• Try to avoid calculation mistake

• Highlights final answers

• use Headings

MADE EASY

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Leading Institute for ESE, GATE & PSUs

ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-1 : Electric Circuits + Engineering Mathematics

Name :

Roll No :

Test Centres

Delhi ☑ Bhopal ☐ Jaipur ☐

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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	30
Q.2	
Q.3	43
Q.4	51
Section-B	
Q.5	36
Q.6	27
Q.7	
Q.8	
Total Marks Obtained	187

Signature of Evaluator: Sourabh kumar

Cross Checked by:

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

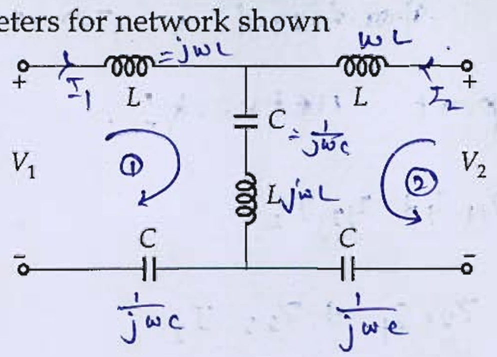
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Electric Circuits

1 (a) Determine Z-parameters for network shown



[12 marks]

Sol^n:-

in loop - ①:-

$$V_1 - j\omega L I_1 - \left(j\omega L + \frac{1}{j\omega C}\right)(I_1 + I_2) - \frac{1}{j\omega C} I_1 = 0$$

$$V_1 - j\omega L I_1 - \left(j\left(\omega L - \frac{1}{\omega C}\right)\right)(I_1 + I_2) - \frac{1}{j\omega C} I_1 = 0$$

$$V_1 - j\omega L I_1 - \left(j\left(\omega L - 1/\omega C\right)\right) I_1 - j\left(\omega L - 1/\omega C\right) I_2 + \frac{j}{\omega C} I_1 = 0$$

$$V_1 - j\omega L I_1 - j\omega L I_1 + \frac{j}{\omega C} I_1 + \frac{j}{\omega C} I_1 - j\left(\omega L - 1/\omega C\right) I_2 = 0$$

$$V_1 - 2j\omega L I_1 + \frac{2j}{\omega C} I_1 - j\left(\omega L - 1/\omega C\right) I_2 = 0$$

$$\boxed{V_1 = \left(2j\omega L - 2j/\omega C\right) I_1 + j\left(\omega L - 1/\omega C\right) I_2}$$

in loop ②:-

$$V_2 - j\omega L I_2 - \left(j\left(\omega L - 1/\omega C\right)\right)(I_1 + I_2) + j/\omega C\, I_2 = 0$$

$$V_2 - j\omega L I_2 - j\left(\omega L - 1/\omega C\right) I_1 - j\left(\omega L - 1/\omega C\right) I_2 + j/\omega C\, I_2 = 0$$

$$V_2 - j\left(\omega L - 1/\omega C\right) I_1 - 2j\omega L_2 I_2 + 2j/\omega C\, I_2 = 0$$

$$\boxed{V_2 = j\left(\omega L - 1/\omega C\right) I_1 + \left(2j\omega L_2 - 2j/\omega C\right) I_2}$$

comparing standard z-parameters of two-port network:-

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$$\therefore [Z] = \begin{bmatrix} 2j\omega L - 2j/\omega C & j\left(\omega L - \frac{1}{\omega C}\right) \\ j\left(\omega L - \frac{1}{\omega C}\right) & 2j\left(\omega L - \frac{1}{\omega C}\right) \end{bmatrix}$$

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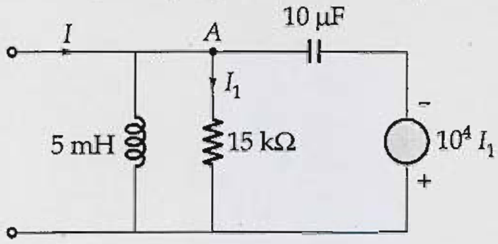
Good Approach

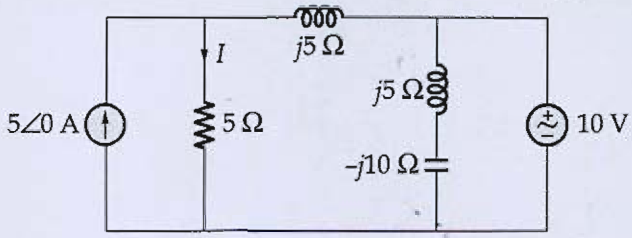
Do not write in this margin

1 (b) Determine the resonant frequency, quality factor Q_0 and bandwidth of the given circuit.



[12 marks]

1 (c) (i) Determine the current 'I' in the network shown using principle of superposition.

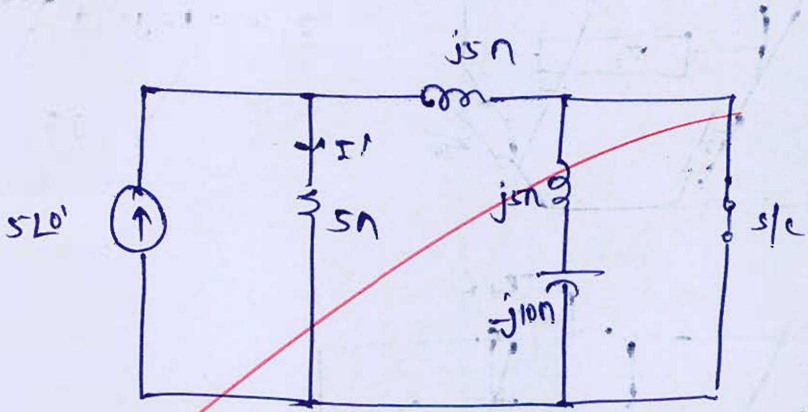


[6 marks]

M1.

current I' in 5Ω resistor due to current source $5\angle 0°$ A.

deactive other voltage and current sources.



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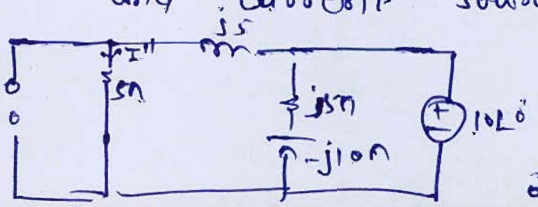
current $I' = \dfrac{5 \times j5}{5+j5}$

$I' = \dfrac{j5}{1+j1} = \underline{3.5355\angle 45°}$

current I'' in 5Ω resistor due to voltage source 10 V,

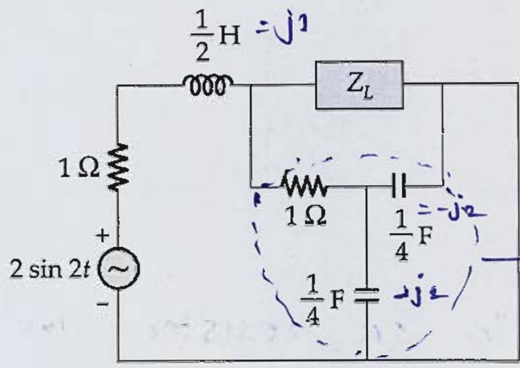
deactive other independent voltage and current sources:-



$\therefore I'' = \dfrac{10}{5+j5} = \dfrac{2}{1+j1}$
$= \sqrt{2}\angle -45°$

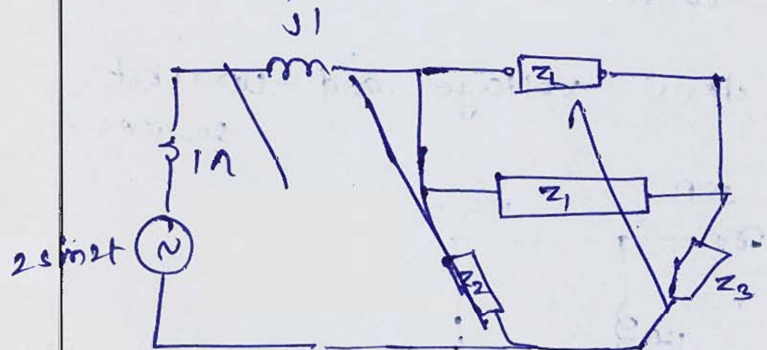
$\therefore I = I' + I'' = 3.5355\angle 45° + \sqrt{2}\angle -45°$
$= 3.8078\angle 23.20$ Ans.

Q.1 (c) **(ii)** Determine the value of impedance Z_L for maximum power transfer, in Z_L, in the given network.



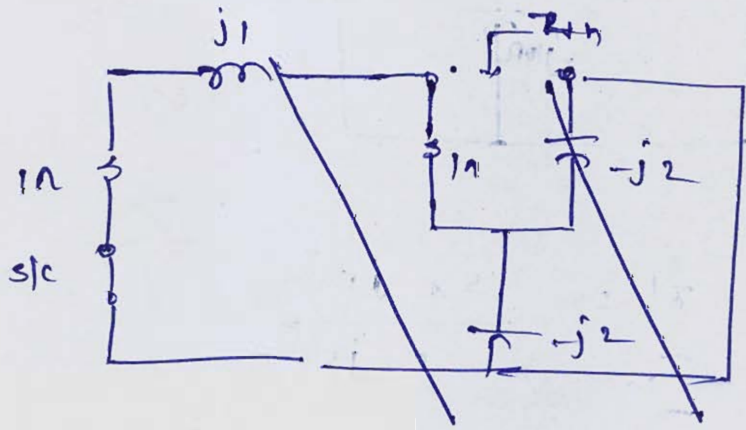
[6 marks]

Sol^n:



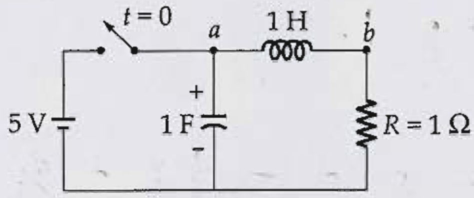
~~$Z_1 = 1 - j2 + \frac{1 \times -j2}{-j2}$~~

~~$= 2 - j2$~~



$Z_{th} =$ ~~$-j\tfrac{1}{2} \times -j$~~ $-j2 - j2 + 1$

Q.1 (d) Consider the following circuit:

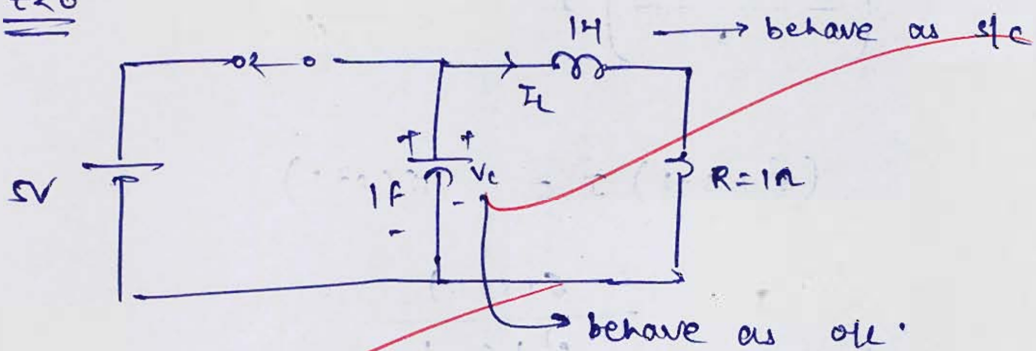


The switch is initially closed. After steady state is reached the switch is opened. Determine the nodal voltage $V_a(t)$ and $V_b(t)$.

