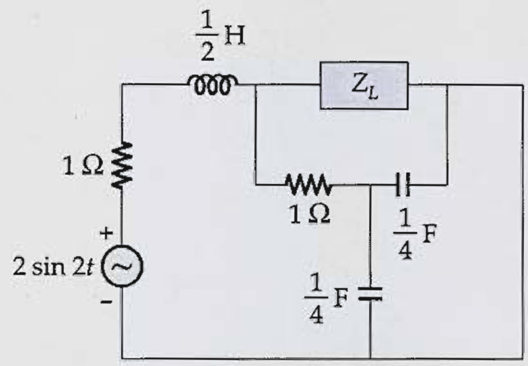
∴ Total current I by superposition theorem

$I = I_1 + I_2$

$= 3.535\angle 45^\circ + 1.414\angle -45^\circ$

$$\boxed{I = 3.807\angle 23.2^\circ\ A}$$

Q.1 (c) **(ii)** Determine the value of impedance Z_L for maximum power transfer, in Z_L, in the given network.

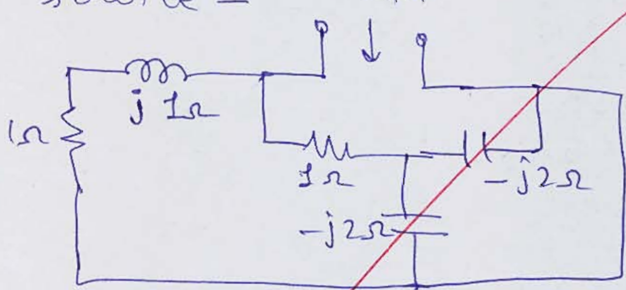


[6 marks]

$\therefore \omega = 2$ r/s $\quad X_L = \omega L = 2 \times \frac{1}{2} = 1\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2 \times 1/4} = 2\Omega$

For Z_{th} across Z_L — ~~open~~ short voltage source —



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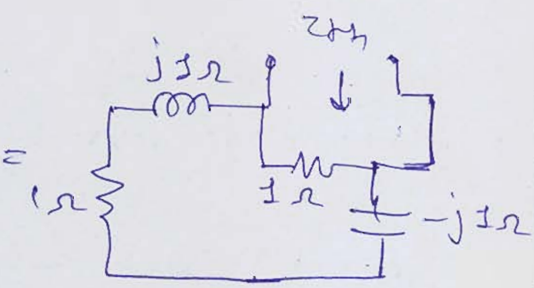


$Z' = -j2\Omega \parallel j2\Omega$

$Z' = \frac{-j2 \times j2}{-j2 + (-j2)} = -j1\Omega$

2

$Z_{th} = \frac{1 \times (1 + j1 - j1)}{1 + 1 + j1 - j1}$

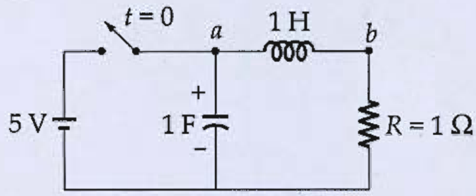
$Z_{th} = \frac{1 \times 1}{2} = 0.5\Omega$

$\therefore$ For maximum power transfer to Z_L

$\therefore Z_L = Z_{th}^*$

$\boxed{Z_L = 0.5\Omega}$

(d) Consider the following circuit:

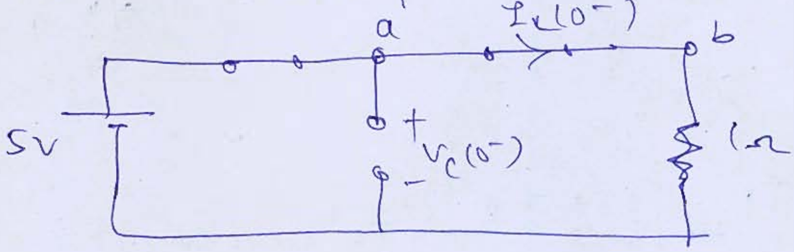


The switch is initially closed. After steady state is reached the switch is opened. Determine the nodal voltage $V_a(t)$ and $V_b(t)$.

[12 marks]

$\underline{t < 0}$

$t = 0^-$ steady state is reached -



$I_L(0^-) = \frac{5}{1} = 5\,A$

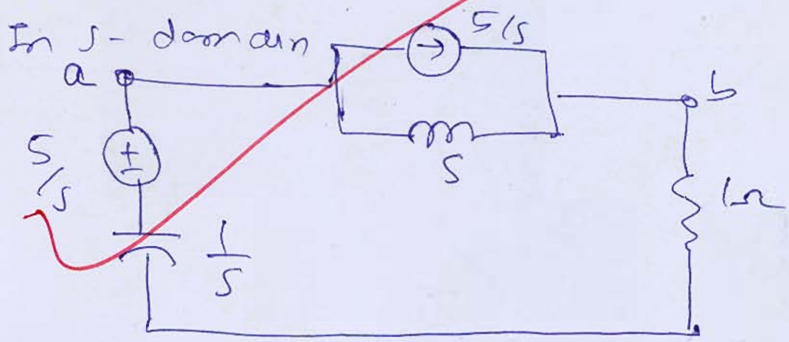
$V_C(0^-) = 5\,V$

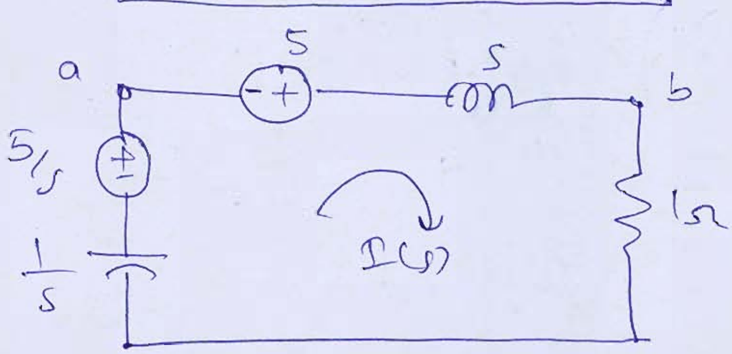
$t > 0$ — switch is opened

$V_C(0^+) = V_C(0^-) = 5\,V$

$i_L(0^+) = i_L(0^-) = 5\,A$

In s - domain





$$I(s) = \frac{5 + 5/s}{s + 1 + \frac{1}{s}} = \frac{5(s+1)}{s^2+s+1}$$

$$\therefore V_b(s) = 1 \times I(s)$$

$$= \frac{5(s+1)}{(s^2+s+1)} = \frac{5(s+1)}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$V_b(s) = \frac{5\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\frac{5}{2} \times \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

Taking inverse laplace transform

$$V_b(t) = 5\, e^{-t/2} \cos \frac{\sqrt{3}}{2} t + \frac{5}{\sqrt{3}}\, e^{-t/2} \sin \frac{\sqrt{3}}{2} t$$

$$\boxed{V_b(t) = 5\, e^{-t/2}\left[\cos \frac{\sqrt{3}}{2} t + \frac{1}{\sqrt{3}} \sin \frac{\sqrt{3}}{2} t\right] V}$$

Now,

$$0 - \frac{1}{s} \times I(s) + \frac{5}{s} = V_a(s)$$

$$V_o(s) = \frac{5}{s} - \frac{1}{s} \times \frac{5(s+1)}{(s^2+s+1)}$$

$$= \frac{5}{s} - \left[\frac{A}{s} + \frac{Bs+C}{s^2+s+1}\right]$$

$$As^2 + As + A + Bs^2 + Cs = 5s + 5$$

$A = 5$ $\quad A + B = 0$ $\quad A + C = 5$

$B = -5$ $\quad C = 0$

$$= \frac{5}{s} - \frac{5}{s} \cancel{s} + \frac{5s}{s^2+s+1}$$

$$= \frac{5\left(s+\frac{1}{2}-\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$V_a(s) = \frac{5\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} - \frac{\frac{5}{2}\times\frac{\sqrt{3}}{2}\times\frac{2}{\sqrt{3}}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

Taking inverse laplace transform -

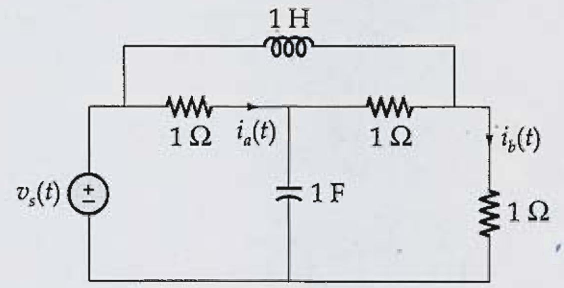
$$V_a(t) = 5e^{-t/2}\cos\frac{\sqrt{3}}{2}t - \frac{5}{\sqrt{3}}e^{-t/2}\sin\frac{\sqrt{3}}{2}t$$

$$\boxed{V_a(t) = 5e^{-t/2}\left\{\cos\frac{\sqrt{3}}{2}t - \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right\} V}$$

11

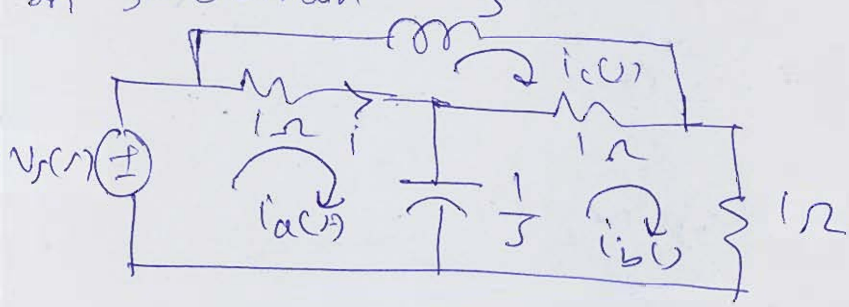
Good Approach

Q.1 (e) For the bridged T-network of the figure given below the source voltage is $v_s(t) = 2\cos t$. The circuit is in steady state condition. Determine: **(i)** $i_a(t)$ and **(ii)** $i_b(t)$.



