DETAILED SOLUTIONS

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ESE 2025 : Prelims Exam
CLASSROOM TEST SERIES

ELECTRICAL ENGINEERING

Test 4

Section A : Control Systems [All Topics] + Engineering Mathematics [All Topics]

Section B : Electrical Circuits - 1 [Part Syllabus]

Section C : Digital Electronics - 1 [Part Syllabus] + Microprocessors - 1 [Part Syllabus]

ANSWER KEY

1.	(d)	**16.**	(b)	**31.**	(c)	**46.**	(b)	**61.**	(b)
2.	(d)	**17.**	(a)	**32.**	(c)	**47.**	(d)	**62.**	(b)
3.	(a)	**18.**	(a)	**33.**	(a)	**48.**	(a)	**63.**	(b)
4.	(c)	**19.**	(a)	**34.**	(c)	**49.**	(d)	**64.**	(c)
5.	(b)	**20.**	(a)	**35.**	(c)	**50.**	(b)	**65.**	(c)
6.	(b)	**21.**	(c)	**36.**	(b)	**51.**	(b)	**66.**	(a)
7.	(d)	**22.**	(a)	**37.**	(a)	**52.**	(b)	**67.**	(c)
8.	(c)	**23.**	(b)	**38.**	(d)	**53.**	(b)	**68.**	(a)
9.	(d)	**24.**	(b)	**39.**	(d)	**54.**	(b)	**69.**	(a)
10.	(a)	**25.**	(d)	**40.**	(a)	**55.**	(c)	**70.**	(c)
11.	(b)	**26.**	(c)	**41.**	(a)	**56.**	(c)	**71.**	(c)
12.	(d)	**27.**	(c)	**42.**	(c)	**57.**	(a)	**72.**	(d)
13.	(a)	**28.**	(c)	**43.**	(c)	**58.**	(b)	**73.**	(a)
14.	(b)	**29.**	(a)	**44.**	(d)	**59.**	(d)	**74.**	(a)
15.	(c)	**30.**	(a)	**45.**	(a)	**60.**	(d)	**75.**	(b)

DETAILED EXPLANATIONS
Section A : Control Systems + Engineering Mathematics

1. (d)

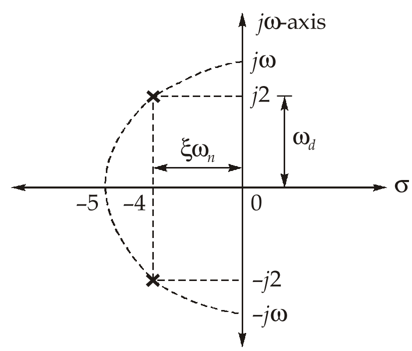
$$\%M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$

So, normalized ($\%M_p$) peak overshoot will remain same as 'ξ' is constant.

2. (d)

Closed loop poles location, $s = -\xi\omega_n \pm j\omega_d$

Also, $s = -4 \pm j2$



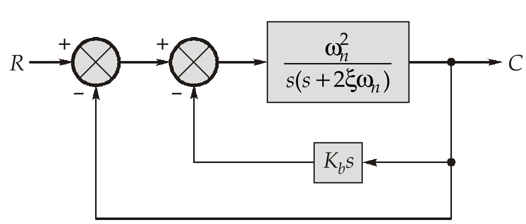
$\Rightarrow$ $\xi\omega_n = 4$

and damped frequency of oscillation,

$$\omega_d = 2 \text{ rad/sec}$$

Settling time (5% tolerance), $t_s = \dfrac{3}{\xi\omega_n} = \dfrac{3}{4} = 0.75 \text{ sec}$

3. (a)



$$\frac{C}{R} = \frac{\omega_n^2}{s^2 + 2\xi'\omega_n s + \omega_n^2} = \frac{\omega_n^2}{s^2 + (2\xi\omega_n + K_b\omega_n^2)s + \omega_n^2}$$

$$= \frac{\omega_n^2}{s^2 + 2\left(\xi + \dfrac{K_b\omega_n}{2}\right)\omega_n s + \omega_n^2}$$

- ω_n remains same i.e., R.L. intersect at same point on $j\omega$-axis.
- ξ increases which reduces peak overshoot.

- Time constant, $\tau = \dfrac{1}{\xi\omega_n}$ as $\xi\uparrow \Rightarrow \tau\downarrow$
- Type of the system is unchanged.

4. (c)

Both the statements are correct.

5. (b)

Routh-Hurwitz array is

$$\begin{array}{c|ccc}
s^4 & b_0 & b_2 & b_4 \\
s^3 & b_1 & b_3 & \\
s^2 & \dfrac{(b_1b_2 - b_0b_3)}{b_1} & b_4 & \\
s^1 & \dfrac{\dfrac{b_1b_3b_3 - b_0b_3^2}{b_1} - b_1b_4}{\dfrac{(b_1b_2 - b_0b_3)}{b_1}} & 0 & \\
s^0 & b_4 & &
\end{array}$$

Necessary condition for stability

$$b_0 > 0, \quad b_1 > 0, \quad b_2 > 0, \quad b_3 > 0 \text{ and } b_4 > 0$$

From R-H array (sufficient conditions) are

$$\frac{b_1b_2 - b_0b_3}{b_1} > 0$$

$$\Rightarrow \quad (b_1b_2 - b_0b_3) > 0$$

and

$$\frac{\dfrac{b_1b_2b_3 - b_0b_3^2}{b_1} - b_1b_4}{\dfrac{b_1b_2 - b_0b_3}{b_1}} > 0$$

$$\Rightarrow \quad (b_1b_2b_3 - b_0b_3^2 - b_1^2b_4) > 0$$

6. (b)

If any coefficient of the characteristic equation is complex or contains power of '*e*', this criterion cannot be applied.

