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PTQ

Prelims Through Questions

for

ESE 2021

General Studies & Engineering Aptitude

Day 2 of 11

Q.51 - Q.90

(Out of 500 Questions)

Engineering Aptitude + Engineering Mathematics

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Q.51 If a car went the 1st one-third of the distance at 80 kmph, the 2nd one-third of the distance at 24 kmph, and the last one-third at 48 kmph, what was the average speed of the car for the entire trip?

(a) 36 kmph
(b) 40 kmph
(c) 42 kmph
(d) 44 kmph

51. (b)

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Let the total distance be $3x$, then:

$$\text{Total time} = \frac{x}{80}+\frac{x}{24}+\frac{x}{48}=\frac{18x}{240};$$

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}=\frac{3x}{18x/240}=\frac{240\times 3}{18}=40 \text{ kmph}$$

Q.52 Rajni can do a piece of job in 6 days, Sonal in 8 days and Manisha in 12 days. Sonal and Manisha worked together for 2 days and then Rajni replaced Manisha. Now, how long the new partner will have to work to complete the job?

(a) 10 days
(b) 7 days
(c) 4 days
(d) 2 days

52. (d)

$$2(S+M) = 2\left(\frac{1}{8}+\frac{1}{12}\right)=2\left(\frac{3+2}{24}\right)=\frac{5}{12}$$

$$\text{Balance work} = 1-\frac{5}{12}=\frac{7}{12}$$

$$(R+S)\text{'s 1 days work} = \frac{1}{6}+\frac{1}{8}=\frac{4+3}{24}=\frac{7}{24}$$

∴ Balance work will be done by R and S in

$$\frac{7/12}{7/24} = 2 \text{ days}$$

Q.53 Consider the following matrix:

5	22	4
7	30	4
4	A	9
9	29	3

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What is the value of 'A' in the above matrix?

(a) 25 (b) 38

(c) 36 (d) None of these

53. (b)

Because in each row, First column × Third column + 2 = Second column.

So, $9 \times 4 + 2 = 38$

Q.54 A man bought a cycle and sold it at a gain of 10%. If he had bought the cycle at 20% less and sold it for ₹100 more than the previous selling price, he would have made a profit of 40%. The C.P of cycle is:

(a) ₹2000 (b) ₹4000

(c) ₹5000 (d) ₹6000

54. (c)

Let x be the CP and y be the SP of the cycle,

Now, $$\text{SP} = \left(\frac{100 + P\%}{100}\right)CP$$

$$y = \frac{110}{100}x \quad \text{... (i)}$$

and $$y + 100 = \frac{140}{100}\left(\frac{80}{100}x\right) \quad \text{... (ii)}$$

From equation (i) and (ii), we get

$$x = ₹5000$$

Alternative:

Taking option (c) i.e. ₹5000 as the CP

Initial CP = ₹5000

Initial SP = 1.1 × 5000 = ₹5500

New CP = 0.8 × 5000 = ₹4000

New SP = Initial SP + 100 = ₹5600

$$\text{Profit} = \frac{5600 - 4000}{4000} \times 100 = 40\%$$

Q.55 In a shooting competition, probability of A hitting target is 2/5, by B is 2/3 and C is 3/5. If all of them fire independently, what is the probability that only one will hit target?

(a) $\frac{3}{25}$ (b) $\frac{4}{25}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

55. (c)

P(x) = Probability of hitting target,

P(x") = Probability of not hitting target

= 1 – P(x), using this:

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$$P(A) = \frac{2}{5}, \text{ so, } P(A'') = \frac{3}{5}$$

$$P(B) = \frac{2}{3}, \text{ so, } P(B'') = \frac{1}{3}$$

$$P(C) = \frac{3}{5}, \text{ so, } P(C'') = \frac{2}{5}$$

We need: $P(A$ hits & B doesn't & C doesn't) OR P(A doesn't & B hits & C doesn't) OR P (A doesn't & B doesn't & C hits) which is: P(A)P(B")P(C") + P(A")P(B)P(C") + P(A")P(B")P(C)

$$= \left(\frac{2}{5}\right)\times\left(\frac{1}{3}\right)\times\left(\frac{2}{5}\right)+\left(\frac{3}{5}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{5}\right)+\left(\frac{3}{5}\right)\times\left(\frac{1}{3}\right)\times\left(\frac{3}{5}\right)$$

$$= \frac{4+12+9}{75} = \frac{25}{75} = \frac{1}{3}$$

Q.56 Three machines, individually, can do a certain job in 4, 5, and 6 hours, respectively. What is the greatest part of the job that can be done in two hours by two of the machines working together at their respective rates?

