NAME:-

ROLL NO:-

TEST 1:- DIGITAL & CONTROL SYSTEM

1)(a)(i) $(N)_2 = (11010100)_2$

~~1's Complement of $(N)_2$ = 00101011~~

~~2's Complement of $(N_2)_2$ = 001010~~

Assuming that the given binary number is a signed number.

2's Complement of the number without sign bit

= 0101100

2's Complement of the number with sign bit

= 1010 1100.

(ii) Given, $(27FCA)_{16} \div (3E)_{16}$.

~~3E) 27FCA~~
~~2E8~~
7
(~~A~~)

~~748.~~
~~784~~
639

3E) 62 27F = 639
26

```
3E ) 27FCA ( A51
     26C
     ----
      13C
      136
      ----
       6A
       3E
       --
       2C
```

Quotient = A51

Remainder = 2C

1)(b)(i)

1. Setup time

It is the time interval immediately preceding the active transition of clock during which synchronous input to be maintained its proper level.

2. Hold time

It is the time interval immediately following the active transition of clock during which synchronous input to be maintained its proper level.

5

(ii)

Given numbers

$$(18.6)_9 = (1\times9 + 8\times9^0 + 6\times9^{-1})_{10}$$
$$= (17.66667)_{10}$$

$$\begin{array}{r|l} 11 & 17 \\ \hline & 1 - 6 \end{array}$$

$(17)_{10} = (16)_{11}$

$0.66667 \times 11 \longrightarrow 7$
$0.33337 \times 11 \longrightarrow 3$
$0.66707 \times 11 \longrightarrow 7$
$0.33777 \longrightarrow 3$

6

The given number cannot be converted exactly to base 11. The approximate equivalent

$$(18.6)_9 = (16.7373\ldots)_{11}$$

(1)(c) (Archana, Vikas)

child child

Saurabh Kunal.

Following Names denoted by the latter as shown

Archana – A
Vikas – V
Saurabh – S
Kunal – K
Burger – B – 0 (let assume)
Chicken – C – 1 (let assume)

Truth table

Archana	Vikas	Kunal	Saurabh	Choosen restaurant
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

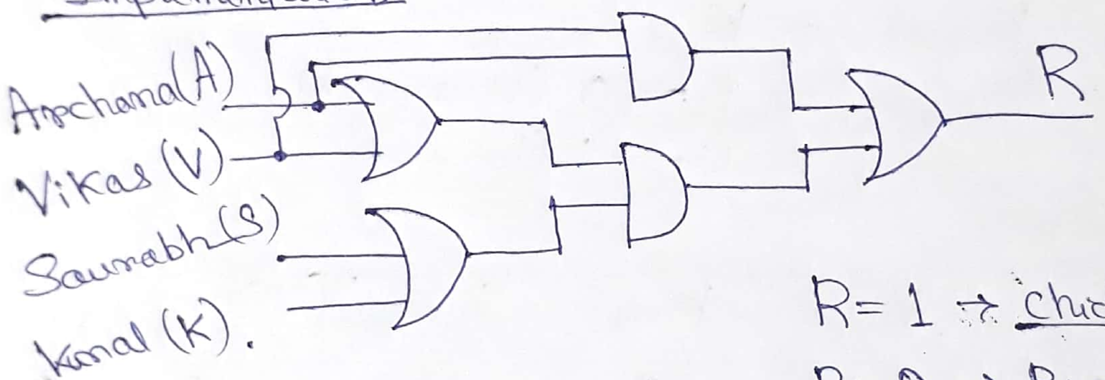
Minimizing by using K-map.

AV \ KS	00	01	11	10
00				
01		1	1	1
11	1	1	1	1
10		1	1	1

$$R_{\text{Chicken}} = AV + VS + VK + AS + AK$$

$$= \cancel{AV + (V+S)(A+K)}\ AV + (V+A)(S+K)$$

Implementation



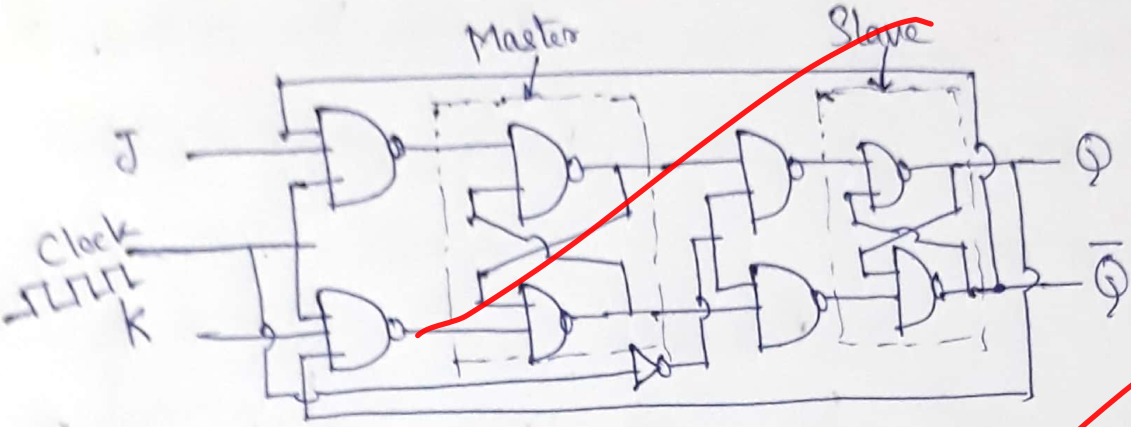
R = 1 → Chicken

R = 0 → Burger

logic '1' → Chicken

logic '0' → Burger.

(1)(d) Master Slave J-K flip-flop



For a master slave flipflop, Clock of positive level given to master and negative level given to slave. As a result ~~both~~ two section are activated for two different interval of clock signal. ~~As a~~ There fore frequent change of output Q from 1 to 0, and 0 to 1 does not happed while both the input J=K=1.

Thus race around avoided.

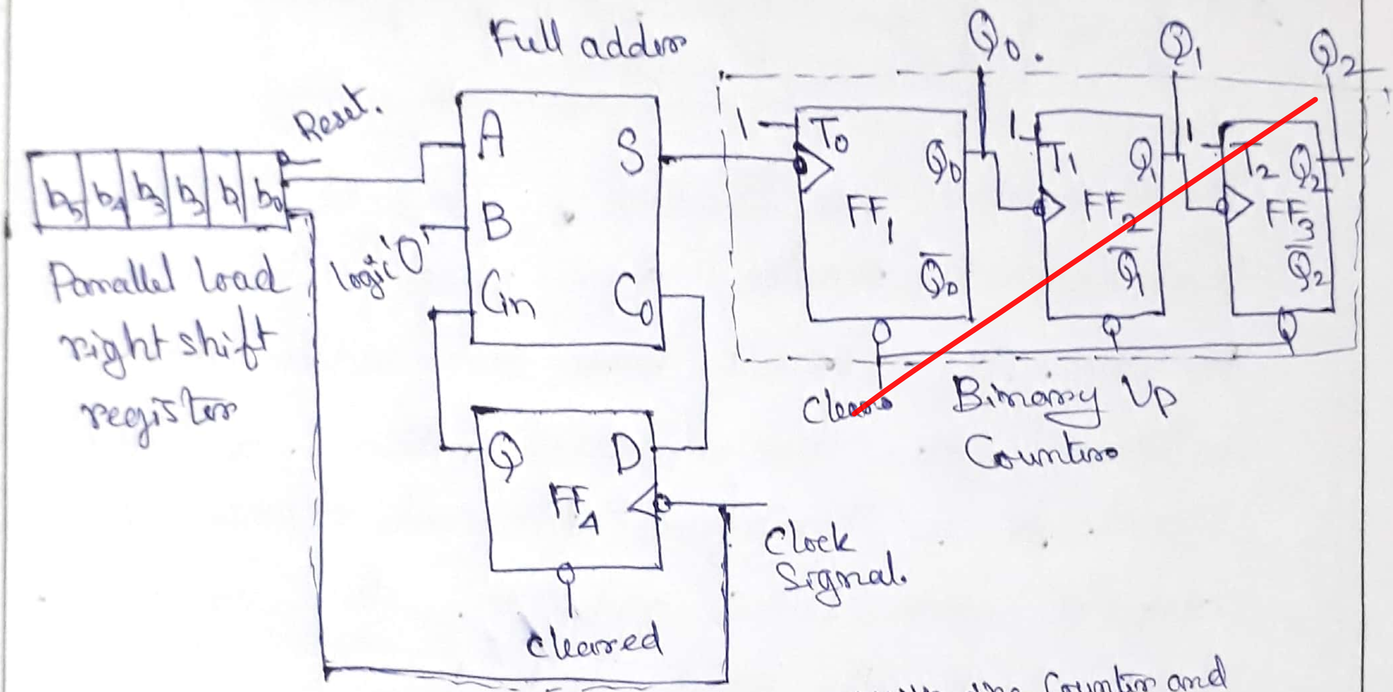
~~Adv~~
Advantages

Race around happens in a standard JK flip flop when J=K=1 for level clock ~~a~~ triggering which is avoided in Master-Slave J-K flipflop.