7. (d)

Characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(1+Ts)} = 0$$

$$\Rightarrow \quad Ts^2 + s + K = 0$$

For s = $-a$ line, replace s by $(s - a)$

$\therefore$ $T(s-a)^2 + (s-a) + K = 0$

$\Rightarrow$ $T(s^2 + a^2 - 2as) + s + (K - a) = 0$

$\Rightarrow$ $Ts^2 + (1 - 2aT)s + (K - a + a^2T) = 0$

For system stability w.r.t. line $s = -a$ i.e. no roots to lie on RHS of $s = -a$ line,

Coefficients must be positive,

i.e., $T > 0, \quad 1 - 2aT > 0$

and $K - a + a^2T > 0$

i.e., $T > 0, \quad T < \frac{1}{2a}$

and $K > a - a^2T$

or $K > a(1 - aT)$

Here, For $T = \frac{1}{2a}$

or $aT = 0.5$

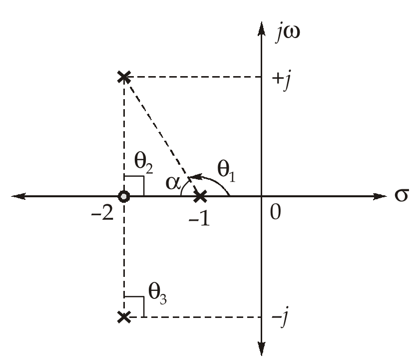
$K > a(1 - 0.5)$

i.e., $K > \frac{a}{2}$

8. (c)

$$G(s)H(s) = \frac{K(s+2)}{(s+1)(s^2+4s+5)}$$

$$= \frac{K(s+2)}{(s+1)(s+2+j)(s+2-j)}$$



Here, $\theta_2 = 90°$

$\theta_3 = 90°$

$\theta_1 = 180° - \alpha$

$$= 180° - \tan^{-1}\left(\frac{1}{1}\right) = 180° - 45° = 135°$$

$\therefore$ Angle of departure, $\phi_D = \pm[180° - (\phi_P - \phi_Z)]$

Where, $\phi_P = \theta_1 + \theta_3 = 135° + 90° = 225°$

and $\phi_2 = \theta_2 = 90°$

$\therefore$ $\phi_D = \pm[180° - (225° - 90°)] = \pm 45°$

The root locus from $(-2 + j)$ will depart at an angle of +45 and at –45° angle at $(-2 - j)$.

9. (d)

$$G(s)H(s) = \frac{K(s+1)}{s(s+2)(s^2+2s+5)}$$

$$= \frac{K(s+1)}{s(s+2)(s+1+j2)(s+1-j2)}$$

Total open loop poles, $P = 4$

Total open loop zeros, $Z = 1$

No. of asymptotes, $P - Z = 4 - 1 = 3$

Angle of asymptotes:

θ	$\frac{(2K+1)}{(P-Z)}\times 180°$ Where, $K = 0,1,2....,(P-Z-1)$
θ_1	60°
θ_2	180°
θ_3	300°

$$\Sigma(\text{real part of poles}) = 0 + (-2) + (-1) + (-1) = -4$$

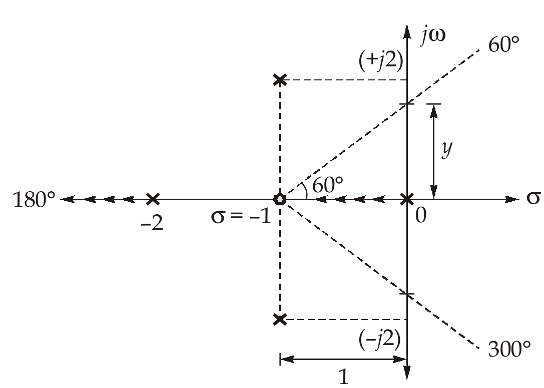
$$\Sigma(\text{Real part of zeros}) = (-1) = -1$$

$\therefore$ Centroid, σ where asymptote meet at real axis is

$$= \frac{\Sigma(\text{Real part of poles}) - \Sigma(\text{Real part of zeros})}{P-Z}$$

$$= \frac{(-4)-(-1)}{4-1}$$

$\Rightarrow$ $\sigma = -1$



From plot, $\tan 60° = \dfrac{y}{1}$

$$y = \sqrt{3} = 1.732$$

∴ The asymptotes along 60° line and 300° line will intersect the $j\omega$-axis at $j1.732$ and $-j1.732$ respectively.

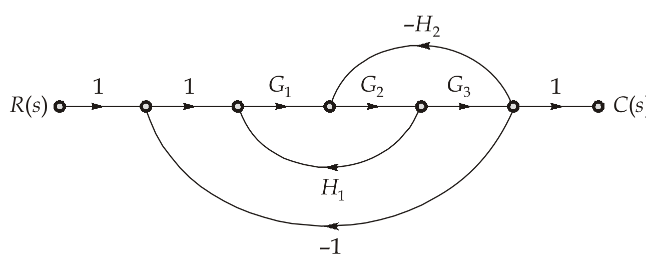
∴ The portion between –1 and 0 and –∞ to –2 will lie on root locus.

10. (a)

- Root locus is plotted when system gain is varied from 0 to ∞ or ∞ to 0.
- Addition of pole to the system transfer function shifts the root locus branches towards $j\omega$-axis.