(a) $\frac{11}{15}$ (b) $\frac{9}{10}$

(c) $\frac{3}{5}$ (d) $\frac{8}{15}$

56. (b)

The greatest part of the work in 2 hours will be done by machines which take the lowest time to get the work done individually. Here machines with individual rates of 4 and 5 hours will do the maximum work in 2 hours.

Work Done in 1 hour will be $\frac{1}{4}+\frac{1}{5} = \frac{9}{20}$

Therefore, Work done in 2 hours will be = $\frac{9}{10}$

Q.57 There are 40 marbles in a jar. 1/5 of the marbles are blue, 1/4 of the remaining marbles are red, and 10 marbles are green. If a marble is selected at random, then what is the probability that the marble will not be blue, red, or green?

(a) $\frac{1}{4}$ (b) $\frac{3}{10}$

(c) $\frac{7}{20}$ (d) $\frac{13}{20}$

57. (c)

Out of the 40 marbles : Blue = 8, Red = 8, and Green = 10.

So, Total = 26

Left over marbles: 40 – 26 = 14

P(of left over marbles) = P(not Blue, Green or Red)

$$= \frac{14}{40} = \frac{7}{20}$$

Alternatively:

P(Blue, Green & Red): $\frac{8}{40}+\frac{8}{40}+\frac{10}{40}=\frac{13}{20}$

So, P(not Blue, Green or Red) $= 1-\frac{13}{40}=\frac{7}{20}$

Q.58 A dessert recipe calls for 50% melted cocoa and 50% blueberry puree to make a particular sauce. A chef accidentally makes 15 cups of the sauce with 40% melted cocoa and 60% blueberry puree instead. How many cups of the sauce does he need to remove and replace with pure melted cocoa to make the sauce the proper 50% of each?

(a) 1.5 (b) 2.5
(c) 3 (d) 4.5

58. (b)

We have 15 cups of sauce with 40% cocoa and 60% blueberry

$$\text{Cups of cocoa} = 0.4 \times 15 = 6$$
$$\text{Cups of blueberry} = 0.6 \times 15 = 9$$

Now lets say we removed x cups of original mix and replaced with x cups of cocoa.

Therefore, final number of cups of cocoa = $6 - 0.4x + x$

Now this number of cocoa cups should be 50% of total = $\frac{15}{2}=7.5$

Therefore $6 - 0.4x + x = 7.5$

On solving, $0.6x = 1.5 \Rightarrow x = 2.5$

Alternate solution:

Replacement method [Form of successive % decrease (change)]

$$\text{Final quantity} = \text{Initial quantity}\left[1-\frac{\text{Taken out quantity}}{\text{Total volume}}\right]^n$$

Initial: 15 cups (40% cocoa, 60% blueberry)
Final: 15 cups (50% cocoa, 50% blueberry)

$$\text{Final blueberry quantity} = \text{Initial blueberry quantity}\left[1-\frac{\text{Taken out quantity}}{\text{Total volume}}\right]^n$$

$$\frac{1}{2}(15) = \frac{3}{5}(15)\left[1-\frac{x}{15}\right]$$

$$\frac{5}{6} = 1-\frac{x}{15}$$

$$\frac{1}{6} = \frac{x}{15}$$

$$x = \frac{15}{6} = 2.5$$

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Q.59 Consider the given statement and the two conclusions that follow:

Statement:

1. Yoga is good for health.

Conclusions:

1. All healthy people practice Yoga.
2. Yoga is essential for maintaining good health.

What is/are the valid conclusion(s)?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

59. (d)

The statement simply says that Yoga is good for health. It is not presented as either the necessary or the sufficient condition for good health. Hence, conclusion 1 is incorrect as all healthy people need not practice Yoga. Similarly, conclusion 2 is also incorrect as Yoga is not an essential condition for good health. Hence, the correct answer is option (d).

Q.60 In a summer camp, there are five trainers A, B, C, D and E. A and B teach Swimming and Guitar. C and B teach Guitar and Singing. D and A teach Dancing and Swimming. E and B teach Painting and Skating. Who teaches maximum number of subjects?

(a) A
(b) B
(c) D
(d) E

60. (b)

Let's list down the activities taught by each of the five trainers,

	Painting	Swimming	Guitar	Singing	Dancing	Skating
A		✓	✓		✓	
B	✓	✓	✓	✓		✓
C			✓	✓		
D		✓			✓	
E	✓					✓

Hence, trainer B teaches the most number of subjects.

