(1)

Q. 1

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Given Data $a = 1.5\,cm$, $b = 0.8\,cm$, $\sigma = 0$, $\mu = \mu_0$, $\epsilon = 4\epsilon_0$

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin(\pi \times 10^{11} t - \beta z) \ A/m.$$

Comparing with

$$H_x = H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(\omega t - \beta z) \ A/m$$

$m = 1$ $n = 3$ $\omega = \pi \times 10^{11}$ Rad/sec

(i) Mode of operation TM_{13}

(ii) cutoff frequency

$$f_c = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{3 \times 10^8}{2\sqrt{4}}\sqrt{\left(\frac{1}{1.5}\right)^2 + \left(\frac{3}{0.8}\right)^2}$$

$f_c = 28.56$ GHz.

(iii) We know

$$\frac{\omega}{\beta} = V_p = \frac{c}{\sqrt{\epsilon_r}} \Rightarrow \beta = \frac{\omega\sqrt{\epsilon_r}}{c}$$

$$\beta = \frac{\pi \times 10^{11}\sqrt{4}}{3 \times 10^8} = 2093.33 \text{ Rad/m.}$$

(iv) Since $\sigma = 0 \Rightarrow \alpha = 0$

$$\gamma = \alpha + j\beta = j\,2093.33$$

04

Scanned with CamScanner

(v) $\eta_{TH} = \frac{\eta}{\cos\theta}$

$N = \pi \times 10^{11}$

$f = 50\ GHz$

$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{4}} = 60\pi$

$\cos\theta = \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \sqrt{1 - \left(\frac{28.56}{50}\right)^2}$

$\eta_{TH} = 229.529\ \Omega$

Wave impedence

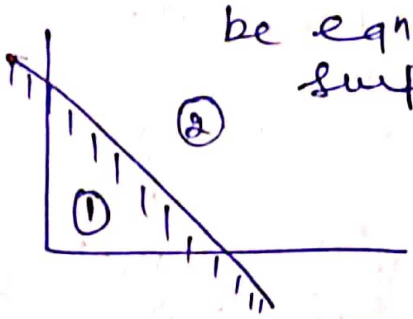
(b) Given

$\mu_1 = 4$

$\mu_2 = 6$

$B_1 = 2\vec{a_x} + \vec{a_y}\ (T)$

let $V = y + z - 1$ be eqn of boundary surface.



Normal to the boundary

$\vec{n} = \pm \nabla V$

$= \hat{a_y} + \hat{a_z}$

$\boxed{B_1 = B_{t1} + B_{n1}}$

$B_{t1} = 2a_x$

$B_{n1} = a_y$

$(\because \vec{n} \cdot B_{t1} = 0)$ B_{t1} ⊥ to Normal to boundary.)

from Boundary condition.

$B_{n1} = \boxed{B_{n2} = a_y}$

$h_{t2} - h_{t1} = \vec{K} \times \hat{n}$
$\Downarrow$
0

$\vec{h_{t2}} = \vec{h_{t1}}$

$\frac{B_{t2}}{\mu_2} = \frac{B_{t1}}{\mu_1}$

$$B_{t2} = B_{t1}\frac{\mu_2}{\mu_1}$$

$$= 2a_x \times \frac{6}{4}$$

$$= 3a_x$$

so, $B_2 = B_{t2} + B_{n2}$

$$= 3\hat{a_x} + \hat{a_y}$$

(ii) $\vec{H_2}$ Since $B = \mu H$

$$\vec{B_2} = \mu \vec{H_2}$$

$$\vec{H_2} = \frac{\vec{B_2}}{\mu_2} = \frac{3\hat{a_x} + \hat{a_y}}{6}$$

$$= \frac{1}{2}\hat{a_x} + \frac{1}{6}\hat{a_y}$$

✓

04

∇

(c)

$W = 10^6$ Rad/sec

~~[illegible]~~ $\nabla \times E = -\mu \frac{\partial H}{\partial t}$

$$\nabla \times \bar{E} = \frac{1}{\rho} \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho E_\phi & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[a_\rho\left(-\frac{\partial}{\partial z}(\rho E_\phi)\right) + 0 + a_z\left(\underbrace{\frac{\partial}{\partial \rho}(\rho E_\phi)}_{0}\right)\right]$$

$$= \frac{1}{\rho}\left[\hat{a_\rho}\left(-\frac{\partial}{\partial z}\left[50\cos(10^6 t + \beta z)\right]\right)\right]$$

$$\nabla \times \vec{E} = -\hat{a_\rho}\left[\frac{1}{\rho} 50\beta\,(-\sin(10^6 t + \beta z)\right] \quad \text{— ①}$$

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial t}\left(\frac{H_0}{\rho}\cos(10^6 t + \beta z)\right)\hat{a_\rho}$$

$$= \frac{H_0}{\rho}\times 10^6\left(-\sin(10^6 t + \beta z)\right)$$

$$-\mu_0\frac{\partial H}{\partial t} = +\frac{\mu_0 H_0}{\rho}\times 10^6\left(\sin(10^6 t + \beta z)\right)\hat{a_\rho} \quad \text{—②}$$

equating ① & ②

$\cancel{\frac{1}{\beta}}(50\beta)\sin(10^6t+\beta z) = \frac{\mu_0 H_0}{\cancel{\beta}} \times 10^6 \cancel{\sin(10^6t+\beta z)}$ (5)

$$H_0 = \frac{50\beta}{\mu_0 \times 10^6}$$

We know
$\beta = \omega\sqrt{\mu_0\epsilon_0}$ [free space]

$$\boxed{\beta = 10^6\sqrt{\mu_0\epsilon_0}}$$

$$= \frac{50 \times \cancel{10^6}\sqrt{\mu_0\epsilon_0}}{\mu_0 \times \cancel{10^6}}$$

$$= 50\sqrt{\frac{\epsilon_0}{\mu_0}} = \frac{50}{\eta_0} = \frac{50}{120\pi} = 0.13269 \text{ A/m}$$

