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ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electric Circuits + Engineering Mathematics

Name : ....

Roll No :

Test Centres	Student's Signature
Delhi ☑ Bhopal ☐ Jaipur ☐ Pune ☐ Kolkata ☐ Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the *answer sheet* (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
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Question No.	Marks Obtained
Section-A	
Q.1	39
Q.2	45
Q.3	
Q.4	48
Section-B	
Q.5	33
Q.6	34
Q.7	
Q.8	
Total Marks Obtained	199

Signature of Evaluator: Sourabh kumar

Cross Checked by: ....

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Name : Rajat Dixit

Roll No : E E 2 5 M T D L A 0 1 1

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1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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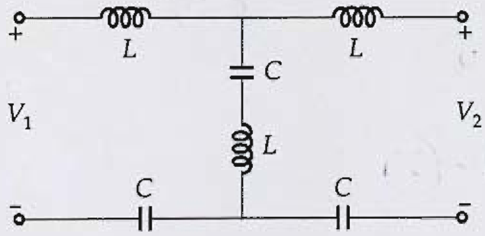
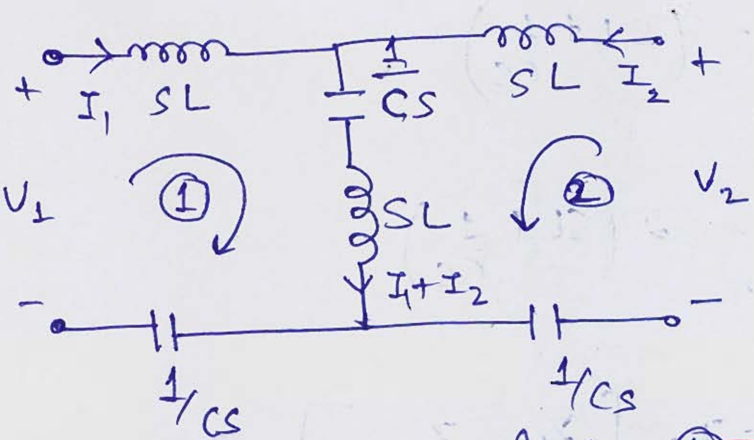
Section A : Electric Circuits

1 (a) Determine Z-parameters for network shown



[12 marks]

Sol^n:-



Applying KVL in loop - ①

$$V_1 = SL\,I_1 + \left(SL + \frac{1}{CS}\right)I_1 + \left(SL + \frac{1}{CS}\right)I_2 + \frac{1}{CS}I_1$$

$$V_1 = 2\left(SL + \frac{1}{CS}\right)I_1 + \left(SL + \frac{1}{CS}\right)I_2 \quad \text{-(1)}$$

IIly, Appling KVL in loop -②, we get

$$V_2 = SL\,I_2 + \left(SL + \frac{1}{CS}\right)I_1 + \left(SL + \frac{1}{CS}\right)I_2 + \frac{1}{CS}I_2$$

$$V_2 = \left(SL + \frac{1}{CS}\right)I_1 + 2\left(SL + \frac{1}{CS}\right)I_2 \quad \text{-(2)}$$

from eqn. (1)

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 2\left(SL + \frac{1}{CS}\right)$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \left(SL + \frac{1}{CS}\right)$$

IIIly, From eqn-(2)

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \left(SL + \frac{1}{CS}\right)$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 2\left(SL + \frac{1}{CS}\right)$$

Z parameter; $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

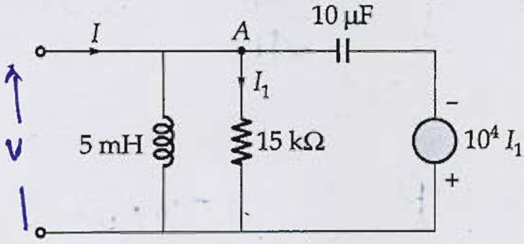
$$\boxed{[Z] = \begin{bmatrix} 2\left(SL + \frac{1}{CS}\right) & SL + \frac{1}{CS} \\ \left(SL + \frac{1}{CS}\right) & 2\left(SL + \frac{1}{CS}\right) \end{bmatrix}}$$

Ans

Good Approach

11

1 (b) Determine the resonant frequency, quality factor Q_0 and bandwidth of the given circuit.



[12 marks]

Sol^n:- from ckt. as shown in figure —

$$I = \frac{V}{j\omega L} + \frac{V}{R} + \frac{V + 10^4 I_1}{\frac{1}{j\omega C}} \quad \text{—(1)}$$

Given, $R = 15k\Omega$
$L = 5mH$
$C = 10\mu F$

from fig:- $I_1 = \frac{V}{R}$

$$I = \frac{V}{j\omega L} + \frac{V}{R} + j\omega C\left[V + 10^4 \frac{V}{R}\right]$$

$$Y_{in} = \frac{I}{V} = \frac{1}{j\omega L} + \frac{1}{R} + j\omega C\left[1 + \frac{10^4}{R}\right]$$

$$= \frac{-j}{\omega L} + \frac{1}{R} + j\omega C\left[1 + \frac{10^4}{R}\right]$$

At resonance $Y_{in}(\text{imag.}) = 0$

$$-\frac{1}{\omega L} + \omega C\left[1 + \frac{10^4}{R}\right] = 0$$

Substituting the values of R, L, C

$$-\frac{1}{5 \times 10^{-3}\,\omega} + \omega \times 10 \times 10^{-6}\left[1 + \frac{10^4}{15 \times 10^3}\right] = 0$$

$$-200 + \omega^2 \times 1.67 \times 10^{-5} = 0$$

$$\omega^2 = \frac{200}{1.67 \times 10^{-8}} \Rightarrow \omega_0 = 3460.64 \text{ rad/sec}$$

$$\boxed{\text{Resonant frequency} = \frac{\omega_0}{2\pi} = 550.77 \text{ Hz}}$$ Ans

$$\text{Quality factor} = \frac{R}{\omega_0 L} = \frac{15\times10^3}{3460.64\times5\times10^{-3}}$$

$$\boxed{Q = 866.89}$$ Ans

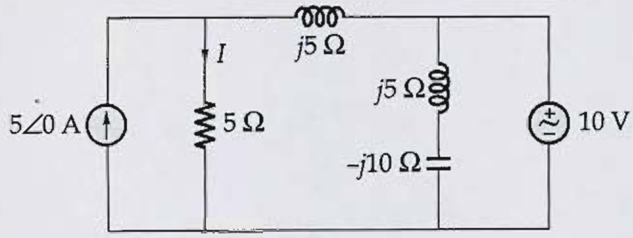
$$\text{Bandwidth} = \frac{\omega_0}{Q} = \frac{3460.64}{866.89} = 3.99 \text{ rad/sec}$$

$$\boxed{B.W = 3.99 \approx 4 \text{ rad/sec}}$$ Ans

11

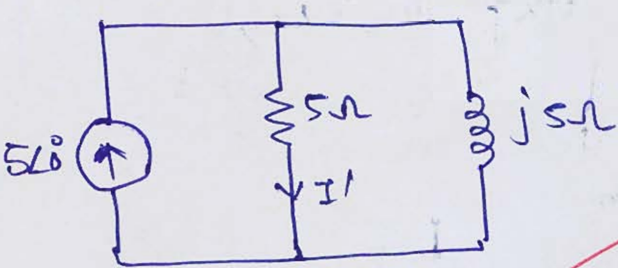
Good Approach

1 (c) (i) Determine the current 'I' in the network shown using principle of superposition.



[6 marks]

Solⁿ:- Current I' in $5\,\Omega$ resistor due to only independent current source $5\angle 0°$ Amp.

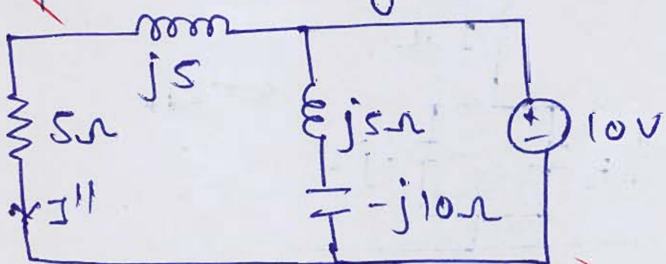


Applying current division rule -

$$I' = \frac{5\angle 0° \times 5\angle 90°}{5+j5}$$

$I' = 3.53\angle 45°$ Amp

Current I'' in $5\,\Omega$ resistance due to only independent voltage source 10V.



$$I'' = \frac{10}{5+j5}$$

$= 1.414\angle -45°$ Amp

According to principle of superposition →

$I = I' + I''$

$= 3.53\angle 45° + 1.414\angle -45°$

$$\boxed{I = 3.80\angle 23.16° \text{ Amp}}$$

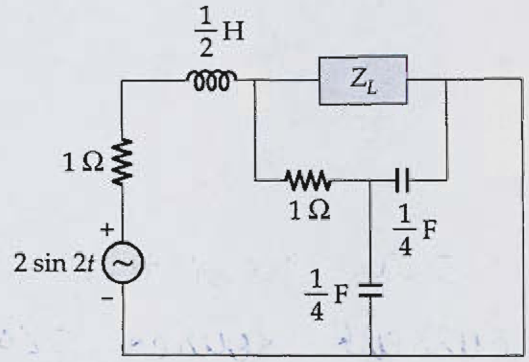
Ans

5

Good Approach

Do not write in this margin

Q.1 (c) (ii) Determine the value of impedance Z_L for maximum power transfer, in Z_L, in the given network.