Q.61 A jar contains 12 marbles. Each is either yellow or green, yellow marbles are twice of green marbles. If two marbles are to be selected from the jar at random, what is the probability that exactly one of each color is selected?

(a) $\frac{8}{33}$
(b) $\frac{16}{33}$
(c) $\frac{1}{2}$
(d) $\frac{17}{33}$

61. (b)

Yellow marbles + Green marbles = 12

Yellow marbles = 2 × Green marbles

So, Green marbles = 4

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$$\text{Yellow marbles} = 8$$

$$\text{Probability} = \frac{{}^4C_1 \times {}^8C_1}{{}^{12}C_2} = \frac{16}{33}$$

Q.62 In how many ways can the letters of the word JUPITER be arranged in a row so that the vowels appear in alphabetic order?

(a) 736
(b) 768
(c) 792
(d) 840

62. (d)

7 letters can be arranged in 7! ways.
Three vowels i.e. E, I and U can be arranged in 3! ways.
But only one combination i.e. EIU is required.
Thus, total number of ways = 7!/ 3! = 840

Q.63 The external fencing of a circular path around a circular plot of land is 66 m longer than its interior fencing. The width of the path around the plot is:

(a) 21 m
(b) 11.5 m
(c) 10.5 m
(d) 12.5 m

63. (c)

In radius of circular plot = r meter
Width of path = x meter
Ex radius = $(r + x)$ m

According to question,

$$2\pi(r+x) - 2\pi r = 66$$
$$2\pi r + 2\pi x - 2\pi r = 66$$
$$2\pi x = 66$$
$$x = \frac{21}{2} = 10.5 \text{ meter}$$

Q.64 The following question consists of two statements and four conclusions. You have to examine these two statements carefully and select your answer:

Statements:
1. Some flowers are apples.
2. Some apples are stones.

Conclusions:
I. No flower is stone.
II. All apples are stones.
III. Some stones are flowers.
IV. No apple is flower.

Codes:
(a) Only either I or III follows
(b) Only I and IV follows
(c) Only II and III follows
(d) None follows

64. (d)

Since both the statements or premises are particular, no conclusion follows. Hence, the answer is (d)

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Q.65 Two clocks are set correctly at 10 AM on Sunday. The clocks gain 6 minutes and 10 minutes, respectively in an hour. What time will the second clock register, if the first clock shows the time as 54 minutes past 7 PM on the same day?

(a) 8:20 PM
(b) 8:30 PM
(c) 8:10 PM
(d) 8:54 PM

65. (b)

The time that elapses from 10AM to 7 PM on the day is 9 hours.

So, The first clock gains = 9 × 6 = 54 minutes and shows the time as 7 : 54 PM, while the second clock gains 9 × 10 = 90 minutes and shows the time as 8 : 30 PM.

Q.66 *A* does half as much work as *B*, in three-fourth of the time taken by *B*. If together they take 18 days to complete the work, how much time shall *B* take to do it?

(a) 10 days
(b) 15 days
(c) 20 days
(d) 30 days

66. (d)

$$\text{Man efficiency } \alpha \ \frac{1}{\text{days}}$$

Let's say B takes *X* days to finish 1 job, then

A will take $\frac{3}{4}(X)$ days to finish 0.5 job.

Now,
$$A : B$$
$$\frac{6}{4} : 1 \quad \text{(days)}$$
$$3 : 2 \quad \text{(days)}$$
$$2 \text{ unit} : 3 \text{ unit} \quad \text{(efficiency)}$$
$$A + B \text{ (18 days)} = 18 \times 5 = 90 \text{ unit work}$$
$$\therefore \quad B \text{ alone} = \frac{90}{3} = 30 \text{ days}$$

Q.67 A coin has two sides. One side has the number 1 on it and the other side has the number 2 on it. If the coin is flipped three times what is the probability that the sum of the numbers on the landing side of the coin will be greater than 4?

(a) $\frac{3}{8}$
(b) $\frac{1}{16}$
(c) $\frac{1}{8}$
(d) $\frac{1}{2}$

67. (d)

Cases for sum to be greater than 4 are as mentioned below

Case 1: (1, 2, 2)
Case 2: (2, 1, 2)
Case 3: (2, 2, 1)
Case 4: (2, 2, 2)

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$$\text{Total Outcomes} = 2 \times 2 \times 2 = 8$$
$$\text{Probability} = \text{Favorable Outcomes/Total Outcomes}$$
i.e. $$\text{Probability} = \frac{4}{8} = \frac{1}{2}$$

Q.68 Marla starts running around a circular track at the same time Nick starts walking around the same circular track. Marla completes 32 laps around the track per hour and Nick completes 12 laps around the track per hour. How many minutes after start will Marla have completed 4 more laps around the track than Nick?