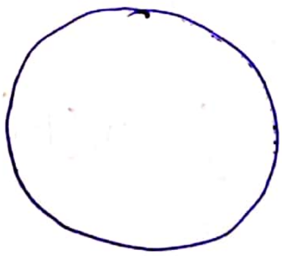
(ii) $\beta = \frac{\omega}{c}$ $\left[\because \frac{\omega}{\beta} = c \text{ (free space)}\right]$

$$\beta = \frac{10^6}{3\times10^8}$$

$$\boxed{\beta = \frac{1}{300} \text{ Rad/m}}$$

10 ✓

(d) Given Data



$l = 30$ cm

$A = 1$ cm2

$\mu_r = 2400$

$N = 2000$

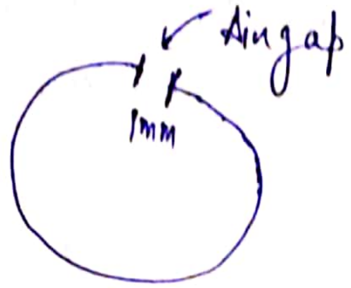
We know that

$$\phi = \frac{NI}{S} = \frac{NI\mu_0\mu_r A}{l}$$

$$\Rightarrow I = \frac{\phi l}{N\mu_0\mu_r A} = \frac{0.2\times10^{-3}\times30\times10^{-2}}{2000\times4\pi\times10^{-7}\times2400\times1\times10^{-4}}$$

$$= 99.52 \text{ mA}$$

(ii) $l_{air} = 1mm$



S_{air} Reluctance $= \frac{l_{air}}{\mu_0 \mu_r A}$

$= \frac{10^{-3}}{4\pi\times10^{-7}\times10^{-4}}$ $\boxed{\mu_r = 1}$ Air

$= \underline{7.96\times10^{6}}$ A/Wb

$l_{iron} = 30cm - 1mm = 0.299m$

$S_{iron} = \frac{0.299}{4\pi\times10^{-7}\times2400\times10^{-4}} = 0.991\times10^{6}$ A/Wb

We know

$$\phi = \frac{NI}{S_{air}+S_{iron}}$$

$$I = \frac{\phi(S_{air}+S_{iron})}{N} = \frac{0.2\times10^{-3}\times(7.96+0.991)\times10^{6}}{2000}$$

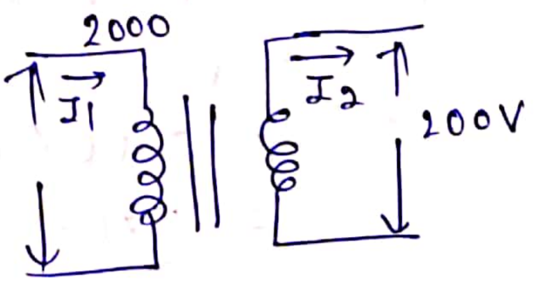
$$= 0.8951\ A$$

Conclusion:

As the Reluctance increases due to Air gap more current Required to maintain the same flux.

(1).

(e) Given 20KVA

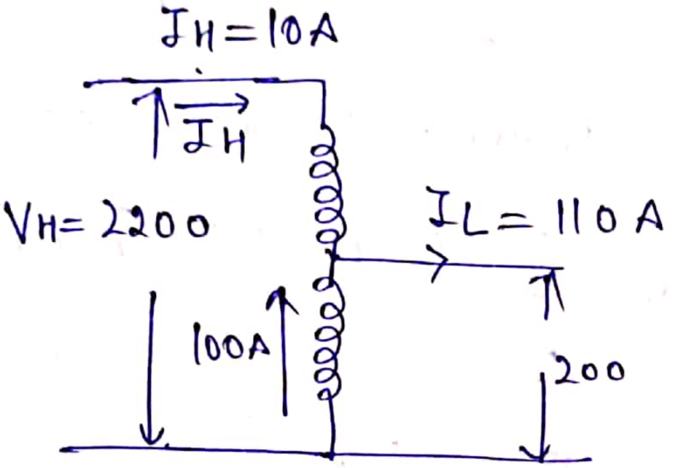


$I_1 = \frac{20\times10^3}{2000} = 10A$

$I_2 = \frac{20\times10^3}{200} = \underline{100A}$

Note current in the windings Remain same in Autotransformer as in Two winding Transformer.

(i) 2200/200 V



$(VA)_{magnetically} =$ O/P voltage × current in opposite branch

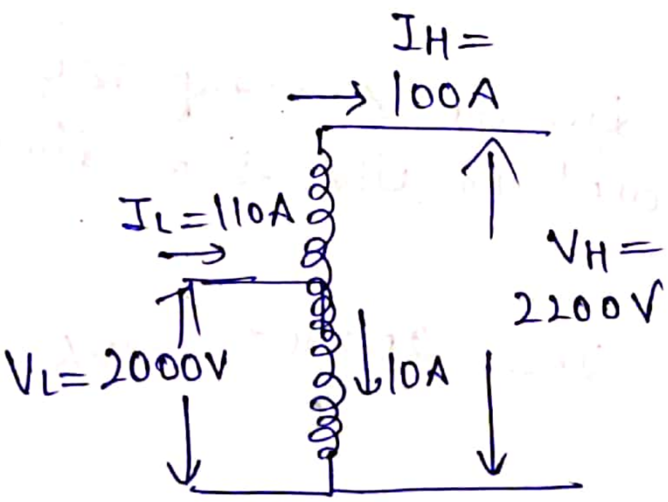
$= 200 \times 10$

$= \underline{2\,KVA}$

$(VA)_{conductively} = 110 \times 200$

$\Rightarrow \underline{22\,KVA}$

(ii) 2000/2200V



$(VA)_{magnetically} = V_H \times (I_L - I_H)$

$= 2200 \times 10$

$= \underline{2.2\,KVA}$

$(VA)_{conductively} = V_H \times I_H$

$= 2200 \times 100$

$= \underline{220\,KVA}$

10

Q.2 (a)

$$\bar{A} = A_0 \cos(\omega t - kz)\,\hat{a_y}$$

we know that $\boxed{B = \nabla \times \bar{A}}$

$$\nabla \times \bar{A} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & A_y & 0 \end{vmatrix}$$

$$\nabla \times \bar{A} = a_x\left[-\frac{\partial (A_y)}{\partial z}\right] + a_z\left[\underbrace{\frac{\partial}{\partial x}(A_y)}_{0}\right]$$

$$\nabla \times \bar{A} = a_x\left[-\frac{\partial}{\partial z}\left(A_0 \cos(\omega t - kz)\right)\right]$$

$$= a_x\left[-A_0(-k)\,\cancel{\cos}\,\sin(\omega t - kz)(-1)\right]$$

$$\nabla \times \bar{A} = \hat{a_x}\left[-A_0 k \sin(\omega t - kz)\right]$$

$$B_x = -A_0 k \sin(\omega t - kz)\, a_x$$

$$B_y = 0, \quad B_z = 0$$

(i) $B = \mu H$

$\cancel{B}\; H = \frac{B}{\mu} \Rightarrow H_x = -\frac{A_0 k}{\mu} \sin(\omega t - kz)\, a_x$