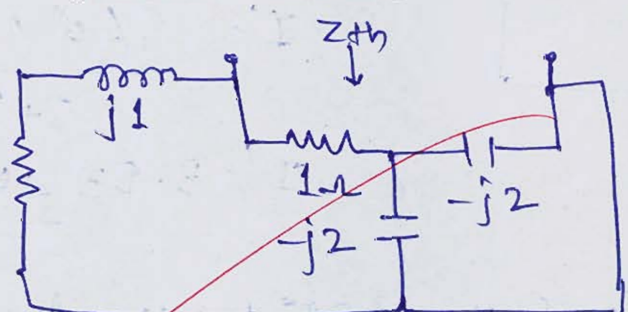


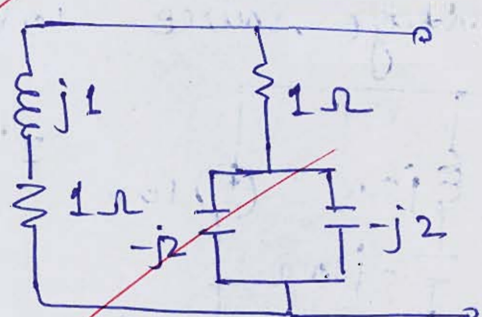
[6 marks]

Sol^n:- For maximum power txf. in Z_L,

$$\boxed{Z_L = Z^*_{th}}$$

$Z_{th} \rightarrow$





5

Good Approach

$Z_{th} = (1+j1) \parallel (1-j1) = \dfrac{\sqrt{2}\angle 45^\circ \times \cancel{4.123\angle -75.9}\ \sqrt{2}\angle -45^\circ}{\sqrt{2}\angle 45^\circ + \cancel{4.123\angle -75.96^\circ}\ \sqrt{2}\angle -45^\circ}$

$= \dfrac{\cancel{5.83\angle -30.96^\circ}}{\cancel{3.60\angle -56.305^\circ}} = \dfrac{2}{2}$

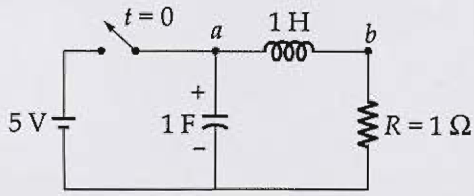
$= \cancel{1.616\angle 25.34^\circ\ \Omega} = 1\Omega$

$\boxed{Z_L = Z^*_{th} = \cancel{1.616\angle -25.34^\circ} = \cancel{1.446 - 0.69j}\ \Omega = 1\Omega}$

Ans

Do not write in this margin

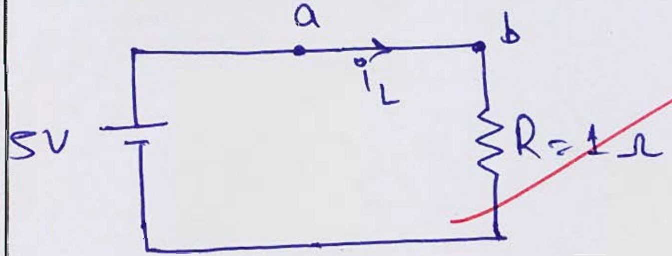
1 (d) Consider the following circuit:



The switch is initially closed. After steady state is reached the switch is opened. Determine the nodal voltage $V_a(t)$ and $V_b(t)$.

[12 marks]

Solⁿ:- At steady state, capacitor acts as O/C and inductor acts as short ckt.



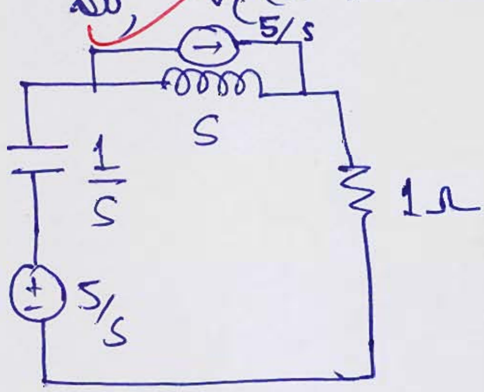
So, $V_a = V_c(0^-) = 5V$

$V_b = 5V$

$i_L(0^-) = 5A$

At instant t = 0, switch is opened, the voltage on capacitor doesn't change instantaneouly.

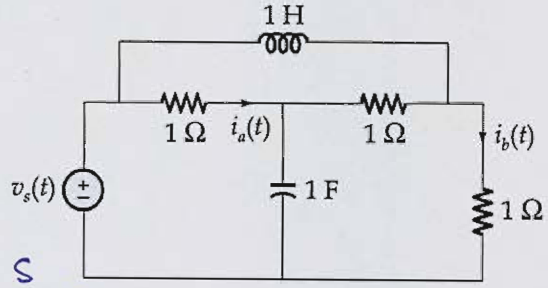
So, $V_c(0^+) = 5V$ ||ly $i_L(0^+) = 5A$



3

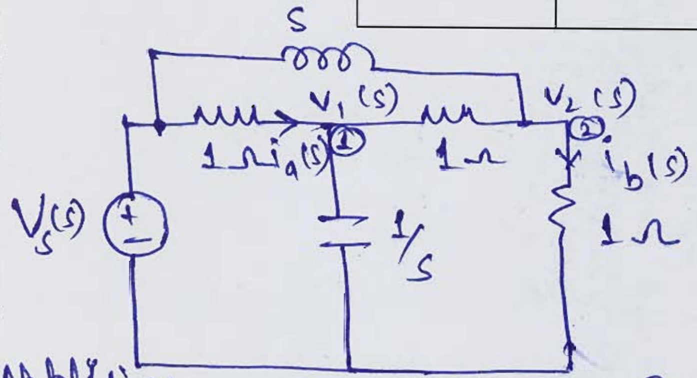
In complete Solution

Q.1 (e) For the bridged T-network of the figure given below the source voltage is $v_s(t) = 2\cos t$. The circuit is in steady state condition. Determine: **(i)** $i_a(t)$ and **(ii)** $i_b(t)$.



[12 marks]

Sol^n:-



$V_s(s) = \frac{2s}{s^2+1}$

Applying KCL at node 1

$$\frac{V_s - V_1}{1} = \frac{V_1}{1/s} + \frac{V_1 - V_2}{1}$$

$$V_s - V_1 = sV_1 + V_1 - V_2$$

$$V_s = (s+2)V_1 - V_2 \quad \text{—(1)}$$

Applying KCL at node 2

$$\frac{V_s - V_2}{s} + \frac{V_1 - V_2}{1} = V_2$$

$$V_s - V_2 + sV_1 - sV_2 = sV_2$$

$$V_s = -sV_1 + V_2 + sV_2 + sV_2 = -sV_1 + (2s+1)V_2 \quad \text{—(2)}$$

from eqn. (1) & (2)

$$(s+2)V_1 - V_2 = -sV_1 + (2s+1)V_2$$

$$(2s+2)V_1 = (2s+2)V_2 \Rightarrow V_1 = V_2$$

Substituting in eqn. (1) & (2)

$V_s = (S+2)V_1 - V_1$

$V_s = (S+2-1)V_1$

$V_1 = \frac{V_s}{(S+1)}$ $\Rightarrow i_a(s) = \frac{V_s - V_1}{1}$

$= V_s - \frac{V_s}{S+1} = V_s\left[\frac{S}{S+1}\right]$

$i_a(S) = \frac{2S^2}{(S^2+1)(S+1)} = \frac{A}{(S+1)} + \frac{BS+C}{S^2+1}$

$2S^2 = AS^2 + A + BS^2 + BS + CS + C$

$A+B=2$; $B+C=0$; $A+C=0$

On solving: $A=1$, $B=1$, $C=-1$

$i_a(S) = \frac{1}{S+1} + \frac{S-1}{S^2+1} \Rightarrow i_a(t) = e^{-t}u(t) + \cos t\, u(t) - \sin(t)\, u(t)$

Inverse Laplace trf. :- $\boxed{i_a(t) = (e^{-t} + \cos t - \sin t)u(t)}$

Ans.

||ly $i_b(S) = \frac{V_2}{1} = \frac{V_1}{1} = \frac{V_s}{S+1} = \frac{2S}{(S^2+1)(S+1)} = \frac{A}{(S+1)} + \frac{BS+C}{S^2+1}$

$2S = AS^2 + A + BS^2 + BS + CS + C$

$A+B=0$; $B+C=2$, $A+C=0$

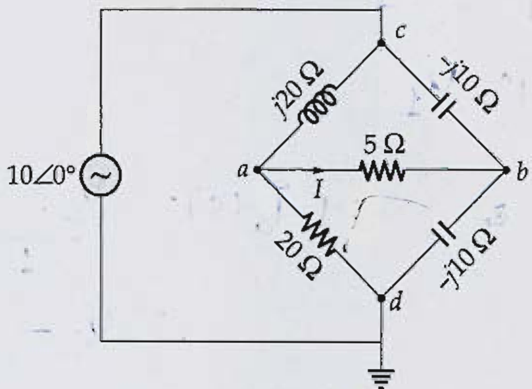
$A=-1$, $B=1$, $C=1$

$i_b(S) = \frac{-1}{S+1} + \frac{S+1}{S^2+1} \longrightarrow \boxed{i_b(t) = (-e^{-t} + \cos t + \sin t)u(t)}$

Ans.