(a) 5 (b) 8
(c) 12 (d) 15

68. (c)

Marla completes 32 – 12 = 20 more laps in 1 hour.

Marla to complete $4\left(\frac{20}{5} = 4\right)$ more laps will need $\frac{1}{5}$ hours, which is 12 minutes.

Q.69 The letters D, G, I, I , and T can be used to form 5-letter strings as DIGIT or DGIIT. Using these letters, how many 5-letter strings can be formed in which the two occurrences of the letter I are separated by at least one other letter?

(a) 12 (b) 18
(c) 24 (d) 36

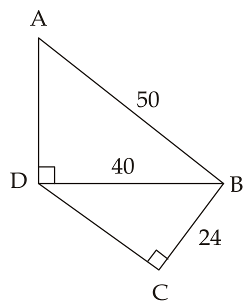
69. (d)

Total no of ways arranging 5 letter with one letter redundant is $\frac{5!}{2!} = 60$

No of ways two I's can be together 4! = 24

No of ways at least one alphabet is between two I's = 60 – 24 = 36

Q.70 What is the perimeter of quadrilateral ABCD given below?



(a) 106 units (b) 114 units
(c) 130 units (d) 136 units

70. (d)

BCD is a right angled triangle. Then DC = $\sqrt{1600 - 576}$ = 32 units

ADB is also a right angled triangle.

Then AD = $\sqrt{2500 - 1600} = \sqrt{900}$ = 30 units

Perimeter = 50+30+32+24 = 136 units

Q.71 Consider the following statements about complementary function (C.F.) of a differential equation:

1. If auxiliary equation has a pair of imaginary roots ($\alpha \pm i\beta$), then CF is of the form $e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x) + ...$
2. If auxiliary equation has two real and equal roots ($\alpha, \alpha, \beta, ...$), then CF is of the form $(C_1 + C_2 x + C_3 x^2)\, e^{\alpha x} + C_4\, e^{\beta x} + ...$
3. If auxiliary equation has real and different roots ($\alpha, \beta, \gamma, ...$), then CF is of the form $(C_1 e^{\alpha x} - C_2 x e^{\beta x} + C_3 x^2 e^{\gamma x}$ ....

Which of the above are INCORRECT statements?

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

71. (b)

If auxiliary equation has two real and equal roots, ($\alpha, \alpha, \beta, ...$), CF is of the form $(C_1 + C_2 x)e^{\alpha x} + C_3 e^{\beta x} + ...$

If auxiliary equation has real and different roots, ($\alpha, \beta, \gamma, ...$), then CF is of the form $C_1 e^{\alpha x} + C_2 e^{\beta x} + C_3 e^{\gamma x} + ...$

Statements 2 and 3 are incorrect.

$\Rightarrow$ Option (b) is correct.

Q.72 Which one of the following differential equations has the same order and degree?

(a) $\dfrac{d^4 y}{dx^4} + 8\left(\dfrac{dy}{dx}\right)^4 + 5y = e^x$

(b) $5\left(\dfrac{d^3 y}{dx^3}\right) + 8\left(\dfrac{dy}{dx} + 1\right)^2 + 5y = x^3$

(c) $\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{2/3} = 4\left(\dfrac{d^3 y}{dx^3}\right)$

(d) $y = x^2\left(\dfrac{dy}{dx}\right) + \sqrt{\left(\dfrac{dy}{dx}\right)^2 + 1}$

72. (c)

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2 = 4^3\left(\frac{d^3 y}{dx^3}\right)^3$$

$\therefore$ Order and degree of this equation, are both 3.

$\Rightarrow$ Option (c) is correct.

Q.73 The existence of the unique solution of the system $2x + 3y - z = 6$, $x + y + z = \lambda$, $5x - y + \mu z = 3$ depends on

(a) Both λ and μ
(b) μ only
(c) λ only
(d) Neither of them

73. (b)

For a unique solution, $\rho[A : B] = \rho(A)$ = Number of unknowns = 3.

$$[A : B] = \begin{bmatrix} 1 & 1 & 1 & \lambda \\ 2 & 3 & -1 & 6 \\ 5 & -1 & \mu & 3 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 5R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & \lambda \\ 0 & 1 & -3 & 6-2\lambda \\ 0 & -6 & \mu-5 & 3-5\lambda \end{bmatrix}$$

$R_3 \to R_3 + 6R_2$

$$= \begin{bmatrix} 1 & 1 & 1 & \lambda \\ 0 & 1 & -3 & 6-2\lambda \\ 0 & 0 & \mu-23 & 39-17\lambda \end{bmatrix}$$

$\rho(A) = 3$ if $\mu \neq 23$

however $\rho[A : B] = 3$ for all values of λ

$\therefore$ Existence of unique solution depend only on μ.