[12 marks]

In s-domain



KVL in loop ①

$$V_S = \left(1+\frac{1}{s}\right) I_a(s) - \frac{1}{s} I_b(s) - I_c(s)$$

2

KVL in loop 2

$$-\frac{1}{s} I_a(s) + \left(\frac{1}{s}+1+1\right) I_b(s) - I_c(s) = 0$$

$$-\frac{1}{s} I_a(s) + \left(\frac{1}{s}+2\right) I_b(s) - I_c(s) = 0 \quad - ②$$

KVL in loop 3 –

$$s I_c(s) + I_c(s) - I_b(s) - I_a(s) = 0$$

$$I_a(s) + I_b(s) = (s+1) I_c(s) \quad ③$$

$$I_a(s) + I_b(s) = (s+1)\left[-\frac{1}{s} I_a(s) + \left(\frac{1}{s}+2\right) I_b(s)\right]$$

$$I_a(s) + I_b(s) = -\frac{(s+1)}{s} I_a(s) + (s+1)\frac{(1+2s)}{s} I_b(s)$$

$$\left[1 + \frac{s+1}{s}\right] I_a(s) = \left(\frac{(s+1)(1+2s)}{s} - 1\right) I_b(s)$$

$$(2s+1)\, I_a(s) = [3s + 2s^2 + 1 - s] I_b(s)$$

$$I_a(s) = \left[\frac{2s^2+2s+1}{2s+1}\right] I_b(s)$$

put in eq-1 -

$$V_s = \left(\frac{s+1}{s}\right) I_a(s) - \frac{1}{s} I_b(s) - \frac{[I_a(s)+I_b(s)]}{s+1}$$

$$= \left(\frac{s+1}{s} - \frac{1}{s+1}\right) I_a(s) - \left[\frac{1}{s} + \frac{1}{s+1}\right] I_b(s)$$

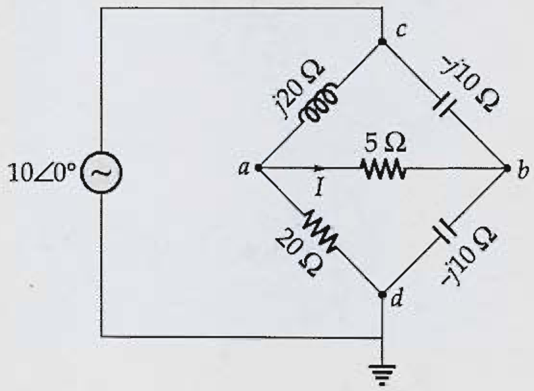
$$= \left[\frac{(s+1)^2 - s}{s(s+1)}\right] I_a(s) - \left[\frac{s+1+s}{s(s+1)}\right] I_b(s)$$

$$= \frac{s^2+s+1}{s(s+1)} \times \left[\frac{2s^2+2s+1}{2s+1}\right]$$

$$- \quad s$$

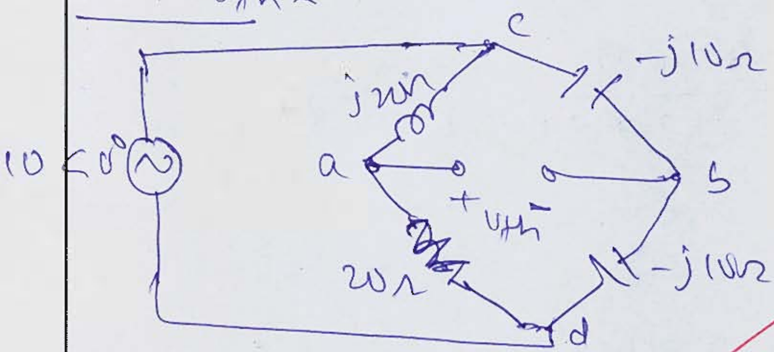
Incomplete solution

Q.2 (a) (i) Determine current I in the network using Thevenin's theorem.



[10 marks]

For V_{th} -



By voltage divider rule -

$V_{ad} = 10\angle 0^\circ \times \frac{20}{20+j20}$

$V_{ad} = 7.07\angle -45^\circ$ V

$\therefore V_{bd} = 10\angle 0^\circ \times \left[\frac{-j10}{-j10-j10}\right]$

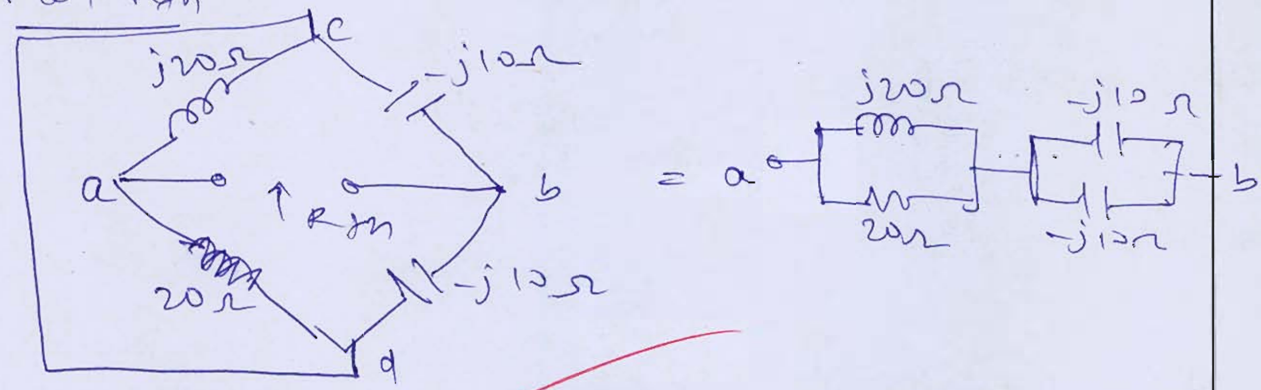
$= 5\angle 0^\circ$ V

$V_{th} = V_{ad} - V_{bd}$

$= 7.07\angle -45^\circ - 5\angle 0^\circ$

$\boxed{V_{th} = 5\angle -90^\circ \text{ V}}$

For R_{th}



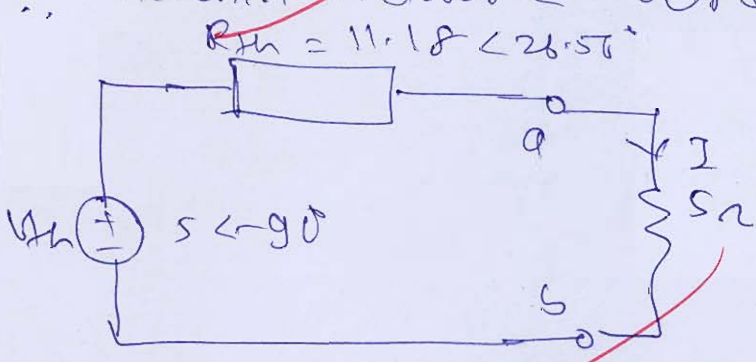
$R_{th} = j20 \parallel 20\Omega + (-j10) \parallel (-j10)$

$$R_{th} = \frac{j20 \times 20}{j20 + 20} + \frac{(-j10) \times (-j10)}{-j10 - j10}$$

$$= 14.14\angle 45^\circ - j5$$

$$\boxed{R_{th} = 11.18\angle 26.56^\circ \Omega}$$

∴ Thevenin network across $R = 5\Omega$



∴ current $I = \dfrac{5\angle -90^\circ}{11.18\angle 26.56^\circ + 5}$

$$\boxed{I = 0.316\angle -108.43^\circ A}$$

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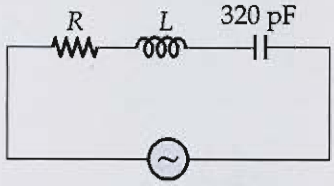
Good Approach

Q.2 (a) **(ii)** For the circuit shown determine the value of inductance for resonance if $Q = 50$ and $f_0 = 175$ kHz. Also find the circuit current, the voltage across the capacitor and the bandwidth of the circuit.