4

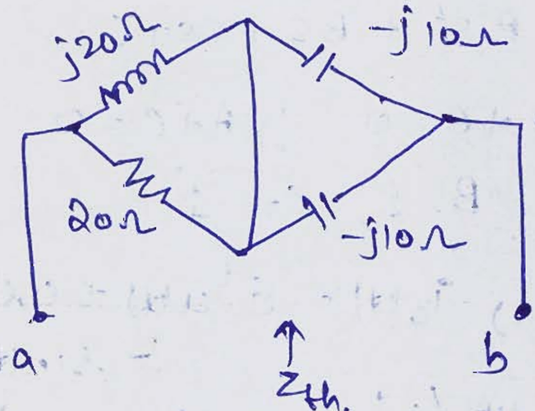
Q.2 (a) (i) Determine current I in the network using Thevenin's theorem.



[10 marks]

Sol^n:

Calculation of Z_{th} →



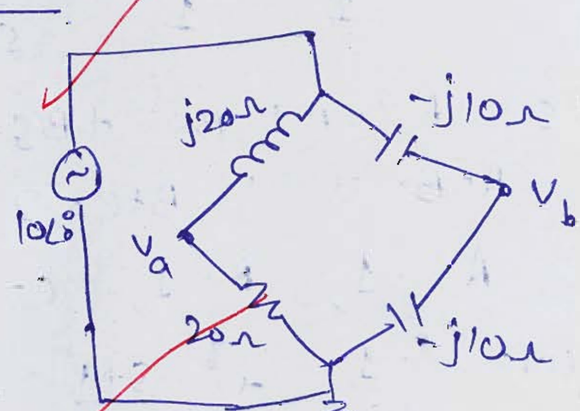
$Z_{th} = (j20 \| 20) + (-j10 \| -j10)$

$= 14.142\angle 45° + 5\angle -90°$

$= 11.180\angle 26.56°\ \Omega$

Calculation of V_{oc} →

$V_{oc} = V_a - V_b$



$V_a = \dfrac{10\angle 0° \times 20}{20 + j20}$

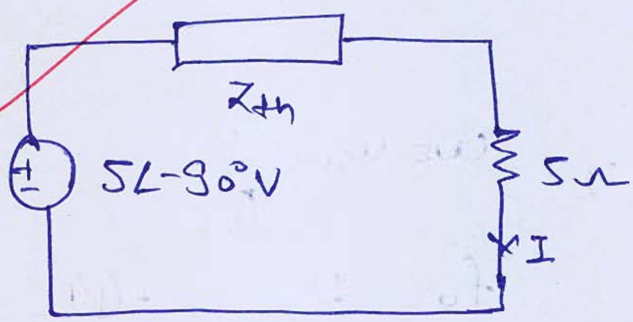
$= 5\sqrt{2}\angle -45°\ V$

IIly $V_b = \dfrac{10\angle 0° \times -j10}{-j10 \ -j10} = \dfrac{10\angle 0° \times \cancel{-j}10}{\cancel{-j}20}$

$= 5\ V$

$$V_{oc} = V_a - V_b = 5\sqrt{2}\angle{-45^\circ} - 5$$
$$= 5\angle{-90^\circ}\ V$$

Thevenin equivalent ckt:-

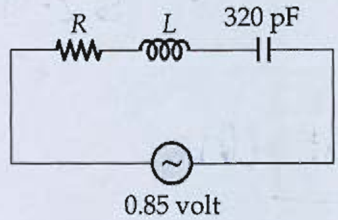


$$\boxed{I = \frac{5\angle{-90^\circ}}{11.180\angle{26.56^\circ} + 5} = 0.316\angle{-108.43^\circ}\ \text{Amp}}$$

9

Good Approach

Q.2 (a) **(ii)** For the circuit shown determine the value of inductance for resonance if $Q = 50$ and $f_0 = 175$ kHz. Also find the circuit current, the voltage across the capacitor and the bandwidth of the circuit.



[10 marks]

Sol^n:-

At resonance

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad -①$$

Given:- $f_0 = 175$ KHz

$Q = 50$, $C = 320$pF

Substituting in eqn - ①

$$175 \times 10^3 = \frac{1}{2\pi\sqrt{L \times 320 \times 10^{-12}}}$$

$$3.0625 \times 10^{10} = \frac{1}{4\pi^2 \times L \times 320 \times 10^{-12}}$$

$L = 2.58 \times 10^{-3}$

$\boxed{L = 2.58 \text{ mH}}$ Ans

$$Q = \frac{\omega_0 L}{R} \Rightarrow R = \frac{\omega_0 L}{Q} = \frac{2\pi \times 175 \times 10^3 \times 2.58 \times 10^{-3}}{50}$$

$\boxed{R = 56.84\,\Omega}$

Circuit current

$$I = \frac{V}{R} = \frac{0.85}{56.84}$$

$\boxed{I = 0.01495 \text{ A}}$ Ans

Voltage across capacitor

$$V_c = QV = 50 \times 0.85$$

$$\boxed{V_c = 42.5V}$$ Ans

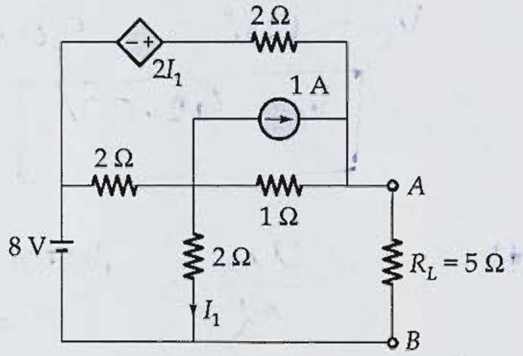
Bandwidth $= \frac{\omega_0}{Q} = \frac{2\pi \times 175 \times 10^3}{50}$

$$\boxed{\begin{aligned} B.W &= 7000\pi \text{ rad/sec} \\ &= 3500 \text{ Hz} \end{aligned}}$$ Ans

9

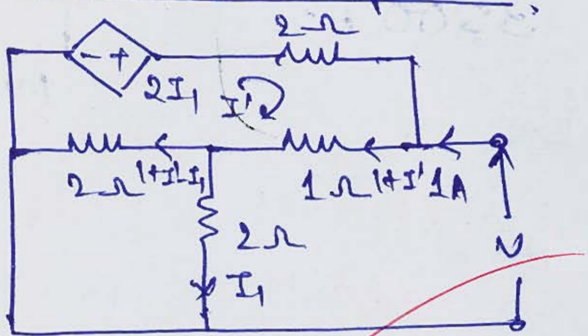
Good Approach

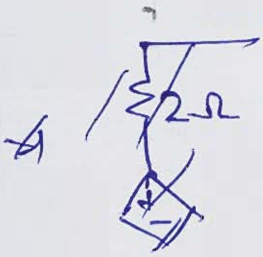
Q.2 (b) Determine the current through the load resistance $R_L = 5\ \Omega$ across the terminals A-B of the circuit shown in figure below, using Thevenin's theorem. Also, find the maximum power that can be transferred to the load resistance R_L.



[20 marks]

Sol^n:- Calculation of $R_{th} \rightarrow$





$V = 1(1+I') + 2I_1 \rightarrow$ (a)

$2I_1 = 2I' + (1+I') + 2(1+I'-I_1)$

$2I_1 = 2I' + 1 + I' + 2 + 2I' - 2I_1$

$-3 = 5I' - 4I_1$ — (1)

Also, $\cancel{2}I_1 = \cancel{2}(1+I'-I_1)$

$I_1 = 1 + I' - I_1$

$2I_1 - I' = 1$ — (2)

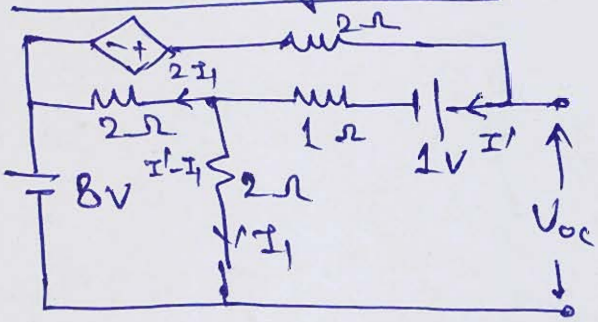
On solving $I' = \frac{-1}{3}$ Amp, $I_1 = \frac{1}{3}$ Amp

Substituting in eqn-(a)