$H_y = 0, \quad H_z = 0$

$$\nabla \times E = -\mu \cdot \frac{\partial H}{\partial t}$$

$$a_x\left[\frac{\partial (E_z)}{\partial y} - \frac{\partial}{\partial z}\cancel{\;}(E_y)\right] = -\frac{A_0 k}{\mu} \times (-\mu) \cos(\omega t - kz) \times \omega \quad a_x$$

Since $\mathcal{E}$ is fn of z only $\frac{\partial \mathcal{E}_z}{\partial y} = 0$

So, $-\frac{\partial(\mathcal{E}_y)}{\partial z} a_x = + A_0 K W \cos(\omega t - Kz)\, a_x$

integrating both sides

$$\mathcal{E}_y = -\frac{A_0 \cancel{K} W \sin(\omega t - Kz)}{\cancel{-K}}\, \hat{a}_y$$

$$\mathcal{E}_y = A_0 W \sin(\omega t - Kz)\, \hat{a}_y$$

$$\boxed{\mathcal{E}_x = 0, \quad \mathcal{E}_z = 0}$$

$$\boxed{V = -\int \vec{\mathcal{E}} \cdot d\vec{l}}$$

$$V = -\int \mathcal{E}_y \vec{y} \cdot dy \vec{y}$$

$$V = -\int \mathcal{E}_y dy$$

$$V = -\int A_0 W \sin(\omega t - Kz) dy$$

$$\boxed{V = -A_0 W y \sin(\omega t - Kz) + C}$$

<u>we know</u>

$$\frac{\mathcal{E}_y}{H_x} = -\eta = -\sqrt{\frac{\mu}{\epsilon}}$$

$$\frac{\cancel{A_0} W}{\frac{-\cancel{A_0} K}{\mu}} = -\sqrt{\frac{\mu}{\epsilon}} \Rightarrow K = \frac{W \times \mu}{\sqrt{\frac{\mu}{\epsilon}}}$$

** $$\boxed{k = \frac{W}{\sqrt{4E}}}$$

08

(b) given Data

$$\boxed{R_a + R_{se} = 0.5\,\Omega}$$

$I_L = 60A = I_{a1}$

$V_L = 500V$

$$\boxed{\tau \propto N^3} \quad —(1)$$

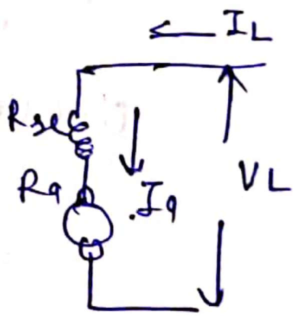
for series Motor

$$\boxed{\tau \propto \phi \cdot I_a}$$

In unsaturation

$$\boxed{\phi \propto I_a}$$

$$\tau \propto I_a^2 \quad —(2)$$



$\Rightarrow$ from (1) & (2)

$$\boxed{N^3 \propto I_a^2}$$

$$\left(\frac{N_2}{N_1}\right)^{3/2} = \frac{I_{a2}}{I_{a1}}$$

$$\boxed{I_{a2} = 0.6495\, I_{a1}}$$

$$E_{b1} = V_L - I_{a1}(R_a + R_{se})$$

$$= 500 - 60 \times 0.5$$

$$= 470\,V$$

let Rex be the Resistance to be Added.

$$E_{b2} = V_L - I_{a2}(R_a + R_{se} + R_{ex}) \quad —(3)$$

Also we know that

$E_b \propto \phi \cdot N \qquad \phi \propto I_a$

$$\frac{E_{b2}}{E_{b1}} = \frac{I_{a2}}{I_{a1}} \times \frac{N_2}{N_1}$$

$= 0.6495 \times 0.75$

$= 0.4871$

$E_{b2} = 0.4871 \times 470$

$= \underline{228.95V}$

from eqn (3)

$228.95 = 500 - 0.6495 \times 60 (0.5 + R_{ex})$

$0.6495 \times 60 (0.5 + R_{ex}) = 271.044$

$R_{ex} + 0.5 = 6.955 \, \Omega$

$\boxed{R_{ex} = 6.455 \, \Omega}$ 18

(c) given $Z_0 = 50\Omega$. $\gamma = 0 + j0.2\pi \; m^{-1}$

$f = 60 MHz$. $\gamma = \alpha + j\beta \Rightarrow \beta = 0.2\pi$ Rad/m

for lossless Transmission line $R = G = 0$

$\Rightarrow \alpha = 0$

$\boxed{\beta = W\sqrt{LC}} \quad \times \quad \boxed{Z_0 = \sqrt{\frac{L}{C}}}$

$\beta \cdot Z_0 = W \times L \Rightarrow \boxed{L = \frac{\beta \cdot Z_0}{W}}$

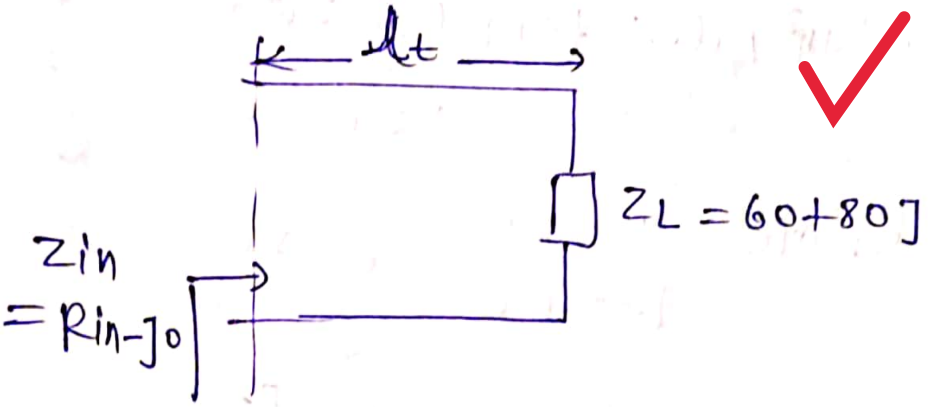
$$L = \frac{0.2\cancel{\pi} \times 50}{2 \times \cancel{\pi} \times 60 \times 10^6} = 8.33 \times 10^{-8} \ \underline{\underline{H/m}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$Z_0^2 = \frac{L}{C} \Rightarrow C = \frac{L}{Z_0^2} = \frac{8.33 \times 10^{-8}}{(50)^2}$$

$$\boxed{C = 83.33 \text{ pF}}$$

(ii)



