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ESE 2025 : Prelims Exam
CLASSROOM TEST SERIES

GS & ENGINEERING APTITUDE

Test 1

Section A : Reasoning & Aptitude **[All Topics]**

Section B : Engineering Mathematics **[All Topics]**

ANSWER KEY

1.	(c)	**11.**	(c)	**21.**	(b)	**31.**	(a)	**41.**	(c)
2.	(c)	**12.**	(c)	**22.**	(b)	**32.**	(a)	**42.**	(a)
3.	(c)	**13.**	(a)	**23.**	(c)	**33.**	(a)	**43.**	(a)
4.	(d)*	**14.**	(a)	**24.**	(a)	**34.**	(a)	**44.**	(b)
5.	(c)	**15.**	(a)	**25.**	(d)	**35.**	(c)	**45.**	(c)
6.	(c)	**16.**	(b)	**26.**	(a)	**36.**	(a)	**46.**	(a)
7.	(b)	**17.**	(a)	**27.**	(c)	**37.**	(b)	**47.**	(a)
8.	(b)	**18.**	(c)	**28.**	(d)	**38.**	(d)	**48.**	(d)
9.	(b)	**19.**	(a)	**29.**	(b)	**39.**	(d)	**49.**	(c)
10.	(b)	**20.**	(b)	**30.**	(a)	**40.**	(a)	**50.**	(b)

***Q.4 [*Answer key has been Updated*]**

Section A : Reasoning & Aptitude

1. (c)

Sum of the n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....$

$$= \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+....$$

$$= n-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}....+n \text{ terms}\right)$$

$$= n-\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} = n+2^{-n}-1$$

2. (c)

Arrange the letters of the word COCHIN in the alphabetical order : CCHINO.

Number of words with the two C's occupying first and second place = 4!

Number of words starting with CH, CI, CN is 4! each.

Thus, number of words before the first word starting with CO = 4! + 4! + 4! + 4! = 96.

The word starting with CO found first in the dictionary is COCHIN. There are 96 words before COCHIN.

3. (c)

LCM of 12, 15 and 25 is 300.

6 digit smallest number divisible by 12, 15 and 25 is 100200.

Required remainder of $\frac{100200}{9}$ is 3.

4. (d)

Replacing symbols and using BODMAS rule, we get

Option (a) : $12 \div 2 \times 3 - 8 + 1 = 6 \times 3 - 7 = 11$

Option (b) : $11 + 2 - 4 \div 2 \times 1 = 13 - 2 = 11$

Option (c) : $7 - 2 + 5 \div 5 \times 2 = 7 - 2 + 2 = 7$

Option (d) : $5 + 6 \div 3 - 3 \times 1 = 5 + 2 - 3 = 4$

5. (c)

We can write, $7^{13} = (6 + 1)^{13}$

In binomial expansion, each term except 1^{13} is divisible by 6. Thus, when 7^{13} is divided by 6, it leaves remainder 1.

When $7^{13} + 1$ is divided by 6, it leaves remainder 1 + 1 = 2

6. (c)

$$|x|^2 - 3|x| + 2 = 0$$

$$\Rightarrow \quad |x|^2 - |x| - 2|x| + 2 = 0$$

$$\Rightarrow \quad [|x| - 2][|x| - 1] = 0$$

$$\Rightarrow \quad |x| = 1 \text{ or } 2$$

or $\quad x = \pm 1, \quad x = \pm 2$

$\therefore \quad$ Number of solutions $= 4$

7. (b)

$$x^2 + 11x + 50 = 0$$

If roots are α and β, we have

$$\alpha + \beta = -11, \alpha\beta = 50$$

$$\therefore \quad \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(11)^2 - 2\times 50}{50} = \frac{121 - 100}{50}$$

$$= \frac{21}{50} = 0.42$$

8. (b)