[10 marks]

For series RLC circuit resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow 175 \times 10^3 = \frac{1}{2\pi\sqrt{L \times 320 \times 10^{-12}}}$$

$$\Rightarrow (175 \times 10^3)^2 = \frac{1}{4\pi^2 \times L \times 320 \times 10^{-12}}$$

$$\boxed{L = 2.584 \text{ mH}}$$

Current at resonance –

$$I = \frac{V}{R}$$

$\therefore$ For series RLC circuit –

$$Q = \frac{\omega L}{R} = \frac{2\pi \times 175 \times 10^3 \times 2.584 \times 10^{-3}}{R}$$

$$\Rightarrow 50 = \frac{2841.25}{R}$$

$$\boxed{R = 56.825\ \Omega}$$

$\therefore$ current I –

$$I = \frac{0.85}{56.825} = 14.96 \text{ mA}$$

At resonance voltage across capacitor

$$V_C = Q V$$
$$= 50 \times 0.85$$
$$= 42.5 V$$

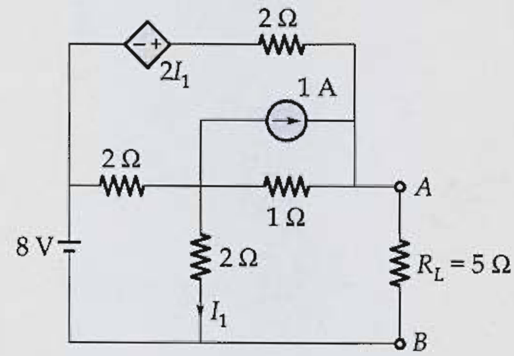
∴ Band width of series RLC circuit

$$BW = \frac{\omega_0}{Q}$$
$$= \frac{f_0}{Q}$$
$$= \frac{175 \times 10^3}{50}$$
$$BW = 3500 \; Hz$$

or 557.04 r/s

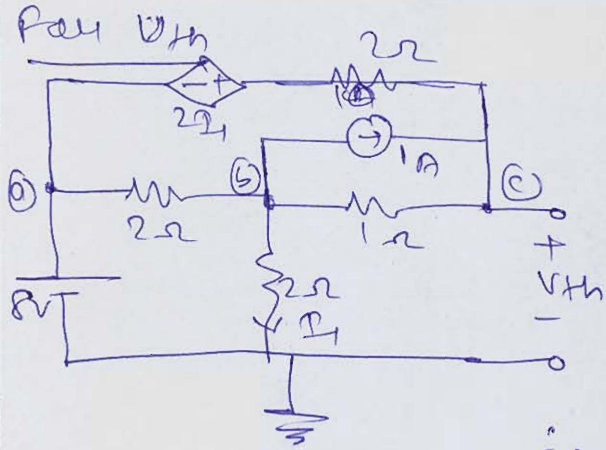
9

Q.2 (b) Determine the current through the load resistance $R_L = 5\ \Omega$ across the terminals A-B of the circuit shown in figure below, using Thevenin's theorem. Also, find the maximum power that can be transferred to the load resistance R_L.



[20 marks]

For V_{th}



voltage at node

(a) – $V_a = 8V$

$V_b = V$ (let)

$V_c = V_{th}$

∴ current $I_1 = \frac{V}{2}$

KCL at node (c) –

$$\frac{V_{th} - 2I_1 - 8}{2} + \frac{V_{th} - V}{1\Omega} = 1$$

$$V_{th} - 2I_1 - 8 + 2V_{th} - 2V = 2$$

$$3V_{th} - 2 \times \frac{V}{2} - 2V = 10$$

$$3V_{th} - 3V = 10 \quad - (1)$$

KCL at node (b) –

$$\frac{V}{2} + \frac{V - 8}{2} + \frac{V - V_{th}}{1} + 1 = 0$$

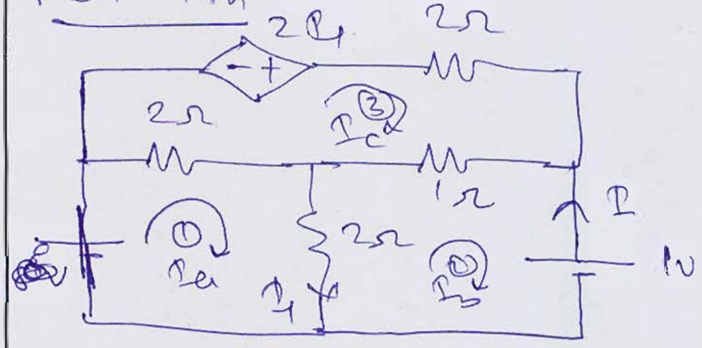
$$V + V - 8 + 2V - 2V_{th} + 2 = 0$$

$$-2V_{th} + 4V = 6 \quad - (2)$$

Solving eq-① and ② we get

$$\boxed{V_{th} = 9.67 V}$$

For R_{th}



~~KVL in loop - ①~~

~~$2(I_a - I_1) + 2(I_a - I_b) = 0$~~

~~$4I_a - 2I_b = 0$~~

KVL in loop ① -

$2(I_a - I_c) + 2(I_a - I_b) = 0$

$4I_a - 2I_b - 2I_c = 0$ — ③

KVL in loop ② -

$2(I_b - I_a) + 1(I_b - I_c) + 1 = 0$

$-2I_a + 3I_b - I_c = -1$ — ④

KVL in loop ③ -

$1(I_c - I_b) + 2(I_c - I_a) - 2I_1 + 2I_c = 0$

$-2I_a - I_b + 5I_c - 2I_1 = 0$

$\therefore I_1 = I_a - I_b$

$-2I_a - I_b + 5I_c - 2(I_a - I_b) = 0$

$-4I_a + I_b + 5I_c = 0$ — ⑤

solving eq - (3), (4), (5) -

$I_a = -0.5\ A$

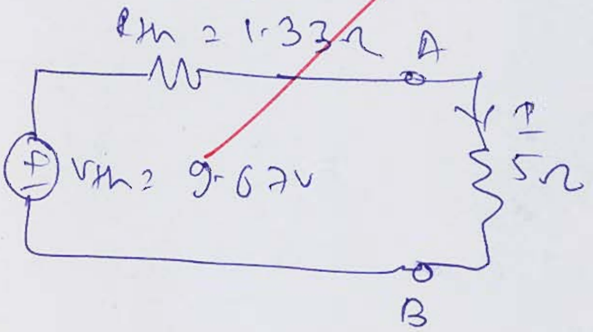
$I_b = -0.75\ A$

$I_c = -0.25\ A$

$\therefore$ current $I = -I_b = 0.75\ A$

$R_{th} = \dfrac{1}{I} = 1.33\ \Omega$

Thevenin equivalent circuit across AB



current

$I = \dfrac{9.67}{1.33+5}$

$I = 1.527\ A$

For maximum power transfer to load R_L

$\therefore R_L = R_{th} = 1.33\ \Omega$

And maximum power -

$P_{max} = \dfrac{V_{th}^2}{4R_{th}} = \dfrac{(9.67)^2}{4 \times 1.33}$

$\boxed{P_{max} = 17.576\ W}$

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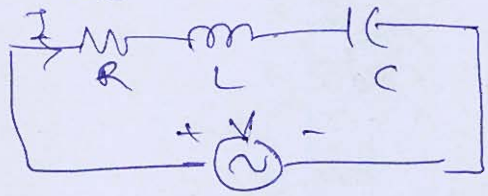
Good Approach

(c) (i) Derive expression for frequency for maximum voltage across inductor in series RLC resonant circuit.

(ii) Calculate the maximum voltage across the inductor using result of **part (i)** with constant voltage and variable frequency. Assume $R = 50\ \Omega$, $L = 0.05\text{H}$, $C = 20\ \mu\text{F}$ and $V = 100$ V.

[13 + 7 marks]

i) series RLC



current $I = \dfrac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$

$\therefore \quad |I| = \dfrac{V}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$

Voltage across inductor L

$V_L = I \times \omega L$

$V_L = \dfrac{V \omega L}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}}$

For V_L to be maximum $\dfrac{dV_L}{d\omega} = 0$

$\dfrac{dV_L}{d\omega} = VL \left[\dfrac{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2} - \omega \cdot \frac{1}{2}\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2} 2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} \right.$

$\dfrac{dV_L}{d\omega} = 0$

$\Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 - \omega\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right) = 0$

$$R^2 + \left[\frac{\omega^2 LC - 1}{\omega C}\right]^2 - \omega\left(\frac{\omega^2 LC - 1}{\omega C}\right)\left[\frac{\omega^2 LC + 1}{\omega^2 C}\right] = 0$$

$$R^2 + \frac{(\omega^2 LC - 1)^2}{\omega^2 C^2} - \frac{(\omega^2 LC - 1)(\omega^2 LC + 1)}{\omega^2 C^2} = 0$$

$$R^2\omega^2 C^2 + \omega^4 L^2 C^2 + 1 - 2\omega^2 LC - [\omega^4 L^2 C^2 - 1] = 0$$

$$R^2\omega^2 C^2 - 2\omega^2 LC + 2 = 0$$

$$\omega^2 \{2LC - R^2C^2\} = 2$$

$$\omega^2 = \frac{2}{2\left[LC - \frac{R^2C^2}{2}\right]}$$

$$\omega = \frac{1}{\left[LC - \frac{R^2C^2}{2}\right]^{1/2}}$$

$\therefore$ Above frequency at which voltage across inductor is maximum.