We know

$$Z(x) = Z_0 \left[\frac{Z_L + j Z_0 \tan(\beta l)}{Z_0 + j Z_L \tan(\beta l)} \right]$$

$$Z_L = R + jX$$

$$Z(x) = Z_0 \left[\frac{R + j(X + Z_0 \tan(\beta l))}{Z_0 + j(R + jX)\tan(\beta l)} \right]$$

$$= Z_0 \left[\frac{R + j(X + Z_0 \tan(\beta l))}{Z_0 - X\tan\beta l + jR\tan\beta l} \right]$$

Rationalizing

$$= Z_0 \left[\frac{R + j(X + Z_0 \tan\beta l)}{(Z_0 - X\tan\beta l) + jR\tan\beta l} \right] \times \frac{(Z_0 - X\tan\beta l) - jR\tan\beta l}{(Z_0 - X\tan\beta l) - jR\tan\beta l}$$

(13)

$Z(x) = Z_{in}$ must be Real at $x = l_t$

Put Imaginary part $= 0$.

$$-R^2 \tan\beta l_t + (X + Z_0 \tan\beta l_t)(Z_0 - X\tan\beta l_t) = 0$$

$$-R^2 \tan\beta l + XZ_0 - X^2 \tan\beta l + Z_0^2 \tan\beta l - XZ_0 \tan^2\beta l = 0$$

$$XZ_0 \tan^2\beta l_t + \tan\beta l_t [-Z_0^2 + X^2 + R^2] - XZ_0 = 0$$

Substitute $X = 80$, $R = 60$, $Z_0 = 50$

$$4000 \tan^2\beta l_t + \tan\beta l_t [7500] - 4000 = 0$$

$$\tan\beta l_t = 0.4332$$

$$l_t = \frac{1}{\beta} \tan^{-1}(0.4332)$$

$$l_t = \frac{0.4085}{0.2 \times 3.14} = 0.65\ m$$

18

Q.3 (a)

(1) given Data (Poles) $P = 4$ $f = 50 Hz$

$V_L = 200 V$ (line voltage) $\quad V_1 = \frac{200}{\sqrt{3}}$ (per phase)

Rotor Resistance $(R_2) = 0.1 \Omega$

" Reactance $(X_2) = 0.9 \Omega$

let V_2 be Rotor Voltage

V_1 be stator voltage per phase

$\frac{(N_2) \text{ Rotor turns}}{(N_1) \text{ Stator turns}} = 0.67$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = 0.67$$

$$V_2 = 0.67 \times \frac{200}{\sqrt{3}} = \underline{77.364 V}$$

(i) when slip $s = 4\%$

we know Torque $\tau = \frac{60 \times 3}{2 \pi N_s} \times \frac{s V_2^2 R_2}{R_2^2 + s X_2^2}$

$$N_s = \frac{120 \times f}{P}$$

$$N_s = \frac{120 \times 50}{4}$$

$$= \underline{1500 RPM}$$

So, $\tau = \frac{60 \times 3}{2 \pi \times 1500} \times \frac{0.04 \times (77.364)^2 \times 0.1}{(0.1)^2 + (0.04 \times 0.9)^2}$

$$\boxed{\tau = 40.49 \text{ N-m}}$$

(2) $\tau_{max} = \frac{3 \times 60}{2 \pi N_s} \times \frac{V_2^2}{2 X_2} = \frac{3 \times 60}{2 \times \pi \times 1500} \times \frac{(77.364)^2}{2 \times 0.9}$

$$= \underline{63.537 \text{ N-m}}$$

(ii) Given Data

$f = 50 \quad P = 4 \quad V_L = 11 \times 10^3\ V$

$N = 1440$ rpm

$P_{out} = 100\ hp$ $\quad \boxed{1\ hp = 746\ Watt}$

$P_{out} = 100 \times 746 = \underline{74.6\ kW}$

Rotational losses $= 2500\ W = 2.5\ kW$

Stator cu losses $= 2000\ W$

$\boxed{\eta = ?}$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500$$

$$\text{Slip} = \frac{N_s - N}{N_s} = \frac{1500 - 1440}{1500} = \frac{60}{1500} = \frac{2}{50} = \frac{1}{25}$$

P_{out} + Rotational losses $= P_m$ (mechinical power Developed at rotor O/P)

$P_m = 74.6 + 2.5$

$= \underline{\quad} \ 77.1\ kW$

$\boxed{P_m = P_2(1-s)}$ where P_2 = power at ~~stat~~ Rotor Input

$$P_2 = \frac{P_m}{1-s} = \frac{77.1}{1 - \frac{1}{25}} = \underline{\quad\ kW} \quad 80.3125$$

Input power $= P_{in} = P_2$ + Stator cu loss

$= 80.3125 + 2$

$= 82.3125\ kW$

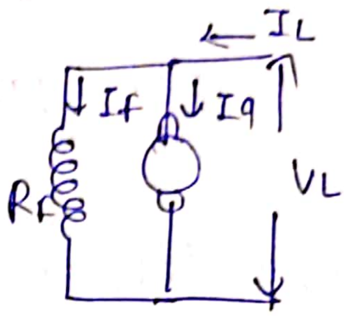
$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{74.6 \times 100}{82.3125} = \underline{90.63\%}$$

(b) given DC shunt motor

$r_a = 0.5\,\Omega$ $\quad N = 1000\,RPM$

$R_f = 220\,\Omega$ $\quad I_L = 20\,A$

$V_L = 220\,V$



R_f Increased by 5%

$$\Rightarrow R_f = 220 + 5\% \cdot 220 = \underline{231\,\Omega}$$

field current $I_f = \frac{V_L}{R_f} = \frac{220}{231} = \underline{0.9523\,A}$

$I_L = I_a + I_f$

So, $I_a = I_L - I_f = 20 - 0.9523$

$= \underline{19.047\,A}$

we know that E_b (back emf) $\propto \phi N$

In shunt motor $\phi \propto I_f$

So, $\boxed{\frac{E_{b1}}{E_{b2}} = \frac{I_{f1}}{I_{f2}} \times \frac{N_1}{N_2}}$

$I_{f1} = \frac{220}{220} = 1A \quad I_{f2} = 0.9523$

$$\boxed{E_{b1} = V_L - I_{a1} r_a}$$

$$I_{a1} = I_L - I_{f1}$$
$$= 20 - 1 = 19$$

$$E_{b1} = 220 - 19 \times 0.5 = \underline{210.5\,V}$$

$$E_{b2} = V_L - I_{a2} \times 0.5$$
$$I_{a2} = 19.047\,A$$

$$E_{b2} = 220 - 19.047 \times 0.5$$
$$= 210.4765$$

$$\frac{E_{b1}}{E_{b2}} = \frac{I_{f1}}{I_{f2}} \times \frac{N_1}{N_2}$$

$$\Rightarrow N_2 = \frac{I_{f1}}{I_{f2}} \times \frac{E_{b2}}{E_{b1}} \times N_1$$

$$= \frac{1}{0.9523} \times \frac{210.4765}{210.5}$$

$$N_2 = 1.0499\,N_1$$

$$N_2 = 1.0499 \times 1000\ \text{RPM}$$
$$= 1049.97 \simeq \underline{1050\ \text{RPM}}$$