$V = \left(1 - \frac{1}{3}\right) + 2 \times \frac{1}{3} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$ V

$R_{th} = \frac{V}{1} = \frac{4}{3}\,\Omega$

Calculation of V_{oc} -



$V_{oc} = 1 + I' + 2I_1$ —(6)

$2I_1 = 2I' + 1 + I' + 2I' - 2I_1$

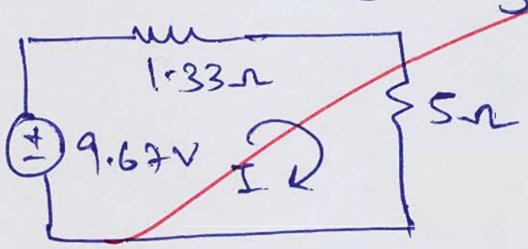
$\Rightarrow -1 = 5I' - 4I_1 \rightarrow (3)$

$8 = -2(I' - I_1) + 2I_1$

$8 = -2I' + 2I_1 + 2I_1 = -2I' + 4I_1 \rightarrow (4)$

On solving:- $I' = \frac{7}{3}\,A$, $I_1 = \frac{19}{6}\,A$

$V_{th} = V_{oc} = 1 + \frac{7}{3} + \frac{19}{3} = 9.67\,V$



$\boxed{I = \frac{9.67}{1.33+5} = 1.526\,A}$

Maxm power transferred $= \frac{V_{th}^2}{4R_{th}} = \frac{(9.67)^2}{4 \times 1.33}$

$\boxed{P_{max} = 17.53\,W}$

18

Good Approach

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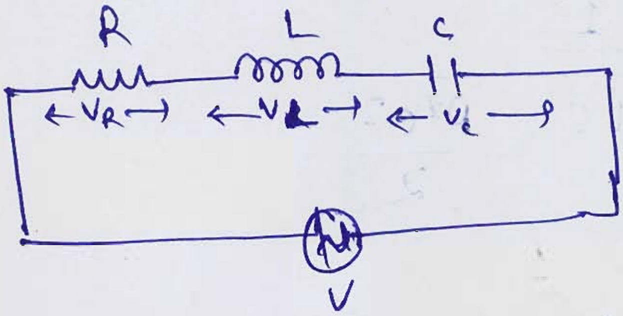
Q.2 (c) (i) Derive expression for frequency for maximum voltage across inductor in series RLC resonant circuit.

(ii) Calculate the maximum voltage across the inductor using result of **part (i)** with constant voltage and variable frequency. Assume $R = 50\ \Omega$, $L = 0.05\text{H}$, $C = 20\ \mu\text{F}$ and $V = 100$ V.

[13 + 7 marks]

Soln:-



Applying voltage division rule -

$$V_L = V \times \frac{j\omega L}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$V_L = V \times \frac{j\omega L}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \times R - j\left(\omega L - \frac{1}{\omega C}\right).$$

$$V_L = V \times \left[\frac{j\omega L R + \omega L\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\right]$$

$$= V \times \left[\frac{j\omega L R + \omega^2 L^2 - \frac{L}{C}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\right] \quad \text{—(1)}$$

Differentiating V_L with respect to ω and equating to zero

i.e $\frac{dV_L}{d\omega} = 0$

On differentiating eqn-① & equating it to zero, we get

$$\omega = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}$$

i.e; $\omega = \frac{1}{\sqrt{LC}\sqrt{1 - \frac{R^2C}{2L}}}$

or $\boxed{f_L = \frac{1}{2\pi\sqrt{LC}\sqrt{1 - \frac{R^2C}{2L}}}}$

8

expression for frequency for maximum voltage across inductor.

(ii) Given $R = 50\Omega$

$L = 0.05H$

$C = 20\ \mu F$

1

we know that $Q = \frac{\omega_0 L}{R}$

$$Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

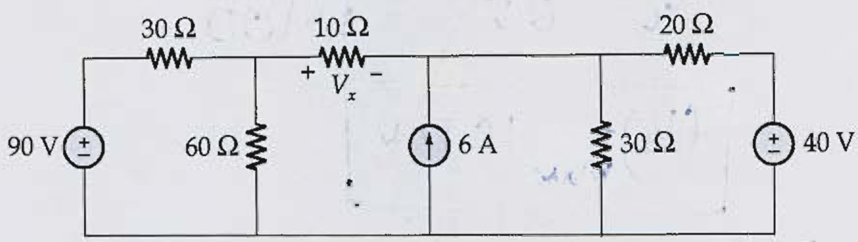
So, $Q = \frac{1}{50}\sqrt{\frac{0.05}{20\times10^{-6}}} = 1$

Maximum voltage across inductor

$$V_L = QV = 1 \times 100$$

$$\boxed{(V_L)_{max} = 100\,V}$$

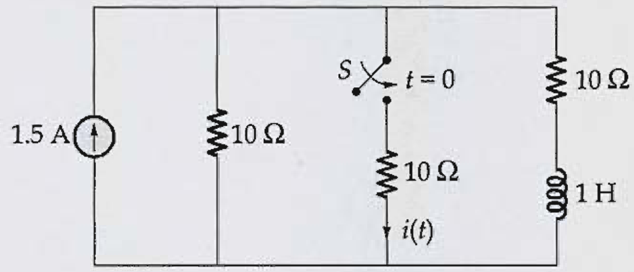
Q.3 (a) (i) The circuit shown in the figure below consists of three independent sources. Determine the value of the voltage across 10 Ω resistance using superposition theorem.



[10 marks]

7

Q.3 (a) (ii) Consider the network shown below:



If switch S is closed at $t = 0$, calculate $i(t)$ for $t > 0$ by using Laplace transform approach.

[10 marks]

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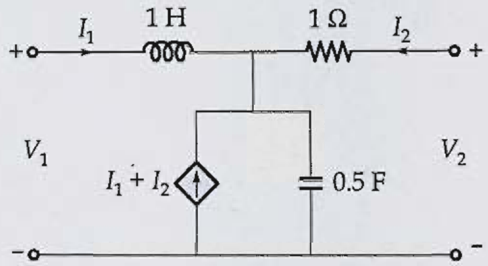
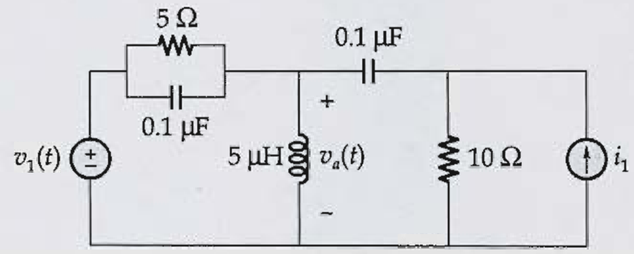
Q.3 (b) Determine the transmission parameters matrix for the two port network shown below.



[20 marks]

Q.3 (c) (i) For the circuit shown below, $v_1(t) = 10 \sin 10^6 t$ V and $i_1(t) = 10 \cos 10^6 t$ A and the circuit is operating in steady state condition. Determine the node to datum voltage $v_a(t)$.



[10 marks]

Q.3 (c) (ii) A certain practical dc voltage source can provide a current of 2.5 A when it is (momentarily) short circuited and can provide a power of 80 W to 20 Ω load.

Find:

1. The open circuit voltage.
2. The maximum power it could deliver to a well-chosen R_L.
3. What is the value of that R_L?

[10 marks]

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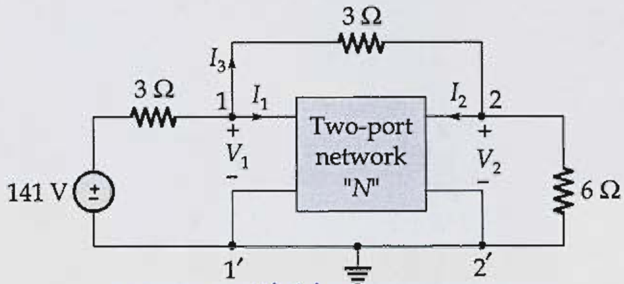
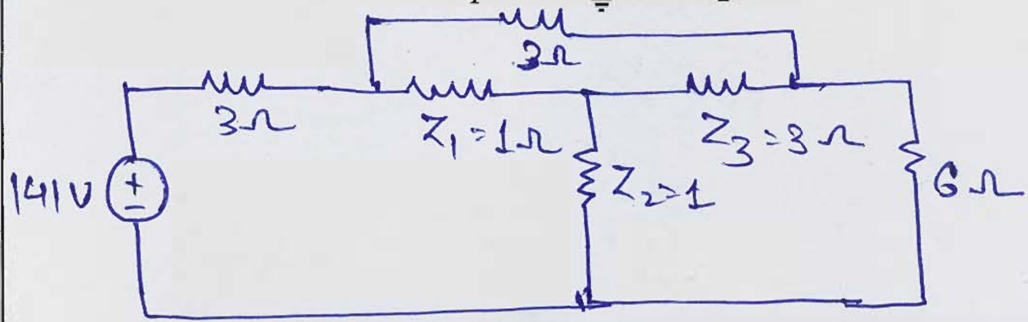
Q.4 (a) (i) The z-parameters of the two port network-N shown in the figure below are given as $z_{11} = 2\ \Omega$, $z_{12} = z_{21} = 1\ \Omega$ and $z_{22} = 4\ \Omega$. Find the values of currents I_1, I_2 and I_3.