(ii) For Given $R = 50\,\Omega$ $L = 0.05\,H$
$C = 20\,\mu F$

$$\omega = \frac{1}{\left[0.05 \times 20\times10^{-6} - \frac{(50)^2(20\times10^{-6})^2}{2}\right]^{1/2}}$$

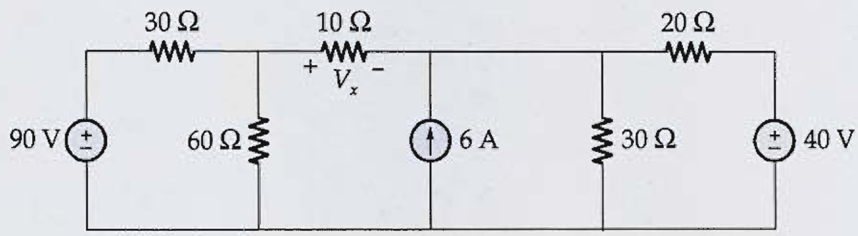
$$= \frac{1}{\left[1\times10^{-6} - 5\times10^{-7}\right]^{1/2}}$$

$$\omega = 1.414\, \times 10^{3} \; r/s$$

$$\omega = 225.08 \; Hz$$

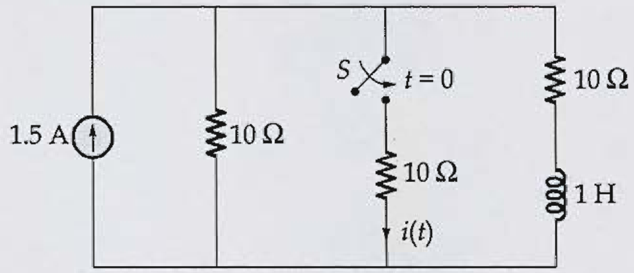
5

Q.3 (a) **(i)** The circuit shown in the figure below consists of three independent sources. Determine the value of the voltage across 10 Ω resistance using superposition theorem.



[10 marks]

Q.3 (a) (ii) Consider the network shown below:

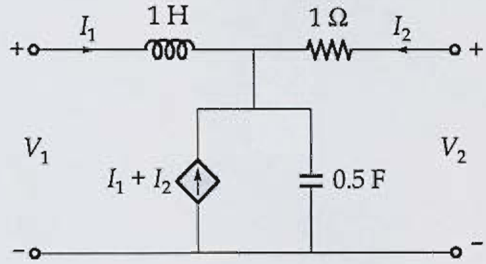


If switch S is closed at $t = 0$, calculate $i(t)$ for $t > 0$ by using Laplace transform approach.

[10 marks]

Do not write in this margin

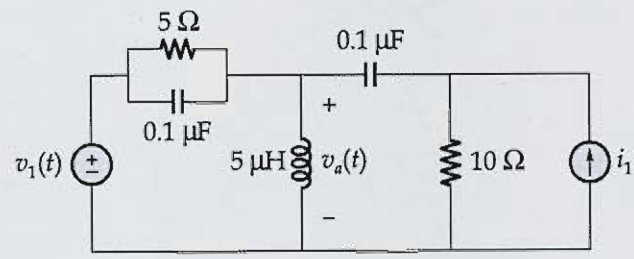
Q.3 (b) Determine the transmission parameters matrix for the two port network shown below.



[20 marks]

Do not write in this margin

Q.3 (c) (i) For the circuit shown below, $v_1(t) = 10 \sin 10^6 t$ V and $i_1(t) = 10 \cos 10^6 t$ A and the circuit is operating in steady state condition. Determine the node to datum voltage $v_a(t)$.



[10 marks]

Do not write in this margin

Q.3 (c) (ii) A certain practical dc voltage source can provide a current of 2.5 A when it is (momentarily) short circuited and can provide a power of 80 W to 20 Ω load.

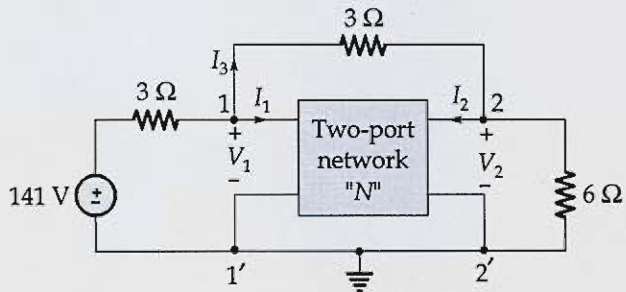
Find:

1. The open circuit voltage.
2. The maximum power it could deliver to a well-chosen R_L.
3. What is the value of that R_L?

[10 marks]

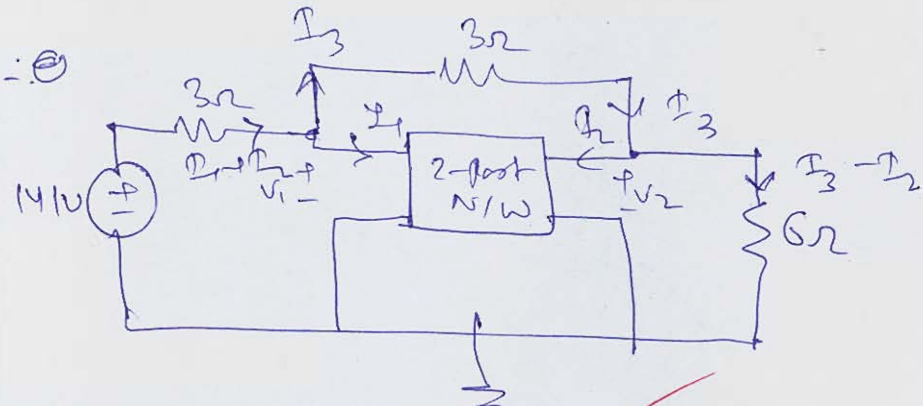
Do not write this margin

Q.4 (a) (i) The z-parameters of the two port network-N shown in the figure below are given as $z_{11} = 2\ \Omega$, $z_{12} = z_{21} = 1\ \Omega$ and $z_{22} = 4\ \Omega$. Find the values of currents I_1, I_2 and I_3.



[12 marks]

Sol:



For 2-port n/w

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = 2I_1 + I_2$$
$$V_2 = I_1 + 4I_2$$

Input KVL

$$141 = 3(I_1 + I_3) + V_1$$
$$= 3(I_1 + I_3) + 2I_1 + I_2$$
$$5I_1 + I_2 + 3I_3 = 141 \quad — (1)$$

Output KVL

$$V_2 = 6(I_3 - I_2)$$
$$I_1 + 4I_2 = 6I_3 - 6I_2$$
$$I_1 + 10I_2 - 6I_3 = 0 \quad — (2)$$

$\therefore$ KVL input to output –

$V_1 - 3I_3 - V_2 = 0$

$\Rightarrow 2I_1 + I_2 - 3I_3 - (I_1 + 4I_2) = 0$

$I_1 - 3I_2 - 3I_3 = 0 \quad - (3)$

solving eq – (1), (2) and (3)

$I_1 = 24 A$

$I_2 = 1.5 A$

$I_3 = 6.5 A$

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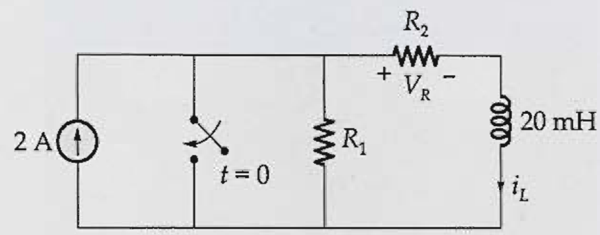
Good Approach

Do not write in this

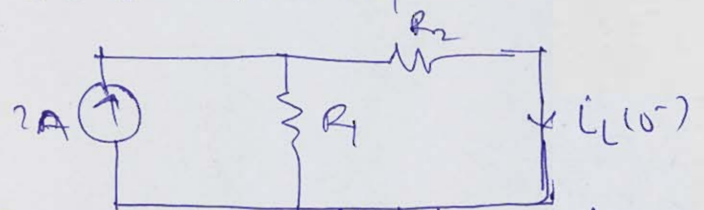
Q.4 (a) **(ii)** Determine values of R_1 and R_2 in the circuit of figure such that $V_R(0^+) = 10$ V and $V_R(1 \text{ msec}) = 5$ V.



