DETAILED SOLUTIONS

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ESE 2025 : Prelims Exam
CLASSROOM TEST SERIES

ELECTRICAL ENGINEERING

Test 6

Section A : Electrical Machines [All Topics]
Section B : Control Systems-1 + Engineering Mathematics-1 [Part Syllabus]
Section C : Electrical Circuits-2 + Digital Electronics-2 [Part Syllabus]

ANSWER KEY

1.	(d)	**16.**	(b)	**31.**	(b)	**46.**	(a)	**61.**	(b)
2.	(b)	**17.**	(c)	**32.**	(c)	**47.**	(b)	**62.**	(d)
3.	(b)	**18.**	(a)	**33.**	(c)	**48.**	(a)	**63.**	(b)
4.	(a)	**19.**	(a)	**34.**	(a)	**49.**	(b)	**64.**	(a)
5.	(b)	**20.**	(d)	**35.**	(d)	**50.**	(c)	**65.**	(b)
6.	(d)	**21.**	(c)	**36.**	(c)	**51.**	(a)	**66.**	(c)
7.	(d)	**22.**	(d)	**37.**	(c)	**52.**	(d)	**67.**	(c)
8.	(a)	**23.**	(a)	**38.**	(d)	**53.**	(a)	**68.**	(c)
9.	(c)	**24.**	(c)	**39.**	(b)	**54.**	(a)	**69.**	(a)
10.	(d)	**25.**	(c)	**40.**	(a)	**55.**	(a)	**70.**	(b)
11.	(c)	**26.**	(a)	**41.**	(c)	**56.**	(d)	**71.**	(c)
12.	(d)	**27.**	(d)	**42.**	(b)	**57.**	(c)	**72.**	(c)
13.	(a)	**28.**	(a)	**43.**	(c)	**58.**	(d)	**73.**	(c)
14.	(d)	**29.**	(d)	**44.**	(b)	**59.**	(a)	**74.**	(d)
15.	(b)	**30.**	(a)	**45.**	(b)	**60.**	(c)	**75.**	(c)

DETAILED EXPLANATIONS
Section A : Electrical Machines

1. (d)

Point S is minimum voltage regulation which occurs at leading power factor. And the minimum voltage regulation occurs at ϕ = 90° leading.

i.e., Power factor $= \cos\phi = \cos(-90°) = 0$

= zero power factor (ZPF) leading

2. (b)

$$\phi = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$$

During open delta operation,

p.f. angle of transformer –1, $\phi_1 = 30° + \phi = 30° + 30° = 60°$

p.f. angle of transformer –2, $\phi_2 = 30° - \phi = 30° - 30° = 0°$

p.f. of transformer $-1 = \cos\phi_1 = \cos 60° = 0.5$

p.f. of transformer $-2 = \cos\phi_2 = \cos 0° = 1.0$

3. (b)

$\because R_C$ and X_m are very large so these parameters may be neglected

At starting, $s = 1$

to get maximum torque at starting,

$$s_{mT} = 1$$

$$s_{mT} = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{R_2'}{\sqrt{(\sqrt{84})^2 + (10+10)^2}}$$

$$1 = \frac{R_2'}{\sqrt{84+400}}$$

$$\Rightarrow \quad R_2' = \sqrt{484} = 22\ \Omega$$

4. (a)

$$R_{\text{p.u.}} = 0.04 \text{ p.u.}$$

$$X_{\text{p.u.}} = 0.6 \text{ p.u.}$$

$$\phi = -\cos^{-1}(0.6) = -53.13°$$

$$\Rightarrow \quad \sin\phi = -0.8$$

$$x = 0.75 \text{ or } \frac{3}{4}\text{th of full load}$$

$$\% V_{reg} = x[R_{\text{p.u.}}\cos\phi + X_{\text{p.u.}}\sin\phi] \times 100$$

$$= 0.75[0.04 \times 0.6 + 0.6 \times (-0.8)] \times 100$$

$$= -34.2\%$$

5. **(b)**

At, 600 V, 50 Hz

$$P_{e1} = 250 \text{ W}$$
$$P_{h1} = 500 \text{ W}$$

We know, $$P_h \propto \frac{V^2}{f} \text{ and } P_e \propto V^2$$

Since, the supply voltage is constant so eddy current loss will be same at 25 Hz.

But, $$\frac{P_{h2}}{P_{h1}} = \left(\frac{V^2}{f}\right) \times \left(\frac{f/2}{V^2}\right) = 2$$

$$P_{h2} = 2P_{h1} = 2 \times 500 = 1000 \text{ W}$$

∴ Total core loss at 600 V, 25 Hz is

$$P_i = P_{h2} + P_{e2} = 1000 + 250 = 1250 \text{ W}$$

6. **(d)**

$$X_d = \frac{(V_{\max})_{\text{ph}}}{I_{\min}} = \frac{120}{\sqrt{3} \times 8} = 8.66\,\Omega$$

$$X_q = \frac{(V_{\min})_{\text{ph}}}{I_{\max}} = \frac{90}{\sqrt{3} \times 15} = 3.46\,\Omega$$

7. **(d)**

- Leakage fields present in a transformer induce eddy currents in conductors, tanks, channels, bolts etc. and these eddy currents give rise to stray load loss.
- Dielectric loss occurs in the insulating materials, i.e., in the transformer oil and the solid insulation of h.v. transformers.