1(e)

~~Desig~~

Desire circuit can be design by using a Full adder folled by binary Counter. Which is shown below



Initially, the D-filp flop cleared with the counter and the register, and 6 bit of number loaded to ~~register~~ parallel load right shift register. By application of clock ~~pr~~ first bit procced to the full adder ~~ac~~ and output of the adder ~~added to~~ ~~A~~ feed into the clock of filpflop 1. Same way the process continued upto six pulse.

(2)(a) (i)

$F_1 = \overline{A+B} + AB\overline{C}$

$= \overline{A}\,\overline{B} + AB\overline{C}$

$= AB\overline{C} + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$

$= \sum m(6,1,0)$.

$F_2 = \overline{A+B} + ABC$

$= \overline{A}\,\overline{B} + ABC$

$= ABC + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$

$= \sum m(7,1,0)$

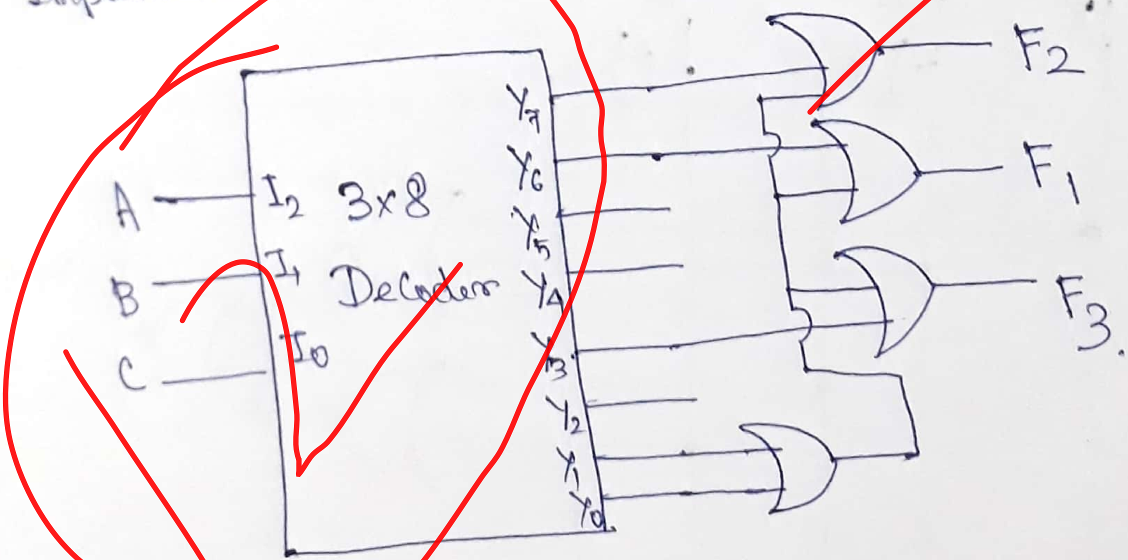
$F_3 = \overline{A}(\overline{B} + C)$

$= \overline{A}\,\overline{B} + \overline{A}C$

$= \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + \overline{A}BC + \overline{A}\,\overline{B}C$

$= \sum m(3,1,0)$.

Implementation



(2) (a) (ii) Given

$$f(x,y,z) = \Sigma m(0,3,4,5,7)$$
$$= \Pi M(1,2,6).$$
$$= (X+Y+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+\bar{Y}+Z).$$

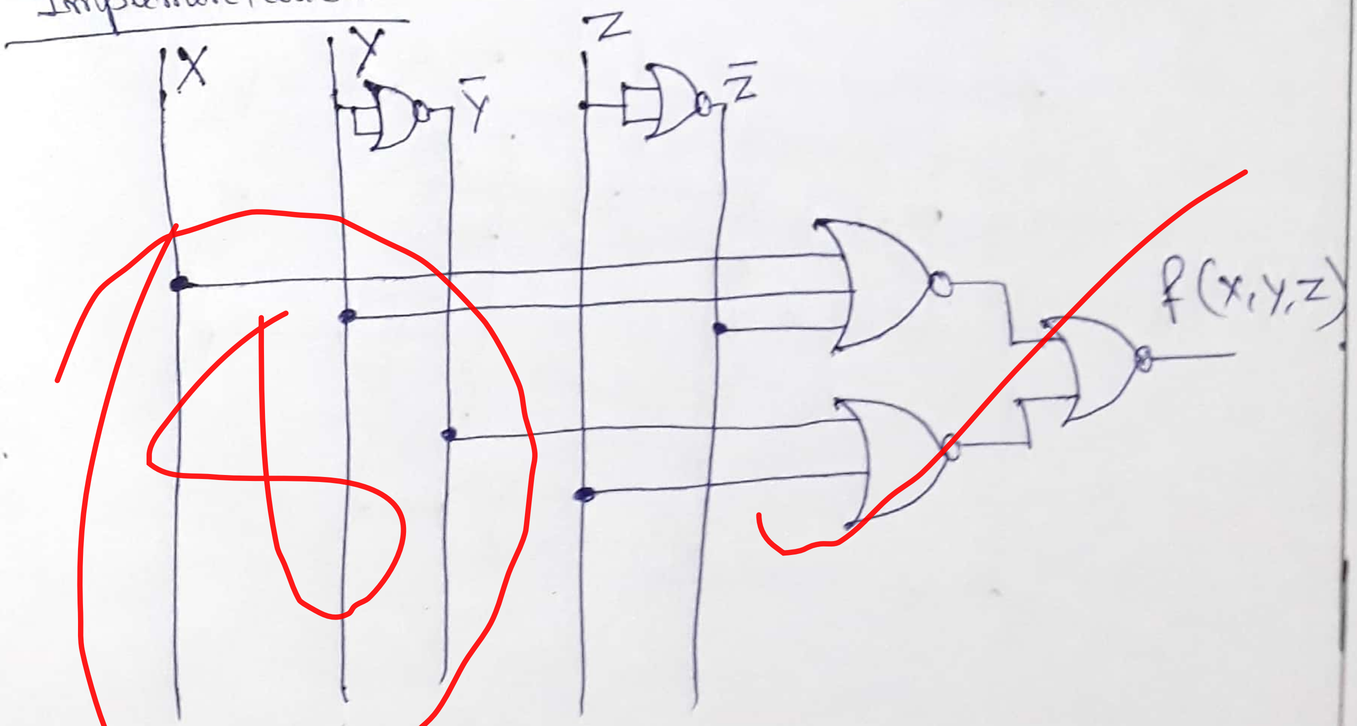
Minimisation by using k-map.