$\Rightarrow$ Option (b) is correct.

Q.74 Let C denotes the positive oriented circle $|z| = 2$ then $\oint_C \tan z dz =$

(a) $2\pi i$
(b) $4\pi i$
(c) $-4\pi i$
(d) 0

74. (c)

$$I = \oint_C \tan z dz$$

where, $C : |z| = 2$

There is a singularity at $\cos z = 0$, iff $z = (2n+1)\frac{\pi}{2}$

$\therefore$ $z = -\frac{\pi}{2}, \frac{\pi}{2}$ are the simple poles that lie inside C.

$$Res|_{z=\pi/2} = \lim_{z\to\pi/2}\left[(z-\pi/2)\frac{\sin z}{\cos z}\right]$$

($\frac{0}{0}$ form, applying L' hospital rule)

$$= \lim_{z\to\pi/2}\left[\frac{\sin z + \left(z-\frac{\pi}{2}\right)\cos z}{-\sin z}\right] = -1$$

$$Res|_{z=-\pi/2} = \lim_{z\to-\pi/2}\left[(z+\pi/2)\frac{\sin z}{\cos z}\right]$$

$$= \lim_{z\to-\pi/2}\left[\frac{\sin z+\left(z+\frac{\pi}{2}\right)\cos z}{-\sin z}\right] = \frac{-1}{-(-1)} = -1$$

$$\therefore \quad \oint_C \tan z dz = 2\pi i\left[R_{\pi/2}+R_{-\pi/2}\right] = 2\pi i[-1-1]$$

$$= -4\pi i$$

⇒ Option (c) is correct.

Q.75 For the differential equation,

$\cos^2\theta u_{xx} + \sin 2\theta\, u_{xy} + \sin^2\theta\, u_{yy} - 2u = 0$, consider the following:

1. Order of the given partial differential equation is 2.
2. Degree of the given partial differential equation is 2.
3. Given partial differential equation is hyperbolic.
4. Given partial differential equation is parabolic.

Select the correct set of statements using the codes given below:

(a) 1 and 3 only (b) 2 and 3 only
(c) 1 and 4 only (d) 2 and 4 only

75. (c)

'Order' of a PDE is defined as the order of highest derivative present,

∴ Order of given PDE = 2

Degree is the power of highest order derivative present,

∴ Degree of given PDE = 1

Comparing with general linear PDE.

$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + FU = G,$

$A = \cos^2\theta,\ B = \sin 2\theta,\ C = \sin^2\theta,$

Now, $B^2 - 4AC = \sin^2 2\theta - 4(\cos^2\theta \sin^2\theta)$

∵ $\sin 2\theta = 2\sin\theta\cos\theta$

$B^2 - 4AC = 0$

∴ Given PDE is parabolic.

Statements 1 and 4 are correct.

⇒ Option (c) is correct.

Q.76 If a matrix A is given as $A = \begin{bmatrix}1 & 9\\1 & 2\end{bmatrix}$ and a matrix I is defined as $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, then A^3 can be expressed in terms of A and I as:

(a) $3A + 7I$ (b) $9A + 49I$
(c) $16A + 21I$ (d) $19A + 12I$

76. (c)

Characteristic equation,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 9 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)(2 - \lambda) - 9 = 0$$

$$2 - 3\lambda + \lambda^2 - 9 = 0$$

$$\lambda^2 - 3\lambda - 7 = 0$$

According to Cayley-Hamilton theorem,

$$A^2 - 3A - 7I = 0 \quad \text{... (i)}$$

$\therefore$ $$A^2 = 3A + 7I$$

Multiplying with A, $$A^3 = 3A^2 + 7A$$

$$= 3(3A + 7I) + 7A$$

$$A^3 = 16A + 21I$$

$\Rightarrow$ Option (c) is correct.