[12 marks]

Solⁿ:-



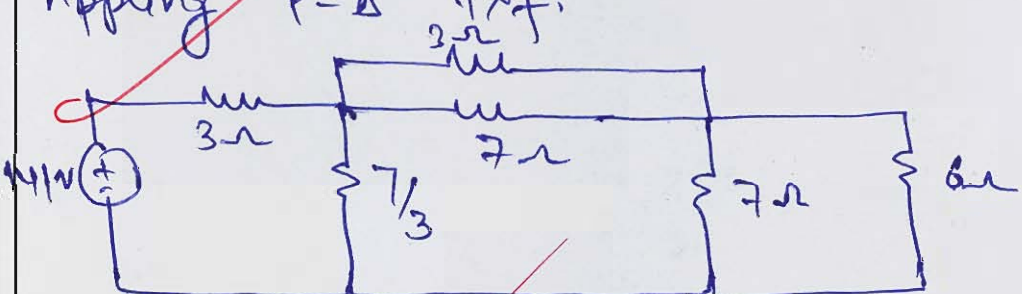
$Z_{11} = 2 \Rightarrow Z_1 + Z_2 = 2$

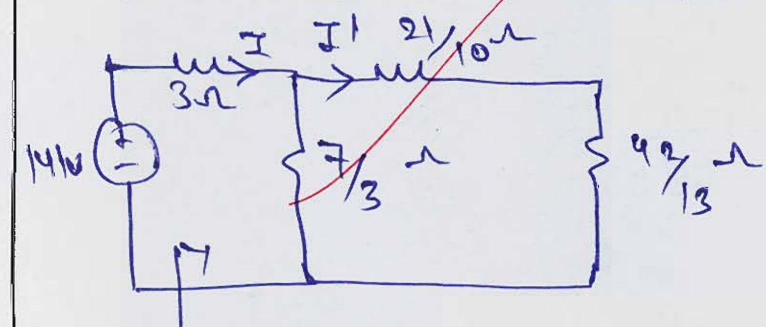
$Z_2 = Z_{12} = Z_{21} = 1\Omega$

So, $Z_1 = 1\Omega$

$Z_{22} = Z_3 + Z_2 = 4 \Rightarrow Z_3 = 3\Omega$

Appling Y-Δ txf.





Z_{eq} $I = \frac{141}{Z_{eq}} = 4.622$ Amp.

$\frac{693}{130}$

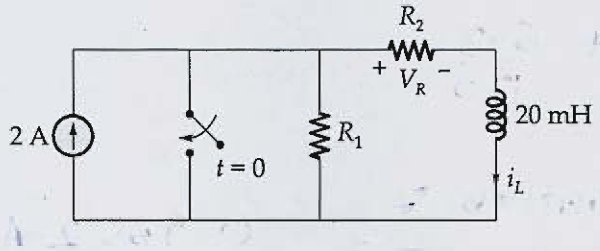
$$I' = \frac{\cancel{4.9}^{4.622} \times 7/3}{\frac{7}{3} + \frac{693}{130}} = \cancel{1.4074 \text{ Amp}}$$

$$I_3 = \frac{1.4074 \times 7}{10} = \cancel{0.9852 \text{ Amp}}$$

$$I_1 = I' - I_3 = \cancel{0.422 \text{ Amp}}$$

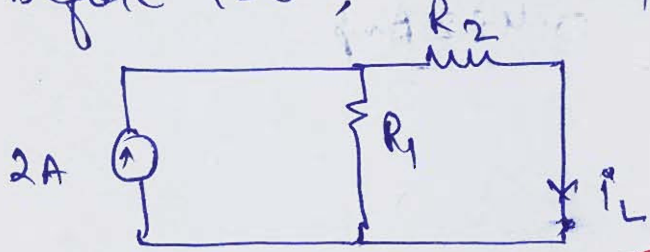
3

Q.4 (a) (ii) Determine values of R_1 and R_2 in the circuit of figure such that $V_R(0^+) = 10$ V and $V_R(1 \text{ msec}) = 5$ V.



[8 marks]

Solⁿ:- Before t=0, at steady state



$i_L = \frac{2 \times R_1}{R_1 + R_2} = \frac{2R_1}{R_1 + R_2}$ [By current division rule]

$V_R = \frac{2 R_1 \cdot R_2}{R_1 + R_2}$ —①

at t=0, switch closed, $i_L(0^-) = i_L(0) = i_L(0^+)$

$= \frac{2R_1}{R_1 + R_2}$

$V_R(0^+) = \frac{2R_1R_2}{R_1 + R_2} = 10$

$i_L = i_L(0) \cdot e^{-t/\tau}$

$i_L = \frac{2R_1}{R_1 + R_2} \cdot e^{-t/L/R_1} = \frac{2R_1}{R_1 + R_2} \cdot e^{-\frac{R_1 t}{L}}$

$V_R(t) = R_2 \cdot i_L = \frac{2R_1 \cdot R_2}{R_1 + R_2} \cdot e^{-\frac{R_1 t}{L}}$ —②

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$$V_R(t) = V_R(0) \cdot e^{-\frac{R_1 t}{L}} \quad [\text{from } (1) \,\&\, (2)]$$

$$5 = 10\, e^{-\frac{R_1 \times 10^{-3}}{20 \times 10^{-3}}}$$

$$-0.693 = -\frac{R_1 \times \cancel{10^{-3}}}{20 \times \cancel{10^{-3}}}$$

$$\boxed{R_1 = 13.86\ \Omega}$$

Substituting R_1 in eqn (1)

$$10 = \frac{2 \times 13.86 \times R_2}{13.86 + R_2}$$

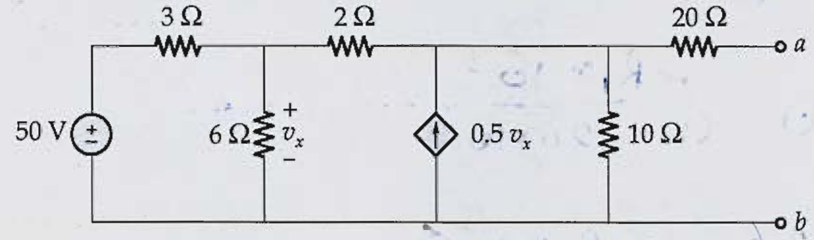
$$13.86 = 1.77\, R_2$$

$$\boxed{R_2 = 7.819\ \Omega}$$

8

Good Approach

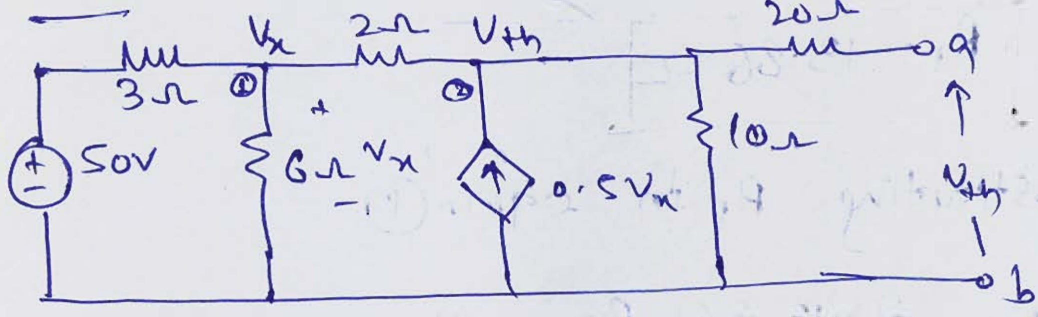
Q.4 (b) Determine the Thevenin's equivalent network and Norton's current at terminals *a-b* for the circuit shown below and draw the two equivalent circuits.



