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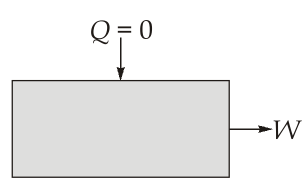
Detailed Solutions

SSC-JE 2018 Mains Test Series (PAPER-II)

Mechanical Engineering Test No : 3

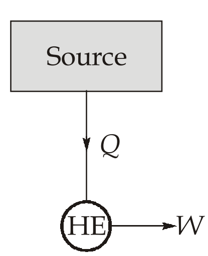
Q.1 (a) Solution:

PMM1: An imaginary device which would produce work continuously without absorbing any energy from its surrounding is called a perpetual motion machine of first kind (PMM1)



PMM2: Second law of thermodynamics restrict the thermal efficiency of a heat engine to less than one. It stipulates that some portion of the energy absorbs as heat from a source must always be rejected to a low temperature sink.

So, PMM2 violates this second law of thermodynamics. It is a device which would perform solely by absorbing energy as heat from a body without rejecting any heat to low temperature.

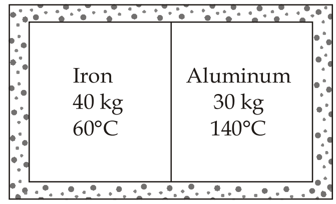


PMM3: It is impossible to construct a device which runs completely in the absence of friction.

Q.1 (b) Solution:

Here the system is well-insulated and hence there is no heat transfer.

- As the system is stationary, so KE and PE are neglected.



Energy balance for this system can be expressed as:

Ne energy transfer by heat, work and mass = change in internal, kE, PE etc.

$$E_{in} - E_{out} = \Delta U$$

$$\Delta U = 0, E_{out} = 0 \qquad [\Delta U = m_c(T_2 - T_1)_{Fe}]$$

$$30 \times 0.949(T_2 - 140) + 40 \times 0.45(T_2 - 60) = 0$$

$$46.47\ T_2 = 28.47 \times 140 + 18 \times 60$$

$$T_2 = 109°C \text{ or } 382 \text{ K} \qquad \text{Answer}$$

Total entropy change for this process can be determined as below:

$$\Delta S_{iron} = mc_{avg} \ln\left(\frac{T_2}{T_1}\right) = 40 \times 0.45 \ln\left(\frac{382}{333}\right) = 2.472 \text{ kJ/K}$$

$$\Delta S_{aluminium} = 30 \times 0.949 \ln\left(\frac{382}{413}\right) = -2.221 \text{ kJ/K}$$

$$\Delta S_{total} = \Delta S_{iron} + \Delta S_{aluminium}$$

$$= 2.472 - 2.221 = 0.251 \text{ kJ/K} \qquad \text{Answer}$$

Q.1 (c) Solution:

$$T_2 = 127°C = 273 + 127 = 400 \text{ K}$$

$$T_1 = 17°C = 273 + 17 = 290 \text{ K}$$

$$T_0 = 7°C = 273 + 7 = 280 \text{ K}$$

$$\text{Work, } w = (h_2 - h_1) - T_0(S_2 - S_1) + \frac{V_2^2 - V_1^2}{2000}$$

$$= mc_p(T_2 - T_1) - mTo\left[c_p \ln\frac{T_2}{T_1} - R\frac{P_2}{P_1}\right] + \frac{V_2^2 - V_1^2}{2000}$$

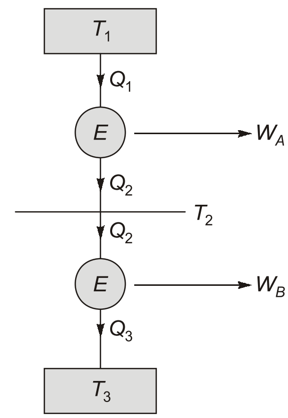
$$= 1.005(400 - 290) - 280\left[1.005 \ln\frac{400}{290} - 0.287 \ln\frac{350}{140}\right] + \frac{110^2 - 70^2}{2000}$$

$$= 110.55 - 16.86 + 3.6 = 97.29 \text{ kJ/kg}$$

$$W_{actual} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000} = 110.55 + 3.6 = 114.15 \text{ kJ}$$

$$\text{Irreversibility, } I = T_0(S_2 - S_1) = 16.86 \text{ kJ/kg}$$

Q.1 (d) Solution:



$$\eta_A = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$W_A = Q_1 - Q_2$$