Q.77 Value of $\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^{x+2}$ is equal to

(a) e (b) e^{-1}

(c) e^{-2} (d) None of these

77. (c)

$$\lim_{x\to\infty}\left(1+\frac{-2}{x+1}\right)^{(x+1)\frac{x+2}{x+1}} = \left[\lim_{x\to\infty}\left(1+\frac{-2}{x+1}\right)^{(x+1)}\right]^{\frac{x+2}{x+1}} \quad \left\{\lim_{n\to\infty}\left(1+\frac{p}{n}\right)^n = e^p\right\}$$

$$= \left(e^{-2}\right)^{\lim_{x\to\infty}\frac{1+2/x}{1+1/x}} = \left(e^{-2}\right)^1 = e^{-2}$$

$\Rightarrow$ Option (c) is correct.

Q.78 The type of the partial differential equation $\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$ is

(a) Parabolic (b) Elliptic

(c) Hyperbolic (d) Circular

78. (a)

Comparing with general partial differential equation,

$$A\frac{\partial^2 f}{\partial x^2} + B\frac{\partial^2 f}{\partial x \partial t} + C\frac{\partial^2 f}{\partial t^2} + D\frac{\partial f}{\partial x} + E\frac{\partial f}{\partial t} + F = 0$$

$A = 1, B = 0, C = 0, E = -1$

Now, since, $$B^2 - 4AC = 0$$

$\therefore$ Given partial differential equation is parabolic.

$\Rightarrow$ Option (a) is correct.

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Q.79 Suppose 5 men out of 100 and 25 women out of 10000 are color blind. A color blind person is chosen at random. What is the probability of the person being a male? (Assume male and female to be in equal numbers)

(a) 0.95
(b) 0.50
(c) 0.19
(d) 0.05

79. (a)

Probability of choosing a male $= P(M) = \frac{1}{2}$

Probability of choosing a female $= P(W) = \frac{1}{2}$

If B represents a blind person,

$$P\left(\frac{B}{M}\right) = \frac{5}{100} = 0.05$$

and

$$P\left(\frac{B}{W}\right) = \frac{25}{10000} = 0.0025$$

$\therefore$

$$P\left(\frac{M}{B}\right) =$$

$$\frac{P\left(\frac{B}{M}\right)\cdot P(M)}{P(M)\cdot P\left(\frac{B}{M}\right) + P(W)P\left(\frac{B}{W}\right)} = \frac{0.05\times 0.5}{(0.05\times 0.5)+(0.5\times 0.0025)}$$

$$= 0.95$$

$\Rightarrow$ Option (a) is correct.

Q.80 Order of integration in the double integral $I = \int_0^2\int_0^{4y} f(x,y)dxdy$ was changed such that the integral becomes $I = \int_p^q\int_r^s f(x,y)dydx$. What is the value of $\frac{r-p}{q-s}$?

(a) $\frac{24}{x}$
(b) $\frac{x}{24}$
(c) $\frac{x-8}{32}$
(d) $\frac{x-32}{8}$

80. (b)

When, $I = \int_0^2\int_0^{4y} f(x,y)dxdy$

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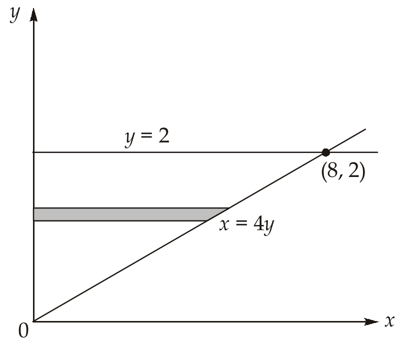
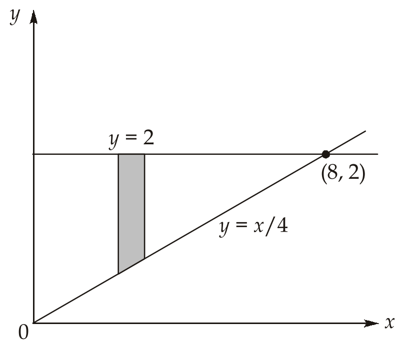


Now,



then, $$I = \int_0^8 \int_{x/4}^2 f(x,y)\,dy\,dx$$

∴ $p = 0 \qquad q = 8$

$r = \dfrac{x}{4}, \qquad s = 2$

⇒ $$\frac{r-p}{q-s} = \frac{x/4-0}{8-2} = \frac{x}{24}$$

⇒ Option (b) is correct.