[12 marks]

When switch is closed :-

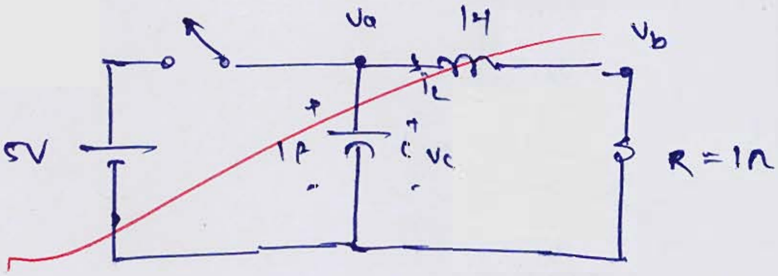
$\underline{t<0}$



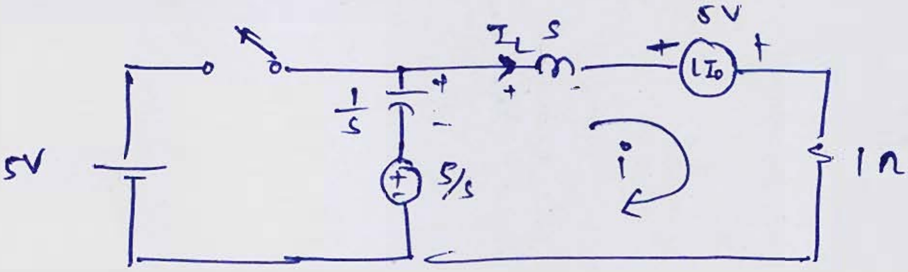
$\therefore \quad V_c = 5V$

$\& \quad I_L = 5A;$

$\underline{\text{at } t=0}$ switch is opened :-



↓ in s-domain



current in loop :-

$$-sI + 5 - I = -\frac{1}{s}I - \frac{5}{s}$$

$$\left(\frac{1}{s} - s - 1\right) I = -5 - \frac{5}{s}$$

$$\left(\frac{-s^2 - s + 1}{\cancel{s}}\right) I = \frac{-5s - 5}{\cancel{s}}$$

$$(s^2 + s - 1)\, I = 5(s+1)$$

$$I = \frac{5(s+1)}{s^2 + s - 1}$$

$$I =$$

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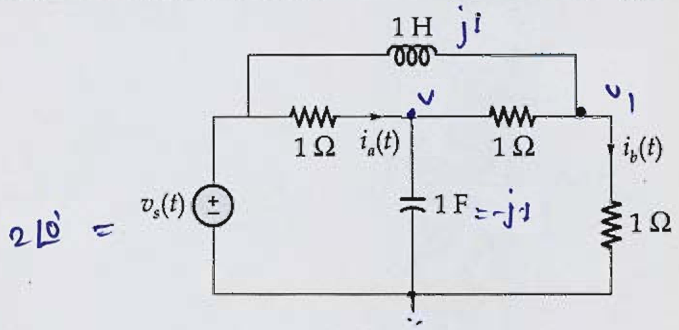
Q.1 (e) For the bridged T-network of the figure given below the source voltage is $v_s(t) = 2\cos t$. The circuit is in steady state condition. Determine: **(i)** $i_a(t)$ and **(ii)** $i_b(t)$.



[12 marks]

Solⁿ :-

$$\frac{V - 2\angle 0^\circ}{1} + \frac{V}{-j1} + \frac{V - V_1}{1} = 0$$

$$V\left[1 + \frac{1}{-j1} + 1\right] = 2 + V_1 \quad \text{--- (1)}$$

$$\frac{V_1 - 2\angle 0^\circ}{j1} + \frac{V_1}{1} + \frac{V_1 - V}{1} = 0$$

$$V_1\left[\frac{1}{j1} + 2\right] = \frac{2}{j1} + V$$

$$V_1\left[2 + \frac{1}{j1}\right] = \frac{2}{j1} + V \quad \text{--- (2)}$$

by eqⁿ (1)

$$V(2 + i) = 2 + V_1 \quad \text{---- (3)}$$

by eqⁿ (2)

$$V_1(2 - i) = \frac{2}{j1} + V$$

Do not write in this margin

$$v_1 (2-j) = \frac{2}{j1} + \left(\frac{2+v_1}{2+j}\right)$$

$$v_1 \left[(2-j) - \frac{1}{2+j}\right] = \frac{2}{j1} + \frac{2}{2+j}$$

$$\boxed{V_1 = 1-j = \sqrt{2}\angle -45^\circ}$$

$$\therefore \quad v = \frac{2+V_1}{2+j}$$

$$\boxed{V = \sqrt{2}\angle -45^\circ}$$

$$\therefore \quad \boxed{i_b(t) = \frac{V_1}{1} = \sqrt{2}\angle -45^\circ}$$

$$i_a(t) = \frac{2\angle 0^\circ - \sqrt{2}\angle 45^\circ}{1}$$

$$\boxed{i_a(t) = \sqrt{2}\angle 45^\circ}$$

$$\boxed{i_a(t) = \sqrt{2}\cos(t+45^\circ)}$$

10

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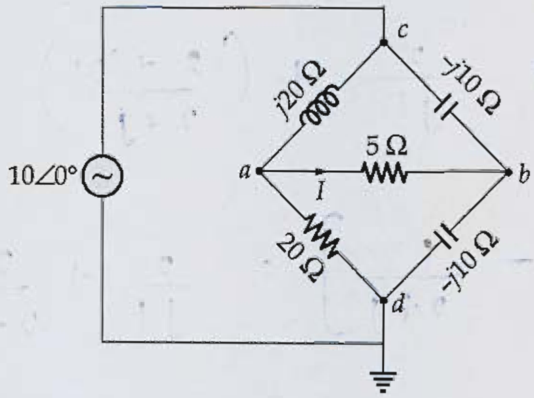
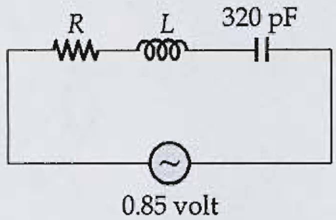
Q.2 (a) (i) Determine current I in the network using Thevenin's theorem.



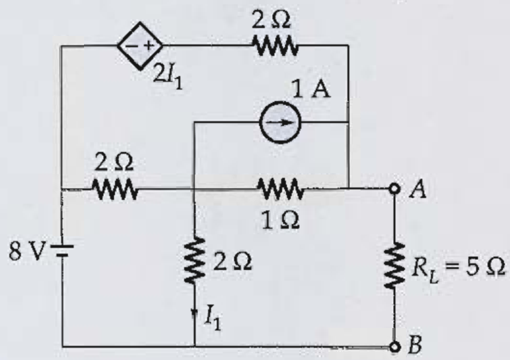
[10 marks]

Q.2 (a) **(ii)** For the circuit shown determine the value of inductance for resonance if $Q = 50$ and $f_0 = 175$ kHz. Also find the circuit current, the voltage across the capacitor and the bandwidth of the circuit.



[10 marks]

Q.2 (b) Determine the current through the load resistance $R_L = 5\ \Omega$ across the terminals A-B of the circuit shown in figure below, using Thevenin's theorem. Also, find the maximum power that can be transferred to the load resistance R_L.

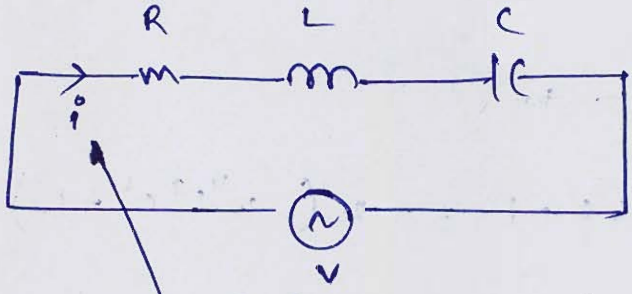


[20 marks]

Q.2 (c) (i) Derive expression for frequency for maximum voltage across inductor in series RLC resonant circuit.

(ii) Calculate the maximum voltage across the inductor using result of **part (i)** with constant voltage and variable frequency. Assume $R = 50\ \Omega$, $L = 0.05$H, $C = 20\ \mu$F and $V = 100$ V.