$$\eta_B = \frac{Q_2 - Q_3}{Q_2} = \frac{T_2 - T_3}{T_2}$$

$$W_B = Q_2 - Q_3$$

1. When $\eta_A = \eta_B$

$\therefore$ $$\frac{T_1 - T_2}{T_1} = \frac{T_2 - T_3}{T_2}$$

or $$T_1T_2 - T_2^2 = T_1T_2 - T_1T_3$$

or $$T_2^2 = T_1T_3 \text{ or } T_2 = \sqrt{T_1T_3} \text{ expression (i)}$$

2. $$W_A = W_B$$

or $$Q_1 - Q_2 = Q_2 - Q_3$$

or $$2Q_2 = Q_1 - Q_3$$

$$Q_2 = \frac{Q_1 + Q_3}{2}$$

For reversible process using temperature scale,

$$2 = \frac{Q_1}{Q_2} + \frac{Q_3}{Q_2}$$

or $$2 = \frac{T_1}{T_2} + \frac{T_3}{T_2}$$

$\Rightarrow$ $$T_2 = \frac{T_1 + T_3}{2} \qquad \text{Expression (ii)}$$

Q.2 (a) Solution:

S.I. engine:

1. **Engine Speed**: Increase in the engine speed increase the turbulence of the mixture resulting in increased flame speed and the time available for end-gas reaction decreases. Hence, the tendency to knock is decreased at higher speeds.

Engine speed: $\Uparrow$	Tendency to knock: $\Downarrow$

2. **Compression ratio**: Increasing the compression ratio, increases both the temperature and pressure. Increase in temperature reduces the delay period. Hence, the tendency to knock is increased with compression ratio.

Compression ratio: $\Uparrow$	Tendency to knock: $\Uparrow$

3. **Jacket temperature**: The effect of jacket temperature and inlet temperature are the same. Decrease the heat loss from the burned gases and therefore the end-gas temperature will increase (also delay period decreases) and so the knocking tendency will increase.

Jacket temperature: $\Uparrow$	Tendency to knock: $\Uparrow$

4. **Load**: An increase in the load, increases the temperature of cylinder resulting mixture and end gas temperatures. Pressure of the end gas is increased also. Hence, the tendency to knock increases.

Load : $\Uparrow$	Tendency to knock: $\Uparrow$

5. **Inlet pressure**: If the inlet pressure is increased, the end gas pressure and temperature will also increase. The high pressure and temperature will reduce the delay period and increase the tendency to knock.

Inlet pressure : $\Uparrow$	Tendency to knock: $\Uparrow$

Q.2 (b) Solution:

The function of condenser in a refrigeration system is to de-superheat and condense the compressor discharge vapour and frequently to subcool the resultant liquid while introducing a minimum pressure drop.

Condensers are of three types.

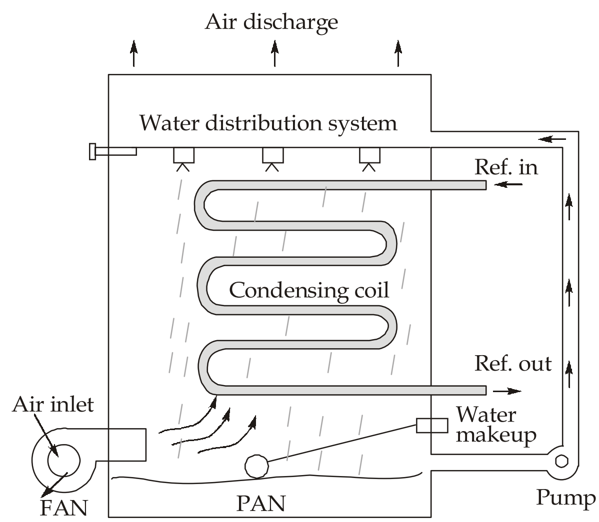
i) Water cooled condenser: Water cooled condensers are classified into three types:
   a. Shell and tube condensers
   b. Shell and coil condenser
   c. Double pipe condenser

ii) Air cooled condenser

iii) Evaporative condenser

Evaporative condenser: Evaporative condenser comprises of a coil in which the refrigerant is flowing and condensing inside, and its outer surface is wetted with water, exposed to stream of air to which heat is rejected principally by evaporation of water.

Description: The coils are generally made of copper or steel in multiple circuits and passes. The external surfaces are sometimes finned to increase heat transfer. The coil should have arrangements for cleaning under fouling water condition.



The wetting of coil is done by re-circulating system comprising water pan, a pump and water distribution system. The water distribution system mainly comprises nozzles for spray of atomized water on the coils. The pan catches the drainage of all coils. There is a float valve to admit make up water and maintain the correct level in the pan. Centrifugal pumps of moderate head are necessary. Such pumps are not affected much by extranous matter found in such open re-circulating systems.

Most evaporative condensers employ forced circulation of air with a fan to either blow or to draw air through the unit. Effective elimination of moisture from the leaving air stream by eliminators is essential to prevent projection of mist which can deposit moisture on the surrounding surfaces. The eliminator plates work on the simple principle of abrupt changes in flow direction. Moisture particles being heavier get deposited on these eliminator plates and get drained back to the sump or the pan.