X \ YZ	00	01	11	10
0		0		0
1				0

$$f(X,Y,Z) = (X+Y+\bar{Z})(\bar{Y}+Z)$$
$$= \overline{\overline{(X+Y+\bar{Z})(\bar{Y}+Z)}}$$
$$= \overline{\overline{(X+Y+\bar{Z})} + \overline{(\bar{Y}+Z)}}$$

~~X~~ ~~Y~~

Implementation



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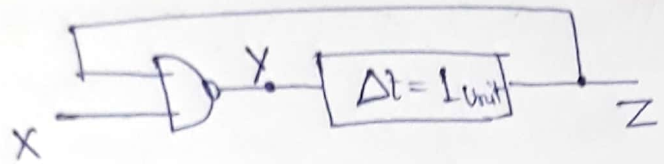
(2)(b) (i) Given Circuit

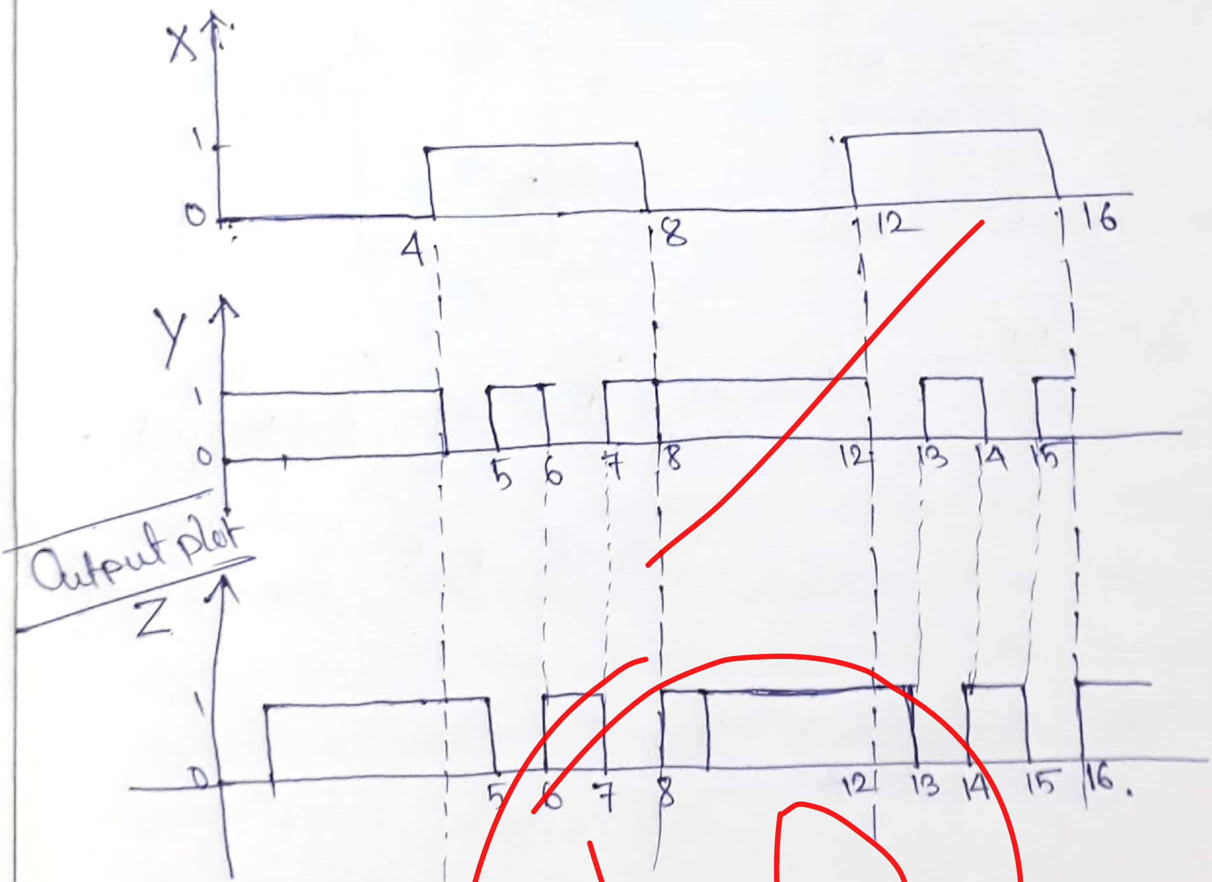


Let assume ~~ø~~ delay of the ~~on~~ NAND gate is zero and initial state of Z = logic '0'.

Therefore, $Y = \overline{XZ}$.

~~Output plot~~.

Output plot



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2(b) (ii)

Given $f(A,B,C,D,E) = \prod M(0,2,4,11,14,15,16,20,24,30,31)$

Minimising by using K-Map

For $\bar{A}$

BC \ DE	00	01	11	10
00	0			0
01	0			
11			0	0
10			0	

For A

BC \ DE	00	01	11	10
00	16 0			
01	20 0			
11	28		0	0
10	24 0			

$$f(A,B,C,D,E) = (B+D+E)(\bar{B}+\bar{C}+\bar{D})(A+B+C+E)\cdot(A+\bar{B}+\bar{D}+\bar{E})(\bar{A}+C+D+E)$$

2(c) (i)

(1) Its resolution $= 1\,mV$

$$= \frac{1\times10^{-3}}{10}\times 100\,\%$$

$$= 0.01\,\%.$$

(2) Total number of count $= \dfrac{10\,V}{10^{-3}\,V} = 10^4 = 1000$

If number of bit n

then $n \geq \log_2(10000)$

$n \geq 13.28$

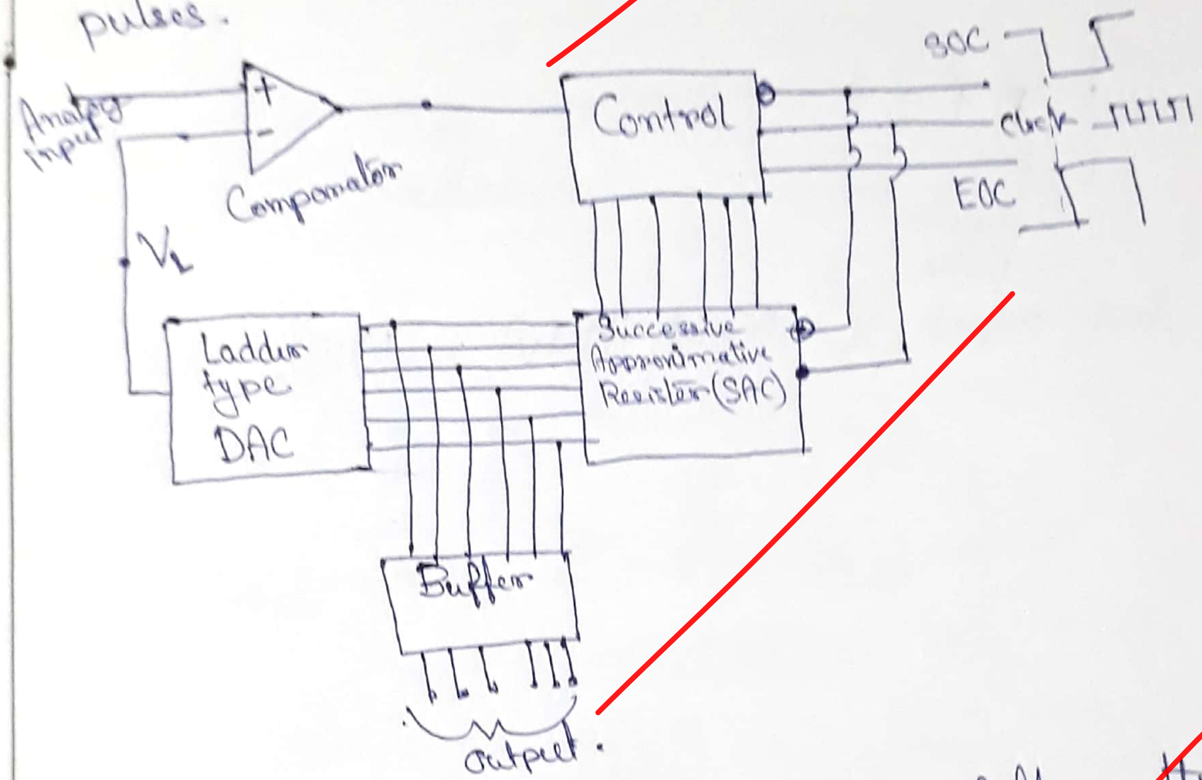
Minimum $= 14$

Therefore minimum 14 bit ADC required.