[13 + 7 marks]



maximum voltage across inductor :-

$$V_L = I X_L$$

$$V_L = \frac{I}{\sqrt{R^2 + (X_L - X_C)^2}} \times X_L$$

$$V_L = \frac{I \cdot \omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

freq^n at maximum voltage :-

$$\frac{dV_L}{d\omega} = 0 \Rightarrow \frac{\omega L\left(\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\right) - \omega L \cdot \frac{2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$L\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right) - \omega L \cdot 2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right) = 0$$

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2\omega\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)$$

$$\frac{R^2\omega^2 C^2 + (\omega^2 LC - 1)}{\omega^2 C^2} = \frac{2\omega(\omega^2 LC - 1)(\omega^2 LC + 1)}{\omega C \cdot \omega^2 C}$$

$$\frac{R^2\omega^2c^2 + \omega^2LC - 1}{\omega^2\cancel{c^2}} = \frac{2\cancel{\omega}(\omega^2LC-1)(\omega^2LC+1)}{\cancel{\omega}C \cdot \omega^2\cancel{C}}$$

$$\cancel{R^2\omega^3c^2 + \omega^3LC - \omega}$$

$$R^2\omega^2c^2 + \omega^2LC - 1 = 2(\omega^4L^2c^2 - 1)$$

$$R^2\omega^2c^2 + \omega^2LC = 2\omega^4L^2c^2 - 1$$

$$\omega^2(R^2c^2 + LC) + 1 = 2\omega^4L^2c^2$$

$$2\omega^4L^2c^2 - \omega^2(R^2c^2 + LC) + 1 = 0$$

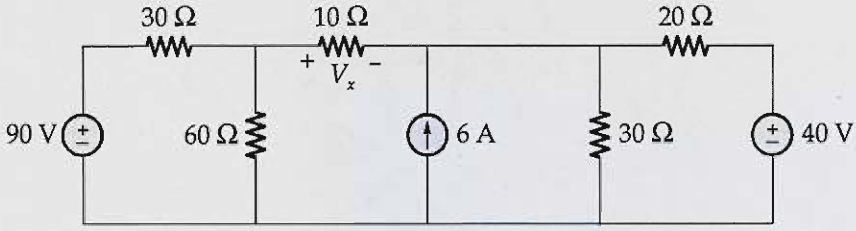
$\boxed{\omega^2 = x}$:-

$$\omega^2 = \frac{(R^2c^2 + LC) \pm \sqrt{(R^2c^2+LC)^2 + 8L^2c^2}}{2 \cdot L^2c^2}$$

$$\omega^2 = \frac{R^2}{2L^2} + \frac{1}{2LC} \pm \sqrt{\left(\frac{R^2c^2+LC}{2L^2c^2}\right)^2 + \frac{8L^2c^2}{4L^4c^4}}$$

$$\omega^2 = \frac{R^2}{2L^2} + \frac{1}{2LC} \pm \sqrt{\left(\frac{R^2}{2L^2} + \frac{1}{2LC}\right)^2 + \frac{2}{L^2C^2}}$$

Q.3 (a) (i) The circuit shown in the figure below consists of three independent sources. Determine the value of the voltage across 10 Ω resistance using superposition theorem.

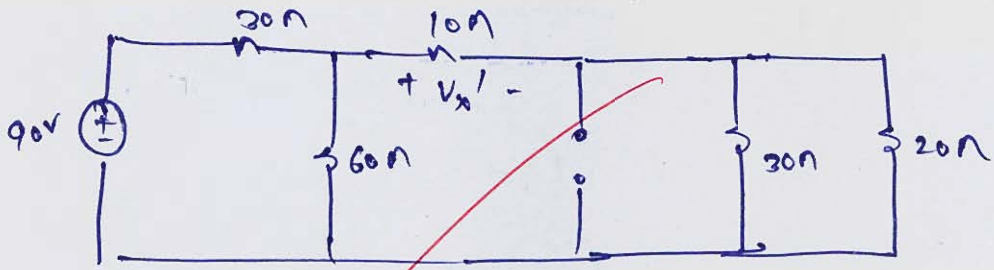


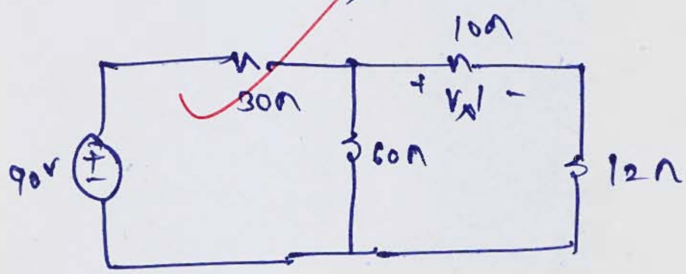
[10 marks]

Sol^n:-

superposition theorem:-

① V_x' due to 90V :-

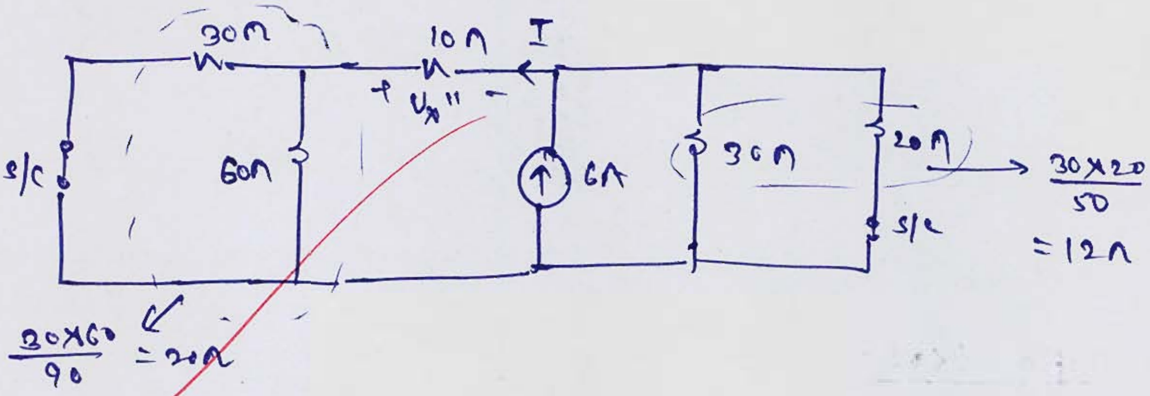




$$V_x' = \left(\frac{90}{30 + (60||22)}\right) \times \frac{60}{22} \times 10$$

$$\boxed{V_x' = 53.25\ V}$$

(2) V_x'' due to 6 A :-

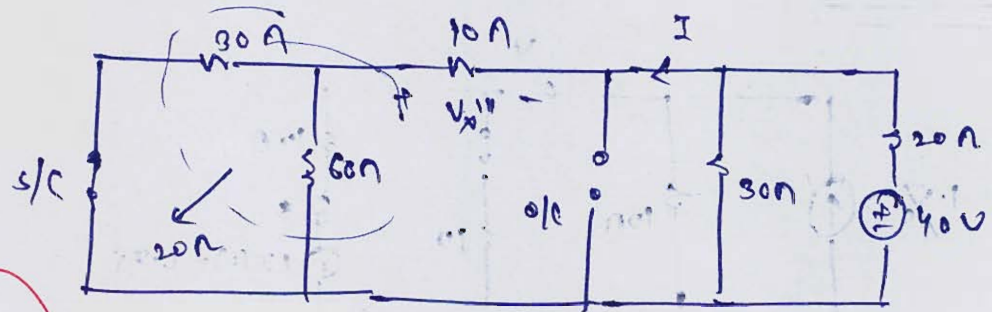


$I = 6 \times \frac{12}{30+12}$

$I = \frac{72}{42}$

$\therefore \boxed{V_x'' = -10 \times \frac{72}{42} = -17.143\text{ V}}$

(3) V_x''' due to 40 A :-



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$I = \left(\frac{40}{35}\right) \times \frac{30}{60}$

$I = \frac{20}{35} = 4/7$ A.

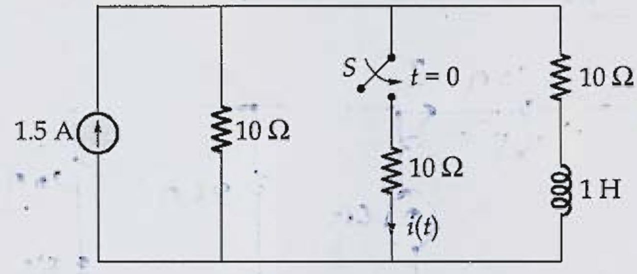
$\therefore V_x''' = -\frac{40}{7} = -5.7143$ V

$\therefore V_x = V_x' + V_x'' + V_x'''$

$= 53.25 - 17.143 - 5.7143$

$\boxed{V_x = 30.393\text{ V}}$

Q.3 (a) (ii) Consider the network shown below:

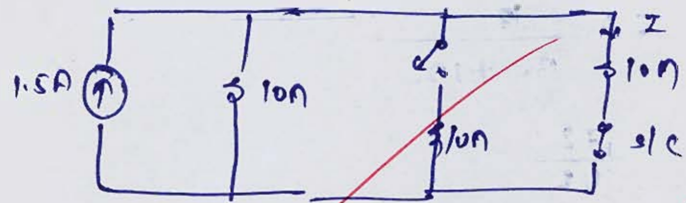


If switch S is closed at $t = 0$, calculate $i(t)$ for $t > 0$ by using Laplace transform approach.

[10 marks]

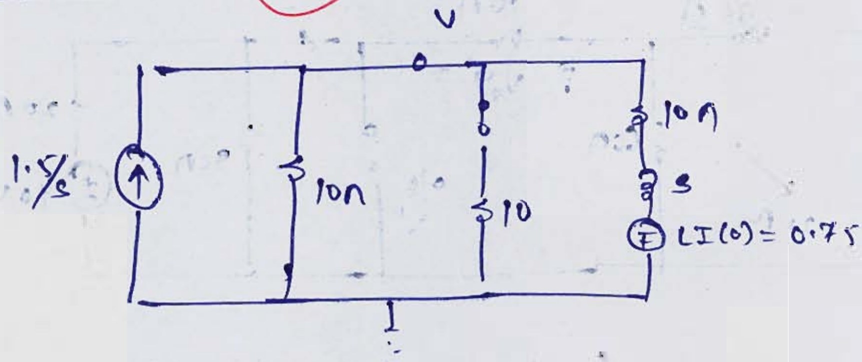
Sol^n :-

for $t<0$:-



$$\boxed{I_L(0^-) = 0.75 \text{ A.}}$$

for $t>0$:-



$$\frac{V}{10} + \frac{V}{10} + \frac{V+0.75}{10+s} = 1.5/s$$

$$\frac{V}{5} + \frac{V+0.75}{s+10} = \frac{1.5}{s}$$

$$\frac{V(s+10)+5V+3.75}{5(s+10)} = \frac{1.5}{s}$$

$$(s+15)V + 3.75 = \frac{1.5 \times 5(s+10)}{s}$$

$$(s+15)V = \frac{-3.75s + 7.5(s+10)}{s}$$

$$(s+15)V = \frac{3.75s + 75}{s}$$

$$V = \frac{3.75s + 75}{s(s+15)}$$

$$V = \frac{A}{s} + \frac{B}{s+15}$$

$$\boxed{A = 5, \quad B = -5/4}$$

$$V = \frac{5}{s} - \frac{5}{4(s+15)}$$

$$v(t) = 5 - \frac{5}{4}e^{-15t}$$

$$\therefore \quad i = \frac{v(t)}{10}$$

$$\boxed{i(t) = \frac{1}{2} - \frac{1}{8}e^{-15t}}$$

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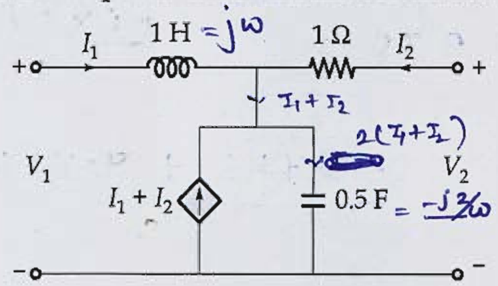
Good Approach