[8 marks]

$\underline{t < 0}$

$t = 0^+$ steady state reached ~



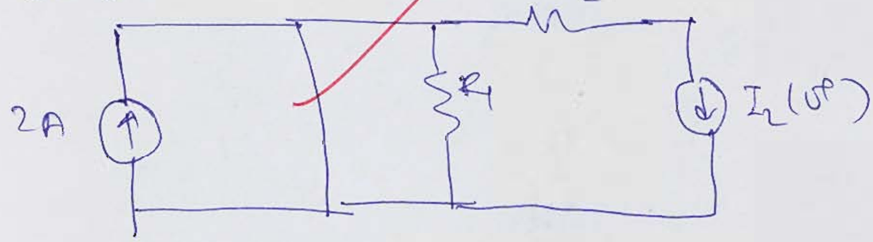
By current divider rule ~

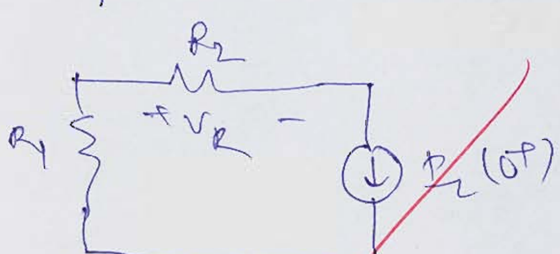
$$i_L(0^-) = 2 \times \frac{R_1}{R_1 + R_2}$$

2

$\underline{t > 0}$

$t = 0^+$





$$I_L(0^+) = I_L(0^-)$$

$$= \frac{2R_1}{R_1 + R_2}$$

$$V_R(0^+) = I_L(0^+) \times R_2 \quad \text{— (1)}$$

$$\Rightarrow \quad 10 = \frac{2R_1}{R_1 + R_2} \times R_2$$

$$\frac{R_1 R_2}{R_1 + R_2} = 5 \quad \text{— (1)}$$

at $t = \infty$

$i_L(\infty) = 0$

$\therefore \; i(t) = i(\infty) + \{i(0) - i(\infty)\} e^{-t/\tau}$

$= 0 + [I_L(0^+) - 0] e^{-t/\tau}$

$i(t) = I_L(0^+) e^{-t/\tau}$

$\tau = \frac{L}{R_{eq}} = \frac{L}{R_1 + R_2}$

$V_R(t) = i(t) \times R_2$

$V_R(t) = I_L(0^+) e^{-t/\tau} \times R_2 = 10 e^{-t/\tau}$

$\therefore t = 1 msec$

$5 = 10 e^{-t/\tau} = 10 e^{-\frac{1 msec}{\tau}}$

$-\frac{1 \times 10^{-3}}{\tau} = -0.693$

$\tau = \frac{10^{-3}}{0.693} = \frac{L}{R_1 + R_2} = \frac{20 \times 10^{-3}}{R_1 + R_2}$

$\Rightarrow \quad R_1 + R_2 = 13.92 \quad - (3)$

from eq-(1) $\quad R_1 R_2 = 69.6 \quad - (4)$

$R_1 - R_2 = [(R_1 + R_2)^2 - 4 R_1 R_2]^{1/2}$

$= [\underbrace{(13.92)^2 - 4 \times 69.6}_{-ve}]^{1/2}$

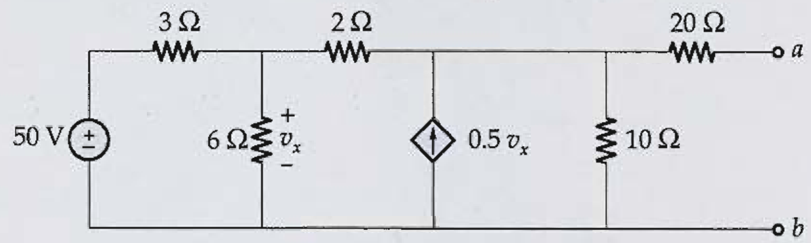
$\therefore \quad R_2 - R_1 = (84.63)^{1/2} = 9.2 \quad [R_1 < R_2]$

$R_2 - R_2 = 9.2 \quad - (5)$

solving (3) & (4) we get -

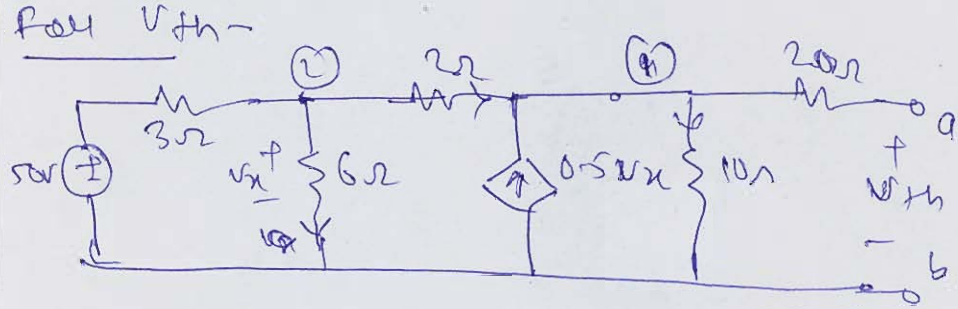
$R_1 = 2.36\,\Omega \qquad R_2 = 11.56\,\Omega$

Q.4 (b) Determine the Thevenin's equivalent network and Norton's current at terminals *a-b* for the circuit shown below and draw the two equivalent circuits.



[20 marks]

<u>For Vth –</u>



$I_{6\Omega} = \frac{v_x}{6}$ $\quad I_{10\Omega} = \frac{V_{th}}{10}$

<u>KCL at ①</u>

$\therefore I_{2\Omega} + 0.5 v_x = \frac{V_{th}}{10}$

$I_{2\Omega} = \frac{V_{th}}{10} - 0.5 v_x$

<u>KCL at ②</u>

$I_{3\Omega} = \frac{v_x}{6} + I_{2\Omega} = \frac{v_x}{6} + \frac{V_{th}}{10} - 0.5 v_x$