Q.2 (c) Solution:

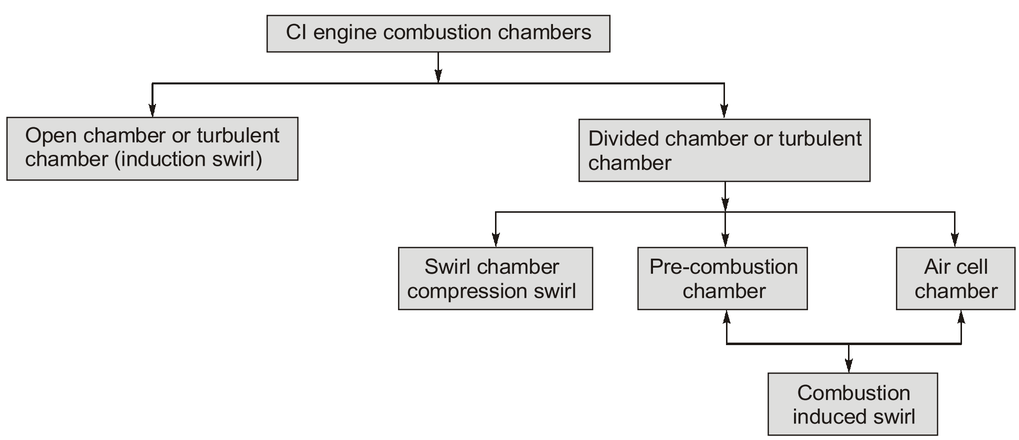
Advantages of open combustion chamber:

1. The high excess air allows lower average combustion chamber temperatures. This coupled with low turbulence and low heat losses permits engine indicated thermal efficiency to approach the ideal air cycle efficiency.

2. The intensity of swirl is low. Therefore, the heat loss to the chamber walls is relatively low, resulting in easier cold starting.
3. No additional work is done in producing swirl as the swirl is generated by induction stroke. This coupled with high indicated thermal efficiency means high brake thermal efficiency and therefore lower fuel combustion.

Disadvantages of open combustion chamber:

1. As the swirl induced by induction is generally weak in intensity, so multi-orifice nozzle with high injection pressure are required. The small nozzle openings are more frequently clogged or cause change of fuel spray pattern by carbon deposits with consequent higher maintainance cost.
2. The use of shrouded valve lowers the volumetric efficiency.
3. Weak swirl necessitates excess air i.e. low air utilization which results in lower mean effective pressure and hence large size engine for a given power.
4. One of the major disadvantage of induction swirl is that it is not proportional to speed and hence efficiency is not maintained over a wide range in a variable speed engine.



Q.2 (d) Solution:

Difference between impulse and reaction turbines.

Impulse turbine	Reaction turbine
i) Pressure drops only in nozzles and not in moving blade channels.	i) Pressure drops in fixed blades (nozzles) as well as in moving blade channels.
ii) Constant blade channel area.	ii) Varying blade channel area (converging types)
iii) Profile type blades.	iii) Aerofoil type blades.
iv) Not all around or complete admission of steam	iv) All around or complete admission of steam.
v) Lesser blade efficiency.	v) Higher blade efficiency.
vi) Diaphragm contains the nozzles.	vi) Fixed blades similar to moving blades attached to casing serve as nozzles and guide the steam.
vii) Occupies less space for same power.	vii) Occupies more space for same power.
viii) Suitable for small power requirements.	viii) Suitable for medium and higher power requirements.

Q.3 (a) Solution:

Compounding is a method for reducing the rotational speed of the impulse turbine to practical limits. If high velocity of steam is allowed to flow through one row of moving blades, it can produce a rotor speed of about 30000 rpm, which is too high for practical use.

It is therefore essential to incorporate some improvements in the simple impulse turbine for practical use and also to achieve high performance. This is possible by making use of more than one set of nozzles, blades, rotors in a series, keyed to a common shaft, so that either the steam pressure or the jet velocity is absorbed by the turbine in stages. The leaving loss will also be less. This process is called compounding of steam turbines.

There are three main types of compounding:

a. Pressure-compounded impulse turbine.

b. Velocity-compounded impulse turbine.

c. Pressure and velocity compounded impulse turbine.