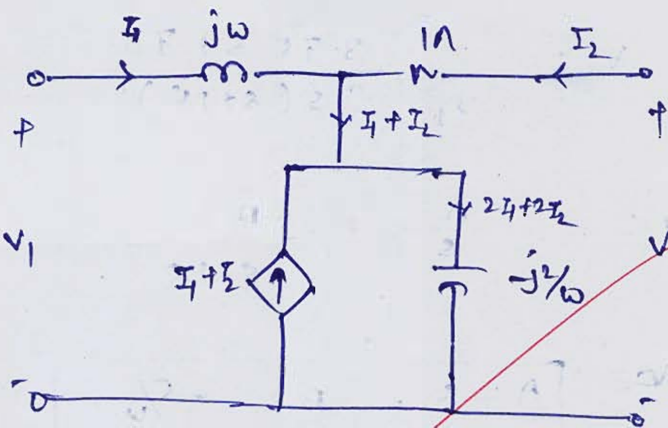
Q.3 (b) Determine the transmission parameters matrix for the two port network shown below.



[20 marks]

Sol^n :-

$\Downarrow$



$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}\right\} \text{— Transmission parameter}$$

$$V_1 - j\omega I_1 + j\frac{2}{\omega}(2I_1 + 2I_2) = 0$$

$$V_1 - j\omega I_1 + j\frac{4}{\omega} I_1 + j\frac{4}{\omega} I_2 = 0 \quad \text{---(i)}$$

$$V_2 - I_2 + j\frac{2}{\omega}(2I_1 + 2I_2) = 0$$

$$V_2 - I_2 + j\frac{4}{\omega} I_2 + j\frac{4}{\omega} I_1 = 0$$

$$j\frac{4}{\omega} I_1 = -V_2 + I_2 - j\frac{4}{\omega} I_2$$

$$\boxed{I_1 = -\frac{\omega}{j4} V_2 + \left(\frac{(\omega - j4)\omega}{j4}\right) I_2} \text{ — (2)}$$

put eqn (ii) value of I_1 in eqn (1):-

$$V_1 = \left(j\omega - j4/\omega\right) I_1 - j4/\omega \, I_2$$

$$V_1 = \left(j\omega - j4/\omega\right)\left[-\frac{\omega}{4j} V_2 + \frac{(\omega - j4)\omega}{j4} I_2\right] - j\frac{4}{\omega} I_2$$

$$V_1 = -\frac{\omega^2}{4} V_2 + V_2 + \frac{\omega^2(\omega - j4)}{4} I_2 - (\omega - j4) I_2 - j\frac{4}{\omega} I_2$$

$$V_1 = \left(1 - \frac{\omega^2}{4}\right) V_2 + \left(\frac{\omega^2(\omega - j4)}{4} - (\omega - j4) - j\frac{4}{\omega}\right) I_2 \quad \text{--- (3)}$$

compare eqn (3) & (2) with standard equation:-

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 - \omega^2/4 & -\left\{\frac{\omega^2(\omega - j4)}{4} - (\omega - j4) - j\frac{4}{\omega}\right\} \\ -\frac{\omega}{j4} & -\left\{\frac{\omega(\omega - j4)}{j4}\right\} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Ans.

12

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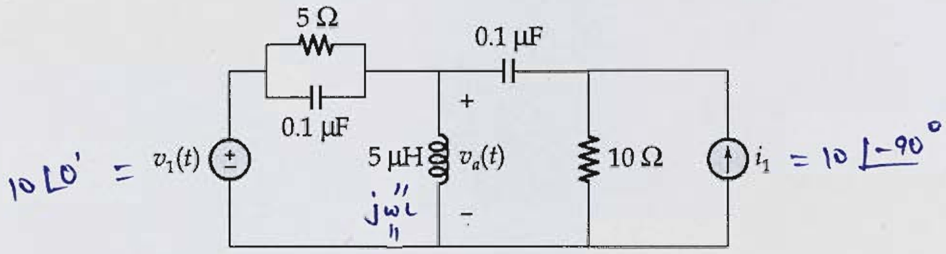
Q.3 (c) (i) For the circuit shown below, $v_1(t) = 10 \sin 10^6 t$ V and $i_1(t) = 10 \cos 10^6 t$ A and the circuit is operating in steady state condition. Determine the node to datum voltage $v_a(t)$.

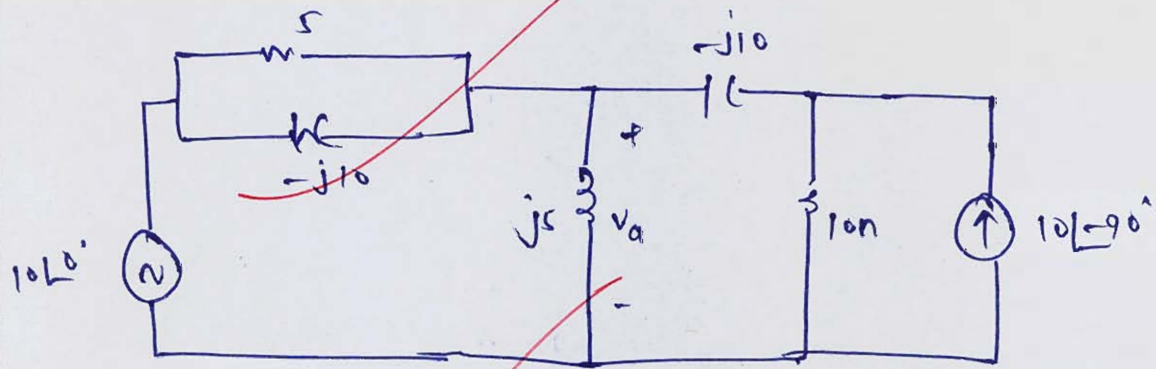


[10 marks]

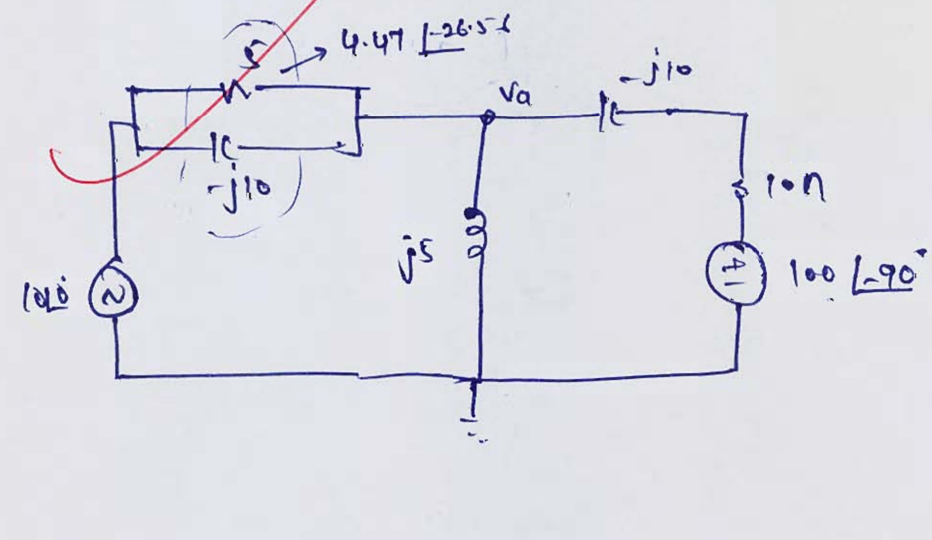
Sol^n:-

$$j\omega L = j \times 10^6 \times 5 \times 10^{-6} = j5$$

$$X_C = \frac{1}{j\omega C} = \frac{1}{j \times 10^6 \times 0.1 \times 10^{-6}} = -j10$$



⇓ source transformation:-



By Nodal Analysis :-

$$\frac{V_a}{j5} + \frac{V_a - 10\angle 0^\circ}{4.47\angle 26.56} + \frac{V_a - 100\angle -90^\circ}{10 - j10}$$

$$V_a\left[\frac{1}{j5} + \frac{1}{4.47\angle -26.56} + \frac{1}{10 - j10}\right] = \frac{10}{4.47\angle -26.56} + \frac{100\angle -90^\circ}{10 - j10}$$

$$V_a\left[0.255\angle -11.30\right] = 8.063\angle -29.74$$

$$V_a = \frac{8.063\angle -29.74}{0.255\angle -11.30}$$

5

$$\boxed{V_a = 31.62\angle -18.44^\circ}$$

$$\therefore \boxed{V_a = 31.62\sin(10^6 t - 18.44)}$$

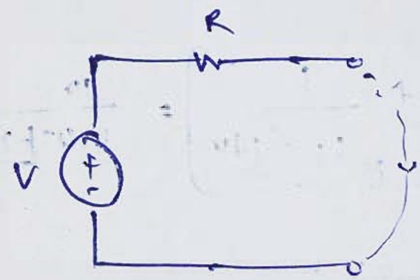
Q.3 (c) (ii) A certain practical dc voltage source can provide a current of 2.5 A when it is (momentarily) short circuited and can provide a power of 80 W to 20 Ω load.