11. (b)

The SFG of the system is



Forward path gain is, $P_1 = G_1G_2G_3$

Loop gains, $L_1 = -G_1G_2G_3$

$$L_2 = G_1G_2H_1$$

$$L_3 = -G_2G_3H_2$$

$$\Delta = 1 - [L_1 + L_2 + L_3]$$

$$= 1 - [-G_1G_2G_3 + G_1G_2H_1 - G_2G_3H_2]$$

$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

and $\Delta_1 = 1$

By Mason's gain formula

∴ $$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2G_3 \times 1}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

$$= \frac{G_1G_2G_3}{1 - \dfrac{G_1G_2G_3H_1}{G_3} + G_2G_3H_2 + G_1G_2G_3}$$

$$= \frac{\delta G_1}{\left(1 - \dfrac{G_1\delta H_1}{G_3} + \delta H_2 + \delta G_1\right)}$$

∴ $\delta = G_2G_3$

12. (d)

When the phase margin (P.M.) is less than zero then the system is said to be unstable.

13. (a)

The given compensator is a lag compensator which reduces the bandwidth there by increasing the rise time.

14. (b)

$$\frac{(1+Ts)}{(1+\alpha Ts)} = \frac{1+2s}{1+2(7-4\sqrt{3})s}$$

$\therefore$ $$T = 2$$

$$\alpha = 7-4\sqrt{3}$$

Maximum phase, $$\phi_m = \sin^{-1}\left[\frac{1-\alpha}{1+\alpha}\right] = \sin^{-1}\left[\frac{1-7+4\sqrt{3}}{1+7-4\sqrt{3}}\right]$$

$$= \sin^{-1}\left[\frac{4\sqrt{3}-6}{8-4\sqrt{3}}\right] = \sin^{-1}\left[\frac{(4\sqrt{3}-6)(8+4\sqrt{3})}{8^2-(4\sqrt{3})^2}\right]$$

$$= \sin^{-1}\left[\frac{\sqrt{3}}{2}\right] = 60°$$

Frequency, $$\omega_m = \frac{1}{T\sqrt{\alpha}} = \frac{1}{2\sqrt{7-4\sqrt{3}}} = \frac{1}{2\sqrt{7-4\times 1.73}}$$

$$= \frac{0.5}{\sqrt{0.08}} = \frac{0.5\times 10}{\sqrt{8}} = \frac{5}{2\sqrt{2}}$$

$$= 2.5 \times 0.707 = 1.8 \text{ rad/sec}$$

15. (c)

All the statements given are correct.

16. (b)

- The plot Q represents an all pass function whose phase at high frequencies approaches to –180°.
- The plot R represents a non-minimum phase function.

17. (a)

At gain crossover frequency, the magnitude has 0 dB value and at this ω_{gc},
Phase of the system is,

$$\phi\big|_{\omega_{gc}} = -240°$$

$\therefore$ $$\text{P.M.} = 180° + \phi\big|_{\omega_{gc}}$$

$$= 180° + (-240°)$$

$$= -60°$$

$\because$ P.M. < 0, system is unstable.

18. (a)

Corner frequency are, 10, 50 and 2000 rad/sec

$$G(s)H(s) = \frac{sK}{\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_2}+1\right)\left(\frac{s}{\omega_3}+1\right)}$$

$$= \frac{sK}{\left(\frac{s}{10}+1\right)\left(\frac{s}{50}+1\right)\left(\frac{s}{2000}+1\right)} = \frac{10^6 Ks}{(s+10)(s+50)(s+2000)}$$

From the starting slope,

$$y = mx + C$$

$$0 = 20\log\sqrt{10} + 20\log K$$

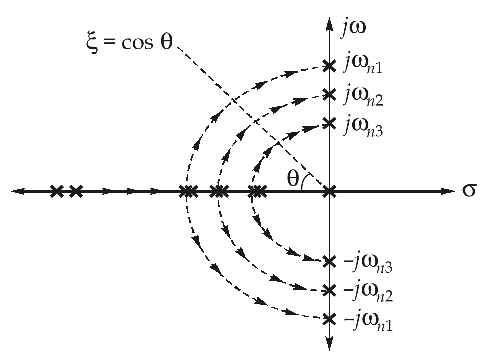
$$0 = \frac{20}{2}\log 10 + 20\log K$$

$$K = 10^{-0.5} = \frac{1}{\sqrt{10}} = 0.316$$

$$\therefore \quad G(s)H(s) = \frac{10^6 \times 0.316 s}{(s+10)(s+50)(s+2000)} \approx \frac{316\times 10^3 s}{(s+10)(s+50)(s+2000)}$$

19. (a)

For constant damping ratio, the root locus will be a family of concentric semi-circle in the left half of s-plane with undamped natural frequency of oscillation.



20. (a)

$$Q_C = [B \;\; AB \;\; .....]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$\Rightarrow \quad |Q_C| = 1 \times (-2) - 0 \times 0 = -2 \neq 0$$

Hence, the system is controllable.

$$Q_C = \begin{bmatrix} C \\ CA \\ \vdots \end{bmatrix}$$

$$CA = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & -4 \end{bmatrix}$$

$\therefore$
$$Q_0 = \begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix}$$

$$|Q_0| = 1 \times (-4) - (-2) \times 1 = -4 + 2 = -2 \neq 0$$

$\therefore$ System is observable.