[20 marks]

Sol^n:

$V_{th} \rightarrow$



$$\frac{50 - V_x}{3} = \frac{V_x}{6} + \frac{V_x - V_{th}}{2} \longrightarrow \text{KCL at } ①$$

$$100 - 2V_x = V_x + 3V_x - 3V_{th}$$

$$100 = 6V_x - 3V_{th} \quad — ①$$

$$\frac{V_x - V_{th}}{2} + 0.5\,V_x = \frac{V_{th}}{10} \longrightarrow \text{KCL at } ②$$

$$V_x - V_{th} + V_x = \frac{V_{th}}{5}$$

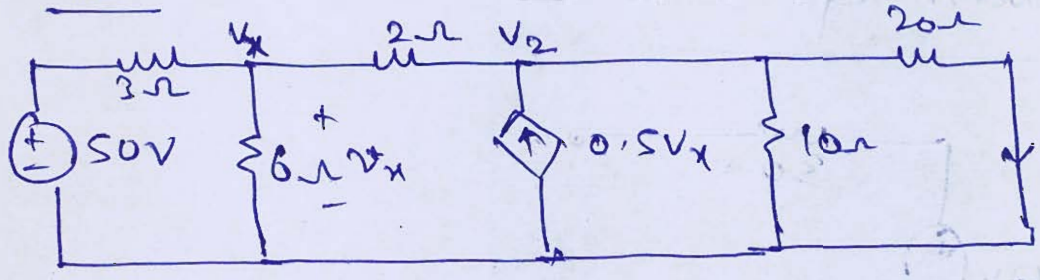
$$5V_x - 5V_{th} + 5V_x = V_{th}$$

$$10V_x - 6V_{th} = 0 \quad — ②$$

Solving eqn ① & ②

$$V_x = 100\,V, \quad V_{th} = 166.67\,V$$

$I_N \rightarrow$



$$\frac{50-V_x}{3} = \frac{V_x}{6} + \frac{V_x - V_2}{2}$$

$$100 - 2V_x = V_x + 3V_x - 3V_2$$

$$100 = 6V_x - 3V_2 \quad -(1)$$

$$\frac{V_x - V_2}{2} + 0.5\,V_x = \frac{V_2}{10} + \frac{V_2}{20} = \frac{2V_2 + V_2}{20}$$

$$10V_x - 10V_2 + 10V_x = 3V_2$$

$$20V_x - 13V_2 = 0 \quad -(2)$$

On solving –

$$V_x = \frac{650}{9}\,V \quad , \quad V_2 = \frac{1000}{9}\,V$$

$$I_N = \frac{V_2}{20} = \frac{50}{9} = 5.55 \text{ Amp}$$

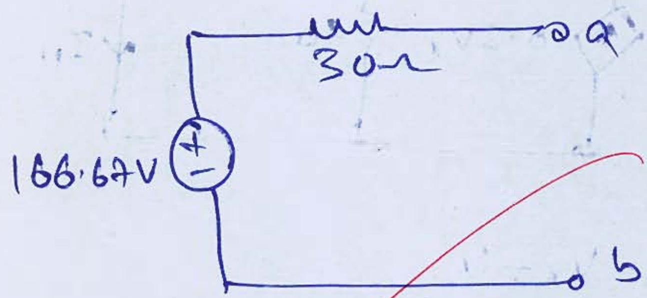
18

Good Approach

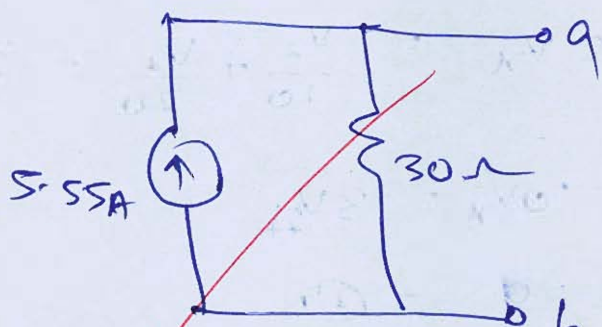
$R_{th} \rightarrow$

$$R_{th} = \frac{V_{th}}{I_N} = \frac{166.67}{5.55} = 30\Omega$$

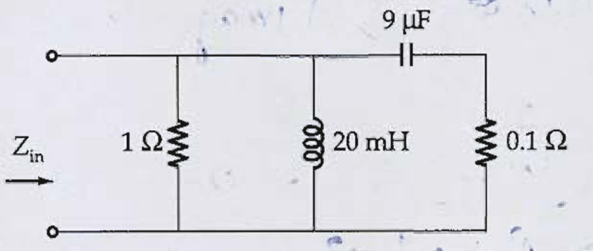
Thevenin eq. N/W –



Norton eq. N/W –



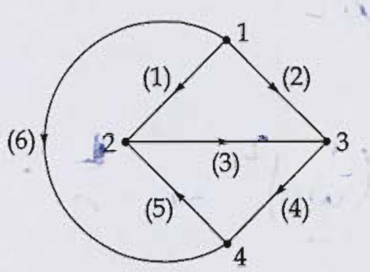
Q.4 (c) (i) For the circuit shown in the figure below:



Determine:

1. The resonant frequency, ω_0.
2. Input impedance at resonant frequency, $Z_{in}(\omega_0)$.

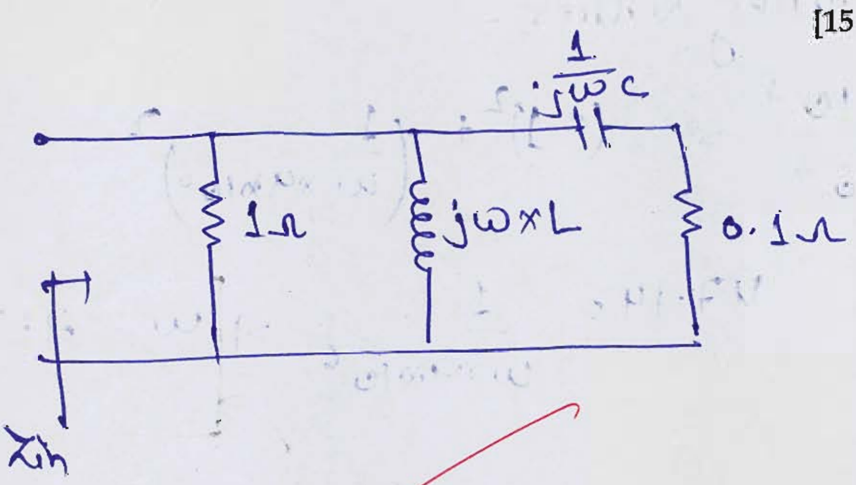
(ii) For the graph shown below:



Find the number of possible trees.

[15 + 5 marks]

Sol^n: (i)



$$Z_{in} = (1\Omega) \parallel (j\omega L) \parallel \left(0.1 + \frac{1}{j\omega C}\right)$$

$$Y_{in} = \frac{1}{1} + \frac{1}{j\omega L} + \frac{1}{0.1 + \frac{1}{j\omega C}} = 1 + \frac{1}{j\omega L} + \frac{1}{0.1 - \frac{j}{\omega C}}$$

$$= 1 - \frac{j}{\omega L} + \frac{0.1 + \frac{j}{\omega C}}{(0.1)^2 + \left(\frac{1}{\omega C}\right)^2}$$

For resonance, $Y_{imag} = 0$

$$-\frac{1}{\omega L} + \frac{\frac{1}{\omega C}}{(0.1)^2 + \left(\frac{1}{\omega C}\right)^2} = 0$$

$$\frac{\frac{1}{\omega C}}{(0.1)^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{1}{\omega L}$$

$$\frac{1}{C} = \frac{(0.1)^2 + \left(\frac{1}{\omega C}\right)^2}{L} \Rightarrow \frac{L}{C} = (0.1)^2 + \left(\frac{1}{\omega C}\right)^2$$

Substituting values -

$$\frac{20\times10^{-3}}{9\times10^{-6}} = (0.1)^2 + \left(\frac{1}{\omega\times9\times10^{-6}}\right)^2$$

$$47.14 = \frac{1}{\omega\times9\times10^{-6}} \Rightarrow \boxed{\omega = 2357.02 \text{ rad/sec}}$$

Resonant frequency 'ω_0'.

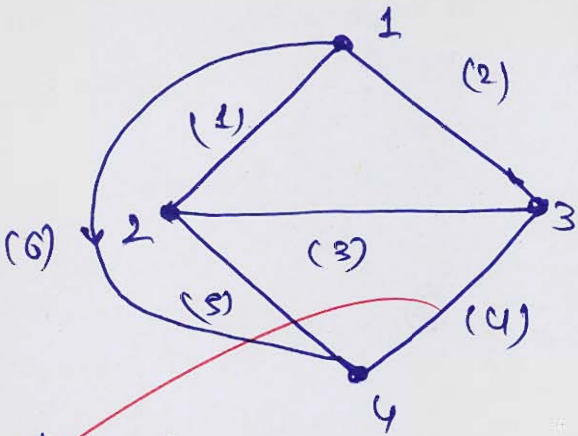
Y_{in} at resonance

$$= 1 + \frac{0.1}{(0.1)^2 + \left(\frac{1}{2357.02\times9\times10^{-6}}\right)^2}$$

$$\approx 1\ \mho$$

$\boxed{Z_{in} \text{ at resonance } = 1\,\Omega}$

(ii)



No. of nodes = 4

No. of possible trees = n^{n-2}

$= (4)^{4-2} = 4^2 = 16$

$\boxed{\text{No. of possible trees} = 16}$

19

Good Approach

[illegible]

[illegible]

[illegible]

[illegible]

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Section B : Engineering Mathematics

Q.5 (a) Find the solution of the differential equation $(y - x + 1)dy - (y + x + 2)dx = 0$.