Q.3 (b) Solution:

The continuity equation for incompressible fluid is given by equation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0$$

$$u = x^2 + y^2 + z^2$$

$$\therefore \quad \frac{\partial u}{\partial x} = 2x$$

$$v = xy^2 - yz^2 + xy$$

$$\therefore \qquad \frac{\partial v}{\partial y} = 2xy - z^2 + x$$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation

$$2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$$

or
$$\frac{\partial w}{\partial z} = -3x - 2xy + z^2$$

or
$$\partial w = (-3x - 2xy + z^2)\partial z$$

Integration of both sides gives:

$$\int dw = \int \left(-3x - 2xy + z^2\right)dz$$

or
$$w = \left(-3xz - 2xyz + \frac{z^3}{3}\right) + \text{Constant of integration,}$$

where constant of integration cannot be a function of z, but it can be a function of x and y that is $f(x, y)$.

$$\therefore \qquad w = \left(-3xz - 2xyz + \frac{z^3}{3}\right) + f(x, y)$$

Q.3 (c) Solution:

Priming of a centrifugal pump:

- The operation of filling the suction pipe, casing of the pump and a portion of the delivery pipe completely from outside source with the liquid to be raised, before starting the pump, to remove any air, gas or vapour from these parts of the pump is called priming of a centrifugal pump. If a centrifugal pump is not primed before starting, air pockets inside the impeller may give rise to vortices and cause discontinuity of flow. Further, dry running of the pump may result in rubbing and seizing of the wearing rings and cause serious damage.
- Small pumps are usually primed by pouring liquid into the funnel provided for the purpose. While doing priming, the air vent valve provided in the pump casing is opened, the air escape through the valve. The priming is continued till all air from the suction pipe, impeller and casing has been removed.
- Large pumps are primed by evacuating the casing and the suction pipe by vacuum pump or by an ejector; the liquid is thus drawn up the suction pipe from the sump and the pump is filled with liquid.
- The internal construction of some pumps is such that special arrangements containing a supply of liquid are provided in the suction pipe due to which automatic priming of the pump occurs, such pumps are known as 'self priming pumps'.

Q.3 (d) Solution:

Given data:

P = 8 MW = 8×10^6 W, H = 6 m, K_u = 2.09, K_f = 0.68, η_0 = 90% = 0.90

$$\frac{d}{D} = \frac{1}{3}$$

or $$D = 3d$$

Now, overall efficiency, $$\eta_0 = \frac{\text{Shaft power}}{\text{Water power}} = \frac{P}{\rho Q g H}$$

$$0.90 = \frac{8\times10^6}{1000\times Q\times9.81\times6}$$

or $$Q = 151.01 \text{ m}^3/\text{s}$$

also discharge, $$Q = \frac{\pi}{4}(D^2 - d^2)V_{fi}$$

$$= \frac{\pi}{4}\left((3d)^2 - d^2\right)K_f\sqrt{2gH}$$

$$= 2\pi d^2 \times 0.68 \times \sqrt{2\times9.81\times6}$$

or $$d^2 = 3.259$$

$$d = 1.80 \text{ m}$$

Diameter of the runner, $$D = 3d = 3 \times 1.80 = 5.4 \text{ m}$$

Speed of the runner, $$N = \frac{60K_u\sqrt{2gH}}{\pi D}$$

$$= \frac{60\times2.09\times\sqrt{2\times9.81\times6}}{3.14\times5.4} = 80.24 \text{ rpm}$$

Specific speed of the turbine, $$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

where N in rpm P in kW, H in m. Then N_s in SI unit.

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{80.245\sqrt{8000}}{6^{5/4}} = 764.376 \quad \text{(SI units)}$$

Q.4 (a) Solution:

Effect of Alloying Elements on Steel:

Carbon : Carbon content in steel affects

- Hardness
- Tensile strength
- Machinability
- Melting point

Nickel :

- Increases toughness and resistance to impact
- Acts as graphitizer
- Lowers the critical temperatures of steel and widens the range of successful heat treatment
- Improves fatigue strength.
- Renders high-chromium iron alloys austenitic

Chromium :

- It improves the corrosion resistance of steel.

Silicon :

- Improves oxidation resistance
- Strengthens low alloy steels
- Acts as a deoxidizer.
- Promotes graphitization

Titanium :

- Forms hard and stable carbide and raises creep strength.
- Prevents formation of austenite in high chromium steels
- Reduces martensitic hardness and hardenability in medium chromium steels.

Molybdenum :

- Promotes hardenability of steel
- Makes steel fine grained
- Makes steel unusually tough at various hardness levels
- Counteracts tendency towards temper brittleness
- Raises tensile and creep strength at high temperatures
- Enhances corrosion resistance in stainless steels
- Forms abrasion resisting particles.

Vanadium :

- Promotes fine grains in steel
- Increases hardenability (when dissolved)
- Imparts strength and toughness to heat-treated steel
- Forms carbides and nitrides

Q.4 (b) Solution:

Casting area, $A = 200 \times 200 = 4 \times 10^4\ \text{cm}^2$

Height of mould cavity, $H = 50$ cm

Manometric height, $h_t = 50$ cm

Gating area, $A_g = 5\ \text{cm}^2$

TOP GATE

Time of filling,
$$t_1 = \frac{AH}{A_g\sqrt{2gh_t}}$$

$$t_1 = \frac{4\times10^4 \times 50}{5\times\sqrt{2\times981\times50}} = 1277.1 \text{ sec}$$