$$^{2n}C_3 : {}^nC_2 = 12 : 1$$

$$\because \quad {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$\frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{2!(n-2)}} = \frac{12}{1}$$

$$\frac{\frac{2n(2n-1)(2n-2)}{3\times 2\times 1}}{\frac{n(n-1)}{2\times 1}} = \frac{12}{1}$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)} = 12 \times 3$$

$$\frac{2n\times(2n-1)\times 2(n-1)}{n(n-1)} = 36$$

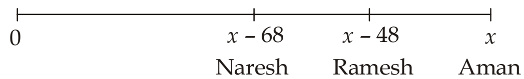
$$2n = 10$$

$$n = 5$$

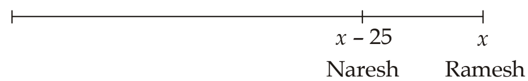
9. (b)

Let the length of race is x

Case I



Case II



Let V_A, V_R and V_N be the speed of Aman, Ramesh and Naresh respectively. Thus,

$$\frac{v_A}{v_R} = \frac{x}{x-48} \text{ and } \frac{v_A}{v_N} = \frac{x}{x-68}$$

$$\therefore \quad \frac{v_R}{v_N} = \frac{x-48}{x-68} \quad \text{...(i)}$$

Also, we have

$$\frac{v_R}{v_N} = \frac{x}{x-25} \quad \text{...(ii)}$$

From equation (i) and (ii), we can write

$$\frac{x}{x-25} = \frac{x-48}{x-68}$$

$$\therefore \quad x = 240 \text{ m}$$

The length of the race is 240 meters.

10. (b)

Let the initial bank balance of A, B and C be $10x$, $12x$ and $5x$, respectively. A transfer of ₹60,000 makes a change of $3x$ in the ratio. Thus,

$$3x = 60000$$

$$x = 20000$$

$$10x = ₹200000$$

11. (c)

Given : Radius of cone, r = 5 cm

Height, h = 24 m

Radius of cylinder, r_c = 10 cm

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{\pi}{3} \times 25 \times 24$$

Volume of water in the cylinder = Volume of cone

$$\pi r_c^2 d = \frac{\pi}{3} \times 25 \times 24$$

$$\text{Depth of water, } d = \frac{\frac{1}{3}\pi \times 25 \times 24}{\pi \times 100}$$

$$= \frac{25 \times 8}{100} = 2 \text{ cm}$$

12. (c)

Probability of an individual suffering from a particular disease,

$$p = \frac{20}{100} = 0.2$$

$$q = 1 - p = 0.8$$

Using binomial probability distribution, the probability that 2 individuals in a sample of 5 individuals would be suffering from disease is given by

$$^5C_2 \times q^{5-2} \times p^2 = \frac{4 \times 5}{2} \times (0.512) \times (0.04) = 0.2048$$

13. (a)

Let rate upstream = x km/h

and rate downstream = y km/hr

Then $$\frac{40}{x} + \frac{55}{y} = 13 \quad \text{...(i)}$$

and $$\frac{30}{x} + \frac{44}{y} = 10 \quad \text{...(ii)}$$

Multiplying (ii) by 4 and (i) by 3 and subtracting, we get

$$\frac{11}{y} = 1$$

$$y = 11$$

Substituting $y = 11$ in equation (i), we get

$$x = 5$$

$$\text{Rate in still water} = \frac{1}{2}(11 + 5) = 8 \text{ kmph}$$

$$\text{Rate of current} = \frac{1}{2}(11 - 5) = 3 \text{ kmph}$$

14. (a)

$$\angle ADC = 90° \text{ and } \angle CDE = 60°$$

$\therefore$ $$\angle ADE = 90° + 60° = 150°$$

ΔEDC is equilateral triangle

$\therefore$ $$AD = DE$$

Since angles opposite to equal sides of a triangle are equal. Thus, in ΔADE,

$$\angle DAE = \angle DEA = x$$

$$\therefore \quad \angle ADE = \angle DAE + \angle DEA = 180°$$

$$150 + x + x = 180°$$

$$\therefore \quad x = 15°$$

15. (a)