21. (c)

$$\phi(t) = e^{At} = I + (At) + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + ...$$

$$= I + (At) + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \frac{A^4t^4}{4!} + ...$$

22. (a)

$$A = \begin{bmatrix} 0 & 1 \\ -p & 3 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ \beta \end{bmatrix};$$

$$C = [\alpha \quad 2]$$

$$Q_C = [B \quad AB \quad .....]$$

Where,
$$AB = \begin{bmatrix} 0 & 1 \\ -p & 3 \end{bmatrix}\begin{bmatrix} 1 \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ -p + 3\beta \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 1 & \beta \\ \beta & -p + 3\beta \end{bmatrix}$$

For the system to be controllable, $|Q_C| \neq 0$

i.e., $(-p + 3\beta) \times 1 - \beta^2 \neq 0$

$\Rightarrow$ $3\beta - p - \beta^2 \neq 0$

$\Rightarrow$ $p \neq 3\beta - \beta^2$

and for the system to be unobservable

$$|Q_0| = 0$$

Where,
$$Q_0 = \begin{bmatrix} C \\ CA \\ \vdots \end{bmatrix}$$

$$CA = [\alpha \quad 2]\begin{bmatrix} 0 & 1 \\ -p & 3 \end{bmatrix} = [-2p \quad (\alpha+6)]$$

$$\therefore \quad Q_0 = \begin{bmatrix} \alpha & 2 \\ -2p & (\alpha+6) \end{bmatrix}$$

$$|Q_0| = 0$$

$$\alpha(\alpha + 6) + 4p = 0$$

$$\Rightarrow \quad p = \frac{-\alpha}{4}(\alpha+6) = -\alpha\left(\frac{\alpha}{4}+\frac{3}{2}\right)$$

23. (b)

Transfer function, $\quad \text{T.F} = C[sI - A]^{-1} B + D$

$$[sI - A] = \begin{bmatrix} s & -1 \\ 4 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{s(s+2)+4}\begin{bmatrix} (s+2) & 1 \\ -4 & s \end{bmatrix}$$

$$\Rightarrow \quad \text{T.F.} = [1 \quad 0]\frac{1}{s(s+2)+4}\begin{bmatrix} (s+2) & 1 \\ -4 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s(s+2)+4}[1 \quad 0]\begin{bmatrix} 1 \\ s \end{bmatrix} = \frac{1}{s^2+2s+4}$$

$$2\xi\omega_n = 2$$

and $\quad \omega_n = \sqrt{4} = 2 \text{ rad/sec}$

$$\Rightarrow \quad 2 \times \xi \times 2 = 2$$

$$\Rightarrow \quad \xi = 0.5$$

24. (b)

$$A = \begin{bmatrix} 2 & -3 & 4 \\ 3 & -2 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 0$$

$$\therefore \quad \rho(A) < 3$$

Minor of the element 2 $= \begin{vmatrix} -2 & 5 \\ 1 & 1 \end{vmatrix} = -7 \neq 0$

$$\therefore \quad \rho(A) = 2$$

Alternate Solution:

Apply row transformation

$$R_2 \to R_2 - (R_1 + R_3)$$

$$A = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3$$

$$A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

25. (d)

If PQ is defined then number of rows in Q must be equal to number of column in P. If QP is defined then number of columns in Q must be equal to number of row in P.

$\therefore$ Order of $Q = n \times m$

26. (c)

$$f(x) = 3x^2 + 5x + 8 \text{ in } \left[\frac{11}{2}, \frac{13}{2}\right]$$

For any second degree polynomial function $f(x)$, the mean value C in the interval (a, b) is $\frac{a+b}{2}$

$$\therefore \quad C = \frac{1}{2}\left(\frac{13}{2} + \frac{11}{2}\right) = 6$$

27. (c)

The maximum value of

$$f(x) = x^3 - 9x^2 + 24x + 5 \text{ in } [1, 6]$$

$$f(1) = 1 - 9 + 24 + 5 = 21$$

$$f(6) = 216 - 36 \times 9 + 24 \times 6 + 5$$

$$= 216 - 324 + 144 + 5 = 41$$

$$f'(x) = 3x^2 - 18x + 24 = 0$$

$$3(x^2 - 6x + 8) = 0$$

$$x = 2, 4$$

$$f''(x) = 6x - 18$$

$$f''(2) = 12 - 18 = -6$$

$$f''(4) = 24 - 18 = 6$$

$$f(2) = 25$$

But $f(6) > f(1)$

Maximum is 41.

28. (c)

$$\bar{A} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$$

$$\text{curl } \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & -2xz & 2yz \end{vmatrix}$$

$$\text{curl } \vec{A} = (2z + 2x)\hat{i} - (0-0)\hat{j} + (-2z - x^2)\hat{k}$$

$$= (2z + 2x)\hat{i} + (-2z - x^2)\hat{k}$$

$$\text{curl(curl } \bar{A}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2x + 2z) & 0 & -2z - x^2 \end{vmatrix} = (2x + 2)\hat{j}$$

At (1, –1, 2)

$$\text{curl(curl } \bar{A}) = 4\hat{j}$$

29. (a)

Evaluate $\int_c \bar{r} \cdot \overline{dr}$ along the curve

$$x^2 + y^2 = 4$$

$$z = 0$$

So that, $dz = 0$

$$\int_c \bar{r} \cdot \overline{dr} = \int_c (x\,\hat{i} + y\,\hat{j} + z\,\hat{k}) \cdot (dx\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= \int_c x\,dx + y\,dy$$

$$= \iint_R (0-0)dx\,dy$$

$$= 0 \qquad (\because \text{ by Green's theorem})$$

30. (a)

$$\text{Required probability} = P(C/M)$$

$$= \frac{P(M \cap C)}{P(M)} = \frac{10}{15} = \frac{2}{3}$$

31. (c)

If the two lines of regerssion are perpendicular to each other then correlation coefficient is zero.