(c) given Data

$f = 4.5\ GHz$

group velocity $V_g = 1.8 \times 10^8\ m/sec$

$\epsilon_r = 2.5$

$$V_g = V \times \cos\theta$$

Here $= \boxed{V = \frac{c}{\sqrt{\epsilon_r}}}$ $\qquad \cos\theta = \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$ — (1)

$$\cos\theta = \frac{V_g}{V} = \frac{V_g \times \sqrt{\epsilon_r}}{c}$$

$$= \frac{1.8 \times \cancel{10^8} \times \sqrt{2.5}}{3 \times \cancel{10^8}}$$

$$= 0.9486$$

from (1)

$$\cos^2\theta = 1 - \left(\frac{f_c}{f}\right)^2$$

$$f_c = f\sqrt{1 - \cos^2\theta}$$

$$f_c = 4.5 \times 10^9 \sqrt{1 - (0.9486)^2}$$

$$= 1.423\ GHz$$

$\boxed{f_c = f_{c10}}$

Assuming Dominant mode as TE_{10}

$$f_{c_{mn}} = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_{c10} = \frac{c}{2\sqrt{\epsilon_r}} \times \frac{1}{a}$$

$$\boxed{a = \frac{c}{2\sqrt{\epsilon_r} \times f_{c10}}}$$

$$a = \frac{3 \times 10^8}{2\sqrt{2.5} \times 1.423 \times 10^9}$$

$$\boxed{a = 6.67\ cm}$$

Since waveguide is a square $\boxed{a = b = 6.67\ cm}$

18

Q.5 (a)

$$y(n) = \sum_{K=n-n_0}^{n+n_0} x(K)$$

(i)

$$a_1 x_1(n) \xrightarrow{sys} \sum_{K=n-n_0}^{n+n_0} a_1 x_1(K) = a_1 y_1(n)$$

$$a_2 x_2(n) \xrightarrow{sys} \sum_{K=n-n_0}^{n+n_0} a_2 x_2(K) = a_2 y_2(n)$$

$$a_1 x_1(n) + a_2 x_2(n) \xrightarrow{sys} \sum_{K=n-n_0}^{n+n_0} a_1 x_1(K) + a_2 x_2(K)$$

$$= a_1 \sum_{K=n-n_0}^{n+n_0} x_1(K) + a_2 \sum_{K=n-n_0}^{n+n_0} x_2(K)$$

$$= a_1 y_1(n) + a_2 y_2(n)$$

$\Rightarrow$ system is linear. ✓

(ii)

$$x(n) \xrightarrow{sys} y(n)$$

$$x(n-n_0) \xrightarrow{sys} \sum_{K=n-n_0}^{n+n_0} x(K-n_0)$$

Put $K - n_0 = P \Rightarrow K = n_0 + P$

$$\sum_{P=n-2n_0}^{n} x(P) = \sum_{K=n-2n_0}^{n} x(K)$$

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$$y(n-n_0) = \sum_{k=n-n_0-n_0}^{n-n_0+n_0} x(k) = \sum_{k=n-2n_0}^{n} x(k)$$

since

$$x(n-n_0) \xrightarrow{sys} y(n-n_0)$$

$\Rightarrow$ system is time invariant

(iii) given $|x(n)| < B_x$ for all n

$$y(n) = \sum_{k=n-n_0}^{n+n_0} x(k)$$

Since $|x(n)| < B_x$

$$y(n) \leq \sum_{k=n-n_0}^{n+n_0} B_x$$

$$y(n) \leq 2n_0 B_x$$

given $|y(n)| \leq C$

$$\Rightarrow \boxed{C \leq 2n_0 B_x}$$

08

5(b) (i) $x(t)$ is Real & Odd
so X_n is odd & purely imaginary.

(ii) $x(t)$ is periodic with $T=2$

(iii) $X_n = 0$ for $|n| > 1$

X_0, X_1, X_{-1}

Since X_n is odd & purely imaginary.

so, $\boxed{X_0 = 0}$

$\boxed{X_1 = -X_{-1}}$ let $\boxed{X_1 = aJ}$

$X_{-1} = -aJ$

(iv) $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$

$$x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{Jwnt}$$

$$= C_1 e^{Jwt} + C_{-1} e^{-Jwt}$$

$$x(t) = J[a e^{Jwt} - a e^{-Jwt}]$$

$$= aJ\left[\frac{e^{Jwt} - e^{-Jwt}}{2J}\right] \times 2J$$

$$x(t) = -2a\sin(wt) \quad \text{—— (1)}$$

$$\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1 \Rightarrow \frac{1}{2}\int_0^2 4a^2\sin^2 wt \, dt = 1$$

$$\frac{1}{2}\int_0^2 4a^2\frac{(1-\cos 2wt)}{2}\,dt = 1$$

$T = 2$

$w = \frac{2\pi}{T}$

$\boxed{w = \pi}$

$$2a^2\left[\frac{1}{2}t - \frac{\sin 2wt}{2w\times 2}\right]_0^2 = 1$$

$$2a^2(\cancel{2}\times\frac{1}{\cancel{2}} - 0) = 1$$

$$a^2 = \frac{1}{2} \Rightarrow \boxed{a = \pm\frac{1}{\sqrt{2}}}$$

Substitute in eqn 1

so, $x(t) = -2\times\left(\pm\frac{1}{\sqrt{2}}\right)\sin(\pi t)$

so, $\boxed{x(t) = -\sqrt{2}\sin(\pi t)\ \cancel{\times} \\ +\sqrt{2}\sin(\pi t)}$

Both these satisfies all condition.