$$\text{Ways to choose first group} = {}^{15}C_1 \times {}^{15}C_1 = 225$$

$$\text{Ways to choose second group} = {}^{14}C_1 \times {}^{14}C_1 = 196$$

$$\text{Therefore, ways to make 15 groups} = 15^2 + 14^2 + 13^2 + ... + 1^2$$

$$= \frac{15(15+1)(2\times15+1)}{6} = \frac{15\times16\times31}{6} = 1240$$

Note, sum of square of first 'n' natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

16. (b)

Arranging the data in ascending order : 2, 4, 6, 8, 10, 12, 14, 16

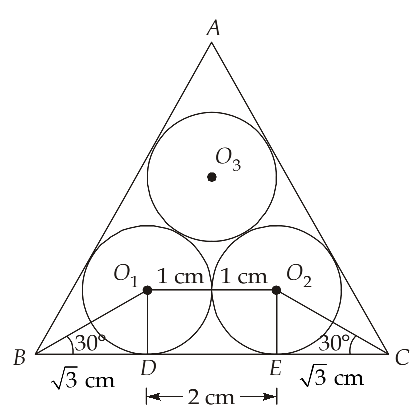
$$\because \quad N = 8$$

$$\therefore \quad \text{Median} = \frac{\left(\frac{N}{2}\right)^{th}\text{ term} + \left(\frac{N}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \frac{4^{th} + 5^{th}}{2} = \frac{8+10}{2} = 9$$

17. (a)

Since, tangents drawn from an external point to the circle subtends equal angle at the centre.



$\therefore$ $\angle O_1BD = 30°$

In ΔO_1BD, $\tan 30° = \frac{O_1D}{BD}$

$\Rightarrow$ $BD = \sqrt{3}$ cm [$\because$ O_1D = Radius of coin = 1 cm]

Also, $DE = O_1O_2 = 2$ cm and $EC = \sqrt{3}$ cm

Thus, $BC = BD + DE + EC$

$= 2 + 2\sqrt{3}$

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4}(BC)^2 = \frac{\sqrt{3}}{4} \times 4\left(1+\sqrt{3}\right)^2$$

$$= \left(6 + 4\sqrt{3}\right) \text{ square cm}$$

18. (c)

Part of cistern filled by A, B and C together in 1 hour $= \frac{1}{6}$

$\therefore$ For the first 2 hours, filled portion $= 2 \times \frac{1}{6} = \frac{1}{3}$

The remaining part of cistern filled by A and B together in 1 hour $= \frac{1}{8}$

$\therefore$ $1 - \frac{1}{3} = \frac{2}{3}$ part of cistern is filled in 8 hours

$\therefore$ $A + B$ can fill the cistern in $\frac{8 \times 3}{2} = 12$ hours

We have, $\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = \frac{1}{6}$

$\Rightarrow$ $\frac{1}{12} + \frac{1}{C} = \frac{1}{6}$

$\Rightarrow$ $C = 6$ hours

19. (a)

Number of candidates who failed in at most two subjects = Candidates failed in one subject + candidates failed in two subjects

$= (25 + 50 + 25) + (12 + 15 + 15) = 142$

$$\text{Required percentage} = \left(\frac{142}{300} \times 100\right)\% = 47.33\%$$

20. (b)

$$\text{Required percentage} = \frac{70+80}{95+110} \times 100 = \frac{150}{205} \times 100 = 73.17$$

21. (b)

Average sales of branches B_1, B_2 and B_3 in 2001

$$= \frac{1}{3}\times(105+65+110) = \frac{280}{3}$$

Average sales of branches B_1, B_3 and B_6 in 2000

$$= \frac{1}{3}\times(80+95+70) = \frac{245}{3}$$

$$\text{Required percentage} = \frac{245/3}{280/3}\times 100 = \frac{700}{8} = 87.5\%$$