BOTTOM GATE

Time of filling,
$$t_1 = \frac{2A}{A_g\sqrt{2g}}\left[\sqrt{h_t} - \sqrt{h_t - H}\right]$$

$$t_2 = \frac{2\times4\times10^4}{5\times\sqrt{1962}}\left[\sqrt{50} - \sqrt{50-50}\right]$$

$$t_2 = 2554.2 \text{ sec.}$$

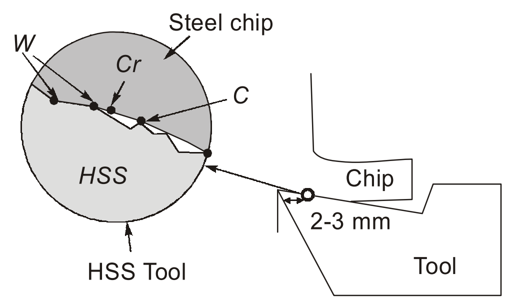
$$\frac{t_2}{t_1} = \frac{2554.2}{1277.1} = \mathbf{2}$$

Time of filling for bottom gate is twice that of top gate. This will be the case when manometric height is equal to the height of casting but in all the other case $t_2 > t_1$.

Q.4 (c) Solution:

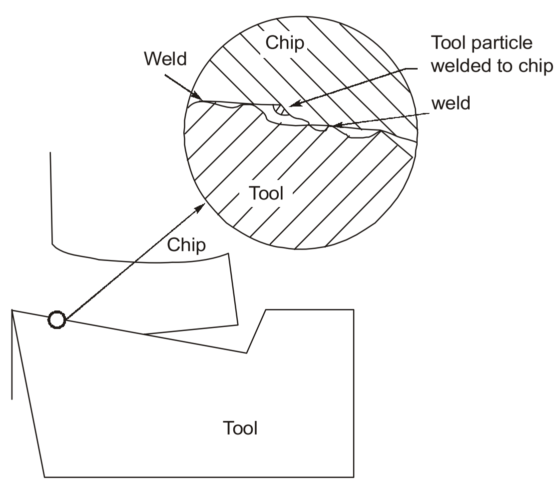
1. **Diffusion wear** : The favourable condition for the diffusion is provided by the localized temperature over the actual area between the chip underside and the tool face. In that condition the metal atoms will transfer from the tool material to the chip material at the points of contact. This weakens the surface structure of the cutting

tool and may ultimately lead to tool failure. The amount of diffusion depends upon temperature, period of contact between tool face and the chip and the bonding affinity between the materials of the tool and chip.



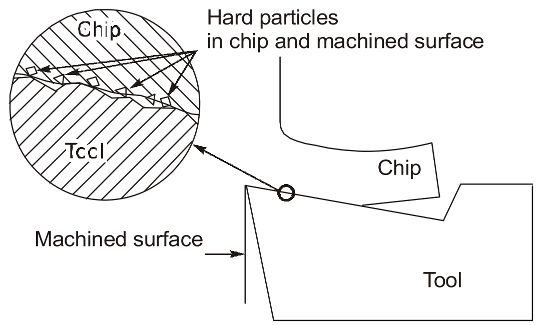
Schematic diagram of Diffusion wear

2. **Adhesion wear** : Due to the excessively high temperature at the chip-tool interface a metallic bond takes place between the chip material and tool material at the contact points, and spot welds are formed. When the chip slides, these small welds are broken and small amount of welded tool carried away by the sliding chip. Thus small particles will continue to separate through this phenomenon and carried away by the chip by adhesion to its underside.



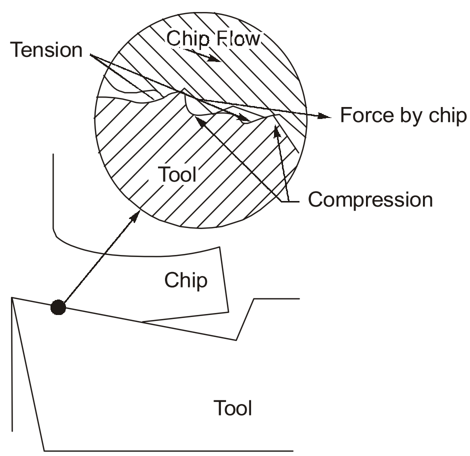
Schematic diagram of Adhesion wear

3. **Abrasion wear** : Since there will be thousands of faylite pockets inside the work material and as these faylite pockets come in contact with the cutting edge, there will be a shock. As a result of that, a portion of cutting edge will be eroded.



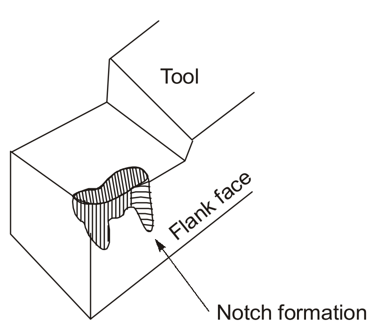
Schematic diagram of Abrasion wear

4. **Fatigue wear** : On a microscopic level, hills of the tool will be eroded and fresh hills will be formed by the interaction with the work piece. This erosion is called fatigue wear.