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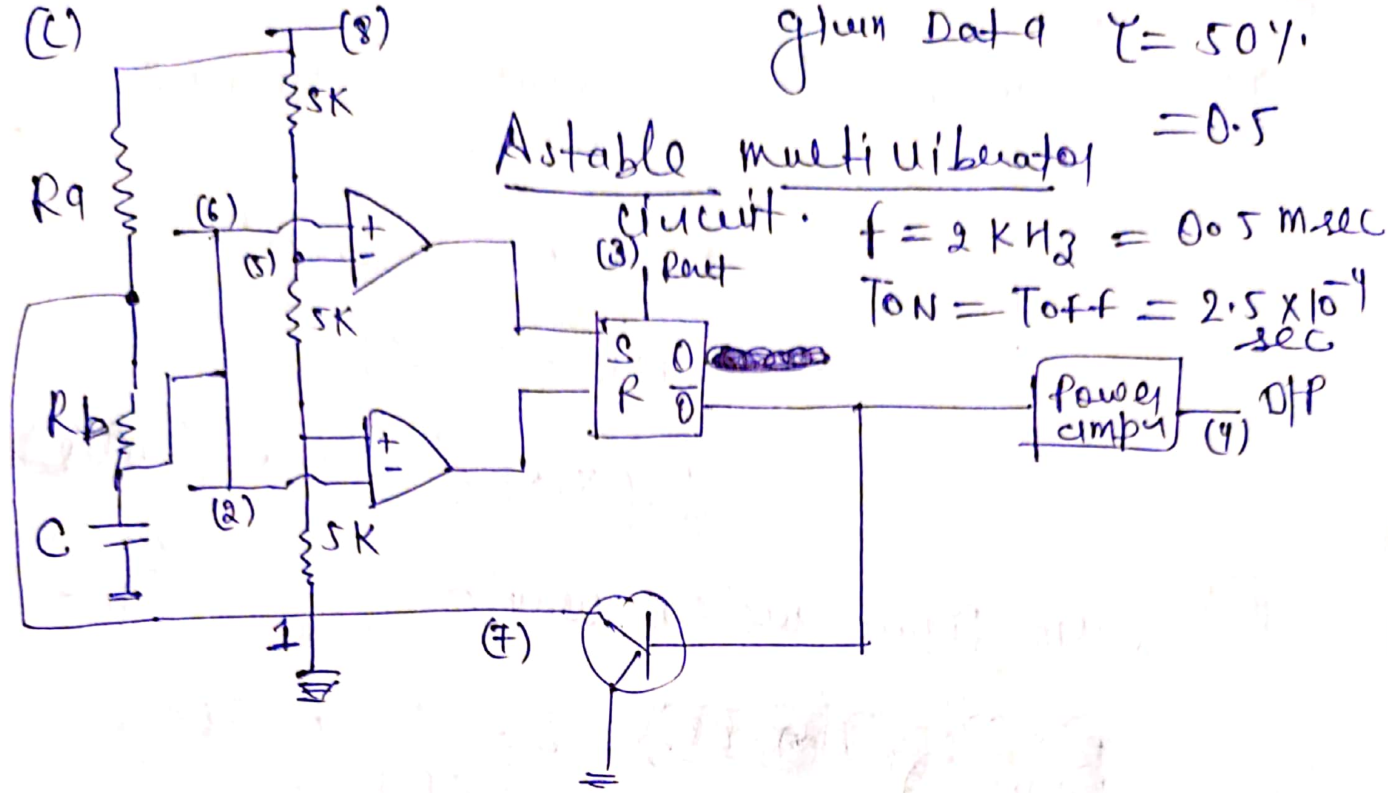
(C)

given Data $\tau = 50\%$
$= 0.5$

Astable multivibrator circuit.

$f = 2\,KHz = 0.5$ m-sec

$T_{ON} = T_{off} = 2.5 \times 10^{-4}$ sec



given $C = 0.1\,\mu F$

$\boxed{f = 2\,KHz}$

$T_{ON} = 0.693(Ra + Rb)\,C$

$T_{off} = 0.693\,Rb\,C$

$T_{ON} = T_{off} \Rightarrow$

$Ra + Rb = Rb$

$\Rightarrow \boxed{Ra = 0}$

$$\boxed{Rb = \frac{T_{off}}{0.693 \times C}} \Rightarrow Rb = \frac{2.5 \times 10^{-4}}{0.693 \times 0.1 \times 10^{-6}}$$

$= 3.607\,K\Omega$

08

(d)

(i) Total Hysteresis loss = Total Area under B–H curve

$= $ Area of parallelogram

$= $ width $\times$ Height

$= 400 \times (2) = 800 \times 10^3 \; \frac{A\text{-}Wb}{m^3}$

(ii) from figure we can write

$$B = \frac{1}{200}(H - 200)$$

$$BH = \frac{1}{200}(H^2 - 200H)$$

Diff. (BH) w.r.t H & put = 0

$$\frac{d(BH)}{dH} = \frac{1}{200}(2H - 200) = 0$$

$$\boxed{H = 100 \, KA/m}$$

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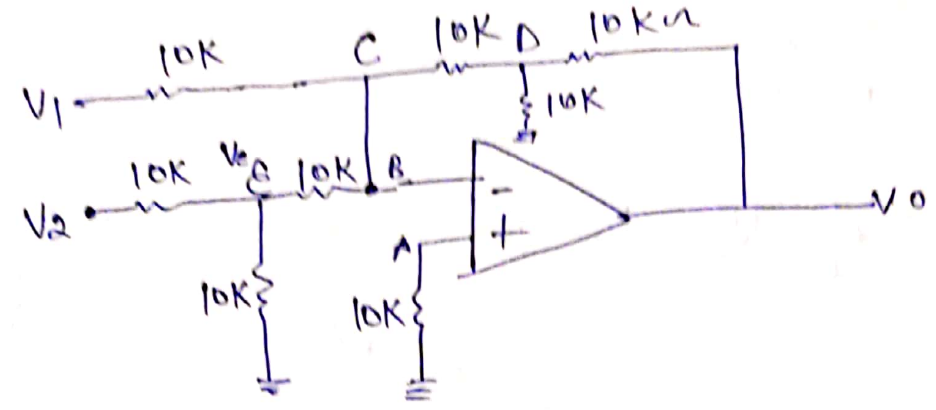
$$\left|(BH)_{max}\right| = \left|\frac{1}{200}\left(100^2 - 200 \times 100\right)\right|$$

$$= 50 \times 10^3 \; \frac{A\text{-}Wb}{m^3}$$

Scanned with CamScanner

(e)



using virtual short

$$\boxed{V_A = V_B} = 0$$

Nodal at Node E

$$\frac{V_E - V_2}{10K} + \frac{V_E}{10K} + \frac{V_E - V_B}{10K} = 0$$

$$V_E - V_2 + V_E + V_E - 0 = 0$$

$$\boxed{3V_E = V_2} \quad - (1)$$

Nodal at C $\quad (V_C = V_B = 0)$

$$\frac{V_C - V_1}{10} + \frac{V_C - V_E}{10} + \frac{V_C - V_D}{10} = 0$$

$$0 - V_1 + 0 - V_E + 0 - V_D = 0$$

$$V_1 + V_E + V_D = 0 \quad - (2)$$

Nodal at D

$$\frac{V_D - V_C}{10} + \frac{V_D}{10} + \frac{V_D - V_0}{10} = 0$$

$$V_D + V_D + V_D - V_0 = 0$$

$$\boxed{V_0 = 3V_D} \quad - (3)$$

put (2) in (3)