22. (b)

The series is:

$$2^3 + 1^2 = 9 \qquad 3^3 + 2^2 = 31$$
$$4^3 + 3^2 = 73 \qquad 5^3 + 4^2 = 141$$
$$6^3 + 5^2 = 241$$

23. (c)

The relative velocity between any two bodies moving in opposite directions is equal to sum of the velocities of two bodies. We have,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\therefore \quad v + 8 = \frac{250}{15}\times\frac{18}{5}$$

$$v = 52 \text{ km/hr}$$

24. (a)

$$\text{Volume of the cylinder} = \pi r^2 h = \frac{22}{7}\times 7\times 7\times 20 = 3080\,\text{cm}^3$$

If 'h' is the height upto which the cuboidal box is filled with liquid, we have

Volume of the liquid transferred to cuboidal box = 3080 × 0.9 = 2772 cm^3

$$15 \times 8 \times h = 2772$$
$$h = 23.1 \text{ cm}$$

25. (d)

If T is son, then it has to be male; (a) and (b) suggest T is a female; while (c) does not tell us whether T is a male or female. As per (d), T is brother of S where T and S are children of Q and R, which implies T is son of Q. Hence, option (d) is correct.

Section B : Engineering Mathematics

26. (a)

$$\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = e^{5x}$$

$$(D^2 - 3D + 2)y = e^{5x}$$

$$(D - 1)(D - 2)y = e^{5x}$$

Thus, the auxiliary equation is $(m - 1)(m - 2)=0$ with roots as $m = 1, 2$.

$$\therefore \quad \text{C.F.} = C_1e^x + C_2e^{2x}$$

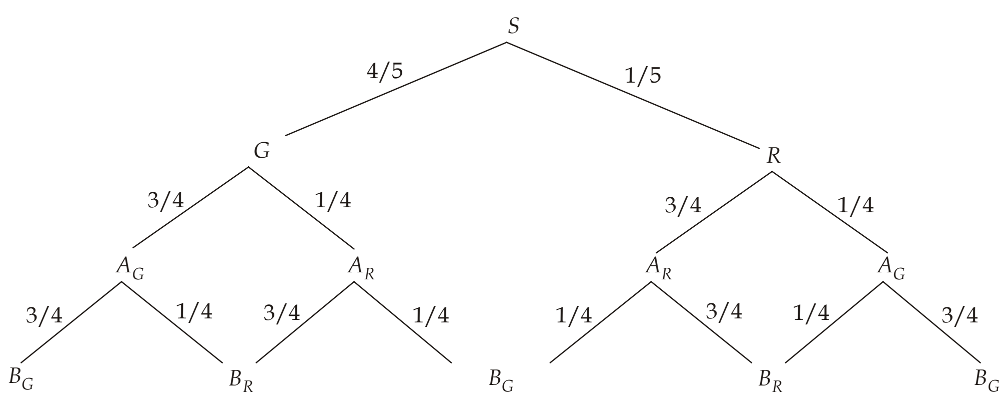
$$\text{P.I.} = \frac{1}{(D-1)(D-2)}e^{5x}$$

$$= \frac{e^{5x}}{(5-1)(5-2)} = \frac{e^{5x}}{12} \qquad \left[\because \text{P.I.} = \frac{1}{f(D)}e^{ax} = \frac{e^{ax}}{f(a)}\right]$$

$$\therefore \quad y = \text{C.F.} + \text{P.I.} = C_1e^x + C_2e^{2x} + \frac{1}{12}e^{5x}$$

27. (c)