Find:

1. The open circuit voltage.
2. The maximum power it could deliver to a well-chosen R_L.
3. What is the value of that R_L?

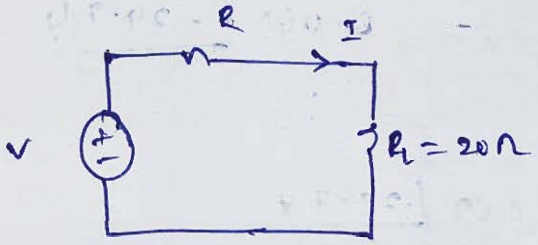
[10 marks]

<u>sol^n:</u>



$\Rightarrow \quad I = \dfrac{V}{R}$

$\boxed{V = 2.5R}$ ---- (1)



$$I = \frac{V}{R+20}$$

$$P = I^2 R_L$$

$$80 = I^2 \cdot 20$$

$$\boxed{I = 2A}$$

$$2 = \frac{V}{\frac{V}{2.5} + 20}$$

$$\frac{V}{2.5} + 20 = \frac{V}{2}$$

$$20 = \frac{V}{2} - \frac{V}{2.5}$$

$$0.1V = 20$$

$$\boxed{V = 200}$$

$$\therefore \quad R = \frac{200}{2.5} = \underline{80\,\Omega}$$

1. ∴ open circuit voltage $\boxed{V = 200\,V}$

2. for maximum power well chosen R_L will be $R_L = 80\,\Omega$.

$$\therefore P_{max.} = \frac{V_{th}^2}{4R_{th}}$$

$$= \frac{(200)^2}{4 \times 80}$$

$$= \underline{\underline{125\,W}}$$

3. value of R_L will be $80\,\Omega$, because at this value of R_L, transferred power will be maximum.

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Good Approach

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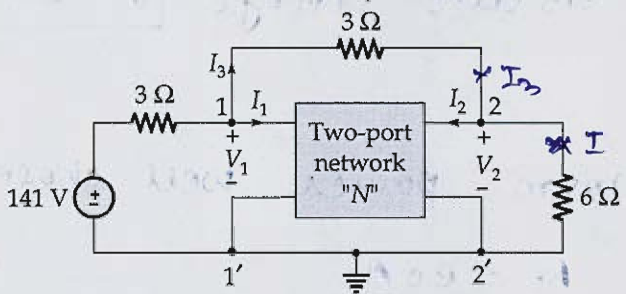
Q.4 (a) (i) The z-parameters of the two port network-N shown in the figure below are given as $z_{11} = 2\ \Omega$, $z_{12} = z_{21} = 1\ \Omega$ and $z_{22} = 4\ \Omega$. Find the values of currents I_1, I_2 and I_3.



$I_3 = I + I_2$

$\boxed{I = I_3 - I_2}$

[12 marks]

sol^n :-

$V_1 = 2I_1 + I_2$

$V_2 = I_1 + 4I_2$

$141 - 3(I_1 + I_3) = 2I_1 + I_2$

$141 - 3I_1 - 3I_3 = 2I_1 + I_2$

$141 = 5I_1 + I_2 + 3I_3 \quad \text{----} \quad (1)$

$\therefore \quad V_2 = 6(I_3 - I_2) = I_1 + 4I_2$

$6I_3 - 6I_2 = 4I_2 + I_1$

$I_1 + 10I_2 - 6I_3 = 0 \quad \text{----} \quad (2)$

$V_1 - 3I_3 - V_2 = 0$

$2I_1 + I_2 - 3I_3 - I_1 - 4I_2 = 0$

$I_1 - 3I_2 - 3I_3 = 0 \quad \text{---} \quad (3)$

by eq^n ①, ② & ③

$$\boxed{\begin{aligned} I_1 &= 24\ A \\ I_2 &= 1.5\ A \\ I_3 &= 6.5\ A \end{aligned}}$$

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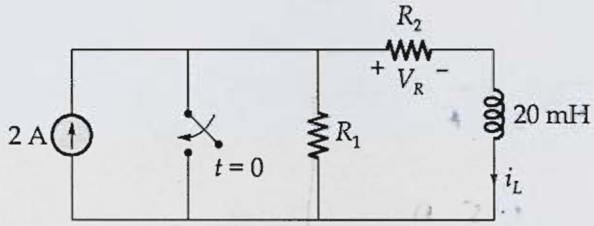
Good Approach

Q.4 (a) **(ii)** Determine values of R_1 and R_2 in the circuit of figure such that $V_R(0^+) = 10$ V and $V_R(1 \text{ msec}) = 5$ V.



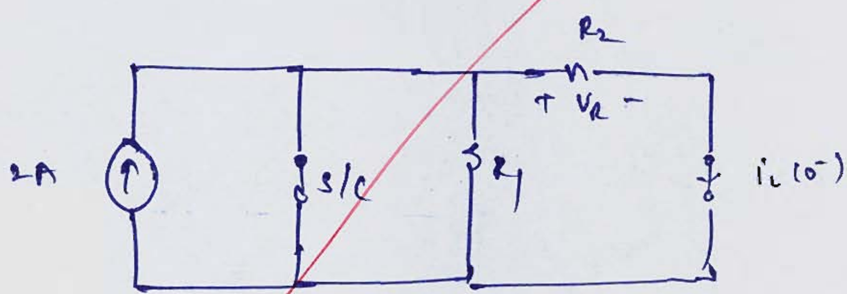
[8 marks]

Sol^n:-

for $\underline{t < 0}$:-

$$i_L(0^-) = \frac{2 \cdot R_1}{R_1 + R_2}$$

for $\underline{t > 0}$



$i_L(t \to \infty) = 0$ A.

$$\therefore \quad i_L(t) = \frac{2 \cdot R_1}{R_1 + R_2} e^{-t/\tau} \quad ; \quad \tau = L/R$$

$$\tau = \frac{20 \times 10^{-3}}{R_2}$$

$$i_L(t) = \frac{2R_1}{R_1 + R_2} e^{-\frac{R_2 t}{20 \times 10^{-3}}}$$

at $t = 0^+$:-

$$V_R(0^+) = 10\,V = \frac{2R_1R_2}{R_1+R_2} = 10$$

$$\boxed{R_1R_2 = 5(R_1+R_2)} \quad ---(1)$$

at $t = 1\,m\,sec.$

$$i_L(t) = i_{R_2}(t) = \frac{2R_1}{R_1+R_2}\,e^{-\frac{R_2\times10^{-3}}{20\times10^{-3}}}$$

$$= \frac{2R_1}{R_1+R_2}\,e^{-\frac{R_2}{20}}$$

8

Good Approach

$$V_R(1\,m\,sec) = \frac{2R_1R_2}{R_1+R_2}\,e^{-\frac{R_2}{20}} = 5$$

$$\Rightarrow \frac{2(5(\cancel{R_1+R_2}))}{\cancel{R_1+R_2}}\,e^{-\frac{R_2}{20}} = 5$$

$$\Rightarrow e^{-\frac{R_2}{20}} = \frac{1}{2}$$

$$-\frac{R_2}{20} = \ln\left(\frac{1}{2}\right)$$

$$-\frac{R_2}{20} = -0.693$$

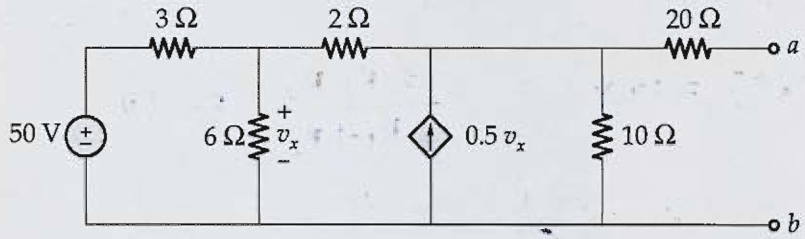
$$\boxed{R_2 = 13.863\,\Omega}$$

by eqn (1)

$$\therefore \quad R_1(13.863) = 5(R_1) + 5\times13.863$$

$$\boxed{R_1 = 7.821\,\Omega}$$

Q.4 (b) Determine the Thevenin's equivalent network and Norton's current at terminals *a-b* for the circuit shown below and draw the two equivalent circuits.

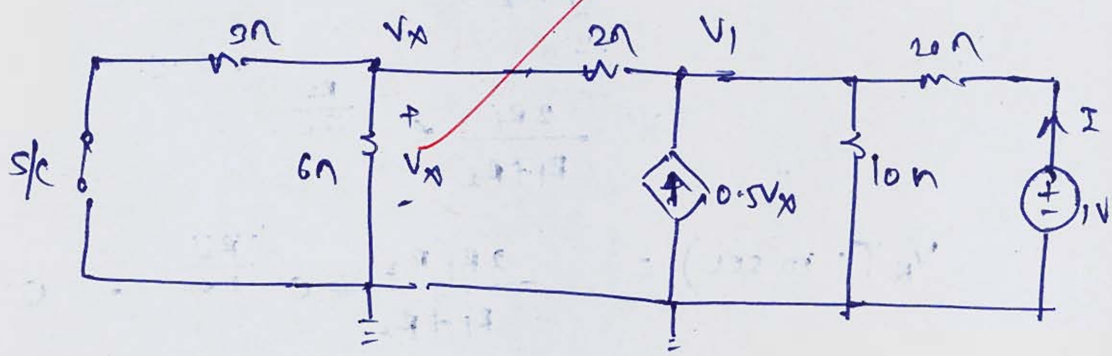


[20 marks]

Solⁿ:-

R_{th} for thevenin & Norton which will be equal for both:-

deactivate all independent sources:-



at node Vx:-

$$\frac{V_x}{3} + \frac{V_x}{6} + \frac{V_x - V_1}{2} = 0$$

$$2V_x + V_x + 3V_x - 3V_1 = 0$$

$$6V_x - 3V_1 = 0$$

$$\boxed{V_x = \frac{1}{2} V_1}$$

at node V_1:-

$$\frac{V_1 - 1}{20} + \frac{V_1}{10} + \frac{V_1 - V_x}{2} = 0.5\,V_x$$

$$\frac{V_1 - 1}{20} + \frac{V_1}{10} + \frac{V_1}{4} = 0.5\left(\frac{1}{2}V_1\right)$$

$$V_1\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{4} - \frac{5}{2}\right) = \frac{1}{20}$$