[12 marks]

Sol^n:- $(y - x + 1)\,dy = ((x+y)+2)\,dx$

$$\frac{dy}{dx} = \frac{x+y+2}{y-x+1}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x + vx + 2}{vx - x + 1}$$

$$= \frac{x(1+v) + 2}{x(v-1) + 1}$$

$(y - x + 1)\,dy - (y + x + 2)\,dx = 0$

Above, equation is of the form $M\,dx + N\,dy = 0$

where $M = -(y+x+2)$

$N = y - x + 1$

$$\frac{\partial M}{\partial y} = -1$$

2

$$\frac{\partial N}{\partial x} = -1$$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, so solution of

the differential equation can be written

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in the form.

$$g = \int M \cdot dx + \int (\text{terms of } N \text{ not containing } x)\, dy + C$$

$$g = \int -(x+y+2)\, dx + \int (y+1)\, dy + C$$

$$= -\frac{x^2}{2} - xy - 2x + \frac{y^2}{2} + y + C$$

$$= -\frac{x^2}{2} - xy - 2x + \frac{y^2}{2} + y + C$$

Ans.

Q.5 (b) Find the value of $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

[12 marks]

Q.5 (c) (i) The vector field $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ is defined over the volume of the cuboid given by $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. Evaluate the surface integral $\iint_S \vec{F} \cdot \overrightarrow{ds}$, where S is the surface of the cuboid.

[6 marks]

Solⁿ:- Given $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$

We know that, according to gauss divergence theorem.

$$\iint_S \vec{F} \cdot \vec{ds} = \iiint_V (\nabla \cdot \vec{F}) \cdot dv$$

$$\nabla \cdot \vec{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2\hat{i} + z\hat{j} + yz\hat{k})$$

$$= 2x + 0 + y = 2x+y$$

$$\iiint_V (\nabla \cdot \vec{F}) \cdot dv = \int_{x=0}^{a}\int_{y=0}^{b}\int_{z=0}^{c} (2x+y)\, dz\, dy\, dx$$

$$= \int_{x=0}^{a}\int_{y=0}^{b} \left[2xz + yz\right]_0^c dy\, dx$$

$$= \int_{x=0}^{a}\int_{y=0}^{b} \left[2xc + yc\right] dy\, dx$$

3

$$= \int_{x=0}^{a} \left[2xyc + \frac{y^2}{2}c\right]_0^b dx$$

$$= \int_{x=0}^{a} \left[2xbc + \frac{b^2c}{2}\right] dx$$

$$= \left[\cancel{2}\frac{x^2}{\cancel{2}}bc + \frac{b^2c}{2}x\right]_0^a$$

$$\iint_S \vec{F}\cdot\vec{ds} = a^2bc + b^2ac$$

$$\boxed{\iint_S \vec{F}\cdot\vec{ds} = abc\,[a+b]}$$

Ans

.5 (c) (ii) Find the absolute maxima and minima of

1. $f(x) = 2x^3 - 9x^2 + 12x - 5$ in $[0, 3]$
2. $f(x) = 12x^{4/3} - 6x^{1/3}, x \in [-1, 1]$

Also, find points of maxima and minima.

[6 marks]

(1) $f(x) = 2x^3 - 9x^2 + 12x - 5$

$f'(x) = 6x^2 - 18x + 12 = 0$

$x = 2, 1 \in [0,3]$

$f(0) = 2\times 0^3 - 9\times 0^2 + 12\times 0 - 5 = -5$

$f(3) = 2\times 3^3 - 9\times 3^2 + 12\times 3 - 5 = 4$

$f(2) = 2\times 2^3 - 9\times 4 + 12\times 2 - 5 = -1$

$f(1) = 2\times 1^3 - 9\times 1^2 + 12\times 1 - 5 = 0$

$f''(x) = 12x - 18$

at $x = 2$ $f''(x) = 6 > 0$

at $x = 1$ $f''(x) = -6 < 0$

$f(x)$ has absolute maxima at $x = 3$

" " absolute minima at $x = 0$

$f(x)_{max} = 4$

$f(x)_{min} = -5$

(2) $f(x) = 12x^{4/3} - 6x^{1/3}$

$f'(x) = \cancel{12}^{4} \times \frac{4}{\cancel{3}} x^{1/3} - \cancel{6}^{2}\times\frac{1}{\cancel{3}} x^{-2/3}$

$f'(x) = 0$

$16x^{1/3} - 2x^{-2/3} = 0$

$8x^{1/3} - x^{-2/3} = 0$

$8x^{1/3} = x^{-2/3}$

$x^{1/3 + 2/3} = \frac{1}{8}$

$x = 1/8 \in [-1, 1]$

$f''(x) = 16 \times \frac{1}{3} x^{-2/3} + \frac{4}{3} x^{-5/3}$

$f''(1/8) > 0$

$f(1) = 12 \times (1)^{4/3} - 6(1)^{1/3}$

$= 12 - 6 = 6$

$f(-1) = 12 \times (-1)^{4/3} - 6(-1)^{1/3}$

$= 12 + 6 = 18$

$f(1/8) = 12 \times (1/8)^{4/3} - 6(1/8)^{1/3}$

$= 12 \times \frac{1}{16} - 6 \times \frac{1}{2} = -9/4$

$f(x) \longrightarrow$ absolute maxima at $x = -1$, $f_{max} = 18$

$f(x) \longrightarrow$ absolute minima at $x = 1/8$, $f_{min} = -9/4$

6

Good Approach

5 (d) Determine the values of x for which the following function fails to be continuous or differentiable.

$$f(x)=\begin{cases} 1-x & , \ x<1 \\ (1-x)(2-x), & 1\le x\le 2 \\ 3-x & , \ x>2 \end{cases}$$

[12 marks]

Soln:- At $x=1$

$f(1^-) = 1-1 = 0$

$f(1^+) = (1-1)(2-1) = 0$

$f(1) = 0$

Since $f(1^-) = f(1) = f(1^+)$ at $x=1$, So $f(x)$ is continuous as well as differentiable at $x=1$

At $x=2$

$f(2^-) = (1-2)(2-2) = 0$

$f(2^+) = (3-2) = 1$

$f(2^-) \neq f(2^+)$

So, $f(x)$ is not continuous at $x=2$

Since, it is not continuous at $x=2$, hence it will not be differentiable at $x=2$.

At $x=1 \longrightarrow$ continuous as well as differentiable

$x=2 \longrightarrow$ Neither continuous nor differentiable.

Ans

11

Good Approach

5 (e) X is a continuous random variable with probability density function given by

$$f(x) = kx\,(0 \le x \le 2)$$
$$= 2k\,(2 \le x < 4)$$
$$= -kx + 6k\,(4 \le x < 6)$$

Find k and mean value of X.

[12 marks]

Solⁿ:- For continuous random variable

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^2 Kx\,dx + \int_2^4 2K\,dx + \int_4^6 (-Kx+6K)\,dx = 1$$

$$K\left[\frac{x^2}{2}\right]_0^2 + 2K\left[x\right]_2^4 + \left[-\frac{Kx^2}{2} + 6Kx\right]_4^6 = 1$$

$$K\left[\frac{4}{2}\right] + 2K \times 2 + \left[-\frac{K}{2} \times 36 + 36K + \frac{K}{2} \times 16 - 24K\right] = 1$$

$$2K + 4K - 18K + 36K + 8K - 24K = 1$$

$$8K = 1 \Rightarrow \boxed{K = \frac{1}{8}}$$

Mean value of $F(x)$

$$= \int_{-\infty}^{\infty} x f(x)$$

$$= \int_0^2 Kx^2\,dx + \int_2^4 2Kx\,dx + \int_4^6 (-Kx^2 + 6Kx)\,dx$$

$$= K\left[\frac{x^3}{3}\right]_0^2 + 2K\left[\frac{x^2}{2}\right]_2^4 + \left[-\frac{Kx^3}{3} + \frac{{}^{3}\cancel{6}Kx^2}{\cancel{2}}\right]_4^6$$

$$= K\left[8/3\right] + 2K\left[\frac{\cancel{16}^{8}}{\cancel{2}} - \frac{\cancel{4}^{2}}{\cancel{2}}\right] + \left[-\frac{K}{3}\times 216 + 108K + \frac{K}{3}\times 64 - 48K\right]$$

$$= K\left[8/3\right] + 12K - 72K + 108K + \frac{64}{3}K - 48K$$

$$= 24K + 12K - 72K + 108K - 48K$$

$$= 24K = \cancel{24}^{3}\times\frac{1}{\cancel{8}} = \underline{\underline{3}}$$

$$\boxed{\text{Mean value of } x = 3}$$

11

Good Approach

Do not write in this margin

6 (a) (i) State Langrange's mean value theorem and explain the theorem in reference to it's geometrical significance.

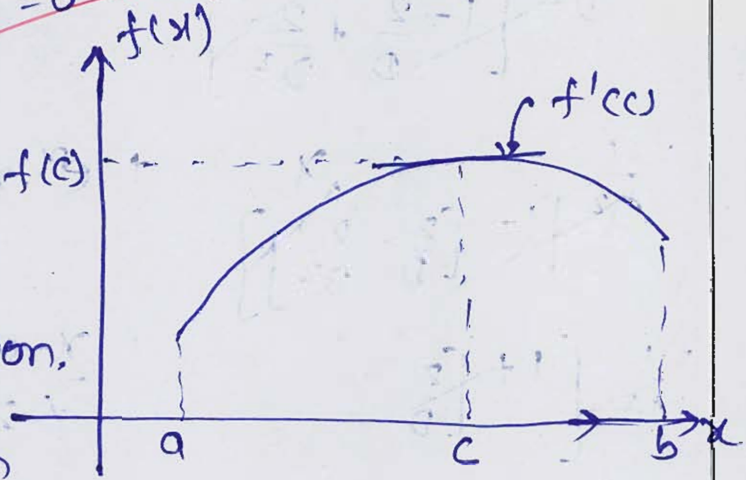
(ii) Find the complete solution of $y^2 - 2y' + 2y = x + e^x \cos x$.