32. (c)

Given, $x\frac{dy}{dx}+y = 0; \quad y(2) = -2$

$$x\frac{dy}{dx} = -y$$

$$\frac{dy}{y} = -\frac{dx}{x}$$

$$\frac{dy}{y}+\frac{dx}{x} = 0$$

Integrating both sides

$$\int\frac{dy}{y}+\int\frac{dx}{x} = \int 0$$

$$\log y + \log x = C'$$

$$xy = C$$

at $x = 2$ and $y = -2$

$$C = -4$$

∴ The solution is $xy = -4$

33. (a)

Let, $x = N^{1/3}$

$$x^3 = N$$

Let, $f(x) = x^3 - N$

$$f'(x) = 3x^2$$

Newton - Raphson's iteration formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n^3 - N)}{3x_n^2}$$

$$x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$$

34. (c)

Singular points obtained by putting

$$z^2 + 2z + 5 = 0$$

$z = -1 + 2i, -1 - 2i$ are singular points but both are lying outside C :

$$|Z| = 1$$

$\therefore$ By Cauchy's integral theorem

$$\int_C \frac{(z+4)}{(z^2+2z+5)}dz = 0$$

35. (c)

Now,

$$y_p = \frac{1}{f(D)}[x \cdot \sin(3x)]$$

$$y_p = x\left[\frac{1}{f(x)}\sin 3x\right] - \left[\frac{2D}{1}\left(\frac{1}{(1+D^2)}\sin(3x)\right)\right]$$

$$y_p = \frac{-x}{8}\sin(3x) - \frac{6}{64}\cos(3x)$$

$$y_p = \frac{-x}{8}\sin(3x) - \frac{3}{32}\cos(3x)$$

36. (b)

Total cases = $6^3 = 216$

Let, X = Number of dice show same face

$$P(X \geq 2) = P(X = 2) + P(X = 3)$$

$$= \frac{{}^3C_2 \times 6 \times 5}{216} + \frac{6}{216} = \frac{96}{216} = \frac{4}{9}$$

37. (a)

The median of 55, 100, 75, 80, 90, 85, 95, 45, 70, 70, 55

Arranging them in increasing order

45, 55, 55, 70, 70, 75, 80, 85, 90, 95, 100

Median = 75 (Middle most observation)

38. (d)

Since, the given system has non-zero solution.

$$\begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

Applying $c_1 \rightarrow c_1 - c_2$, $c_2 \rightarrow c_2 + c_3$

$$\begin{vmatrix} 1+k & -k & -1 \\ 1+k & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix} = 0$$

$$2(k+1) - (k+1)^2 = 0$$

$$(k+1)(2-k-1) = 0$$

$$k = \pm 1$$

39. (d)

Since, three distinct numbers are to be selected from first 100 natural numbers.

$$n(s) = {}^{100}C_3$$

E(favourable events) = All three of them are divisible by both 2 and 3

$$\therefore \quad n(E) = {}^{16}C_3$$

$$\therefore \quad \text{Required probability} = \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$$

40. (a)

Using Cauchy's Integral formula $\int_C \frac{\log z}{(z-1)^3} dz \qquad C : |z-1| = \frac{1}{2}$

Poles are determined by putting denominator equal to zero.

$$(z-1)^3 = 0$$

$$z = 1, 1, 1$$

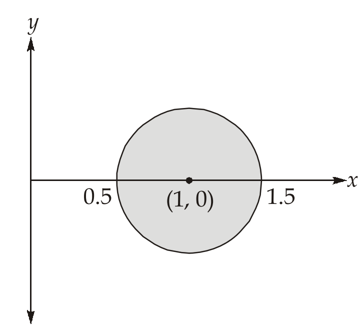
There is one pole of order three at $z = 1$ which is inside the circle C.

$$\int \frac{f(z)}{(z-a)^3} dz = \frac{2\pi i}{2!} f^2(a)$$

$$= \frac{2\pi}{2!}\left[\frac{d^2}{dz^2}\log z\right]_{z=1}$$

$$= \frac{2\pi i}{2}\left(-\frac{1}{z^2}\right)_{z=1}$$

$$= -\frac{2\pi i}{2} = -\pi i$$



41. (a)

Since $\sum_{x=0}^{4} P(x) = 1$

$$c + 2c + 2c + c^2 + 5c^2 = 1$$

$$6c^2 + 5c - 1 = 0$$

$$c = \frac{1}{6}, -1$$

Since $P(x) \geq 0$, the possible value of

$$c = \frac{1}{6}$$

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{5}{36}$
$xP(x)$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{3}{36}$	$\frac{20}{36}$

$$\text{Mean} = \sum_{x=0}^{4} xP(x) = 0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{36} + \frac{20}{36}$$

$$= \frac{59}{36} = 1.638$$

$$\text{Variance} = \sigma^2 = E(X^2) - [E(X)]^2$$

$$= \left[0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + 9\left(\frac{1}{36}\right) + 16\left(\frac{5}{36}\right) - \left(\frac{59}{36}\right)^2\right]$$

$$= 1.45$$

42. (c)

$$\int_0^{0.4} f(x)dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3) + y_4]$$

$$= \frac{0.1}{2}[0 + 2(15 + 60 + 135) + 240]$$

$$= 33$$

43. (c)

$$A_n = \frac{n(n+1)}{2}$$

$$A_n - 1 = \frac{n(n+1)}{2} - 1 = \frac{n^2 + n - 2}{2}$$

$$= \frac{(n+2)(n-1)}{2}$$

$$\frac{A_n}{A_n - 1} = \frac{n(n+1)}{(n+2)(n-1)}$$

$$T_n = \left(\frac{n}{n-1}\right)\left(\frac{n+1}{n+2}\right) = \left(\frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \cdots\cdots \frac{n}{n-1}\right)\left(\frac{3}{4} \times \frac{4}{5} \cdots\cdots \frac{n+1}{n+2}\right)$$

$$= \frac{3n}{n+2}$$

$$\lim_{n\to\infty} T_n = \lim_{n\to\infty} \frac{3n}{n+2} = 3$$

44. (d)