(2)(C)(ii)

Successive approximation type ADC

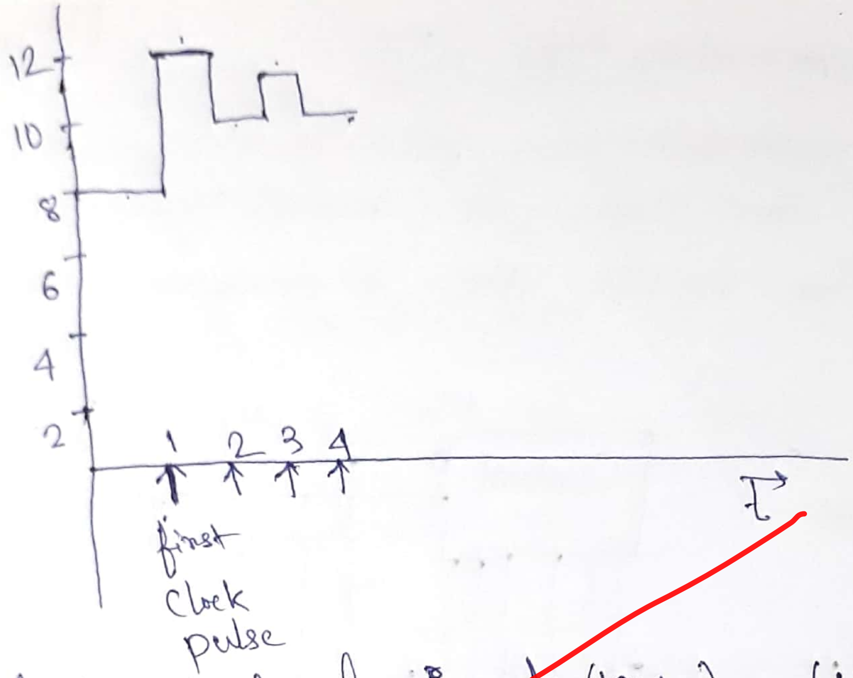
~~To~~ Successive approximative type ADC is comparative fast adder than dual slope and ramp type ADC. ~~For n-type~~ For n-bit ADC it required n-clock pulses.



ADC initially is at reset condition. After getting Low active start of conversion pulse (SOC), SAC register MSB turns into logic '1', where as other bits are remain in '0'. The ladder type of DAC convert the digital equivalent to analog which cap compared with input.

If output of ladder circuit > Analog input then changes to logic '0'. Otherwise it will remain in logic. Same process will continue for the lower order bit by keeping higher bit in same logic.

For 4 bit SAC type ADC with resolution of 1V ~~Vt~~ V_L waveform shown below for the input 10·4V.



And output of input $(10.4V) = (1010)_2$.

(3). Quantization interval

$$= \frac{1}{2^{14}} = 61.035 \, \mu\text{Sec.}$$

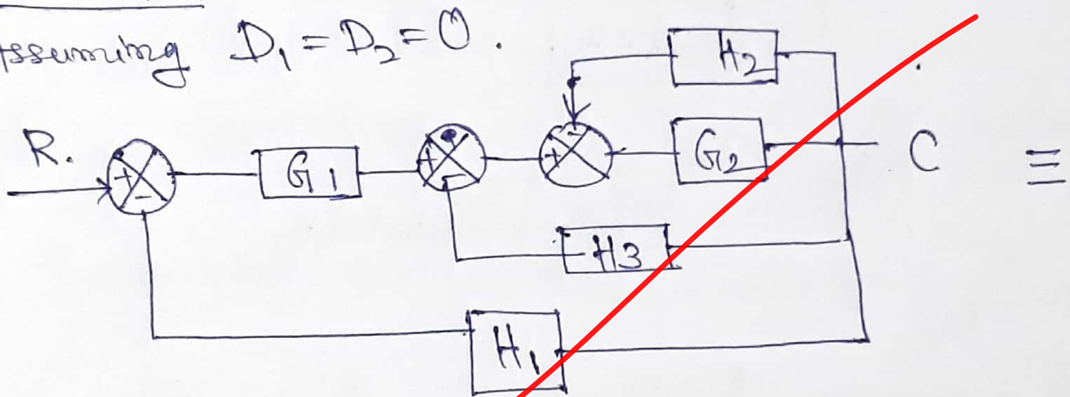
(4). Number of decision level

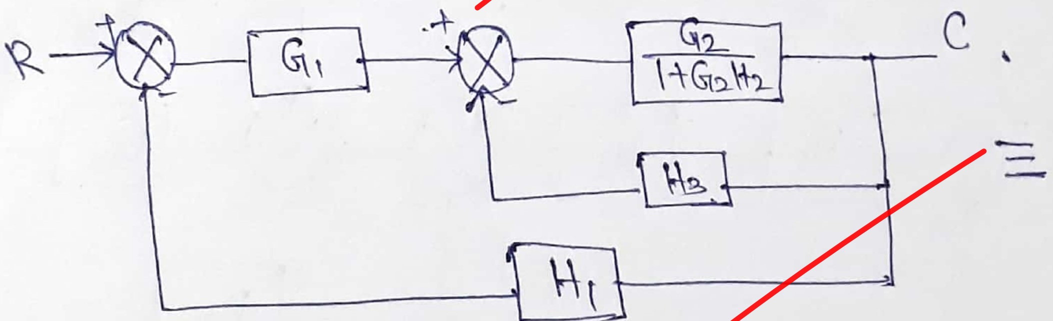
$$= 2^{14} - 1 = 16383.$$

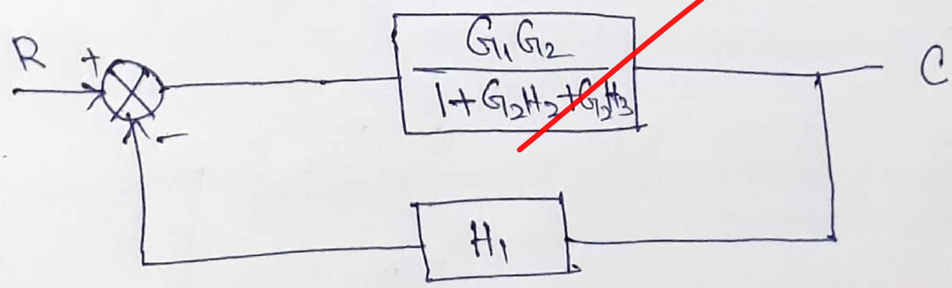
Section - B

(5) (a) (i) Find C/R

Assuming $D_1 = D_2 = 0$.



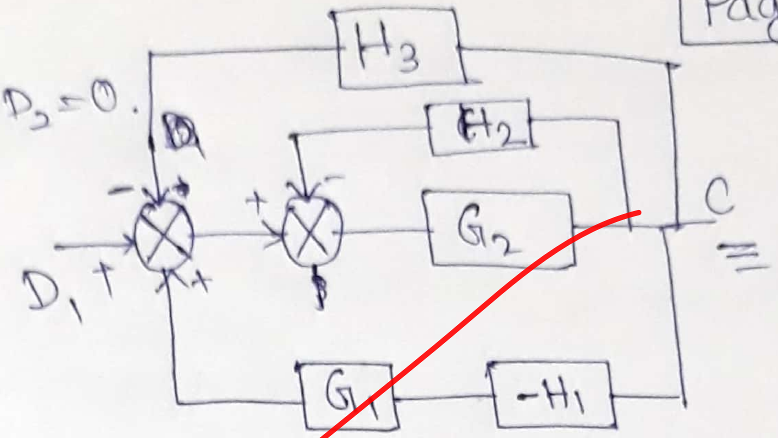


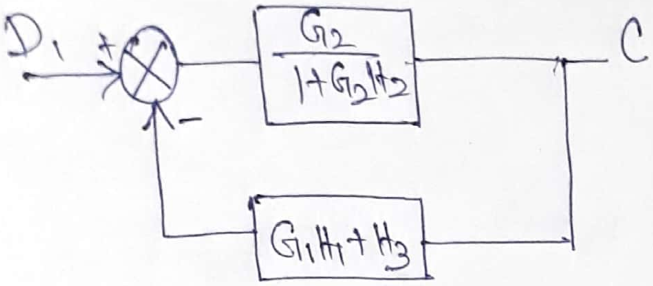


$$\frac{C}{R} = \frac{G_1 G_2}{1 + G_2 H_2 + G_2 H_3 + G_1 G_2 H_1}$$

(5)(a) (ii) Find C/D_1

Assuming $R = D_2 = 0$.