Q.81 The particular integral of $(D^3 + 1)y = \cos(2x - 1)$ is

(a) $-\dfrac{1}{7}\cos(2x-1)$ (b) $\dfrac{2}{7}\sin(2x-1)$

(c) $\dfrac{1}{65}[\cos(2x-1) - 8\sin(2x-1)]$ (d) $\dfrac{1}{64}[\cos(2x-1) + 8\sin(2x-1)]$

81. (c)

$$\text{PI} = \frac{1}{D^3+1}\cos(2x-1) \qquad (\text{Put } D^2 = -2^2 = -4)$$

$$= \frac{1}{D(-4)+1}\cos(2x-1) = \frac{1+4D}{(1-4D)(1+4D)}\cos(2x-1)$$

$$= 1+4D\times\frac{1}{1-16D^2}\cos(2x-1)$$

Put $D^2 = -2^2 = -4$

$$= (1+4D)\times\frac{1}{1-16(-4)}\cos(2x-1)$$

$$= \frac{1}{65}\left[\cos(2x-1)+4D\cos(2x-1)\right]$$

$$= \frac{1}{65}\left[\cos(2x-1)-8\sin(2x-1)\right]$$

$\Rightarrow$ Option (c) is correct.

Q.82 If $u = (x^2 + y^2 + z^2)^{1/2}$, then $\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$ is equal to

(a) $4u$ (b) $2/u$
(c) $2u$ (d) $-u/4$

82. (b)

$\because$
$$u = (x^2 + y^2 + z^2)^{1/2}$$

$$\frac{\partial u}{\partial x} = \frac{2x}{2(x^2+y^2+z^2)^{1/2}} = \frac{x}{u}$$

$$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{u^2}\times\frac{x}{u}\times x+\frac{1}{u} = -\frac{y^2+z^2}{u^3}$$

Similarly,
$$\frac{\partial^2 u}{\partial y^2} = \frac{x^2+z^2}{u^3} \text{ and}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{y^2+z^2}{u^3}$$

$\therefore$
$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = \frac{2(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{3/2}} = \frac{2}{(x^2+y^2+z^2)^{1/2}} = \frac{2}{u}$$

$\Rightarrow$ Option (b) is correct.

Q.83 The residues of $f(z)=\frac{\sin\pi z^2+\cos\pi z^2}{(z-1)^2(z-2)}$ at its simple pole and second order pole are respectively.

(a) $1, 2\pi$ (b) $1, 2\pi+1$
(c) $2\pi+1, 0$ (d) $2\pi, 0$

83. (b)

$z = 1$ is a pole of order 2

$\therefore$
$$\text{Res } f(1) = \frac{1}{1!}\left[\frac{d}{dz}\left\{(z-1)^2 f(z)\right\}\right]_{z=1} = \left[\frac{d}{dz}\left(\frac{\sin\pi z^2+\cos\pi z^2}{(z-2)}\right)\right]_{z=1}$$

$$= \left[\frac{(z-2)(2\pi z\cos\pi z^2 - 2\pi z\sin\pi z^2) - (\sin\pi z^2 + \cos\pi z^2)}{(z-2)^2}\right]_{z=1}$$

$$= (-1)(-2\pi) - (-1) = 2\pi + 1$$

$$\text{Res } f(2) = \lim_{z\to 2}[(z-2)f(z)] = 1$$

$\Rightarrow$ Option (b) is correct.

Q.84 Out of all possible words, that can be made out of the word "MATHS", a random word in picked, what is the probability that the word will start with a "S"?

(a) $\frac{1}{5!}$ (b) $\frac{1}{4}$

(c) $\frac{1}{5}$ (d) $\frac{4}{5}$

84. (c)

$n(S)$ = All possible jumbles of "MATHS" = 5!

$n(E)$ = Jumbles that start with "S" = 4!

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4!}{5!} = \frac{1}{5}$

Q.85 Let $f(z) = u + iv$ is an analytic function. If $u - 2v = y^3 + 6xy^2 - 3x^2y - 2x^3$

Then $f(z)$ =

(a) $-z^3 + c$ (b) $iz^3 + c$

(c) $(1 + i)z^3 + c$ (d) $(-2 + i)z^3 + c$

85. (b)

$f(z) = u + iv$

$2i \times f(z) = -2v + 2iu$

On adding, $(1 + 2i) f(z) = (u - 2v) + i(v + 2u)$

Let, $F(z) = (1 + 2i) f(z)$, $U = u - 2v$ and $V = v + 2u$ ... (i)

Then, $F(z) = U + iV$

$U_x = 6y^2 - 6xy - 6x^2 = -6z^2$

$\therefore \quad U = u - 2v = y^3 + 6xy^2 - 3x^2y - 2x^3$ (Given)

Similarly, $U_y = 3y^2 + 12xy - 3x^2 = -3z^2$

$\therefore \quad F'(z) = U_x - iU_y$

$= -6z^2 + i3z^2$

$F(z) = -2z^3 + iz^3$

From (i) $(1 + 2i)f(z) = (-2 + i)z^3 + c$

$\therefore \quad f(z) = iz^3 + c$

$\Rightarrow$ Option (b) is correct.