Root locus branches always starts from open loop poles and end either at open loop zeros or at infinity as the system gain K varies from 0 to ∞.

45. (a)

Both the statements are correct and statement-II is the correct explanation of statement-I.

Section B : Electrical Circuits-1

46. (b)

Average power absorbed by a load depends only on fundamental components.

$$v(t) = 100\cos(\omega t) = 100\sin(\omega t + 90°)\text{ V}$$

So,
$$P_{\text{avg(load)}} = \frac{V_1 I_1}{2}\cos\phi = \frac{100\times 6}{2}\cos(90° - 30°)\text{ mW}$$
$$= 300\cos(60°) \times 10^{-3} = 150\text{ mW} = 0.15\text{ W}$$

47. (d)

For maximum power transfer,
$$Z_L = Z_s^* = (R_s + jX_s)^* = R_s - jX_s$$
$$I = \frac{V_m\angle 0°}{Z_s + Z_L} = \frac{V_m}{2R_s}$$

∴ Average power,
$$P_{\text{avg}} = \frac{1}{2}I^2 R_s = \frac{1}{2}\frac{V_m^2}{(2R_s)^2}\times R_s = \frac{V_m^2}{8R_s}$$

48. (a)

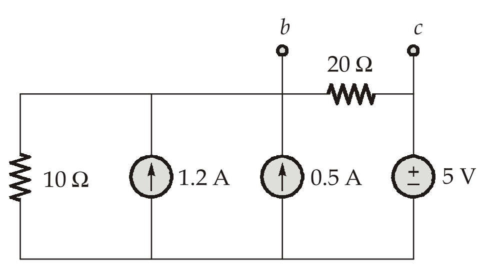
Initially,
$$C_1 = 5\ \mu\text{F and } Q_1 = 19\ \mu\text{C}$$
$$C_2 = C\ \mu\text{F and } Q_2 = 0$$
∴
$$Q_T = Q_1 + Q_2 = 19\ \mu\text{C} \quad \text{...(i)}$$

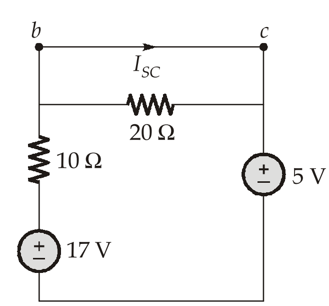
When C_1 is connected to C_2 parallelly, the charge is transferred from C_1 to C_2. However the total charge remains equal to 19 μC due to the law of conservation of charge. For the capacitors connected in parallel.

∴
$$V_1 = V_2$$
$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$
$$\frac{11}{5} = \frac{8}{C}$$
$$C = 3.64\ \mu\text{F}$$

49. (d)

In order to obtain Norton's equivalent across the terminals '*b*' and '*c*' the circuit can be redrawn as



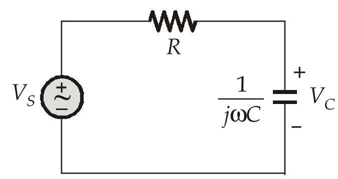


∴ $$I_{SC} = \frac{17-5}{10} = 1.2\text{ A}$$

and $$R_{Th} = \frac{10\times 20}{20+10} = 6.67\ \Omega$$

50. (b)

For a series RC circuit,



Here, $$V_C = \frac{V_S}{R+\frac{1}{j\omega C}}\left(\frac{1}{j\omega C}\right) = \frac{V_S}{j\omega RC+1}$$

Phase difference between V_S and V_C,

$$\phi = \tan^{-1}\omega RC$$

As $\omega\uparrow \phi\uparrow; R\uparrow \phi\uparrow; C\uparrow \phi\uparrow$

As $\omega\downarrow \phi\downarrow$

51. (b)

The time constant for RC circuit is

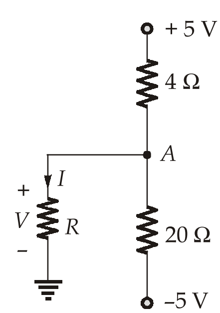
$$\tau = R_{eq}C_{eq}$$

here, $$R_{eq} = 2\ \Omega$$

and $$C_{eq} = \frac{(4\text{ F}+4\text{ F})(10\text{ F})}{(4\text{ F}+4\text{ F})+10\text{ F}} = \frac{8\text{ F}\times 10\text{ F}}{18\text{ F}} = 4.44\text{ F}$$

∴ $$\tau = 2\times 4.44 = 8.88\text{ sec}$$

52. (b)