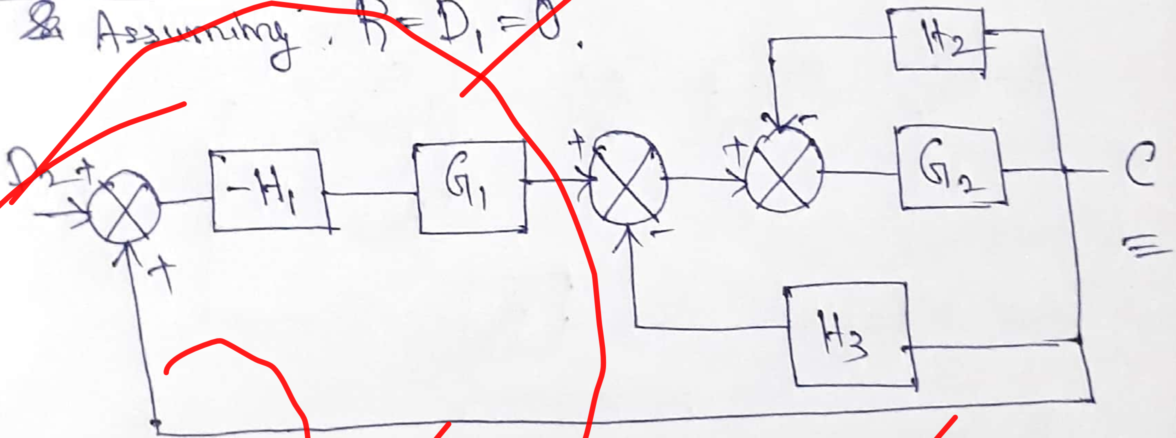


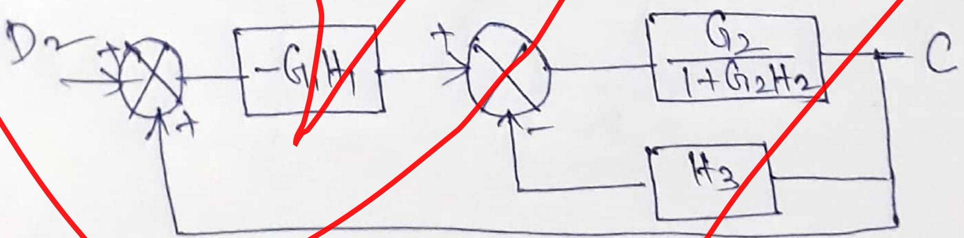


Therefore $$\frac{C}{D_1} = \frac{G_2}{1+G_2H_2+G_1G_2H_1+G_2H_3}$$

(iii) Find C/D_2

Assuming $R = D_1 = 0$.



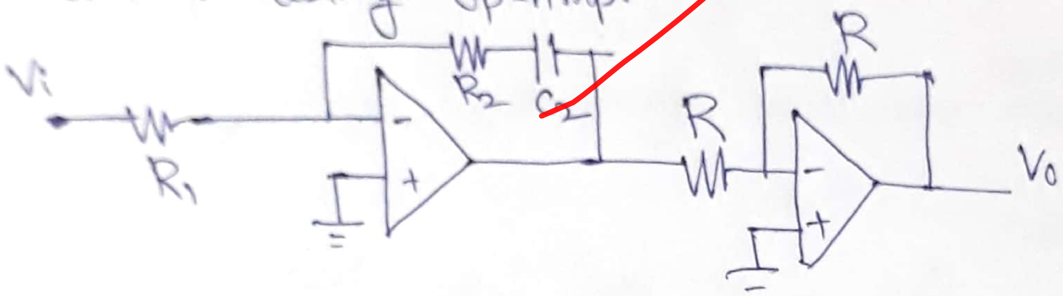


$$\frac{C}{D_2} = \frac{-G_1G_2H_1}{1+G_2H_2+G_2H_3+G_1G_2H_1}$$

(5) (b) $G_c(s) = \frac{10s+4}{s} = \cancel{10+4} = \left(10 + \frac{4}{s}\right)$ ——— (1)

It is a PI Controller.

Standard circuit diagram of a PI Controller using Op-Amp.



$$\frac{V_o}{V_i} = \left(-\frac{R_2 + \frac{1}{C_2 s}}{R_1}\right) \times \left(-\frac{R}{R}\right).$$

$$G_c(s) = \cancel{R_2 C_2 s +} = \left(\frac{R_2}{R_1} + \frac{1}{R_1 C_2 s}\right) \text{ ——— (2)}$$

By Comparing eq^n (1) and (2) we get.

$$\frac{1}{R_1 C_2} = 4. \quad \text{and} \quad \frac{R_2}{R_1} = 10.$$

$$R_1 = \frac{1}{4 \times C_2}$$

$$R_2 = 10 \times R_1 = 100\ k\Omega.$$

$$R_1 = \frac{1}{4 \times 25 \times 10^{-6}}$$

$$R_1 = 10\ k\Omega$$

Therefore values of other parameter

$R_1 = 10\ k\Omega$

$R_2 = 100\ k\Omega$

$R = 10\ k\Omega$ (let assume).

(5)(c) Given transfer function (open loop)

$$G(s) = \frac{k e^{-0.5s}}{(s+1)}.$$

phase at phase cross over frequency.

$$\phi = -0.5\omega - \tan^{-1}(\omega) = -\pi$$

$$\Rightarrow \quad 0.5\omega + \tan^{-1}\omega = \pi.$$

$$\left[\tan^{-1}x = \pi/2 - \frac{1}{x} \text{ for } x>1\right]$$

$$\Rightarrow \quad 0.5\omega + \left(\pi/2 - \frac{1}{\omega}\right) = \pi$$

$$\Rightarrow \quad 0.5\omega - \frac{1}{\omega} = \pi/2$$

$$\Rightarrow \quad 0.5\omega^2 - 1.57\omega - 1 = 0.$$

$$\omega = 3.683, \ -0.543$$

Therefore $\omega_{pc} = 3.683$ rad/s

~~Gain~~

Magnitude $|G(j\omega)| = \frac{k}{\sqrt{1+\omega^2}}$

$$|M|_{\omega=\omega_{pc}} = \frac{k}{\sqrt{1+3.683^2}} = \frac{k}{3.8163}.$$

For the system to be stable

$$\frac{k}{3.8163} < 1$$

$$\Rightarrow \quad k < 3.8163$$

(5) (d)

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \qquad x(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$$

$$[C] = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Transition matrix

$$\phi(s) = [Is - A]^{-1} = \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} s - \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \right]^{-1}$$

$$= \begin{bmatrix} s & -1 \\ 2 & s \end{bmatrix}^{-1} = \frac{1}{s^2+2} \begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}$$

Response

$$Y(s) = C[\phi(s)]\, x(0).$$

$$= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{s+1}{s^2+2} \\ \frac{s-2}{s^2+2} \end{bmatrix}$$

$$= \frac{(s+1-s+2)}{(s^2+2)}$$

$$Y(s) = \frac{3}{s^2+2}$$

$$y(t) = 3\sin(\sqrt{2}t) \quad \text{[By Inverse Laplace transform]}$$

(5)(e) Given $G(s) = \dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}$

At gain crossover frequency.

$$|G(j\omega)| = \frac{\omega_n^2}{\omega\left(\sqrt{\omega^2+(2\zeta\omega_n)^2}\right)}$$

$$|G(j\omega)|_{\omega=\omega_{gc}} = \frac{\omega_n^2}{\omega_{gc}\sqrt{\omega_{gc}^2+(2\zeta\omega_n)^2}} = 1.$$

$$\Rightarrow \quad \omega_n^4 = \omega_{gc}^2\left(\omega_{gc}^2+4\zeta^2\omega_n^2\right).$$

$$\Rightarrow \quad \omega_{gc}^4 + 4\zeta^2\omega_n^2\omega_{gc}^2 - \omega_n^4 = 0.$$

$$\omega_{gc}^2 = \frac{-4\zeta^2\omega_n^2 \pm \sqrt{(4\zeta^2\omega_n^2)^2+4\omega_n^4}}{2}$$

$$= -2\zeta^2\omega_n^2 \pm \omega_n^2\sqrt{\zeta^2+1}$$

And $\omega_{gc}^2 > 0$, $\omega_{gc}^2 = \omega_n^2\left[\sqrt{\zeta^2+1}-\zeta^2\right]$

$$\omega_{gc} = \omega_n\sqrt{\sqrt{\zeta^2+1}-\zeta^2}$$

phase at $\omega = \omega_{gc}$ of the system.