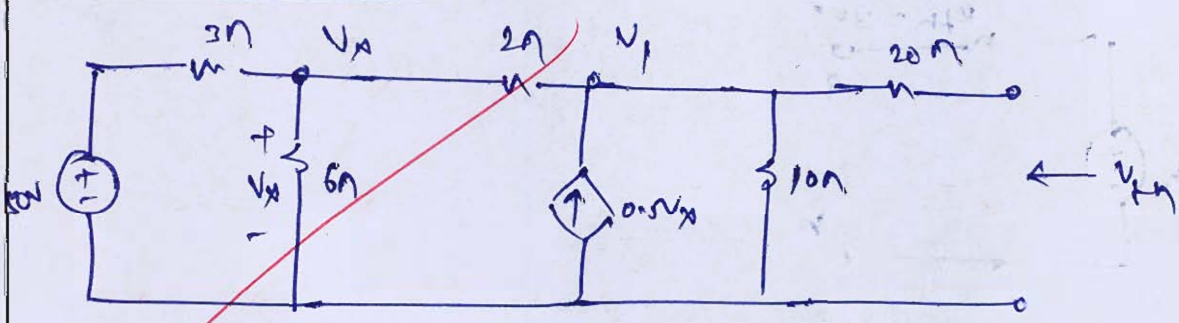
$$V_1 \cancel{\times}\left(\frac{3}{20}\right) = \frac{1}{20}$$

$$\boxed{V_1 = \frac{1}{3}\,V}$$

$$\therefore \quad I = \frac{1 - 1/3}{20} \Rightarrow \frac{2/3}{20} = \frac{1}{30}$$

$$\therefore \quad \boxed{R_{th} = \frac{V}{I} = \frac{1}{1/30} = 30\,\Omega}$$

thevenin voltage:-



$$\frac{V_x - 50}{3} + \frac{V_x}{6} + \frac{V_x - V_1}{2} = 0$$

$$2V_x - 100 + V_x + 3V_x - 3V_1 = 0$$

$$\boxed{6V_x - 3V_1 = 100}$$

$$\frac{V_1 - V_x}{2} + \frac{V_1}{10} = 0.5\,V_x$$

$$\frac{V_1}{2} + \frac{V_1}{10} = (0.5 + 0.5)\,V_x$$

$$\frac{6V_1}{10} = V_x$$

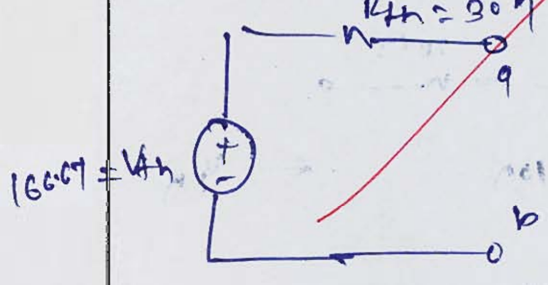
$$\frac{6V_1}{10} = \frac{100 + 3V_1}{6}$$

$$36V_1 = 1000 + 30V_1$$

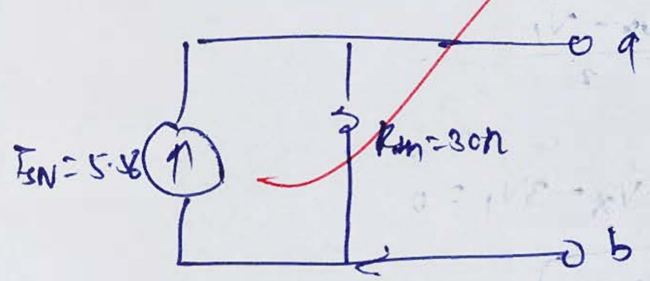
$$6V_1 = 1000 \Rightarrow V_1 = \frac{1000}{6} = \underline{166.67\,V}$$

$$I_{SN} = \frac{V_{th}}{R_{th}} = \frac{166.67}{30} = \underline{5.56}\ A.$$

<u>Thevenin</u> diagram



Norton diagram :-



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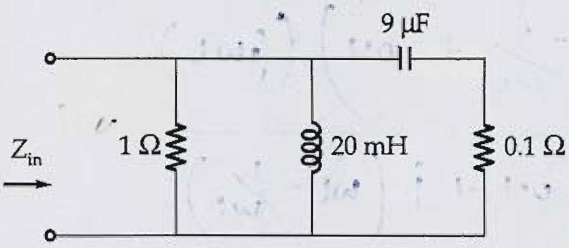
Good Approach

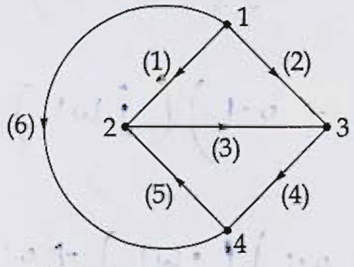
Q.4 (c) (i) For the circuit shown in the figure below:



Determine:

1. The resonant frequency, ω_0.
2. Input impedance at resonant frequency, $Z_{in}(\omega_0)$.

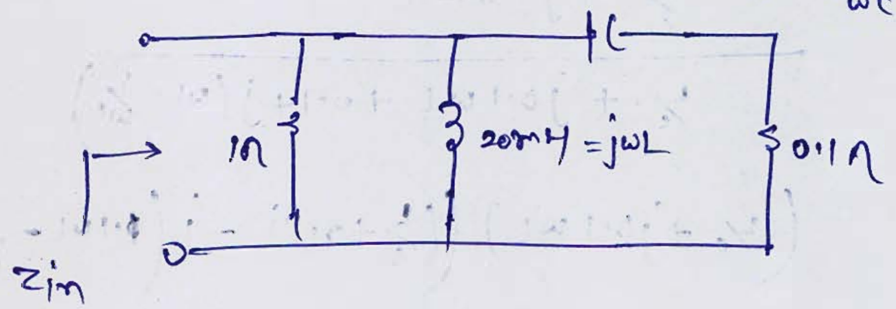
(ii) For the graph shown below:



Find the number of possible trees.

[15 + 5 marks]

$9\mu F \Rightarrow \frac{-j}{\omega C}$



$$Z_1 = \frac{-j}{\omega C} + 0.1$$

$$Z_2 = j\omega L$$

$$Z_{in} = Z_1 \,||\, Z_2 \,||\, 1\Omega$$

$$Z_{in} = \left[\frac{\left(\frac{-j}{\omega C} + 0.1\right)(j\omega L)}{0.1 + j\left(\omega L - \frac{1}{\omega C}\right)}\right] \,||\, 1\Omega$$

$$Z_{in} = \frac{\dfrac{\left(\frac{-j}{\omega C} + 0.1\right)(j\omega L)}{0.1 + j\left(\omega L - \frac{1}{\omega C}\right)} \times 1}{\dfrac{\left(\frac{-j}{\omega C} + 0.1\right)(j\omega L)}{0.1 + j\left(\omega L - \frac{1}{\omega C}\right)} + 1}$$

$$Z_{in} = \frac{\left(\frac{-j}{\omega C} + 0.1\right)(j\omega L)}{\left(\frac{-j}{\omega C} + 0.1\right)(j\omega L) + 0.1 + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$Z_{in} = \frac{\frac{L}{C} + j0.1\omega L}{\frac{L}{C} + j0.1\omega L + 0.1 + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$Z_{in} = \frac{\left(\frac{L}{C} + j0.1\omega L\right)\left(\left(\frac{L}{C} + 0.1\right) - j\left(1.1\omega L - \frac{1}{\omega C}\right)\right)}{\left[\left(\frac{L}{C} + 0.1\right) + j\left(1.1\omega L - \frac{1}{\omega C}\right)\right]\left[\frac{L}{C} + 0.1 - j\left(1.1\omega L - \frac{1}{\omega C}\right)\right]}$$

for resonant freq^n imaginary part should be zero :-

$$\Rightarrow \frac{L}{C}\left(1.1\omega L - \frac{1}{\omega C}\right) = 0.1\omega L\left(\frac{L}{C} + 0.1\right)$$

$$\Rightarrow \frac{1}{c}\left(1.1\omega L - \frac{1}{\omega c}\right) = 0.1\omega\left(\frac{L}{c} + 0.1\right)$$

$$\Rightarrow 1.1\omega L/c - \frac{1}{\omega c^2} = 0.1\omega L/c + .01\omega$$

$$\Rightarrow \omega L/c - \frac{1}{\omega c^2} - 0.01\omega = 0$$

$$\omega^2 Lc - 1 - 0.01\omega^2 c^2 = 0$$

$$\omega^2(Lc - 0.01c^2) = 1$$

$$\omega = \frac{1}{\sqrt{Lc - 0.01c^2}}$$

put value $L = 20mH$

$C = 9\mu f$

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$$\omega = \frac{1}{\sqrt{180\times10^{-9} - 81\times10^{-14}}}$$

$$\boxed{\omega_0 = 2357.03 \text{ rad/sec}}$$

impedance at resonant freq^n:-

Section B : Engineering Mathematics

Q.5 (a) Find the solution of the differential equation $(y - x + 1)dy - (y + x + 2)dx = 0$.

[12 marks]

Sol^n:-

$$\underbrace{(y-x+1)}_{f_1}\,dy - \underbrace{(y+x+2)}_{f_2}\,dx = 0$$

$$\frac{\partial f_1}{\partial x} = -1 \;,\quad \frac{\partial f_2}{\partial y} = -1$$

$$\boxed{\frac{\partial f_1}{\partial x} = \frac{\partial f_2}{\partial y}} \longrightarrow \text{Exact differential equation}$$

$\therefore$

solution for given equation:- 2

$$\int f_2 \cdot dx + \int f_1 \cdot dy = c$$

$\hookrightarrow$ terms free from x

$$\Rightarrow -\int (y+x+2)\cdot dx + \int (y+1)\,dy = c$$

$$\Rightarrow -xy - x^2/2 - 2x + y^2/2 + y = c$$

$$\Rightarrow \boxed{\left(\frac{x^2-y^2}{2}\right) + \cancel{xy} + 2x - y = c}$$

solution

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Q.5 (b) Find the value of $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

[12 marks]

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Q.5 (c) (i) The vector field $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ is defined over the volume of the cuboid given by $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. Evaluate the surface integral $\iint_S \vec{F}\cdot\vec{ds}$, where S is the surface of the cuboid.