∵ $I = \frac{1}{4}$ A ∴ $V = I\times R = R/4$ V

At Node 'A' by using KCL, we get,

$$\frac{5-V}{4}+\frac{-V-5}{20} = I$$

$$\Rightarrow \quad \frac{5 - R/4}{4} + \frac{-R/4 - 5}{20} = \frac{1}{4}$$

$$25 - \frac{5R}{4} - \frac{R}{4} - 5 = 5$$

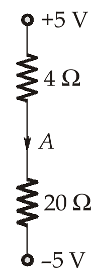
$$20 - \frac{6R}{4} = 5$$

$$\frac{6R}{4} = 15$$

$$R = 10\ \Omega$$

Alternate Solution:

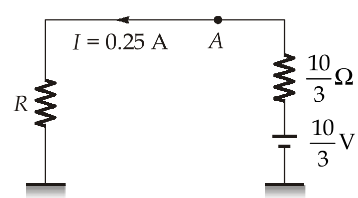
Thevenin's equivalent network across A,



$$V_A = \frac{5 \times 20 - 5 \times 4}{20 + 4}$$

$$V_A = \frac{10}{3}\text{V};$$

$$R_{\text{Th}} = 20 \,||\, 4 = \frac{10}{3}\text{V}$$



$$I = \frac{10/3}{R + 10/3}$$

$$\Rightarrow \quad R + \frac{10}{3} = \frac{10/3}{0.25} = \frac{40}{3}$$

$$R = \frac{40}{3} - \frac{10}{3} = \frac{30}{3} = 10\ \Omega$$

53. (b)

For an ideal transformer, $K = 1$

applying KVL in the primary and secondary loops, we get,

$$V_1 = (j\omega L_1)I_1 + j\omega M I_2$$

$$V_2 = (j\omega L_2)I_2 + j\omega M I_1$$

($\because$ I_1 and I_2 both enters through the dots)

$$\frac{V_1}{V_2} = \frac{L_1 I_1 + M I_2}{L_2 I_2 + M I_1}$$

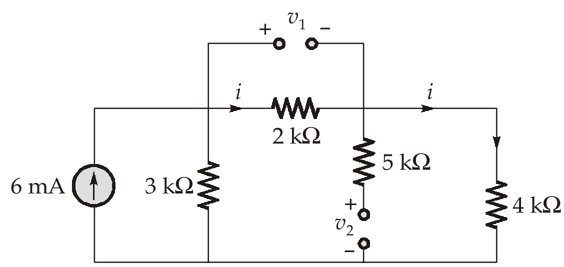
$$\because \quad K = 1$$

$$\therefore \quad M = \sqrt{L_1 L_2}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{L_1}{L_2}}\left[\frac{\sqrt{L_1} \cdot I_1 + \sqrt{L_2} I_2}{\sqrt{L_1} I_1 + \sqrt{L_2} I_2}\right] = \sqrt{\frac{L_1}{L_2}}$$

54. (b)

Under steady state condition,



Capacitor behaves as open circuit,

By current division rule,

$$i = 6\,\text{mA} \times \frac{3\times10^3}{3\times10^3 + 2\times10^3 + 4\times10^3} = 2\text{ mA}$$

Power dissipation in 2 kΩ,

$$P_{2\text{ k}\Omega} = i^2 \times 2\times10^3 = (2\times10^{-3})^2 \times 2\times10^3$$

$$= 8\text{ mW}$$

Voltage,

$$v_2 = \text{i} \times 4\times10^3$$

$$= 2\times10^{-3}\times 4\times10^3$$

$$= 8\text{ volts}$$

∴ Energy stored in capacitor C_2,

$$E_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}\times 4\times10^{-3}\times(8)^2 = 128\text{ mJ}$$

55. (c)

Here,

$$T = 2 \quad \text{and} \quad v(t) = \begin{cases} 4; & 0 < t < 1 \\ 8; & 1 < t < 2 \end{cases}$$

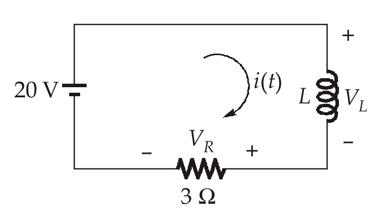
$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 4^2\,dt + \int_1^2 8^2\,dt\right]$$

$$= \frac{1}{2}[16+64] = \frac{80}{2} = 40$$

$$V_{rms} = \sqrt{40} = 2\sqrt{10}\text{ V} = 2\times3.162 = 6.325\text{ V}$$

56. (c)

At $t > 0$



$$V_R = i(t)R = 6\times3 = 18\text{ V}$$

$\therefore \quad V_L = 20 - 18 = 2\text{ V}$

$$V_L = L\frac{di(t)}{dt} \Rightarrow 2 = L(8)$$

$$\Rightarrow \quad L = \frac{2}{8} = 0.25\text{ H}$$

58. (b)

Applying KVL in the circuit;

$$80 - V_x - 2V_x + V_0 = 0$$

$$\Rightarrow \quad 80 = 3V_x - V_0 \qquad \text{...(i)}$$

Also, $$\frac{-V_x}{30} = \frac{V_0}{45}$$

$$\Rightarrow \quad V_x = \frac{-2}{3}V_0 \qquad \text{...(ii)}$$

From (i) and (ii),

$$V_0 = \frac{-80}{3}\text{V} = -26.67\text{ V}$$

59. (d)

Bandwidth of a series RLC circuit is given as:

$$\text{B.W} = \frac{R}{L}$$

$$\text{B.W} = \frac{25}{0.04}$$

$$\text{B.W} = 625\text{ rad/sec} = 0.625\text{ k rad/sec}$$

60. (d)

Power efficiency of the source can be more than 50%. But power efficiency will be 50% when maximum power is being delivered to R_L.