From the tree diagram, it follows that



$$P(B_G) = \frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}=\frac{46}{80}$$

$$P(B_G/G) = \frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{10}{16}=\frac{5}{8}$$

$$P(B_G \cap G) = P\left(\frac{B_G}{G}\right)\times P(G) = \frac{5}{8}\times\frac{4}{5}=\frac{1}{2}$$

$$\therefore \quad P(G/B_G) = \frac{P(B \cap G)}{P(B_G)} = \frac{\frac{1}{2}}{P(B_G)} = \frac{1}{2}\times\frac{80}{46}=\frac{20}{23}$$

28. (d)

Consider the diagonal matrices D_1 and D_2 as below,

$$D_1 = \begin{bmatrix} a_{11} & & \\ & a_{22} & \\ & & \ddots a_{nn} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} I_{11} & & \\ & I_{22} & \\ & & \ddots I_{nn} \end{bmatrix}$$

then $$D_1 + D_2 = \begin{bmatrix} a_{11}+I_{11} & & \\ & a_{22}+I_{22} & \\ & & a_{nn}+I_{nn} \end{bmatrix}$$ is a diagonal matrix

$$D_1 D_2 = \begin{bmatrix} a_{11}I_{11} & & \\ & a_{22}I_{22} & \\ & & \ddots a_{nn}I_{nn} \end{bmatrix}$$ is a diagonal matrix

$$D_1^2 = \begin{bmatrix} a_{11}^2 & & \\ & a_{22}^2 & \\ & & \ddots a_{nn}^2 \end{bmatrix}$$

$\therefore$ $D_1^2 + D_2^2$ is also a diagonal matrix.

29. (b)

Let $$I = \int \frac{dx}{1+\cos x}$$

We know that $$1 + \cos x = 2\cos^2 \frac{x}{2}$$

So the integral simplifies to

$$I = \int \frac{dx}{2\cos^2 \frac{x}{2}}$$

$$I = \frac{1}{2}\int \sec^2 \frac{x}{2} dx$$

Let $u = \frac{x}{2}$, so $du = \frac{dx}{2}$, and the integral becomes

$$I = \frac{1}{2}\int \sec^2 u \cdot 2du$$

Since $\int \sec^2 du = \tan u$, we get

$$I = \tan u$$

Thus, the integral simplifies to

$$I = \tan\frac{x}{2}$$

Evaluating from $x = \frac{\pi}{4}$ to $x = \frac{3\pi}{4}$, we get

$$I = \tan\frac{3\pi}{8} - \tan\frac{\pi}{8}$$

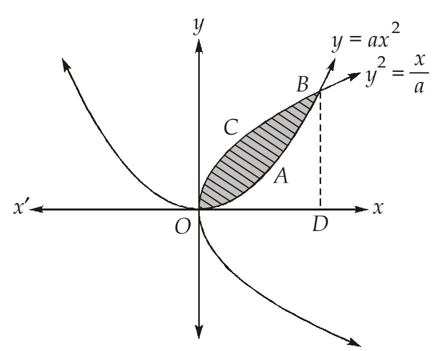
Using the known values:

$$\tan\frac{3\pi}{8} = 1+\sqrt{2},\ \tan\frac{\pi}{8} = \sqrt{2}-1$$

$$I = (1+\sqrt{2}) - (\sqrt{2}-1) = 2$$

30. (a)

Area enclosed between the curves is $OABCO$.



Thus, the point of intersection of $y = ax^2$ and $x = ay^2$ is given by

$$x = a(ax^2)^2$$

$$x = 0, \frac{1}{a}$$

$$\Rightarrow \quad y = 0, \frac{1}{a}$$

So, the points of intersection are (0, 0) and $\left(\frac{1}{a}, \frac{1}{a}\right)$