$$V_0 = 3[-V_1 - V_E] \quad \text{— (4)}$$

put (1) in (4)

$$V_0 = 3\left[-V_1 - \frac{V_2}{3}\right]$$

$$\boxed{\begin{aligned} V_1 &= 2 \\ V_2 &= 3 \end{aligned}}$$

$$V_0 = 3\left[-2 - \frac{3}{3}\right]$$

$$V_0 = 3[-2-1] = \underline{-9V}$$

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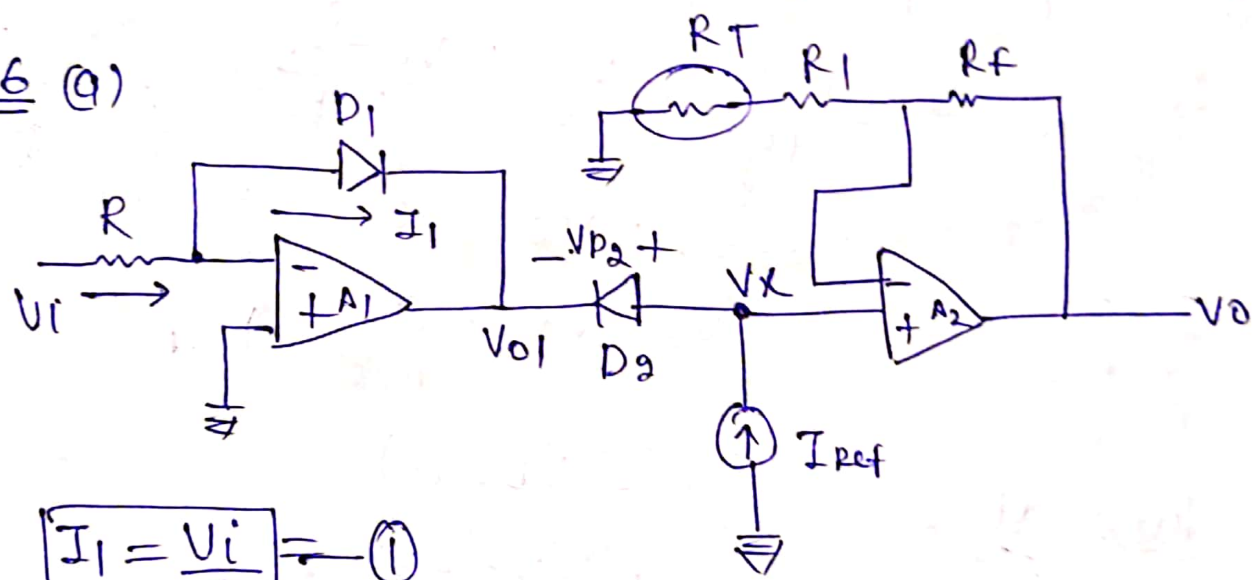
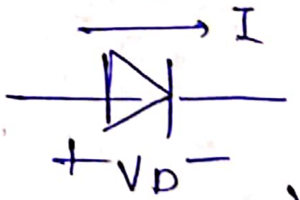
Q.6 (a)



$$\boxed{I_1 = \frac{V_i}{R}} \quad \text{— (1)}$$

from Diode eqn



$$I = I_0 \left(e^{\frac{V_D}{\eta V_T}} - 1\right)$$

$$\boxed{I \simeq I_0 e^{\frac{V_D}{\eta V_T}}}$$

so, $I_1 = I_0 e^{\frac{-V_{o1}}{\eta V_T}}$ — (2)

1 = 2

~~Vo~~ $\frac{V_i}{R} = I_o e^{\frac{-V_{ol}}{\eta V_T}}$

$$\frac{V_i}{I_o R} = e^{\frac{-V_{ol}}{\eta V_T}}$$

Taking log on both sides.

$$V_{ol} = -\eta V_T \ln\left(\frac{V_i}{I_o R}\right) \quad \text{—(3)}$$

where I_o Reverse saturation current.

for Diode D_2

$$\boxed{I_{Ref} = I_o e^{\frac{V_{D2}}{\eta V_T}}}$$

$$V_{D2} = \eta V_T \ln\left(\frac{I_{Ref}}{I_o}\right) \quad \text{—(4)}$$

Also $V_x = V_{D2} + V_{ol} \rightarrow$ (5)

substitute (3) & (4) in (5)

$$V_x = \eta V_T \ln\left(\frac{I_{Ref}}{I_o}\right) - \eta V_T \ln\left(\frac{V_i}{I_o R}\right)$$

$$V_x = -\eta V_T \ln\left(\frac{V_i}{\cancel{I_o} R} \times \frac{\cancel{I_o}}{I_{Ref}}\right)$$

$$V_x = -\eta V_T \ln\left(\frac{V_i}{I_{Ref} R}\right)$$

$$\boxed{V_x = -\eta V_T \ln\left(\frac{V_i}{I_{Ref} \times R}\right)}$$

using Nodal

$$\frac{V_x}{R_T+R_1} + \frac{V_x - V_0}{R_F} = 0$$

$$V_x\left[\frac{1}{R_T+R_1} + \frac{1}{R_F}\right] = \frac{V_0}{R_F}$$

$$V_0 = \left(\frac{R_F + R_T + R_1}{R_T + R_1}\right) V_x$$

$$\boxed{V_0 = -\eta V_T \ln\left(\frac{V_i}{I_{Ref} R}\right)\left[1 + \frac{R_F}{R_T+R_1}\right]}$$

So, we can see as temperature changes. I_0 of Diode changes but I_{Ref} is constant also variation in V_T can be compensated by change in R_T with temperature.

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(b)

We know correction factor for error due to pressure coil inductance

$$C.f = \frac{T.V}{M.V} = \frac{1}{1+\tan\phi . \tan\alpha}$$

where ϕ = load power factor angle.