Section C : Digital Electronics - 1 + Microprocessors - 1

61. (b)

$$\overline{f(A,B,C)} = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$$

$$= \bar{A}C(\bar{B}+B) + A\bar{C}(\bar{B}+B)$$

$$= \bar{A}C + A\bar{C} = A \oplus C$$

$$f(A, B, C) = \overline{A \oplus C} = A \odot C$$

62. (b)

$$Z = x \oplus [y \oplus xy] = x \oplus \left[y\overline{xy} + \overline{y}xy\right]$$
$$= x \oplus \left[y(\overline{x}+\overline{y})+0\right] = x \oplus \left[y\overline{x}+0\right]$$
$$= x(\overline{y}+x)+\overline{x}y\overline{x} = x+\overline{x}y = x + y$$

Now, on representing in K-map, we get,

x \ y	0	1
0		1
1	1	1

Hence, $f(x, y) = \Sigma m(1, 2, 3)$ or $f(x, y) = \Sigma\pi(0)$

63. (b)

$$Y = \overline{C}(A\overline{C}+BC)+C(B\overline{C}+AC)$$
$$= A\overline{C}+AC = A$$

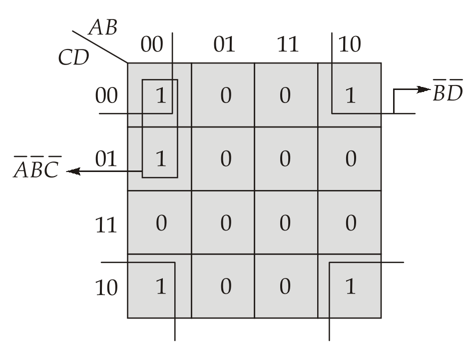
Further,
$$f = Y\overline{D}+\overline{Y}D$$
$$f = A\overline{D}+\overline{A}D = A \oplus D = \overline{A \odot D}$$

64. (c)

$$f = A\left\{B+C\left(\overline{AB+AC}\right)\right\} = AB+AC(\overline{AB}\cdot\overline{AC}) = AB$$

Thus, it behaves as AND gate with respect to A and B.

65. (c)



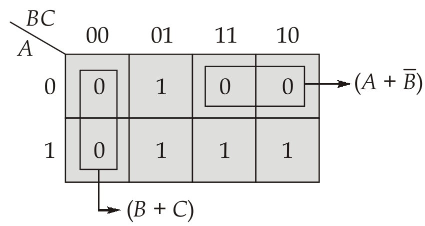
$$f = \overline{A}\overline{B}\overline{C}+\overline{B}\overline{D} = \overline{B}(\overline{A}\overline{C}+\overline{D})$$

66. (a)

$$f = \overline{AB}+A\overline{C}\overline{D}+\overline{A}\overline{B}D+\overline{AB}C\overline{D}$$
$$= \overline{A}+\overline{B}+A\overline{C}\overline{D}+\overline{A}\overline{B}D+(\overline{A}+\overline{B})C\overline{D}$$
$$= \overline{A}+\overline{B}+A\overline{C}\overline{D}+\overline{A}\overline{B}D+\overline{A}C\overline{D}+\overline{B}C\overline{D}$$
$$= \overline{A}(1+\overline{C}\overline{D}+\overline{B}D+C\overline{D})+\overline{B}(1+C\overline{D})$$
$$= \overline{A}+\overline{B} \quad \Rightarrow \quad \text{Two literals}$$

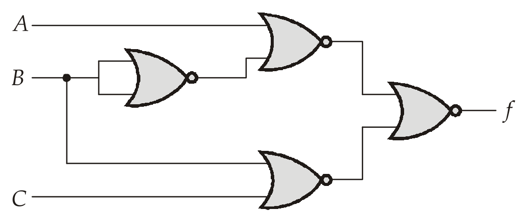
67. (c)

The expression can be written as



$$f = (A+\overline{B})(B+C)$$

Thus, the circuit can be drawn as



So, minimum 4 NOR gates are required.

68. (a)

- Address bus in 8085 microprocessors is unidirectional bus.
- Data bus is used to transfer data between memory, IO and microprocessor.

69. (a)

8085 microprocessor has a multipurpose register called accumulator by which almost all arithmetic and logical operations are performed.

70. (c)

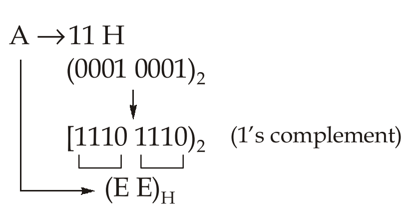
$$f_{\text{clk}} = \frac{f_{\text{crystal}}}{2} = \frac{10}{2}\text{MHz} = 5\text{ MHz}$$

71. (c)

MVI stands for "move immediately". The instruction indicates that the 8-bit data will be immediately stored at a memory location whose address is stored at reference of HL pair register.

72. (d)

CMA operation will complement the content of accumulator

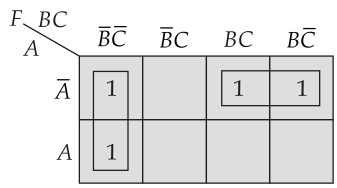


73. (a)

RST 7.5 is a hardware interrupt.

74. (a)

- Grouping process (grouping a pair i.e. 2 cells) minimizes the number of variables in the product term.
- Example:



$$F = \bar{B}\bar{C}(A+\bar{A}) + \bar{A}B(C+\bar{C})$$

$$= \bar{B}\bar{C} + \bar{A}B$$

Hence, variable existing in normal and its complemented form disappears.