$$\phi = -90^\circ - \tan^{-1}\left(\frac{\omega_{gc}}{2\zeta\omega_n}\right).$$

$$= -90^\circ - \tan^{-1}\left(\frac{\sqrt{\sqrt{\zeta^2+1}-\zeta^2}}{2\zeta}\right)$$

phase Margin

$$\phi_{PM} = 180+\phi = 90^\circ - \tan^{-1}\left[\frac{\sqrt{\sqrt{\zeta^2+1}-\zeta^2}}{2\zeta}\right]$$

$$= \cot^{-1}\left[\frac{\sqrt{\sqrt{\zeta^2+1}-\zeta^2}}{2\zeta}\right]$$

$$\boxed{\phi_{pm} = \tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{\zeta^2+1}-\zeta^2}}\right)}$$

(6)(a) Characteristics equation.

$$s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4 = 0.$$

Routh table

s^6	1	-2	-7	-4
s^5	$+1$	-3	-4	
s^4	1	-3	-4	
s^3	2	-3		
s^2	$-3/2$	-4		
s^1	-8.33			
s^0	-4			

As s^5 and s^4 coefficient are same, s^3 row will ~~Come~~ be zero.

We can find the coefficient of s^3 by considering auxilairy equation

$$A(s) = s^4 - 3s^2 - 4 = 0.$$

$$A'(s) = 4s^3 - 6s = 0.$$

$$2s^3 - 3s = 0.$$

As there is a sign change in first column from row s^3 to s^2. Thus, there is a RHS pole in s-plane.

Therefore, the given system is unstable.

There is another way to overcome problem of all element zero for a particular row.

~~Consider~~ s

Assume $s = \frac{1}{z}$ and follow the same procedure as Routh criterion.

By putting $s=\frac{1}{z}$ in the characteristics equation we get.

$$\frac{1}{z^6}+\frac{1}{z^5}-\frac{2}{z^4}-\frac{3}{z^3}-\frac{7}{z^2}-\frac{4}{z}-4=0$$

$$\Rightarrow 4z^6+4z^5+7z^4+3z^3+2z^2-z-1=0.$$

<u>Routh table</u>

z^6	4	7	2	-1
z^5	4	3	-1	
z^4	4	3	-1	
z^3	8	3		
z^2	1.5	-1		
z^1	8.33			
z^0	-1			

As all ~~the~~ elements of z^5 and z^4 are same. row z^3 will be zero

Auxiliary equation

$$4z^4+3z^2-1=0.$$

$$16z^3+6z=0.$$

$$8z^3+3z=0.$$

As there is sign change from row z^1 to z^0. The system is consisting a RHS pole. Thus the system is unstable.

<u>Roots of characteristics equation</u>

$$(s^6+s^5-2s^4-3s^3-7s^2-4s-4)=0 \quad \text{———} (1)$$

$$(\cancel{s-2.469})(\cancel{s^5+3.469s^4+6.5649s^3+13.386s^2+26.055+1.62})=0$$

~~As the s^3 row was~~

As all the coefficient of s^3 row was zero, the system consisting ~~two~~ pairs of conjugate pole on $j\omega$ axis.

$$(s^4+as^2+b)(s^2+cs+d)=0.$$

$s^6 + cs^5 + (a+d)s^4 + acs^3 + (ad+b)s^2 + bcs + bd = 0$ ——— (2)

By Comparing equation (1) and (2)

$c = 1$

$a+d = -2$ By Solving we get

$ac = -3$ $a = -3,\ b = -4,\ c = 1,\ d = 1$

$(ad+b) = -7$

$bc = -4$ Characteristics equation becomes

$bd = -4$ $(s^4 - 3s^2 - 4)(s^2 + s + 1)$.

$(s^2-4)(s^2+1)(s^2+s+1) = 0$.

So, Roots are at

$s = 2, 2, \pm j, -0.5 \pm j0.866$.

(6)(b) ~~Given System~~,

~~Given system transfer function~~,

Characteristics equation of given system,

$$1 + \left(K_P + \frac{K_I}{s} + K_D s\right)\frac{10}{s(s+4)} = 0.$$

$\Rightarrow s^3 + 4s^2 + K_P s + K_I + K_D s^2 = 0$.

$\Rightarrow s^3 + (4+K_D)s^2 + K_P s + K_I = 0$ ——— (1)

Characteristics equation with poles are at $-4, -1 \pm j2$,

$(s+4)\left[(s+1)^2 + 4\right] = 0$.

$\Rightarrow (s+4)(s^2 + 2s + 5) = 0$.

$\Rightarrow s^3 + 6s^2 + 11s + 20 = 0$ ——— (2)

Comparing equation (1) and (2) we get.

$K_I = 20$, $K_P = 11$ and $K_D = 6 - 4 = 2$

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(6)(b)(ii) characteristics equation of close loop system

$1 + \frac{9}{s^3 + (1+k)s^2 + ks + 9} = 0.$

$s^3 + (1+k)s^2 + ks + 10 = 0.$

Routh table

s^3	1	k
s^2	1+k	10
s^1	$\frac{k^2+k-10}{(1+k)}$	
s^0	10	

For the System to be Stable

i) $1+k > 0$

$k > -1$

(ii) $\frac{(k^2+k-10)}{1+k} > 0$

$(k-2.7)(k+3.7) > 0.$

$\underline{k > 2.7}$ ✓ , $\underline{k < -3.7}$ ✗

Combining (i) and (ii) we get the range of k for the system to be stable

$2.7 < k < \infty$

(6)(c) (i) Given $G(s) = \frac{k}{s(s^2+4s+5)}$

poles are at $s = 0, -2 \pm j$

Centroid $\sigma_A = \frac{\Sigma \text{Real part of pole} - \Sigma \text{real part of zeroes}}{P - Z}$

$= \frac{0-2-2-0}{3-0} = -4/3$

Angle of asymptotes

$= \frac{(2q+1)}{P-Z} . 180^\circ = \frac{(2q+1)}{3} \times 180^\circ \quad q = 0, 1, 2 --$

$= 60^\circ, 180^\circ, 300^\circ.$

Scanned with CamScanner

Intersection on $j\omega$ axis

Characteristics equation

$$1+G(s)=0.$$

$$s^3+4s^2+5s+k=0.$$

Routh table

s^3	1	5
s^2	4	k
s^1	$\frac{20-k}{4}$	
s^0	k	

(i) $k>0$

(ii) $\frac{20-k}{4} \geq 0$

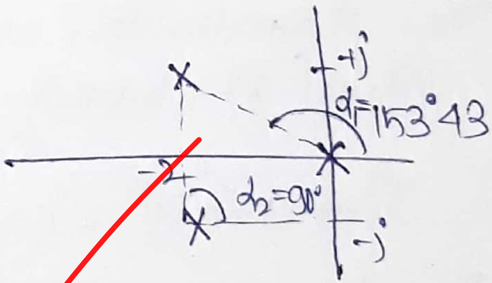
$k \leq 20.$

Marginal value of $k=20$.

Auxilairy equation $4s^2+k=0$

$s=\pm j\sqrt{k/4} = \pm j\sqrt{5}$

Angle of departure from pole $-2+j$



Angle of departure

$$\phi_d = 180+\phi$$

$$= 180^\circ+(\phi_z-\phi_p)$$

$$= 180^\circ+\{0-(\alpha_1+\alpha_2)\}$$

$$= 180-90-153.43^\circ$$

$$= -63.43^\circ$$

By using, property of root locus.