α = pressure coil power factor angle given Data $R_p = 10{,}000\,\Omega$ $f = 50$

$L_p = 100\,mH$

$\boxed{\cos\phi = 0.1}$ $\tan\phi = 9.9498$

$$\tan\alpha = \frac{\omega L_p}{R} = \frac{2\pi \times 50 \times 100 \times 10^{-3}}{10000}$$

$$= \pi \times 10^{-3}$$

we know

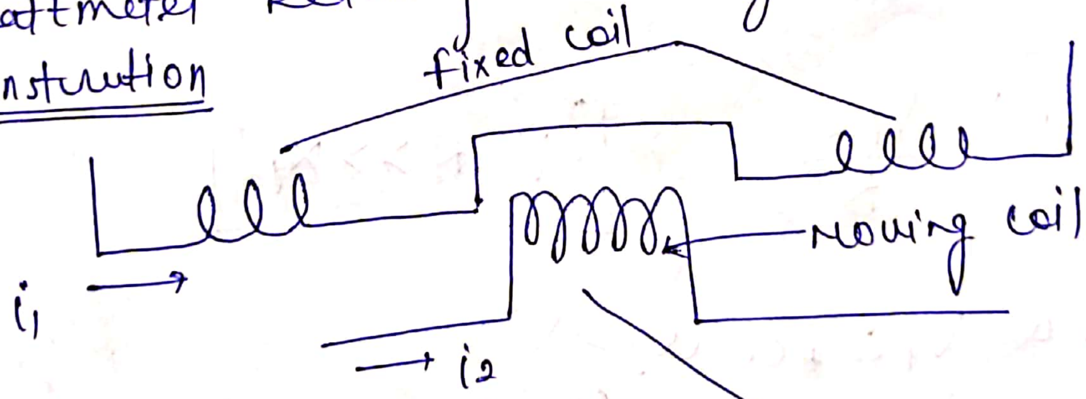
$$\%\ \text{error} = \frac{M.V - T.V}{T.V} \times 100 = \tan\phi \cdot \tan\alpha \times 100$$

$$= \pi \times 10^{-3} \times 9.9498 \times 100$$

$$= 3.1242\ \%$$

<u>Note</u> for lagging load, Due to pressure coil inductance wattmeter Reading show higher value.

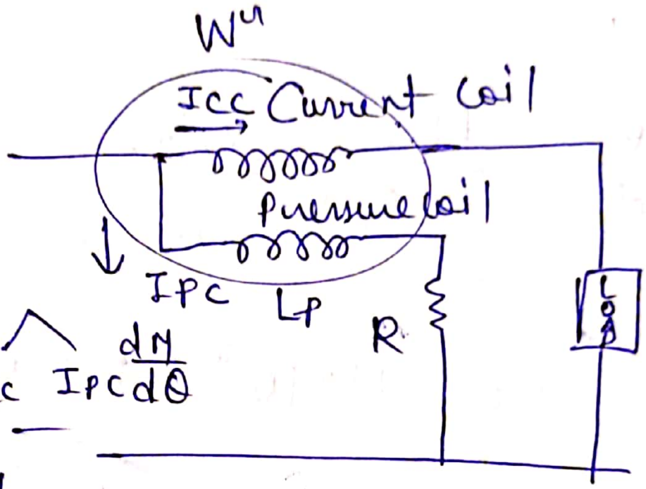
(ii) <u>construction</u>



electrodynamometer type wattmeter consist two coils (ED)
One fixed coil also called current coil & one moving coil also called pressure coil.

Principle: current coil is used to measure the current & pressure coil used to measure the potential.

When current passed through coils, Due to interaction between the flux of pressure coil & current coil torque is generated & which moves the pointer.



We know that

$$T_d = I_{cc} I_{pc} \cos \angle I_{cc} I_{pc} \frac{dM}{d\theta}$$

for the range $\frac{dM}{d\theta}$ = constant

$$T_d = K I_{cc} I_{pc} \cos \angle I_{cc} I_{pc}$$

$$I_{pc} \cong \frac{V_{pc}}{R} \quad [\because R >> X_p]$$

$$T_d \propto V_{pc} I_{cc} \cos \angle I_{cc} I_{pc}$$

$\boxed{T_d \propto \text{power across load}}$

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(C) given input-output Relationship

$$y(n) = \begin{cases} x(n) & ; n \geq 1 \\ 0 & ; n = 0 \\ x(n+1) & ; n \leq -1 \end{cases}$$

(i)

$$x(n) \xrightarrow{sys} y(n)$$

$$a_1 x_1(n) \longrightarrow \begin{cases} a_1 x_1(n) & n > 1 \\ 0 & n = 0 \\ a_1 x_1(n+1) & n \leq -1 \end{cases} = y_1(n)$$

$$a_2 x_2(n) \xrightarrow{sys} \begin{cases} a_2 x_2(n) & n > 1 \\ & n = 0 \\ a_2 x_2(n+1) & n \leq -1 \end{cases} = y_2(n)$$

$$a_1 x_1(n) + a_2 x_2(n) \xrightarrow{sys} \begin{cases} a_1 x_1(n) + a_2 x_2(n) & n > 1 \\ 0 & n = 0 \\ a_1 x_1(n+1) + a_2 x_2(n+1) & n \leq -1 \end{cases}$$

$$= a_1 y_1(n) + a_2 y_2(n)$$

Thus given system is linear.

(ii) Causal :

since $y(-5) = x(-5+1) = x(-4)$

since present value of o/p depend upon future value of Input

so, system non-Causal.

(iii) Time Invariant

$x(n) \xrightarrow{sys} y(n)$

$$x(n-n_0) \xrightarrow{sys} \begin{cases} x(n-n_0) & n \geq 1 \\ 0 & n=0 \\ x(n-n_0+1) & n \leq -1 \end{cases} \quad —①$$

$$y(n-n_0) = \begin{cases} x(n) ; & n-n_0 \geq 1 \\ 0 ; & n-n_0 = 0 \\ x(n+1) ; & n-n_0 \leq -1 \end{cases}$$

$$= \begin{cases} x(n) & n \geq n_0+1 \\ 0 & n = n_0 \\ x(n+1 & n \leq n_0-1 \end{cases}$$

Put $n-n_0 = k$ in ①

$$x(n-n_0) \xrightarrow{sys} \begin{cases} x(k) & k+n_0 \geq 1 \\ 0 & k+n_0 = 0 \\ x(k+1) & k+n_0 \leq -1 \end{cases}$$

Replace k by n

$$x(n-n_0) \longrightarrow \begin{cases} x(n) & n \geq 1-n_0 \\ 0 & n = -n_0 \\ x(n+1) & n \leq -1-n_0 \end{cases}$$

Since ~~x(n-n0)~~. $\neq y(n-n_0)$ so, Time variant system.

(iv) <u>Stable</u>

If $|x(n)| \leq M$ i.e If input is Bounded

$y(n)$ is also bounded

$\Rightarrow$ system is BIBO stable.

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