$\therefore$ Required area $OABCO$ = Area of curve $OCBDO$ – Area of curve $OABDO$

$$\int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx = 1$$

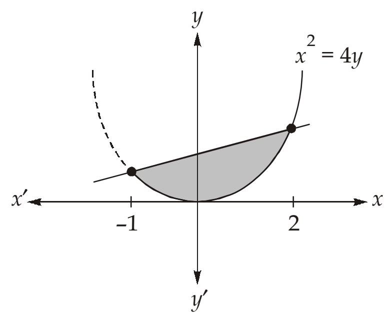
$$\left[\frac{1}{\sqrt{a}}\times\frac{x^{3/2}}{3/2}-\frac{ax^3}{3}\right]_0^{1/a} = 1$$

$$\frac{2}{3a^2}-\frac{1}{3a^2} = 1$$

$$a = \frac{1}{\sqrt{3}}$$

31. (a)

The point of intersection of the curves $x^2 = 4y$ and $x = 4y - 2$ sketched below can be obtained as $x = -1$ and $x = 2$.



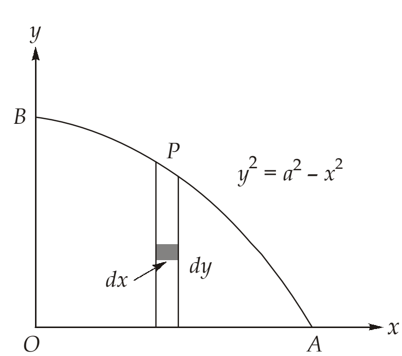
$\therefore$ Required area $= \int_{-1}^{2}\left\{\frac{(x+2)}{4}-\left(\frac{x^2}{4}\right)\right\}dx$

$$= \frac{1}{4}\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[\left(2+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)\right]$$

$$= \frac{1}{4}\left[\frac{10}{3}-\left(-\frac{7}{6}\right)\right] = \frac{1}{4}\times\frac{9}{2}=\frac{9}{8} \text{ square unit}$$

32. (a)



The region of integration is the first quadrant of the circle OAB.

Assume
$$I = \iint_R xy\,dx\,dy$$
$$x^2 + y^2 = a^2$$
$$y = \sqrt{a^2 - x^2}$$

First we integrate w.r.t. y and then w.r.t. x.

The limits for y are 0 and $\sqrt{a^2 - x^2}$ and for x, 0 at a. Thus,

$$I = \int_0^a \int_0^{\sqrt{a^2-x^2}} xydydx$$
$$= \int_0^a x\left[\frac{y^2}{2}\right]_0^{\sqrt{a^2-x^2}} dx$$
$$= \frac{1}{2}\int_0^a x\left(a^2 - x^2\right)dx = \frac{1}{2}\left[\frac{a^2x^2}{2} - \frac{x^4}{4}\right]_0^a$$
$$= \frac{a^4}{8}$$

33. (a)

- The transpose of the product of the two matrices is the product of their transposes taken in the reverse order.
$$(AB)^T = B^TA^T$$
- Every square matrix can be uniquely expressed as a sum of a symmetric and a skew-symmetric matrix.
  Let A be the given square matrix then,
$$A = \frac{1}{2}\left(A + A^T\right) + \frac{1}{2}\left(A - A^T\right)$$

where $A + A^T$ is a symmetrix matrix and $A - A^T$ is a skew-symmetric matrix.

Let B and C be the two inverses of matrix A. We have, $AB = BA = I = AC = CA$, then $B = BI = B(AC) = (BA)C = IC = C$.

- Inverse of a matrix, is unique.
- The inverse of the product of two matrices is the product of their inverse taken in the reverse order i.e.,
$$(AB)^{-1} = B^{-1}A^{-1}$$

Hence, only statements 1 and 2 are correct.