$I_{3\Omega} = -0.333 v_x + 0.1 V_{th}$

$\therefore$ <u>KVL</u>

$50 = 3(-0.333 v_x + 0.1 V_{th}) + v_x$

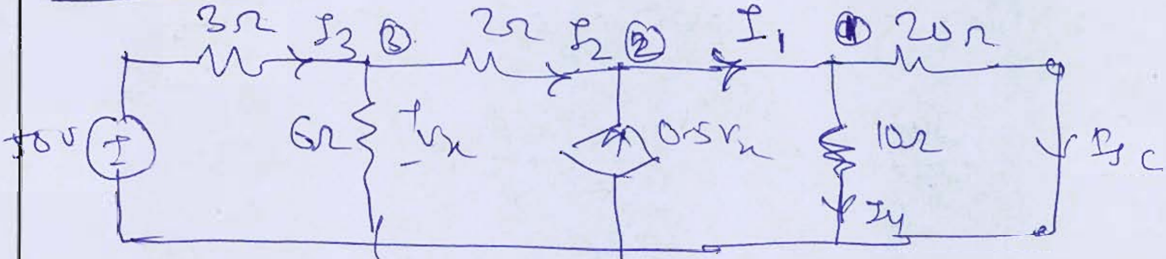
$50 = 0.3 V_{th}$

$\boxed{V_{th} = 166.67 \text{ V}}$

Thevenin equivalent voltage across ab as –

$\boxed{V_{th} = 166.67 \text{ V}}$

For I_{SC}



$\therefore \quad 10 I_4 = 20 I_{SC}$

$I_4 = 2 I_{SC}$

KCL at node ①

$\therefore \quad I_1 = I_4 + I_{SC} = 2I_{SC} + I_{SC} = 3 I_{SC}$

KCL at node ②

$I_2 + 0.5 v_x = I_1 = 3 I_{SC}$

$I_2 = 3 I_{SC} - 0.5 v_x$

KCL at node ③

$I_3 = I_2 + \frac{v_x}{6}$

$= 3 I_{SC} - 0.5 v_x + \frac{v_x}{6}$

$I_3 = 3 I_{SC} - 0.333 v_x$

KVL

$50 = 3 I_3 + v_x$

$= 3 (3 I_{SC} - 0.333 v_x) + v_x$

$= 9 I_{SC}$

$I_{SC} = 5.55 A$

Norton's equivalent current comes as —

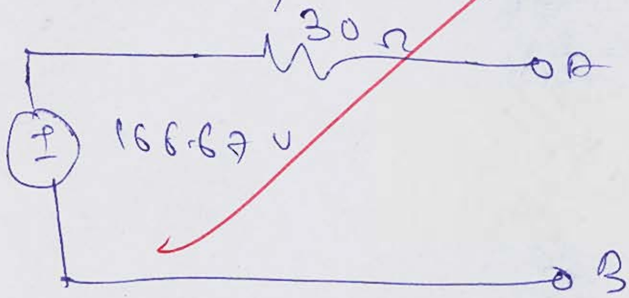
$I_N = I_{SC} = 5.55 A$

$\therefore$ Thevenin ~~equivalent~~ equivalent resistance

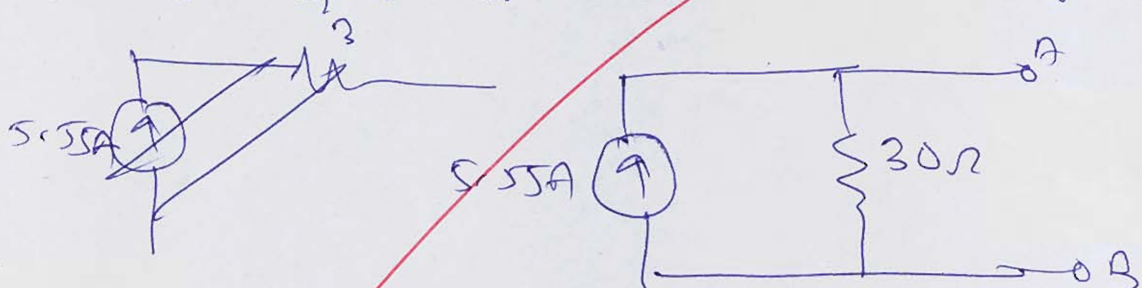
= Norton's equivalent resistance

$$= \frac{V_{th}}{I_{sc}} = \frac{166.67}{5.55} = 30\,\Omega$$

Thevenin equivalent network across AB –



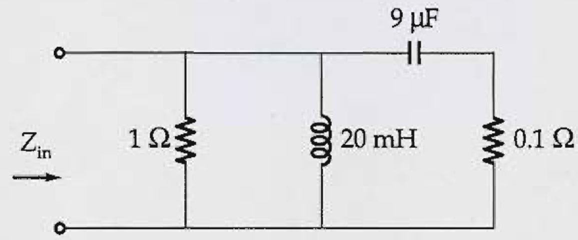
Norton's equivalent network across AB –



18

Good Approach

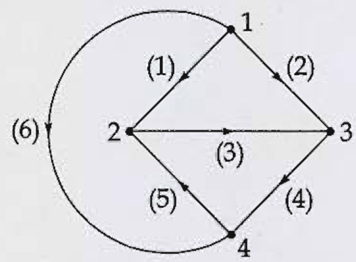
(c) (i) For the circuit shown in the figure below:



Determine:

1. The resonant frequency, ω_0.
2. Input impedance at resonant frequency, $Z_{in}(\omega_0)$.

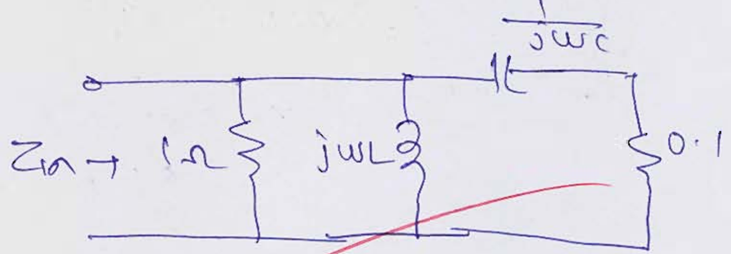
(ii) For the graph shown below:



Find the number of possible trees.

[15 + 5 marks]

(i)



$$Y_{in} = \frac{1}{1} + \frac{1}{j\omega L} + \frac{1}{0.1 + \frac{1}{j\omega C}}$$

$$= 1 - \frac{j}{\omega L} + \frac{j\omega C}{1 + j\omega C \times (0.1)}$$

$$= 1 - \frac{j}{\omega L} + \frac{j\omega C\,(1 - j\omega C \times 0.1)}{1 + \omega^2 C^2 \times 0.01}$$

~~At~~ ~~resonance~~ resonance Y_{in} is real

$$-\frac{1}{\omega L} + \frac{\omega C}{1 + \omega^2 C^2 \times 0.01} = 0$$

$$\frac{1}{\omega L} = \frac{\omega C}{1 + \omega^2 C^2 \times 0.01} \Rightarrow 1 + \omega^2 C^2 \times 0.01 = \omega^2 LC$$

$$\omega^2 [LC - C^2 \times 0.01] = 1$$

$$\omega^2\left[20\times10^{-3}\times 9\times10^{-6} - (9\times10^{-6})^2\times 0.01\right] = 1$$

$$\omega = 2.357 \times 10^3 \text{ r/s}$$

$\therefore$ Resonant frequency

$$\omega_0 = 2.357\times10^3 \text{ r/s} = 2357 \text{ r/s}$$

$\therefore$ At resonant frequency

$$Y_{in} = 1 + \frac{\omega^2 C^2 \times 0.1}{1+\omega^2 C^2 \times 0.01}$$

$$= 1 + \frac{(2357)^2\times(9\times10^{-6})^2\times 0.1}{1+(2357)^2\times(9\times10^{-6})^2\times 0.01}$$

$$Y_{in} = \cancel{\text{...}}\ 1 \text{ ℧}$$

Input impedance at resonance -

$$Z_{in} = \frac{1}{Y_{in}} = 1\,\Omega$$

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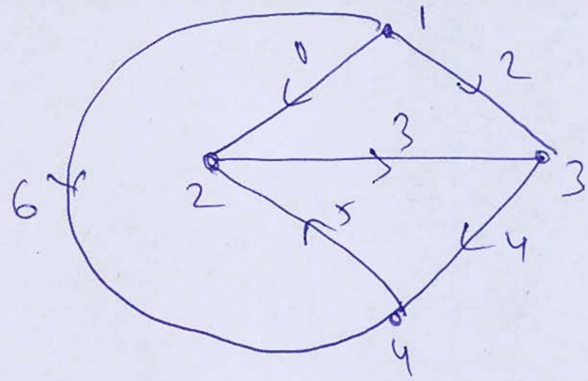
Good Approach

(ii)



Number of nodes = n = 4
Number of branch = b = 6

Since the given graph is completely connected - as a direct path exist between any two node

∴ For completely connected graph
number of trees = $(n)^{n-2}$
where n = number of node
$= (4)^{4-2} = (4)^2 = 16$

5

∴ ~~Rest~~ number of trees = 16

Good Approach

Section B : Engineering Mathematics

.5 (a) Find the solution of the differential equation $(y - x + 1)dy - (y + x + 2)dx = 0$.

[12 marks]

$$(y - x + 1)\,dy - (y + x + 2)\,dx = 0$$

$$\Rightarrow y\,dy - x\,dy + dy - y\,dx - x\,dx + 2\,dx = 0$$

$$y\,dy - [x\,dy + y\,dx] + dy - x\,dx + 2\,dx = 0$$

2

$$\Rightarrow y\,dy - d(xy) + dy - x\,dx + 2\,dx = 0$$

Integrate the above equation we get –

$$\frac{y^2}{2} - xy + y - \frac{x^2}{2} + 2x = c$$

$$\Rightarrow \frac{1}{2}(x^2 + y^2) - xy + y + 2x = c$$

∴ solution of the given differential equation –

$$\frac{1}{2}(x^2 + y^2) - xy + y + 2x = c$$

Go through the made easy solution

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.5 (b) Find the value of $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\,dx$.

[12 marks]

$$I = \int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\,dx$$

$$= \int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+9x^2+x^2+9}\,dx$$

$$= \int_{-\infty}^{\infty} \frac{x^2-x+2}{(x^2+9)(x^2+1)}\,dx$$

$$= \int_{-\infty}^{\infty}$$

$$\frac{x^2-x+2}{(x^2+9)(x^2+1)} = \frac{A}{x^2+9} + \frac{B}{x^2+1}$$

$$= \cancel{Ax^2+9A+Bx^2+9B}$$

$$= Ax^2+A+Bx^2+9B$$

$$A+B = 1$$

$$A+9B = 2$$

$$A = 0.875$$

$$B = 0.125$$

$$= \int_{-\infty}^{\infty} \frac{0.875}{x^2+9} + \frac{0.125}{x^2+1}\,dx$$

$$= \frac{0.875}{3}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_{-\infty}^{\infty} + \frac{0.125}{1}\left(\tan^{-1}(x)\right)_{-\infty}^{\infty}$$

$$= \frac{0.875}{3} \times \pi + 0.125 \times \pi$$

$\boxed{I = 0.416\ \pi}$

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Good Approach

5 (c) (i) The vector field $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ is defined over the volume of the cuboid given by $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. Evaluate the surface integral $\iint_S \vec{F} \cdot \vec{ds}$, where S is the surface of the cuboid.