[6 marks]

Sol^n :-

$$\iint \vec{F}\cdot\vec{ds} = \iiint (\vec{\nabla}\cdot\vec{F})\, dV$$

$$\vec{\nabla}\cdot\vec{F} = (2x + y)$$

$$\therefore \iint \vec{F}\cdot\vec{ds} = \iiint (2x+y)\, dx\, dy\, dz$$

$$= \int_{z=0}^{c}\int_{y=0}^{b}\int_{x=0}^{a} (2x+y)\, dx\, dy\, dz$$

$$= \int_0^c\int_0^b\int_0^a (x^2 + xy)\, dy\, dz$$

$$= \int_0^c\int_0^b \left[x^2 + xy\right]_0^a dy\, dz$$

$$= \int_0^c\int_0^b (a^2 + ay)\, dy\, dz$$

$$= \int_0^c \left[a^2y + \frac{ay^2}{2}\right]_0^b = \int_0^c \left(a^2b + \frac{ab^2}{2}\right) dz$$

$$= \boxed{c\left(a^2b + \frac{ab^2}{2}\right)}$$ Ans.

6

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Q.5 (c) (ii) Find the absolute maxima and minima of

1. $f(x) = 2x^3 - 9x^2 + 12x - 5$ in $[0, 3]$
2. $f(x) = 12x^{4/3} - 6x^{1/3}, x \in [-1, 1]$

Also, find points of maxima and minima.

[6 marks]

Sol^n: 1. $f(x) = 2x^3 - 9x^2 + 12x - 5$ in $[0, 3]$

$\frac{\partial f}{\partial x} = 6x^2 - 18x + 12$

$= x^2 - 3x + 2$

$\frac{\partial f}{\partial x} \Rightarrow 0 \Rightarrow x = 1, 2$

$\frac{\partial^2 f}{\partial x^2} = 2x - 3$

$\left.\frac{\partial^2 f}{\partial x^2}\right|_{at\ x=1} \Rightarrow$ -ve point of maxima

$\left.\frac{\partial^2 f}{\partial x^2}\right|_{at\ x=2} \Rightarrow$ +ve point of minima :-

$f(0) = -5,$

$f(1) = 2 - 9 + 12 - 5 = 0$

$f(2) = 16 - 36 + 24 - 5 = -1$

$f(3) = 54 - 81 + 36 - 5 = +4$

$\therefore$ absolute maxima at $x = 3$ and value is $+4$

absolute minima at $f(0) = \underline{\underline{-5}}$

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Solⁿ: 2. $f(x) = 12x^{4/3} - 6x^{1/3}$; $x \in [-1, 1]$

$$f'(x) = 12 \cdot \frac{4}{3} x^{1/3} - 6 \times \frac{1}{3} \cdot x^{-2/3}$$

$$f'(x) = 0 \Rightarrow 16x^{1/3} - 2x^{-2/3} = 0$$

$$16x - 2 = 0$$

$$x = 1/8$$

$$f''(x) = 16 \cdot \frac{1}{3} x^{-2/3} + 4/3 \, x^{-5/3}$$

$$= \frac{16}{3x^{2/3}} + \frac{4}{3}(x)^{-5/3}$$

$f''(x)_{\text{at } x = 1/8} = 64$ (+ve) point of minima.
↳ relative min.

$$f(-1) = 12(-1)^{4/3} - 6(-1)^{1/3}$$

$$= 18$$

6

$$f(1) = 12(1)^{4/3} - 6(1)^{4/3}$$

$$= 12 - 6 = 6$$

∴ absolute minimal at $x = 1$ & value is 6.

absolute maxima at $x = 1/8$

$f(x) = 64.$

Q.5 (d) Determine the values of x for which the following function fails to be continuous or differentiable.

$$f(x)=\begin{cases} 1-x & , \ x<1 \\ (1-x)(2-x), & 1\le x\le 2 \\ 3-x & , \ x>2 \end{cases}$$

[12 marks]

Sol^n :-

value at $x=1^- \Rightarrow 0$

at $x=1^+ \Rightarrow 0$

$\therefore$ continuous at point $x=1$

value at $x=2^- \Rightarrow 0$

$x=2^+ = 1$

$x=2^- \neq x=2^+$ $\therefore$ not continuous at point $x=2$

for differtiability :-

at $x=1^-$

$\frac{d}{dx}(1-x) = -1$

$\frac{d}{dx}(1-x)(2-x) \Rightarrow -1(2-x) + (1-x)$

$= x-2+x-1$

$= 2x-3$

at $x=1^+ = 2(1)-3 = -1$

$\therefore$ f(x) is not differentiable at $x=1$ & $x=2$

& f(x) is not continuous at $x=2$ while continuous at $x=1$

11

Good Approach

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Q.5 (e) X is a continuous random variable with probability density function given by

$$f(x) = kx\,(0 \le x \le 2)$$
$$= 2k\,(2 \le x < 4)$$
$$= -kx + 6k\,(4 \le x < 6)$$

Find k and mean value of X.

[12 marks]

Sol^n:-

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$ to satisfy probability density function

$$\Rightarrow \int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 -kx+6k = 1$$

$$\Rightarrow \left[\frac{kx^2}{2}\right]_0^2 + (2kx)_2^4 + \left(-\frac{kx^2}{2} + 6kx\right)_4^6 = 1$$

$$2k + 2k\cdot(4-2) + -\frac{k}{2}\,(36-16) + 6k\,(2) = 1$$

$$2k + 4k + -\frac{k}{2}\,(20) + 12k = 1$$

$$6k - 10k + 12k = 1$$

$$8k = 1$$

$$\boxed{k = \frac{1}{8}}$$

mean value of X:-

$$E(X) = \int_{-\infty}^{\infty} x\,f(x)\cdot dx$$

$$\therefore \quad E(x) = \int_0^2 kx^2 \cdot dx + \int_2^4 2kx \cdot dx + \int_4^6 -kx^2 + 6kx \cdot dx$$

$$E(x) = \left(kx^3/3\right)_0^2 + \left(kx^2\right)_2^4 + \left(-kx^3/3 + 3kx^2\right)_4^6$$

$$E(x) = k \cdot 8/3 + k(16-4) + k\left(3x^2 - x^3/3\right)_4^6$$

$$E(x) = k \cdot 8/3 + 12k + k\left[3(36-16) - \frac{1}{3}(216-64)\right]$$

$$E(x) = \frac{8k}{3} + 12k + k\left[60 - \frac{152}{3}\right]$$

$$E(x) = \frac{8k}{3} + 12k + k\left[\frac{28}{3}\right]$$

$$= \frac{8k + 36k + 28k}{3}$$

$$= \frac{72k}{3} = 24k$$

$$E(x) = 24k = 24 \cdot \frac{1}{8} = \underline{3}$$

11

Good Approach

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Q.6 (a) (i) State Langrange's mean value theorem and explain the theorem in reference to it's geometrical significance.

(ii) Find the complete solution of $y^2 - 2y' + 2y = x + e^x \cos x$.

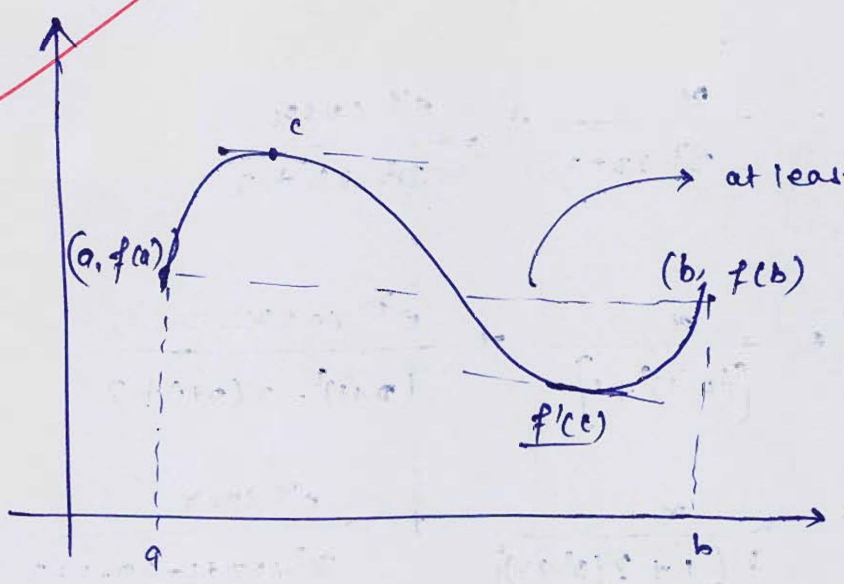
(iii) Prove that the matrix, $A = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$ is unitary and find A^{-1}.