Schematic diagram of Fatigue wear

5. **Oxidation wear** : After machining operation, oxide layer will be formed over the tool which will be removed in the next cut. The formation of grooves or notches at the rake face and the flank is on account of the sliding of portions of the chip and the machined surface which have reacted with the oxygen in the atmosphere to form abrasive oxides. This causes oxidation wear.



Schematic diagram of Oxidation wear

Q.4 (d) Solution:

Given,Voltage arc length relation is given as

$$V = 20 + 4l$$

We know that,

$$V = V_0 - \left(\frac{I_t}{I_s}\right)V_0$$

At,

$$l = 4 \text{ mm}$$
$$V_1 = 20 + 4 \times 4 = 36 \text{ Volt}$$
$$I_1 = 550 \text{ Amp.}$$

(∵ Where voltage is minimum current will be maximum)

At,

$$l = 6 \text{ mm}$$
$$V_2 = 20 + 4 \times 6 = 44 \text{ Volt}$$
$$I_2 = 450 \text{ Amp.}$$

Now,

$$V_1 = 36 \text{ Volt}, I_1 = 550 \text{ Amp.}$$
$$36 = V_0 - \left(\frac{550}{I_s}\right)V_0$$
$$36 = V_o\left[1 - \frac{550}{I_s}\right] \quad \text{...(i)}$$

at

$$V_2 = 44 \text{ Volt}, I_2 = 450\text{A}$$
$$44 = V_0\left[1 - \frac{450}{I_s}\right] \quad \text{...(ii)}$$

Now, equation (i)/(ii),

$$\frac{36}{44} = \left[\frac{1 - \frac{550}{I_s}}{1 - \frac{450}{I_s}}\right]$$

$$36\left[1 - \frac{450}{I_s}\right] = 44\left[1 - \frac{550}{I_s}\right]$$

$$44 \times \frac{550}{I_s} - \frac{36 \times 450}{I_s} = 44 - 36$$

$$\text{Short circuit current, } I_s = \frac{44 \times 550 - 36 \times 450}{8} = 1000 \text{ A}$$

Now, putting value of I_s in equation (1)

$$36 = V_0\left[1 - \frac{550}{I_s}\right]$$

$$36 = V_0\left[1 - \frac{550}{1000}\right]$$

$$\text{Open circuit voltage, } V_o = \frac{36}{0.45} = 80 \text{ Volt}$$

Q.5 (a) Solution:

$$r = 200 \sin 30° + 40 = 140 \text{ mm}$$

$$h = \frac{r}{\tan 30°}$$

$$h = 140 \,/\, \tan 30 = 242.5 \text{ mm}$$

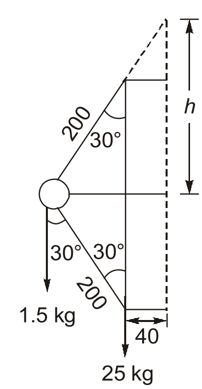
At 30° angle, the sleeve begins to rise; therefore, the friction force is to act downwards.

$$N^2 = \frac{895}{h}\left(\frac{mg + (Mg + f)}{mg}\right)$$

$$(260)^2 = \frac{895}{0.2425}\left(\frac{1.5 \times 9.81 + 25 \times 9.81 + f}{1.5 \times 9.81}\right)$$

$$14.715 + 245.25 + f = 269.523$$

$$f = 9.558 \text{ N}$$



Now, we have to find N,(maximum speed) and N_2(minimum speed) with the data in hand.

$$r = 200 \sin 45° + 40 = 181.4 \text{ mm}$$

$$h = \frac{r}{\tan 45°} = 181.4 \text{ mm}$$

$$N_1{}^2 = \frac{895}{0.1814}\left(\frac{1.5 \times 9.81 + 25 \times 9.81 + 9.56}{1.5 \times 9.81}\right) = 90370$$

$$N_1 = \textbf{300.6 rpm} \qquad \textbf{Ans.}$$

$$N_1{}^2 = \frac{895}{0.1814}\left(\frac{1.5 \times 9.81 + 25 \times 9.81 - 9.56}{1.5 \times 9.81}\right) = 83959.2$$

$$N_2 = 289.76 \text{ rpm} \qquad \text{Ans.}$$

Q.5 (b) Solution:

$$\text{Arc of contact} = 1.75 \times \text{Circular pitch}$$

$$\text{Path of contact} = 1.75 \times \text{Circular pitch} \times \cos\phi$$

$$2\sqrt{R_a^2 - (R\cos\phi)^2} - 2R\sin\phi = 1.75 \times \pi \times m \times \cos\phi$$