[6 marks]

Gauss divergence theorem

$$\oiint \bar{F} \cdot \bar{ds} = \iiint \nabla \cdot \bar{F} \cdot dv$$

$$\bar{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$$

$$\nabla \cdot \bar{F} = 2x + 1 + y$$

$$\therefore \oiint \bar{F} \cdot \bar{ds} = \iiint (2x + 1 + y)\, dx\, dy\, dz$$

where $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$

$$= \int_{x=0}^{a} \int_{y=0}^{b} \int_{z=0}^{c} (2x + 1 + y)\, dx\, dy\, dz$$

3

$$= \int_{x=0}^{a} \int_{y=0}^{b} (2x + 1 + y)(z)_0^c\, dx\, dy$$

$$= c \int_{x=0}^{a} \int_{y=0}^{b} (2x + 1 + y)\, dx\, dy$$

$$= c \int_{x=0}^{a} \left[2xy + y + \frac{y^2}{2}\right]_0^b dx$$

$$= c \int_{x=0}^{a} \left[2xb + b + \frac{b^2}{2}\right] dx$$

Do not write in this margin

$$= c\left[\frac{2x^2 b}{2} + \left(b + \frac{b^2}{2}\right)x\right]_0^a$$

$$= c\left[\cancel{2}\, a^2 b + a\left(b + \frac{b^2}{2}\right)\right]$$

$$= c\left[a^2 b + ab + \frac{ab^2}{2}\right)$$

$$= abc\left[a + 1 + \frac{b}{2}\right]$$

$$\therefore \iint_S \bar{F}\cdot \overline{dS} = abc\left[a + 1 + b/2\right]$$

Do not write in this margin

.5 (c) (ii) Find the absolute maxima and minima of

1. $f(x) = 2x^3 - 9x^2 + 12x - 5$ in $[0, 3]$
2. $f(x) = 12x^{4/3} - 6x^{1/3}, x \in [-1, 1]$

Also, find points of maxima and minima.

[6 marks]

① $f(x) = 2x^3 - 9x^2 + 12x - 5$

$f'(x) = 6x^2 - 18x + 12$

at maxima or minima

$f'(x) = 0$

$6x^2 - 18x + 12 = 0$

$x = 2, 1$

$\therefore$ at $x = 2$ $f(2) = -1$

$x = 1$ $f(1) = 0$

$\therefore$ at $x = 0$ $f(0) = -5$

$x = 3$ $f(3) = 4$

$\therefore$ Absolute maxima $= 4$ at $x = 3$

Absolute minima $= -5$ at $x = 0$

② $f(x) = 12x^{4/3} - 6x^{1/3}$

$f'(x) = 12 \times \frac{4}{3} x^{1/3} - 6 \times \frac{1}{3} x^{-2/3}$

$= 16x^{1/3} - 2x^{-2/3}$

$\Rightarrow 16x^{1/3} - 2x^{-2/3} = 0$

$16x - 2 = 0$

$x = 1/8$

$\therefore x = \frac{1}{8}$ $f(x) = -2.25$

$x = 1$ $f(x) = 6$

$x = -1 \quad f(x) = 18$

Absolute maxima $= \cancel{68}\ 18$ at $x = -1$

Absolute minima $= -2.25$ at $x = \frac{1}{8}$

5

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5 (d) Determine the values of x for which the following function fails to be continuous or differentiable.

$$f(x)=\begin{cases} 1-x & , \ x<1 \\ (1-x)(2-x), & 1\le x\le 2 \\ 3-x & , \ x>2 \end{cases}$$

[12 marks]

check continuity at $x=1$

LHL at $x=1$

$\lim_{h\to 0} f(1-h) = \lim_{h\to 0} 1-(1-h) = 0$

RHL at $x=1$

$\lim_{h\to 0} f(1+h) = \lim_{h\to 0} [1-(1+h)]\{2-(1+h)\}$

$= 0$

at $x=1$

$f(1) = (1-1)(2-1) = 0$

LHL = RHL = $f(1) = 0$

$f(x)$ is continuous at $x=1$

check continuity at $x=2$

LHL at $x=2$

$\lim_{h\to 0} f(2-h) = \lim_{h\to 0} [1-(2-h)][2-(2-h)]$

$= 0$

RHL at $x=2$

$\lim_{h\to 0} f(2+h) = \lim_{h\to 0} 3-(2-h) = 1$

LHL $\neq$ RHL

$\therefore f(x)$ is not continuous at $x=2$

since it is not continuous it is also not differentiable at $x=2$

~~f(x) = lf~~

check differentiability at $x = 1$

LHD at $x = 1$

$$\lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{1-(1-h) - 0}{-h}$$

$$= \lim_{h \to 0} \frac{-h}{h} = -1$$

RHD at $x = 1$

$$\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1-(1+h))[2-(1+h)] - 0}{h}$$

$$= \lim_{h \to 0} \frac{-h \times (1-h)}{h}$$

$$= -1$$

LHD = RHD = -1

$f(x)$ is differentiable at $x = 1$

$\therefore$ Hence, the given fn is fails to be continuous at $x = 2$

and differentiable at $x = 2$

11

(e) X is a continuous random variable with probability density function given by

$$f(x) = kx\,(0 \le x \le 2)$$
$$= 2k\,(2 \le x < 4)$$
$$= -kx + 6k\,(4 \le x < 6)$$

Find k and mean value of X.

[12 marks]

For $f(x)$ to be probability density function $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx+6k)\,dx$$

$$= k\left(\frac{x^2}{2}\right)_0^2 + 2k(x)_2^4 + \left[-\frac{kx^2}{2} + 6kx\right]_4^6$$

$$= 2k + 4k + [-18k + 36k - (-8k + 24k)]$$

$$= 6k + (18k - 16k)$$

$$= 8k$$

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

$8k = 1$

$$\boxed{k = \frac{1}{8}}$$

Mean $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_0^2 kx^2\,dx + \int_2^4 2kx\,dx + \int_4^6 (-kx^2 + 6kx)\,dx$$

$$= \int_0^8 k\left(\frac{x^3}{3}\right)_0^2 + 2k\left(\frac{x^2}{2}\right)_2^4 + \left[-k\frac{x^3}{3} + 6k\frac{x^2}{2}\right]_4^6$$

$$= \frac{8k}{3} + 12k + \left[-72k + 108k - \left(-\frac{64}{3}k + 48k\right)\right]$$

$$= \frac{8k}{3} + 12k + 36k - 26.667k$$

$$E(x) = 24k$$

$$= 24 \times \frac{1}{8}$$

$$\boxed{E(x) = 3}$$

Hence, mean = 3 and $k = \frac{1}{8}$

11

Good Approach

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(a) (i) State Langrange's mean value theorem and explain the theorem in reference to it's geometrical significance.

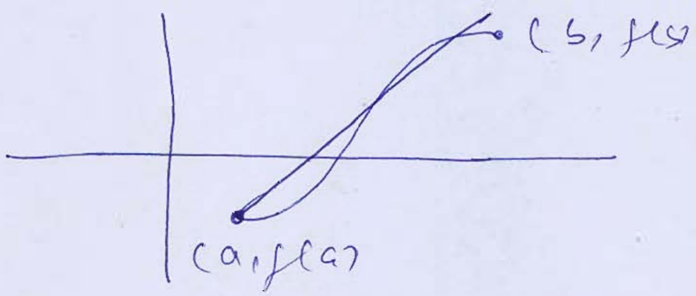
(ii) Find the complete solution of $y^2 - 2y' + 2y = x + e^x \cos x$.

(iii) Prove that the matrix, $A = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$ is unitary and find A^{-1}.

[6 + 6 + 8 marks]

(i) Langrange mean value theorem –
f(x) is continous [a, b], differentiable (a, b) then ∃ a number $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



slope of line joining [a, f(a)] and [b, f(b)]

$$= \frac{f(b) - f(a)}{b - a}$$

∴ point c is the slope of the tangent to the curve f(x)

(ii) $y'' - 2y' + 2y = x + e^{x}\cos x$

Auxiliary equation

$m^2 - 2m + 2 = 0$

$m = 1 \pm j$

CF $= \cancel{c_1 e^{x} [c_2 \cos}$

$= e^{x} [c_1 \cos x + c_2 \sin x)$

$PI = \dfrac{1}{D^2 - 2D + 2} [x + e^{x}\cos x]$

$PI_1 = \dfrac{1}{D^2 - 2D + 2} (x)$

$= \dfrac{1}{2\left(1 + \left(\frac{D^2 - 2D}{2}\right)\right)} (x)$

$= \dfrac{1}{2}\left[1 + \left(\dfrac{D^2 - 2D}{2}\right)\right]^{-1} x$

$= \dfrac{1}{2}\left[1 - \left(\dfrac{D^2 - 2D}{2}\right)\right] x$

$= \dfrac{1}{2}[x + 1] = \dfrac{x+1}{2}$

$PI_2 = \dfrac{1}{D^2 - 2D + 2} e^{x}\cos x$

$= e^{x} \cdot \dfrac{1}{(D+1)^2 - 2(D+1) + 2} \cos x$

$= e^{x} \dfrac{1}{D^2 + 2D + 1 - 2D - 2 + 2} \cos x$

$$= e^{x} \cdot \frac{1}{D^2+1} \cos x$$

$$D^2 = -1^2 = -1$$

$$f(D^2) = 0$$

$$= x e^{x} \cdot \frac{1}{2D} \cos x$$

$$= \frac{x e^{x}}{2} \int \cos x \, dx$$

$$PI_2 = \frac{x e^{x}}{2} \sin x$$

$$\therefore PI = \frac{(x+1)}{2} + \frac{x e^{x}}{2} \sin x$$

complete solution –

$$y = e^{x}(C_1 \cos x + C_2 \sin x) + \frac{(x+1)}{2} + \frac{x e^{x}}{2} \sin x$$