34. (a)

$$I + A = \begin{bmatrix} 1 & 1+2i \\ -1+2i & 1 \end{bmatrix}$$

$$|I + A| = 1 - (-1 - 4) = 6$$

$$(I + A)^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}$$

Also,
$$(I - A) = \begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}$$

Thus,
$$(I - A)(I + A)^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}\begin{bmatrix} 1 & -1-2i \\ 1-2i & 1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} -4 & -2-4i \\ 2-4i & -4 \end{bmatrix} = \frac{1}{6}(-4I - 2A) = -\frac{1}{3}(2I + A)$$

35. (c)

Given that
$$|A^{10}| = 1024$$
$$|A^{10}| = 2^{10}$$
$$|A| = 2$$
$$-x^3 - 25 = 2$$
$$x^3 + 27 = 0$$
$$x^3 + 3^3 = 0$$
$$(x + 3)(x^2 + 3x + 3^2) = 0$$

∴ The real value of x is –3.

36. (a)

$$\lim_{x\to 0}\frac{\sin 2x + a\sin x}{x^3} = b$$

By L'Hospital rule,

$$\lim_{x\to 0}\frac{2\cos 2x + a\cos x}{3x^2} = b$$

For b to be finite,
$$2 + a = 0$$
∴
$$a = -2$$

By L'Hospital rule,

$$\lim_{x\to 0}\frac{-4\sin 2x + 2\sin x}{6x} = b$$

Again applying L'Hospital rule,

$$\lim_{x\to 0}\frac{-8\cos 2x + 2\cos x}{6} = b$$

$$b = -1$$

$\therefore$ $$a = -2 \text{ and } b = -1$$

$$a \times b = -2 \times -1 = 2$$

37. (b)

Since the function ($x\sin x$) is an even function in the interval [–π, π], hence the Trigonometric Fourier series coefficient, $b_n = 0$. Thus, Fourier series expansion of $x\sin x$ contains only cosine terms.

38. (d)

Number of integers between 100 and 999 = 900

In a three digit integer not containing the digit 7, first digit can be chosen in 8 ways (1 – 9 exclusing 7), second and third digit can be chosen in 9 ways (0 - 9 exclusing 7). Thus

Number of three digit integers which do not contain digit 7 = 8 × 9 × 9 = 648

$$\text{Required probability} = \frac{648}{900} = \frac{18}{25}$$

39. (d)

Since the failures of gates 2 and 3 are independent events, therefore

$$\text{Required probability} = P(G_2 \cap G_3) = P(G_2)P(G_3)$$

$$= 0.2 \times 0.2 = 0.04$$

40. (a)

X	1	2	3
$P(x)$	$\frac{2+5P}{5}$	$\frac{1+3P}{5}$	$\frac{1.5+2P}{5}$

We have $$\Sigma P_i = 1$$

$$\frac{2+5P}{5} + \frac{1+3P}{5} + \frac{1.5+2P}{5} = 1$$

$$P = 0.05$$

41. (c)

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2} = \frac{y^2/x^2}{y/x - 1}$$

Since it is a homogenous differential, we put

$$\frac{y}{x} = v$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$x\frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v^2 - v^2 + v}{v-1} = \frac{v}{v-1}$$

$$\left(1-\frac{1}{v}\right)dv = \frac{dx}{x}$$

$$\int dv - \int \frac{1}{v}dv = \log x + \log c$$

$$v - \log v = \log x + \log c$$

$$\frac{y}{x} - \log y + \log x = \log x + \log c$$

$\therefore$
$$\frac{y}{x} = \log (cy)$$

$$y = x \log cy$$

42. (a)

The general from of 2nd order linear partial differential equation is given by

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0$$

The above equation is

- Parabolic if $B^2 - 4AC = 0$
- Elliptic if $B^2 - 4AC < 0$
- Hyperbolic if $B^2 - 4AC > 0$

Given:
$$C^2\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$$

$$A = C^2;\ B = 0,\ C = 0$$

$$B^2 - 4AC = 0 - 4 \times C^2 \times 0 = 0$$

$\therefore$ The equation is parabolic.