Q.86 How many solutions does the following system of linear equations have?

$-x + 5y = -1, \quad x - y = 2, \quad x + 3y = 3$

(a) Infinitely many (b) Two distinct solutions

(c) Unique (d) None of the above

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86. (c)

$$\text{Augmented matrix} = \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right]$$

Using Gauss-elimination,

$$\left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] \xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 + R_1} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 8 & 2 \end{array}\right] R_2 \to R_3 - 2R_2 \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

Rank [A | B] = 2 (Number of non-zero rows in [A | B])

Rank [A] = 2 (Number of non-zero rows in [A])

Rank [A | B] = Rank [A]

= 2 = Number of variables

∴ Unique solution exists.

⇒ Option (c) is correct.

Q.87 The value of integral $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is

(a) $\log \pi$

(b) $\log 2\pi$

(c) $\tan^{-1} e - \frac{\pi}{4}$

(d) $\cos^{-1} e - \frac{\pi}{4}$

87. (c)

$$I = \int_0^1 \frac{dx}{e^x + e^{-x}} = \int_0^1 \frac{e^x dx}{e^{2x} + 1}$$

Let, $e^x = t, \quad e^x dx = dt$

$$I = \int_1^e \frac{dt}{t^2 + 1} = \left[\tan^{-1} t\right]_1^e = \tan^{-1} e - \frac{\pi}{4}$$

⇒ Option (c) is correct.

Q.88 The solution of $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ is

(a) $y = (C_1 + C_2 \log x)x + \log x + 2$

(b) $y = (C_1 + C_2 \log x)x + \frac{1}{2}\log x + 2$

(c) $y = (C_1 + C_2 e^x)x + e^x + 2$

(d) None of the above

88. (a)

This is Cauchy's homogeneous linear equation put, $x = e^t$, $t = \log x$

Such that $x\frac{dy}{dx} = Dy, \quad x^2 \frac{d^2y}{dx^2} = D(D-1)y$ where $D = \frac{d}{dt}$

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Then the given equation becomes $[D(D-1) - D + 1]y = t$ or $(D-1)^2 y = t$ ... (i)
Its AE is $(D-1)^2 = 0$, where $D = 1, 1$.

$$\therefore \quad CF = (C_1 + C_2 t)e^t \text{ and PI} = \frac{1}{(D-1)^2}t$$

$$\Rightarrow \quad (1-D)^{-2}t = (1 + 2D + 3D^2 + ...)t = t + 2$$

Hence, solution of (i) is,

$$y = (C_1 + C_2 t)e^t + t + 2$$

$$\because \quad t = \log x \text{ and } e^t = x,$$

$$\therefore \quad y = (C_1 + C_2 \log x)x + \log x + 2$$

⇒ Option (a) is correct.

Q.89 The Newton-Raphson method is used to find roots of equation $f(x) = x + \sin\pi x$ on $0 \le x \le 1$. If initial value is $x = 0.5$, then value of x after first iteration will be:

(a) 0.5
(b) –1
(c) 1
(d) 1.5

89. (b)

$$f(x) = x + \sin\pi x$$

$$f'(x)' = 1 + \pi\cos\pi x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{0.5+1}{1+\pi\cos\frac{\pi}{2}}$$

$$= 0.5 - 1.5 = -1$$

⇒ Option (b) is correct.

Q.90 If $y = x + 1$ and $x = 3y - 7$ are two regression lines, then co-efficient of correlation is

(a) 1.414
(b) 0.577
(c) 0.707
(d) 1.73

90. (b)

$byx = 1,$ $\quad bxy = 3$

then, $\quad r = \sqrt{byx \cdot bxy} = \sqrt{3} > 1$

$\therefore x = y - 1,$ $\quad e_y = \frac{7+x}{3}$

$bxy = 1,$ $\quad byx = \frac{1}{3}$

$$\text{Coefficient of correlation, } r = \sqrt{1\cdot\frac{1}{3}} = \frac{1}{\sqrt{3}} = 0.577$$

⇒ Option (b) is correct.

■■■■