(iii)
$$A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$$

conjugate

$$A^* = \begin{bmatrix} \frac{1-i}{2} & \frac{-1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{bmatrix}$$

Transpose

$$A^{\Theta} = (A^*)^T = \begin{bmatrix} \frac{1-i}{2} & \frac{1-i}{2} \\ \frac{-1-i}{2} & \frac{1+i}{2} \end{bmatrix}$$

$$A A^{\Theta} = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix} \begin{bmatrix} \frac{1-i}{2} & \frac{1-i}{2} \\ -\frac{1-i}{2} & \frac{1+i}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A A^{\Theta} = I$$

$\therefore$ ~~Inverse~~ Hence, the given matrix A is unitary matrix

$\therefore$ Inverse of matrix A

$$A^{-1} = A^{\Theta} = \begin{bmatrix} \frac{1}{2}(1-i) & \frac{1}{2}(1-i) \\ \frac{1}{2}(-1-i) & \frac{1}{2}(1+i) \end{bmatrix}$$

Good Approach

18

(b) (i) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ show that $A^2 - 4A - 5I = 0$ where $I, 0$ are the identity matrix and the null matrix of order 3 respectively. Use this result to find A^{-1}.

[10 marks]

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

characteristic eqn -

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1-\lambda & 2 & 2 \\ 2 & 1-\lambda & 2 \\ 2 & 2 & 1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)[(1-\lambda)^2 - 4] - 2[2(1-\lambda) - 4] + 2[4 - 2(1-\lambda)] = 0$$

$$(1-\lambda)\{1 + \lambda^2 - 2\lambda - 4\} - 2[2 - 2\lambda - 4] + 2[4 - 2 + 2\lambda] = 0$$

$$(1-\lambda)(\lambda^2 - 2\lambda - 3) + 4 + 4\lambda + 4 + 4\lambda = 0$$

$$\lambda^2 - 2\lambda - 3 - \lambda^3 + 2\lambda^2 + 3\lambda + 8 + 8\lambda = 0$$

$$\lambda^3 - 3\lambda^2 - 9\lambda - 5 = 0$$

$$\lambda = 5, -1, -1$$

eigen value of matrix $A = 5, -1, -1$

$\therefore$ Now $A^2 - 4A - 5I$

$\lambda = 5 \Rightarrow (5)^2 - 4\times 5 - 5$

$= 0.$

$\lambda = -1 \Rightarrow (-1)^2 - 4\times(-1) - 5$

$= 0$

$\therefore A^2 - 4A - 5I = 0$

$A^{-1}A^2 - 4A^{-1}A - 5A^{-1} = 0$

$A - 4I - 5A^{-1} = 0$

$5A^{-1} = A - 4I$

$$5A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$5A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Good Approach

9

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(b) (ii) Examine the following vectors for linear dependence and find the relation if it exists.
$X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$ and $X_4 = (-3, 7, 2)$.

[10 marks]

$x_1 = (1, 2, 4)$

$x_2 = (2, -1, 3)$

$x_3 = (0, 1, 2)$

$x_4 = (-3, 7, 2)$

$A = [x_1 \; x_2 \; x_3 \; x_4]$

$= \begin{bmatrix} 1 \\ 2 \\ \end{bmatrix}$

Incomplete Solution

(c) (i) Solve: $(D^2 - 4D + 4)\,y = 8x^2 e^{2x} \sin 2x$.

(ii) Find the regression line of y on x for the following data and estimate the value of y, when $x = 10$. (Use the least square approximation method)

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

[12 + 8 marks]

(i) $(D^2 - 4D + 4)\,y = 8x^2 e^{2x} \sin 2x$

AE

$m^2 - 4m + 4 = 0$

$m = 2, 2$

$\therefore$ CF $= (C_1 + C_2 x)\,e^{2x}$

PI $= \dfrac{1}{D^2 - 4D + 4}\, 8x^2 e^{2x} \sin 2x$

$= 8e^{2x} \cdot \dfrac{1}{(D+2)^2 - 4(D+2) + 4}\, x^2 \sin 2x$

$= 8e^{2x} \dfrac{1}{D^2 + 4D + 4 - 4D - 8 + 4}\, x^2 \sin 2x$

$= 8e^{2x} \dfrac{1}{D^2}\, x^2 \sin 2x$

$\dfrac{1}{D} x^2 \sin 2x = \int x^2 \sin 2x\, dx$

$= x^2 \left(-\dfrac{\cos 2x}{2}\right) + 2\int x \dfrac{\cos 2x}{2}\, dx$

$= -\dfrac{x^2}{2} \cos 2x + \left[x \dfrac{\sin 2x}{2} + \dfrac{\cos 2x}{4}\right]$

$\cancel{\frac{1}{D^2} \int -\frac{x^2}{2} \cos 2x \, dx}$

$\frac{1}{D} x^2 \sin 2x = -\frac{x^2}{2} \cos 2x + \frac{x}{2} \sin 2x + \frac{\cos 2x}{4}$

$\frac{1}{D^2} x^2 \sin 2x = -\int \frac{x^2}{2} \cos 2x + \int \frac{x}{2} \sin 2x + \int \frac{\cos 2x}{4} \, dx$

$= -\left[\frac{x^2}{2} \sin 2x - \frac{2}{2}\int x \sin 2x \, dx\right] + \left[\frac{x}{2}\left(-\frac{\cos 2x}{2}\right) + \frac{\sin 2x}{8}\right] + \frac{\sin 2 \cdot x}{8}$

$= -\frac{x^2}{2} \sin 2x + \left[-\frac{x \cos 2x}{2} + \frac{\sin 2x}{4}\right] - \frac{x}{4} \cos 2x + \frac{\sin 2x}{8} + \frac{\sin 2x}{8}$

$= -\frac{x^2}{2} \sin 2x - \frac{3}{4} x \cos 2x + \frac{1}{2} \sin 2x$

$\therefore$ PI $= 8 e^{2x} \left[-\frac{x^2}{2} \sin 2x - \frac{3}{4} x \cos 2x + \frac{1}{2} \sin 2x\right]$

∴ complete solution –

$$y = (c_1 + c_2 x) e^{2x}$$

$$\cancel{+ 8 e^{2x} \left[-\frac{x^2}{2} \sin 2x - \frac{3}{4} x \cos 2x + \frac{1}{2} \sin 2x \right]}$$

5

(1)

x	y	xy	x^2
1	1	1	1
3	2	6	9
4	4	16	16
6	4	24	36
8	5	40	64
9	7	63	81
11	8	88	121
14	9	126	196

$\Sigma x = 56$ $\quad \Sigma y = 40$ $\quad \Sigma xy = 364$ $\quad \Sigma x^2 = 524$

regression line of y on x

$$y = a + bx \qquad n = 8$$

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

$$\Rightarrow 40 = 8a + b \times 56 \quad -(i)$$

$$364 = 56a + b \times 524 \quad -(ii)$$

solving (1) & (2) –

$$a = 0.545 \qquad b = 0.636$$

∴ regression line of y on x

$y = 0.545 + 0.636x$

for $x = 10$

$y = 0.545 + 0.636 \times 10$

$= 6.905$

7

Good
Approach

(a) If the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$, then find all possible value of m.

[20 marks]

Q.7 (b) **(i)** If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

(ii) Find the real root of the following equation, correct to three decimal places. (Using Newton-Raphson method)

$$x^3 - 2x - 5 = 0$$

[12 + 8 marks]

Q.7 (c) Show that the vector field $\vec{F} = \dfrac{\vec{r}}{|\vec{r}|^3}$ is irrotational as well as solenoidal. Find the scalar potential.

[20 marks]

Sixteen players $S_1, S_2, ... S_{16}$ play in a tournament. They are divided into eight pairs at random, from each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

Find:

(i) The probability that the player S_1 is among the eight winners.

(ii) The probability that exactly one of the two players S_1 and S_2 is among the eight winners.

[20 marks]

Q.8 (b) **(i)** Assuming that the diameters of 1000 brass plugs taken consecutively from a machine from a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm, how many number of brass plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm? (Given, Area$[Z = -1.75] = 0.4599$ and Area$[Z = 2.25] = 0.4878$).

(ii) A periodic function of time period 4 is defined as $f(x) = |x|$, $-2 < x < 2$. Find its Fourier series expansion.

[8 + 12 marks]

Q.8 (c) Apply Runge-Kutta 4th order method to find an approximate value of y when $x = 0.2$.

Given that $\frac{dy}{dx} = x + y$, $y = 1$ when $x = 0$. (Take step size of 0.1)

[20 marks]

OOOO

Space for Rough Work

Ath – 300

Space for Rough Work