(iii) Prove that the matrix, $A = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$ is unitary and find A^{-1}.

[6 + 6 + 8 marks]

Lagrange's MVT :-

Sol^n :- (i) If $f(x)$ is continuous in $x \in [a, b]$ and differentiable in $x \in (a,b)$, then there exist a point $c \in (a,b)$ such that $f'(c) = 0$



Such, point c in the interval (a,b) is called as point of inflexion, where, the function changes its nature from increasing to decreasing or vice versa,

(ii) $y^2 - 2y' + 2y = x + e^x \cos x$

C.F :-

$D^2 - 2D + 2 = 0$

$m_1 = 1+i$

$m_2 = 1-i$

$C.F = e^x [C_1 \cos x + C_2 \sin x]$

P.I

$$\frac{1}{(D^2-2D+2)}[x+e^x\cos x]$$

$$= \frac{1}{D^2-2D+2}x + \frac{1}{(D^2-2D+2)}e^x\cos x$$

$$= \cancel{\frac{1}{D^2}\left[1-\frac{2}{D}\right.} \quad \frac{1}{2}\left[1+\frac{D^2}{2}-D\right] \qquad \frac{1}{D^2-2D+2}x$$

$$= \cancel{\frac{1}{D^2}\left[1-\frac{2}{D}+\frac{2}{D^2}\right]x} + e^x\cdot\frac{1}{D^2+\cancel{2D}+1-\cancel{2D}-\cancel{2}+\cancel{2}}\cos x$$

$$= \cancel{\frac{1}{D^2}\left[1-\left[\frac{2}{D}-\frac{2}{D^2}\right]\right]x} + e^x\cdot x\,\frac{D}{2D^2}\cos x$$

$$= \cancel{\frac{1}{D^2}\left[1+\left[\frac{2}{D}\right.\right.} \qquad + \frac{x\cdot e^x(-\sin x)}{-2}$$

$$= \frac{1}{2\left(1-\left(D-\frac{D^2}{2}\right)\right)}x \qquad + \frac{1}{2}xe^x\sin x$$

$$= \frac{1}{2}\left(1+D-\frac{D^2}{2}\right)x + \frac{1}{2}x\cdot e^x\sin x.$$

$$= \frac{1}{2}[x+1] + \frac{1}{2}x\cdot e^x\sin x$$

$$y = e^x[C_1\cos x + C_2\sin x] + \frac{1}{2}[x+1] + \frac{1}{2}x\cdot e^x\sin x$$

(iii) $A = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$

For unitary matrix $A \cdot A^{\theta} = I$

$A^{\theta} = (\bar{A})^T$

$\bar{A} = \begin{bmatrix} \frac{1}{2}(1-i) & \frac{1}{2}(-1-i) \\ \frac{1}{2}(1-i) & \frac{1}{2}(1+i) \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} \frac{1}{2}(1-i) & \frac{1}{2}(1-i) \\ \frac{1}{2}(-1-i) & \frac{1}{2}(1+i) \end{bmatrix}$

$A \cdot A^{\theta} = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix} \begin{bmatrix} \frac{1}{2}(1-i) & \frac{1}{2}(1-i) \\ \frac{1}{2}(-1-i) & \frac{1}{2}(1+i) \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{4}(1-i^2+(-1)^2-i^2) & \frac{1}{4}(1-i^2+i^2-1) \\ \frac{1}{4}[1-i^2+(-i)^2-1] & \frac{1}{4}(1-i^2+1-i^2) \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

So, since $A \cdot A^{\theta} = I$, the given matrix is unitary

$A^{\theta} = A^{-1}$

So, $A^{-1} = \begin{bmatrix} \frac{1}{2}(1-i) & \frac{1}{2}(1-i) \\ \frac{1}{2}(-1-i) & \frac{1}{2}(1+i) \end{bmatrix}$ Ans

18

Good Approach

6 (b) (i) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ show that $A^2 - 4A - 5I = 0$ where I, 0 are the identity matrix and the null matrix of order 3 respectively. Use this result to find A^{-1}.

[10 marks]

Solⁿ:- $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Cayley-Hamilton theorem –

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} (1-\lambda) & 2 & 2 \\ 2 & (1-\lambda) & 2 \\ 2 & 2 & (1-\lambda) \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)^2 - 4\right] - 2\left[2(1-\lambda) - 4\right] + 2\left[4 - 2(1-\lambda)\right] = 0$$

On simplifying we get

$$\lambda^2 - 4\lambda - 5 = 0$$

Since, each eigen value satisfies its eqn.

So, $\boxed{A^2 - 4A - 5I = 0}$ Ans

Given, $A^2 - 4A - 5I = 0$

Multiplying both side by A^{-1}

$$A - 4I - 5A^{-1} = 0$$

$$A^{-1} = \frac{1}{5}(A - 4I)$$

$$= \frac{1}{5}\left(\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right)$$

$$\boxed{A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}}$$

Ans.

9

Good Approach

6 (b) (ii) Examine the following vectors for linear dependence and find the relation if it exists.
$X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$ and $X_4 = (-3, 7, 2)$.

[10 marks]

Sol^n:- ~~for linear~~ dependence, determinant ~~of the matrix~~ = 0

Given $x_1 = (1, 2, 4)$

$x_2 = (2, -1, 3)$

$x_3 = (0, 1, 2)$

$x_4 = (-3, 7, 2)$

1

Incomplete solution

6 (c) (i) Solve: $(D^2 - 4D + 4)\, y = 8x^2 e^{2x} \sin 2x$.

(ii) Find the regression line of y on x for the following data and estimate the value of y, when $x = 10$. (Use the least square approximation method)

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

[12 + 8 marks]

(i)

$D^2 - 4D + 4 = 0$

$(D-2)^2 = 0$

$m = 2, 2 \Rightarrow C.F = (C_1 + C_2 x)\, e^{2x}$

P.I.

$\dfrac{1}{D^2 - 4D + 4}\, 8x^2 e^{2x} \sin 2x$

$= 8x^2 e^{2x} \cdot \dfrac{1}{(D+2)^2 - 4(D+2) + 4} \sin 2x$

$= 8x^2 \cdot e^{2x} \cdot \dfrac{1}{D^2 + \cancel{4D} + \cancel{4} - \cancel{4D} - \cancel{4}} \sin 2x$

$= 8x^2 \cdot e^{2x} \cdot \dfrac{1}{(-2^2)} \sin 2x$

$= -2x^2 \cdot e^{2x} \cdot \sin 2x$

$y = C.F + P.I$

$= (C_1 + C_2 x)\, e^{2x} - 2x^2 \cdot e^{2x} \sin 2x$

Ans.

3

(ii)

x	y	y^2	xy
1	1	1	1
3	2	4	6
4	4	16	16
6	4	16	24
8	5	25	40
9	7	49	63
11	8	64	88
14	9	81	126
56	40	256	364

$$\Sigma y = na + b\Sigma x$$

$$\Sigma y^2 = a\Sigma y + b\Sigma xy$$

$$40 = 8a + 56b$$

$$256 = 40a + 364b$$

$$a = \frac{1}{3}, \quad b = 2/3$$

3

7 (a) If the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$, then find all possible value of m.

[20 marks]

Q.7 (b) **(i)** If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}\right)^2 u = \dfrac{-9}{(x+y+z)^2}$.

(ii) Find the real root of the following equation, correct to three decimal places. (Using Newton-Raphson method)

$x^3 - 2x - 5 = 0$

[12 + 8 marks]

Q.7 (c) Show that the vector field $\vec{F} = \frac{\vec{r}}{|\vec{r}|^3}$ is irrotational as well as solenoidal. Find the scalar potential.

[20 marks]

(a) Sixteen players S_1, S_2, ... S_{16} play in a tournament. They are divided into eight pairs at random, from each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

Find:

(i) The probability that the player S_1 is among the eight winners.

(ii) The probability that exactly one of the two players S_1 and S_2 is among the eight winners.

[20 marks]

Do not write in this

Q.8 (b) (i) Assuming that the diameters of 1000 brass plugs taken consecutively from a machine from a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm, how many number of brass plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm? (Given, Area[$Z = -1.75$] = 0.4599 and Area[$Z = 2.25$] = 0.4878).

(ii) A periodic function of time period 4 is defined as $f(x) = |x|$, $-2 < x < 2$. Find its Fourier series expansion.

[8 + 12 marks]

Q.8 (c) Apply Runge-Kutta 4th order method to find an approximate value of y when $x = 0.2$.

Given that $\frac{dy}{dx} = x + y$, $y = 1$ when $x = 0$. (Take step size of 0.1)

[20 marks]

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Space for Rough Work

Alt - 288

Space for Rough Work