Angle of departure from root $-2-j$

$$\phi_d = 63.43^\circ.$$

Scanned with CamScanner

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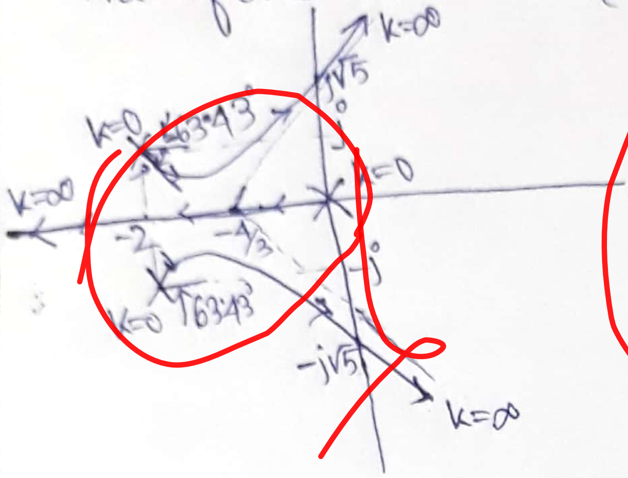
Therefore root locus of the system



(6) (c) (ii) Characteristics equation of the Close loop system

$$1+G(s)=0.$$

$$1+\frac{K}{s(s^2+4s+5)}=0.$$

$$s^3+4s^2+5s+K=0. \qquad \text{—— (1)}$$

The characteristics equation can be expressed as

$$(s+a)(s^2+bs+c)=0.$$

$$s^3+(a+b)s^2+(c+ab)s+ac=0. \qquad \text{—— (2)}$$

By Comparing equation (1) and (2) we get.

$a+b=4 \Rightarrow a=(4-b).$

$c+ab=5$

$ac=K. \Rightarrow a=\frac{K}{c}$

$4-b=K/c$

$\Rightarrow b=4-K/c$

$c+ab=5.$

$c+\frac{K}{c}(4-K/c)=5$

$\Rightarrow c^3+K(4c-K)=5c^2$

$\Rightarrow c^3-5c^2+4Kc-K^2=0.$

(8) (7) (a)

Given $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}$. $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$C = [1\ 1\ 0]$ $X(0) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ input $U(s) = L(e^{-t}) = \frac{1}{S+1}$

Transition matrix

$$\phi(s) = [Is - A]^{-1}$$

$$= \left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} s - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}\right]^{-1}$$

$$= \begin{bmatrix} S & -1 & 0 \\ 0 & S & -1 \\ 24 & 26 & S+9 \end{bmatrix}^{-1} = \begin{bmatrix} s^2+9s+26 & \cancel{s+9s}\ 24 & -24s \\ s+9 & s^2+9s & -(26s+24) \end{bmatrix}$$

(8) (a) (i)

For the given Bode plot,

Inbetween frequency range ω_1 to 8.

Slope, $\frac{0 - 24.1}{\log_{10} 8 - \log_{10} \omega_1} = -40.$

$\Rightarrow \log_{10} 8/\omega_1 = \frac{24.1}{40}.$

$\Rightarrow \omega_1 = 8 \times 10^{-\frac{24.1}{40}} = 1.9979 \cong 2$ rad/s.

Inbetween frequency range 8 to ω_2

Slope $\frac{-12.05 - 0}{\log_{10} \omega_2 - \log 8} = -40.$

$\omega_2 = 8 \times 10^{\frac{12.05}{40}} \cong 16$ rad/sec.

Inbetween frequency ω_2 to ω_3

Slope $\frac{-20.05 - (-12.05)}{\log \omega_3 - \log \omega_2} = -20$

$\omega_3 = \omega_2 \times 10^{\frac{8}{20}} = 16 \times 10^{8/20} \cong 40.2$ rad/sec.

And Gain at $\omega = \omega_1 = 2$ rad/sec

Range of frequency	Net Slope (dB/dec)	Slope Contribution (dB/dec)	Corner frequency (rad/sec)	Factor
1 to $\omega_1 = 2$ rad/sec	-20	-20	None (initial slope)	$\frac{K}{S}$
$\omega_1 = 2$ rad/s to $\omega_2 = 16$ rad/s	-40	-20	$\omega_c = \omega_1 = 2$ $T_c = \frac{1}{\omega_c} = \frac{1}{2}$	$\frac{1}{(0.5S+1)}$
$\omega_2 = 16$ rad/s to $\omega_3 = 40.2$ rad/sec	-20	$+20$	$\omega_c = \omega_2 = 16$ $T_c = \frac{1}{16} = 0.0625$	$(0.0625S+1)$
~~ω_2 to~~ ~~$\omega_2 = 16$ rad/s to~~ ω_3 from $\omega_3 = 40.2$ rad/sc	-40	-20	$\omega_c = \omega_3 = 40.2$ $T_c = \frac{1}{40.2} =$	$\frac{1}{(0.02487S+1)}$

Therefore Open loop transfer function

$$G(S) = \frac{K\,(0.0625S+1)}{S(0.5S+1)(0.02487S+1)}$$

Gain at $\omega = \omega_1 = 2$ rad/sec

$$20\log_{10}\left(\frac{K}{2}\right) = 24.1$$

$$K \cong 32$$

So,

$$G(S) = \frac{32\,(0.0625S+1)}{S(0.5S+1)(0.02487S+1)}$$

$$\boxed{G(S) = \frac{160.8\,(S+16)}{S(S+2)(S+40.2)}}$$

8 (a) (ii) Forward path

$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_1 G_6 G_4 G_5$

$P_3 = G_1 G_2 G_7$

$\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta_3 = 1 - L_1 = 1 + H_1 G_4$

Individual loops

$L_1 = -H_1 G_4$

$L_2 = -G_2 G_3 G_4 G_5 H_2$

$L_3 = -G_2 G_7 H_2$

$L_4 = -G_4 G_5 G_6 H_2$

Two Non touching loop

NiL

$$\Delta = 1 - \left(\begin{matrix}\text{Sum of individual} \\ \text{loop gains}\end{matrix}\right) + \left(\begin{matrix}\text{Sum of two not} \\ \text{touching loop gain}\end{matrix}\right) - \ldots$$

$$= 1 - (L_1 + L_2 + L_3 + L_4) + 0$$

$$= 1 + H_1 G_4 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_4 G_5 G_6 H_2$$

By using Mason's gain formula

$$\frac{C}{R} = \frac{1}{\Delta} \sum_{K=1}^{3} P_K \Delta_K = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_6 G_4 G_5 + G_1 G_2 G_7 (1 + H_1 G_4)}{1 + H_1 G_4 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_4 G_5 G_6 H_2}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_4 G_5 G_6 + G_1 G_2 G_7 + G_1 G_2 G_4 G_7 H_1}{1 + H_1 G_4 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_4 G_5 G_6 H_2}$$

8(b)

~~GH~~
Open loop ~~trans~~ Gain of the system $G(s)H(s) = k(s+0.5) \times \left\{ \dfrac{\frac{1}{s^2(s+1)}}{1+\frac{1}{s^2(s+1)}} \right\}$

$$G(s)H(s) = \frac{k(s+0.5)}{s^3+s^2+1}$$

$$G(j\omega)H(j\omega) = \frac{k(j\omega+0.5)}{-j\omega^3-\omega^2+1}$$

$$= \frac{k(j\omega+0.5)\{(1-\omega^2)+j\omega^3\}}{\{(1-\omega^2)-j\omega^3\}\{(1-\omega^2)+j\omega^3\}}$$

$$= k\frac{0.5(1-\omega^2)-\omega^4}{(1-\omega^2)^2+\omega^6} + jk\frac{\omega(1-\omega^2)+0.5\omega^3}{(1-\omega^2)^2+\omega^6}.$$

$$= k\frac{0.5-0.5\omega^2-\omega^4}{(1-\omega^2)^2+\omega^6} + jk\frac{(-0.5\omega^3+\omega)}{(1-\omega^2)^2+\omega^6}$$