$$2\sqrt{R_a^2 - (R\cos\phi)^2} = \{1.75 \times \pi \times m \times \cos\phi\} + \{2R \sin\phi\}$$

$$= \{1.75 \times \pi \times 6 \times \cos 20\} + \{2 \times \frac{6 \times 60}{2} \times \sin 20\}$$

$$= 30.997 + 123.127 = 154.124$$

$$\therefore \quad \sqrt{R_a^2 - (R\cos\phi)^2} = \frac{154.124}{2}$$

$$R_a^2 - (R\cos\phi)^2 = (77.0622)^2$$

$$R_a{}^2 = (77.0622)^2 + \left(\frac{6 \times 60}{2}\cos 20\right)^2 = 34548.5$$

$$R_a = 185.87 \text{ mm}$$

$$\text{Addendum} = R_a - R = 185.87 - \{6 \times 60/2\}$$

$$= 185.87 - 180 = 5.87 \text{ mm}$$

Q.5 (c) Solution:

As per given information,

$t = 12$ teeth, $T = 20$ teeth, $G = \frac{20}{12} = \frac{5}{3}$

$$\phi = 20°$$

As we know, the minimum number of teeth required on the gear in order to avoid interference/undercutting,

$$T = \frac{2A_G}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)(\sin\phi)^2}-1}$$

$$20 = \frac{2A_G}{\sqrt{1+\frac{3}{5}\left(\frac{3}{5}+2\right)(\sin 20°)^2}-1}$$

$$A_G = 0.874$$

Addendum of gear wheel is 0.874 module.

For pinion or gear wheel with number of teeth is 12.

$$t = \frac{2A_P}{\sqrt{1+G(G+2)\sin^2\phi}-1}$$

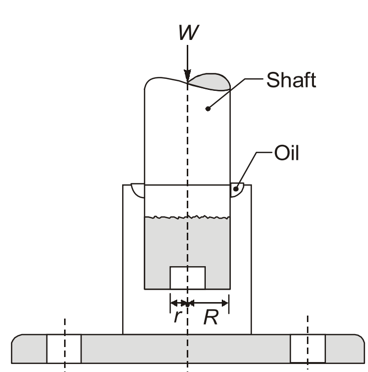
$$12 = \frac{2A_P}{\sqrt{1+\frac{5}{3}\left(\frac{5}{3}+2\right)(\sin 20°)^2}-1}$$

$$A_P = 1.857$$

Hence, addendum of gear wheel with 12 teeths is 1.857 module.

Q.5 (d) Solution:

Given: D = 180 mm or R = 90 mm; d = 60 mm or r = 30 m; P = 1 N/mm^2, N = 120 rpm, μ = 0.015



Let w = load to be supported

Assuming that the pressure is uniformly distributed over the bearing surface, therefore bearing pressure (P),

$$P = \frac{W}{\pi(R^2 - r^2)}$$

$$W = \pi(R^2 - r^2)P$$

$$W = \pi(90^2 - 30^2) = 22619.47 \text{ N} \qquad \text{Ans. (i)}$$

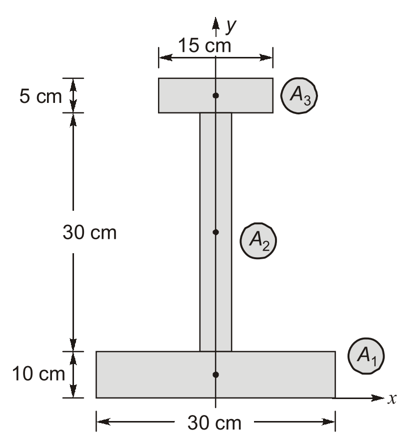
Frictional torque is given as
$$T = \frac{2}{3}\times 0.015\times 22619.47\left(\frac{90^3 - 30^3}{90^2 - 30^2}\right)$$
$$= 22053.98 \text{ N.mm or } 22.054 \text{ N.m}$$

Power lost in friction,
$$P = \frac{2\pi NT}{60} = \frac{2\pi\times 120\times 22.054}{60}$$
$$= 277.14 \text{ W or } 0.277 \text{ kW} \qquad \text{Ans. (ii)}$$

Heat generated at the bearing = Power lost in friction
$$= 0.277 \times 60 = 16.62 \text{ kJ/min} \qquad \text{Ans. (iii)}$$

Q.6 (a) Solution:

As per problem statement configuration is shown in figure. Reference axis are chosen as shown in figure. As the section is symmetrical about y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Given section may be dividing into three parts.