43. (a)

The area bounded by x-axis, lines $x = 1$, $x = 5$ and the curve $y = f(x)$ is given by

$$\int_1^5 y\,dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3) + y_4]$$

$$= \frac{1}{2}[10 + 2(50 + 70 + 80) + 100] = \frac{1}{2}[10 + 400 + 100] = 255$$

44. (b)

Consider $f(z) = z^2 - z + 1,$

Using Cauchy's integral formula, if $f(z)$ is an analytic function within and on curve c and if 'a' is any point within c, we have

$$\frac{1}{2\pi i}\int_c \frac{f(z)}{z-a}dz = f(a)$$

We have $a = 1$

Now, $a = 1$ lies outside the circle $c : |z| = \frac{1}{2}$, so $\frac{z^2 - z + 1}{z - 1}$ is analytic every where within c,

∴ By Cauchy's theorem,

$$\int_c \frac{z^2 - z + 1}{z - 1} dz = 0$$

Hence, option (b) is correct.

45. (c)

$$f(z) = \sin z, \quad f'(z) = \cos z,$$
$$f''(z) = -\sin z, \quad f'''(z) = -\cos z,$$
$$\therefore \quad f(0) = 0, \quad f'(0) = 1,$$
$$f''(0) = 0, \quad f'''(0) = -1$$

∴ By Taylor's series expansion about, $z = 0$,

$$f(z) = f(0) + \frac{(z-0)}{1!} f'(0) + \frac{(z-0)^2}{2!} f''(0) + \frac{(z-0)^3}{3!} f'''(0) + \ldots..$$

$$\Rightarrow \quad \sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} \ldots\ldots\ldots..$$

$$\therefore \quad \text{Coefficient of } z^2 = 0$$

46. (a)

$$\frac{\partial z}{\partial x} = a^2 \qquad \frac{\partial z}{\partial y} = 2ay$$

$$\Rightarrow \quad \left(\frac{\partial z}{\partial y}\right)^2 = 4a^2y^2$$

$$\left(\frac{\partial z}{\partial y}\right)^2 = 4y^2\left(\frac{\partial z}{\partial x}\right)$$

$$\Rightarrow \quad 4y^2\left(\frac{\partial z}{\partial x}\right) - \left(\frac{\partial z}{\partial y}\right)^2 = 0$$

Hence, option (a) is correct.

47. (a)

$$\text{Div}\vec{V} = \nabla \cdot \vec{V}$$
$$= \frac{\partial}{\partial x}(xy \sin z) + \frac{\partial}{\partial y}(y^2 \sin x) + \frac{\partial}{\partial z}(z^2 \sin xy)$$
$$= y \sin z + 2y \sin x + 2z \sin xy$$

Now, at the given point

$$x = 0, \; y = \frac{\pi}{2}, \; z = \frac{\pi}{2}$$

We have,

$$\text{Div}\,\vec{V} = \frac{\pi}{2}\sin\frac{\pi}{2} + 2\times\frac{\pi}{2}\sin 0 + 2\cdot\frac{\pi}{2}\cdot\sin 0$$

$$= \frac{\pi}{2}$$

48. (d)

The auxiliary equation is

$$D^3 - 2D^2 + 4D - 8 = 0$$

$$(D-2)(D^2+4) = 0$$

$$D = 2, \pm 2i$$

The solution of differential equation is $y = C_1e^{2x} + C_2\sin 2x + C_3\cos 2x$

49. (c)

Residue at $z = 0$ will be,

$$\text{Residue} = \underset{z\to 0}{lt}\; z \times \frac{1+e^z}{z\left(\frac{\sin z}{z} + \cos z\right)} = \frac{1+e^0}{1+1} = 1$$

50. (b)

For Newton Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x = \sqrt{28}$$

$$f(x) = x^2 - 28$$

$\Rightarrow$ $x^2 - 28 = 0$ is the equation which is to be solved

$$f'(x) = 2x$$

$$x_1 = 5.6 - \left.\frac{x^2-28}{2x}\right|_{x_0}$$

$$= 5.6 - \frac{5.6^2 - 28}{2\times 5.6} = 5.30$$

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