[6 + 6 + 8 marks]

Q. (i) Langrange's mean value theorem:-

If any function $f(x)$ is continuous in $x \in [a, b]$ and differentiable in interval $x \in (a, b)$ then there will be a point $c \in (a, b)$, at which

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$



at least there will be a point c which slope is equal to slope of this line $= \frac{f(b) - f(a)}{b - a}$

soln:- (ii) $y^2 - 2y' + 2y = x + e^x \cos x$

for C.F.:-

$$(m^2 - 2m + 2)y = 0$$

$$(m^2 - 2m + 2) = 0$$

$$(m-1)^2 + 1 = 0$$

$$m = 1 \pm j1$$

$\therefore$ ~~$CF = e^x (C_1 \cos p$~~

$$\boxed{C.F. = e^x (C_1 \cos x + C_2 \sin x)}$$

for P.I.:-

$$(D^2 - 2D + 2)y = x + e^x \cos x$$

$$= \frac{x}{D^2 - 2D + 2} + \frac{e^x \cos x}{D^2 - 2D + 2}$$

$$= \frac{x}{[(D-1)^2 + 1]} + \frac{e^x \cos x}{(D+1)^2 - 2(D+1) + 2}$$

$$= \frac{x}{2\left[1 + \frac{1}{2}(D^2 - 2D)\right]} + \frac{e^x \cos x}{D^2 + 2D + 1 - 2D - 2 + 2}$$

$$= \frac{x}{2}\left[1 + \frac{1}{2}(D^2 - 2D)\right]^{-1} + \frac{e^x \cos x}{D^2 + 1}$$

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$$\Rightarrow \quad \frac{x}{2}\left[1-\frac{1}{2}(D^2-2D)+\cdots\right]+\frac{e^x\cos x}{D^2+1}$$

$D^2 = -\alpha^2 \rightarrow$ condition fail

$$\Rightarrow \quad \frac{x}{2}-\frac{1}{4}\left[\frac{d^2}{dx^2}(x)-2\cdot\frac{d}{dx}(x)\right]+\frac{e^x \cdot x\cos x}{2D}$$

$$\Rightarrow \quad \frac{x}{2}-\frac{1}{4}\left[0-2\right]+\frac{e^x \cdot x\cdot(-\sin x)}{2D^2}$$

$$\Rightarrow \quad \frac{x}{2}+\frac{1}{2}+\frac{e^x \cdot x\,(-\sin x)}{2(-1)}$$

$$\Rightarrow \quad \frac{x+1}{2}+\frac{e^x\cdot x\sin x}{2}$$

$$\boxed{P.I = \frac{x+1+e^x\cdot x\sin x}{2}}$$

$\therefore$ complete soln! -

$$y(x) \Rightarrow e^x\left(c_1\cos x + c_2\sin x\right)+\frac{x+1+e^x\cdot x\sin x}{2}$$

(iii) <u>for unitary matrix</u> :-

$$A A^\theta = I$$

$$\therefore \quad A^\theta = (\bar{A})^T = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \\ \frac{1}{2}(-1-i) & \frac{1}{2}(1+i) \end{bmatrix}$$

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$$\Rightarrow A A^{\theta} = \begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix} \begin{bmatrix} \frac{1}{2}(1-i) & \frac{1}{2}(1-i) \\ \frac{1}{2}(-1-i) & \frac{1}{2}(1+i) \end{bmatrix}$$

$$AA^{\theta} = \begin{bmatrix} \frac{1}{4}(1+i)(1-i) + \frac{1}{4}(-1+i)(-1-i) & \frac{1}{4}(1+i)(1-i) + \frac{1}{4}(1+i)(-1+i) \\ \frac{1}{4}(1+i)(1-i) + \frac{1}{4}(1-i)(-1+i) & \frac{1}{4}(1+i)(1-i) + \frac{1}{4}(1-i)(1+i) \end{bmatrix}$$

$$AA^{\theta} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore$ A is a unitary matrix.

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$adj(A) = \begin{bmatrix} \frac{1}{2}(1-i) & -\frac{1}{2}(-1+i) \\ -\frac{1}{2}(1+i) & \frac{1}{2}(1+i) \end{bmatrix}$$

$$|A| = \frac{1}{4}(1-i)(1+i) - \frac{1}{4}[(-1+i)(1+i)]$$

$$= \frac{1}{4}[2] - \frac{1}{4}[-2] = 1$$

$$A^{-1} = \frac{adj(A)}{|A|} = \begin{bmatrix} \frac{1}{2}(1-i) & -\frac{1}{2}(-1+i) \\ -\frac{1}{2}(1+i) & \frac{1}{2}(1+i) \end{bmatrix}$$

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Good Approach

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Q.6 (b) (i) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ show that $A^2 - 4A - 5I = 0$ where I, 0 are the identity matrix and the null matrix of order 3 respectively. Use this result to find A^{-1}.

[10 marks]

Ans:

for characteristic equation:-

$$|A - \lambda I| = 0$$

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 & 2 \\ 2 & 1-\lambda & 2 \\ 2 & 2 & 1-\lambda \end{bmatrix} = (1-\lambda)\left[(1-\lambda)^2 - 4\right] - 2\left[2(1-\lambda) - 4\right] + 2\left[4 - 2(1-\lambda)\right]$$

$$\Rightarrow (1-\lambda)\left[\lambda^2 - 2\lambda - 3\right] - 2\left[2 - 2\lambda - 4\right] + 2\left[4 - 2 + 2\lambda\right]$$

$$\Rightarrow (1-\lambda)(\lambda^2 - 2\lambda - 3) - 2\left[-2 - 2\lambda\right] + 2\left(2 + 2\lambda\right)$$

$$\Rightarrow \lambda^2 - 2\lambda - 3 - \lambda^3 + 2\lambda^2 + 3\lambda + 4 + 4\lambda + 4 + 4\lambda = 0$$

$$\Rightarrow -\lambda^3 + 3\lambda^2 + 9\lambda + 5 = 0$$

$$\Rightarrow \lambda^3 - 3\lambda^2 - 9\lambda - 5 = 0$$

$$\lambda^3 - 3\lambda^2 - 4\lambda - 5\lambda - 5 + \lambda^2 - \lambda^2 = 0$$

$$\lambda^3 - 4\lambda^2 - 5\lambda + \lambda^2 - 4\lambda - 5 = 0$$

$$\lambda(\lambda^2 - 4\lambda - 5) + 1(\lambda^2 - 4\lambda - 5) = 0$$

$$(\lambda + 1)(\lambda^2 - 4\lambda - 5) = 0$$

$\lambda^2 - 4\lambda + 5 = 0$

matrix A will satisfy this characteristic equation:-

$$\boxed{A^2 - 4A + 5I = 0}$$

by using this:- A^{-1}:-

$$\lambda^2 - 4\lambda - 5 = 0$$

$$A^2 - 4A - 5I = 0$$

$$A - 4 - 5A^{-1} = 0$$

$$A^{-1} = \frac{1}{5}[A - 4I]$$

$$A^{-1} = \frac{1}{5}\left\{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\}$$

$$\boxed{A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}}$$

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.6 (b) **(ii)** Examine the following vectors for linear dependence and find the relation if it exists.

$X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$ and $X_4 = (-3, 7, 2)$.

[10 marks]

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.6 (c) (i) Solve: $(D^2 - 4D + 4)\, y = 8x^2 e^{2x} \sin 2x$.

(ii) Find the regression line of y on x for the following data and estimate the value of y, when $x = 10$. (Use the least square approximation method)

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

[12 + 8 marks]

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.7 (a) If the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$, then find all possible value of m.

[20 marks]

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Q.7 (b) (i) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

(ii) Find the real root of the following equation, correct to three decimal places. (Using Newton-Raphson method)

$$x^3 - 2x - 5 = 0$$

[12 + 8 marks]

Q.7 (c) Show that the vector field $\vec{F} = \dfrac{\vec{r}}{|\vec{r}|^3}$ is irrotational as well as solenoidal. Find the scalar potential.

[20 marks]

(a) Sixteen players $S_1, S_2, \ldots.. S_{16}$ play in a tournament. They are divided into eight pairs at random, from each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

Find:

(i) The probability that the player S_1 is among the eight winners.

(ii) The probability that exactly one of the two players S_1 and S_2 is among the eight winners.

[20 marks]

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Q.8 (b) (i) Assuming that the diameters of 1000 brass plugs taken consecutively from a machine from a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm, how many number of brass plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm? (Given, Area[$Z = -1.75$] = 0.4599 and Area[$Z = 2.25$] = 0.4878).

(ii) A periodic function of time period 4 is defined as $f(x) = |x|$, $-2 < x < 2$. Find its Fourier series expansion.

[8 + 12 marks]

Q.8 (c) Apply Runge-Kutta 4th order method to find an approximate value of y when $x = 0.2$. Given that $\frac{dy}{dx} = x + y$, $y = 1$ when $x = 0$. (Take step size of 0.1)

[20 marks]

oooo

Space for Rough Work

$(1+x)^{-1}$

$\Rightarrow (1-x)^{-1}$

$= 1 + x + x^2 + x^3$

$1 - x + x^2 - x^3$

$2 + (i+1) + (i-1)$

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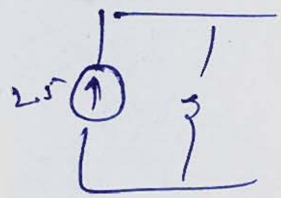
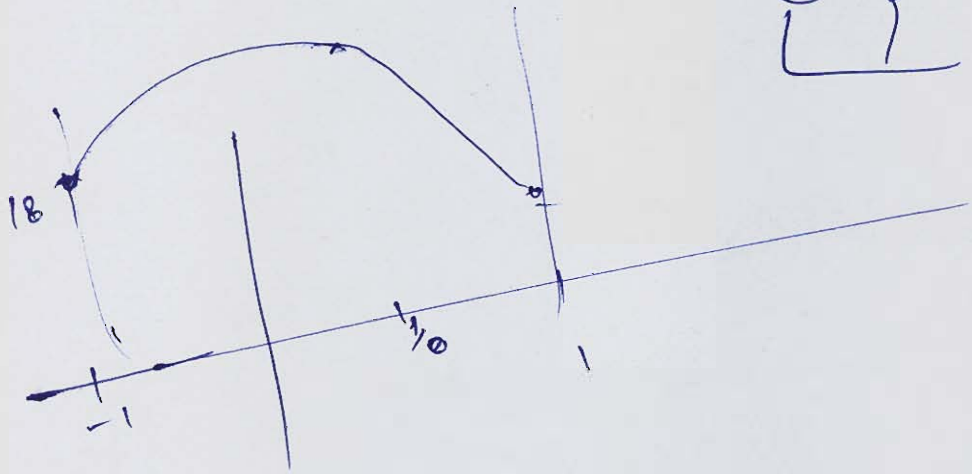
Space for Rough Work





$x^2+1 + 1-x$

$\frac{\cancel{3}\cdot 4\cdot 2\cancel{0}}{8\cancel{0}}$

12 22

$\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\,dx$

$x^2-2x+x+2$

$x(x-2)-1(x$

$(x-1)^2+1+x$

x^2-2x+1

$(x-1)^2+$

$x^4+6x^2+4+6x^2+5$

$x^4+9x^2+x^2+9$

$x^2(x^2+9)+1(x^2+9)$

$(x^2+1)(x^2+9)$

$i = C\frac{dv}{dt}$

$I = sC\,V(s) - CV_0$

$\frac{I}{sC} + \frac{V_0}{s}$

$\frac{1}{x^2+9} + \frac{1-x}{(x^2+9)(x}$

$\frac{-1 \pm \sqrt{1+4}}{2}$

$\frac{-1 \pm \sqrt{5}}{2}$