1. A_1 – Bottom flange (30 × 10) cm^2
2. A_2 – Web (30 × 5) cm^2
3. A_3 – Top flange (15 × 5) cm^2

To determine the location of the centroid of the plane figure we have the following table:

A	Area (cm^2) (A)	$\bar{y}$ from x (cm)	$A\bar{y}$ (cm^3)
A_1	30 × 10 = 300	5	1500
A_2	30 × 5 = 150	10 + 30/2 = 25	3750
A_3	15 × 5 = 75	10 + 30 + 5/2 = 42.5	3187.5
	ΣA = 525		$\Sigma A\bar{y}$ = 8437.5

Thus $$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8437.5}{525} = 16.07 \text{ cm}$$

Q.6 (b) Solution:

Given: Mass of bullet, m = 40 gm = 0.04 kg, Initial velocity of bullet, u_1 = 170 m/s
Mass of wooden cuboid, M = 900 gm = 0.9 kg, Initial velocity of cuboid, u_2 = 0 m/s

Combined final velocity = v, By law of conservation of momentum

$$mu_1 + Mu_2 = (m + M)v$$
$$(0.04)(170) + 0 = (0.04 + 0.9)v$$
$$v = 7.234 \text{ m/s}$$

Let R_f be the friction force

$$R_f = \mu(M + m)g$$
$$= 0.22\,(0.9 + 0.04) \times 9.81$$
$$= 2.03 \text{ N}$$

$$R_f s = \frac{1}{2}(m + M)v^2$$

Where, s = Distance travelled by combined mass

$$\frac{1}{2}(0.9 + 0.04) \times 7.234^2 = 2.03 \times s$$
$$s = 12.11 \text{ m}$$

Q.6 (c) Solution:

$$P_c = \frac{\pi^2 EI}{l_e^2}$$

$$\therefore \quad P = \frac{\pi^2 \times E \times I}{3^2} \quad \text{(For both end hinged)} \quad ...(1)$$

$$[P + 300] = \frac{\pi^2 \times E \times I}{(l/2)^2} \quad \text{(For both end fixed)}$$

$$[P + 300] = \frac{\pi^2 \times E \times I}{(1.5)^2} \quad \text{...(2)}$$

Taking ratio (1) and (2)

$$\frac{P}{[P+300]} = \frac{1.5^2}{3^2}$$

$$\frac{P}{P+300} = \left(\frac{1.5}{3}\right)^2$$

$$4P = P + 300$$

$$\mathbf{P = 100\ kN}$$

$$P = \frac{\pi^2 EI}{l^2}$$

$$100 = \frac{\pi^2 \times (100 \times 10^9)}{1000} \times \frac{I}{3^2}$$

$$I = \frac{100 \times 1000 \times 9}{\pi^2 \times 100 \times 10^9} = 9.1189 \times 10^{-7}\ \text{m}^4$$

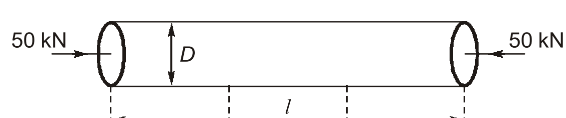
$$\frac{\pi}{64}[D_0^4 - D_i^4] = 9.1189 \times 10^{-7}$$

$$\frac{\pi}{64}[D_0]^4\left[1 - \left(\frac{1}{1.25}\right)^4\right] = 9.1189 \times 10^{-7}$$

$$D_0 = 74.89\ \text{mm} \approx 75\ \text{mm}$$

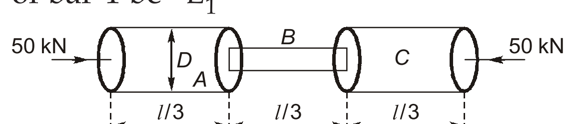
$$D_i = 59.92\ \text{mm} \approx 60\ \text{mm}$$

Q.6 (d) Solution:



Bar-1

Let the strain energy of bar-1 be "E_1"



Bar-2

Let the strain energy of bar-2 be "E_2"

$\therefore$ $$E_2 = E_A + E_B + E_C$$

$$= \frac{E_1}{3} + E_B + \frac{E_1}{3} = \frac{2E_1}{3} + E_B$$

($E_A = E_1/3 = E_C$ Because strain energy is directly proportional to length)

we have $E_2 = 1.5\ E_1$

$$\frac{2E_1}{3} + E_B = \frac{3}{2}E_1$$

$$E_B = \left(\frac{3}{2} - \frac{2}{3}\right)E_1$$

$$E_B = \frac{5}{6} \times \frac{1}{2}\frac{P^2L}{AE}$$

$$\frac{1}{2}\frac{P^2 \times L}{3d^2E} = \frac{5P^2 \times L}{12D^2E} = \frac{1}{6}\cdot\frac{1}{d^2} = \frac{5}{12} \times \frac{1}{D^2}$$

$$d^2 = \frac{1}{6} \times \frac{12}{5} \times D^2$$

$$d = D \times \sqrt{\frac{1 \times 12}{5 \times 6}} = 60 \times \sqrt{\frac{12}{30}} = 37.95 \text{ mm}$$

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