For inter section on ~~jω axi~~ real axis imaginary part to be zero,

$$\frac{k(-0.5\omega^3+\omega)}{(1-\omega^2)^2+\omega^6} = 0.$$

$$\Rightarrow \quad \omega - 0.5\omega^3 = 0.$$

$$\omega = 0, +\cancel{0.707}\ {+1.414}, \cancel{-0.707}\ {-1.414}$$

~~Gain at~~ Real part $\omega = 0.707$ rad/sec

$$= k.\frac{0.5-0.5\omega^2-\omega^4}{(1-\omega^2)^2+\omega^6}\Bigg|_{\omega=\cancel{0.707}\ -1.414}$$

$$= -0.5k$$

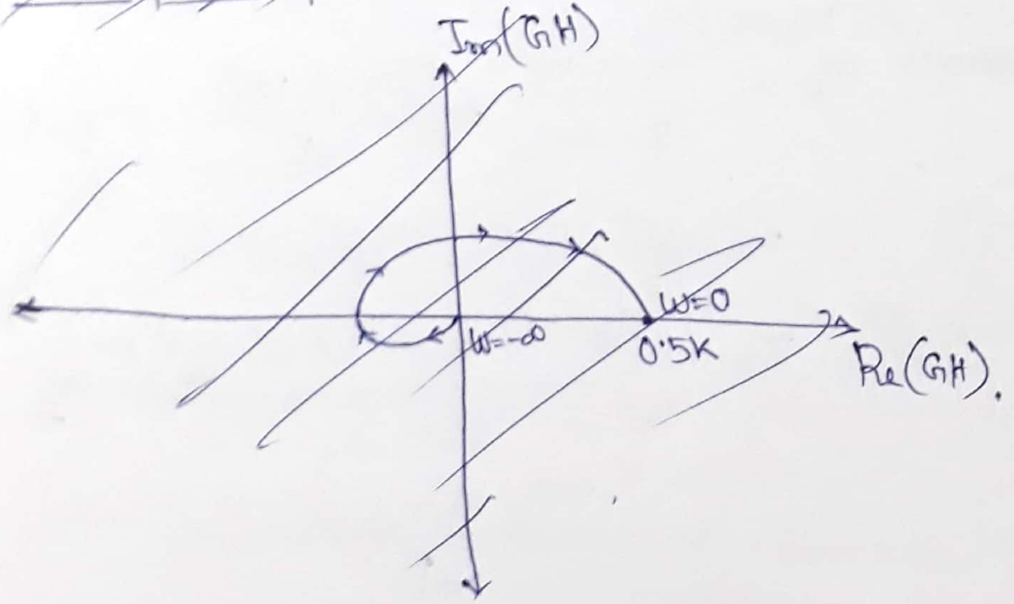
~~Gain of~~ Magnitude of the system $M = |G(j\omega)H(j\omega)| = \dfrac{k\sqrt{0.5^2+\omega^2}}{\sqrt{(1-\omega^2)^2+\omega^6}}$

phase $\phi = \angle G(j\omega)H(j\omega) = \tan^{-1}\left(\frac{\omega}{0.5}\right) - \tan^{-1}\left(\frac{-\omega^3}{-\omega^2+1}\right)$

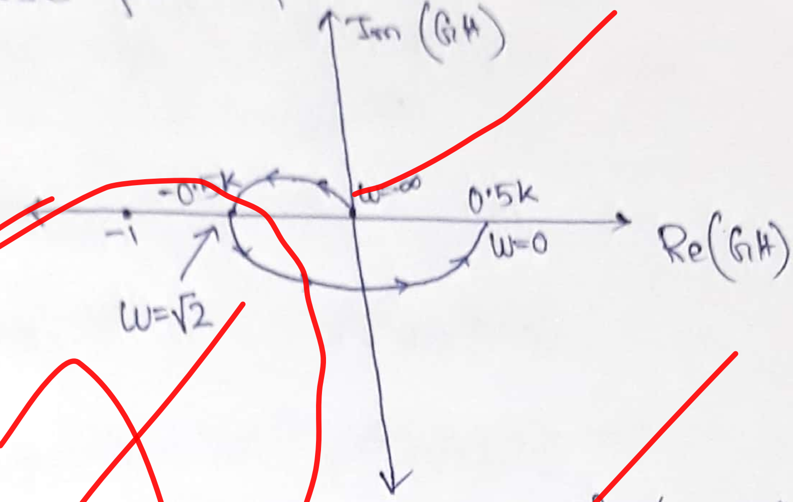
For inverse polar plot ω varies from $-\infty$ to 0.

ω	$M = \frac{K\sqrt{0.5^2+\omega^2}}{\sqrt{(1-\omega^2)^2+\omega^6}}$	$\phi = \tan^{-1}\frac{\omega}{0.5} - \tan^{-1}\left(\frac{-\omega^3}{-\omega^2+1}\right)$
$-\infty$	0	~~180°~~ ~~0°~~ −180°
−100	$9.99\times10^{-5}K$	−180.28°
−10	$9.96\times10^{-3}K$	−182°
−5	0.0394 K	−185°
−3	0.108 K	−187°
−2	0.24 K	−186
−1.5	0.439 K	−181°
−1	1.11 K	~~26.56°~~ −153.43°
−0.75	1.483K	−100.26°
−0.5	0.92K	−54.46
0	0.5K	0°

~~Inverse polar plot.~~



Inverse polar plot



For the system to be stable if $(-1+j0)$ point is left side of $(-0.5k, 0)$ point.

Therefore, System to stable if

$$0.5k < 1$$

$$\Rightarrow \quad k < 2$$

8) (c)

Given open Loop transfer function

$$G(s)\,H(s) = \frac{k}{s(s+4)(s^2+4s+13)}$$

poles are, $s = 0, -4, -2+j3, -2-j3$

Angle of asymptotes

~~Centroid~~ $\sigma_A = \dfrac{(2q+1)180^\circ}{P-Z} = \dfrac{(2q+1)\,180^\circ}{4} \quad (q = 0,1,2,3 ..)$

$$= 45^\circ, 135^\circ, 225^\circ, 315^\circ.$$

Centroid $\sigma_A = \dfrac{\sum \text{real part of poles} - \sum \text{real part of zeros}}{P-Z}$

$$= \frac{+0-4-2-2-0}{4}$$

$$= -2$$

Break away point

$1 + G(s)H(s) = 0$

$\Rightarrow \quad 1 + \dfrac{k}{s(s+4)(s^2+4s+13)} = 0.$

$\Rightarrow \quad k = -(s^4 + 8s^3 + 29s^2 + 52s)$

$\dfrac{dk}{ds} = -(4s^3 + 24s^2 + 58s + 52) = 0$

$s = -2, \; -2 \pm j1.58$

$s = -2$ is valid break away point.

Intersection on imaginary axis

Characteristics equation

$1 + G(s)H(s) = 0.$

$\Rightarrow \quad 1 + \dfrac{k}{s(s+4)(s^2+4s+13)} = 0.$

$\Rightarrow \quad \cancel{4s^3 + 24}$

$s^4 + 8s^3 + 29s^2 + 52s + k = 0.$

Routh table

s^4	1	29	k
s^3	8	52	
s^2	22.5	k	
s^1	$\dfrac{1170-8k}{22.5}$		
s^0	k		

(i) $k > 0$

(ii) $\dfrac{1170-8k}{22.5} > 0 \Rightarrow k < \dfrac{1170}{8}$

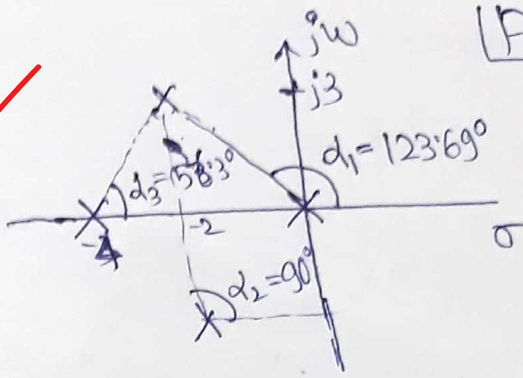
Marginal value of $k = \dfrac{1170}{8} = 146.25.$

Auxiliary equation $22.5 s^2 + k = 0.$

$s = \pm j\sqrt{\dfrac{k}{22.5}} = \pm j2.55$

Angle of departure

From the pole $-2+j3$



$$\phi_d = 180 + \phi$$
$$= 180 + (\phi_z - \phi_p)$$
$$= 180^\circ + (0 - \alpha_1 - \alpha_2 - \alpha_3)$$
$$= 180 - (123.69^\circ + 56.69^\circ + 90^\circ)$$
$$= -90^\circ$$

Angle of departure from the pole $-2-j3 \Rightarrow +90^\circ$ (by using symmetric property of root locus).

Therefore root locus plot is