8. **(a)**

Zero voltage regulation of the transformer occurs when load p.f. is $\frac{x_{e2}}{Z_{e2}}$ leading.

9. **(c)**

Given, $V_t = 1.0$, $X_d = 1.5$ p.u.,

$X_q = 0.8$ $I_a = 1.0$ p.u.

$$\cos\phi = 1$$

$$\Rightarrow \quad \phi = 0°$$

We know that, $$\tan\psi = \frac{V_t \sin\phi + I_a X_q}{V_t \cos\phi + I_a R_a}$$

$$\tan\psi = \frac{1 \times 0 + 1 \times 0.8}{1 \times 1 + 1 \times 0} = 0.8$$

$$\psi = \tan^{-1}(0.8) \approx 38.65°$$

Now, $\psi = \delta + \phi$

$\Rightarrow$ $\delta = \psi - \phi = 38.65° - 0 = 38.65°$

10. **(d)**

The Δ–Δ connection is satisfactory for unbalanced loading. The closed delta configuration allows circulating current to dampen out the third harmonic components.

11. **(c)**

$$V = 5 \text{ Volts}$$
$$I = 1 \text{ A}$$
$$X_s = 4\ \Omega$$

DC resistance of one phase,

$$r_{dc} = \frac{1}{2}\left(\frac{V}{I}\right) = \frac{1}{2}\left(\frac{5}{1}\right) = 2.5\Omega$$

Effective armature resistance,

$$r_a = 1.2 \times r_{dc} = 1.2 \times 2.5\ \Omega = 3\ \Omega$$

Synchronous impedance,

$$Z_s = \sqrt{r_a^2 + X_s^2} = \sqrt{3^2 + 4^2} = 5\Omega$$

At $I_a = 100$ A, internal machine drop will be

$$\text{Drop} = I_a Z_s = 100 \times 5 = 500 \text{ V}$$

12. **(d)**

$$E_{f1} = 1.65 \text{ p.u.}$$
$$\delta_1 = 30°$$
$$\delta_2 = 60°$$
$$V_t = 1.0 \text{ p.u.}$$

$$\text{Power, } P = \frac{V_t E_f}{X_s}\sin\delta$$

$$P \propto E_f \sin\delta$$

$\because$ Power is constant

$\therefore$ $E_{f1}\sin\delta_1 = E_{f2}\sin\delta_2$

$$1.65 \times \sin 30° = E_{f2}\sin 60°$$

$$1.65 \times \frac{1}{2} = E_{f2} \times \frac{\sqrt{3}}{2}$$

$$E_{f2} = 0.952 \text{ p.u.}$$

13. **(a)**

Field current at rated open circuit voltage of 232 V is 1.8 A.

At $I_f = 1.8$ A, $I_{Sc} = 59.0$

$\therefore$ $X_s = \left.\frac{V_{oc}}{I_{sc}}\right|_{I_f = 1.8A} = \frac{232}{59.0} = 3.93\Omega$

14. (d)

$$E_f = \frac{11}{\sqrt{3}}\text{kV} \quad \text{per phase}$$

$$V = \frac{11}{\sqrt{3}}\text{kV} \quad \text{per phase}$$

$$\delta = 30°$$

$$Z_S = 100\angle 75°\ \Omega = |Z_S|\angle\beta\ \Omega$$

$$P = \frac{VE_f}{|Z_S|}\cos(\beta-\delta) - \frac{V^2}{|Z_s|}\cos\beta$$

$$= \frac{\left(11\times10^3\times11\times10^3\right)/(\sqrt{3})^2}{100}\cos(75°-30°) - \frac{\left(11\times10^3/\sqrt{3}\right)^2}{100}\times\cos 75°$$

$$= 180.29 \text{ kW per phase}$$

$$\text{Total power } (3-\phi) = 3\times P = 3\times 180.29 = 540.84 \text{ kW}$$

15. (b)

$$T_{\text{st}} \propto \frac{V^2\cdot r_2}{r_2^2+x_2^2}$$

$$\frac{T_{st(\text{inner})}}{T_{st(\text{outer})}} = \left(\frac{r_2}{r_2^2+x_2^2}\right)_{\text{inner}} \times \left(\frac{r_2^2+x_2^2}{r_2}\right)_{\text{outer}} = \frac{0.2}{0.2^2+4^2}\times\left(\frac{1^2+1^2}{1}\right)$$

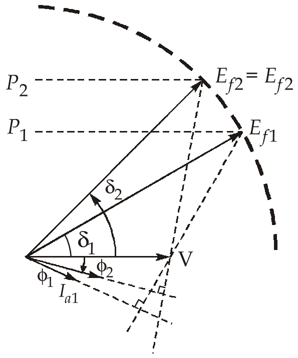
$$= \frac{0.2\times2}{16.04} = \frac{10}{401} \approx 1:40$$

16. (b)

$$P_1 = 80 \text{ MW}$$

$$P_2 = 100 \text{ MW}$$

$$E_f = E_{f1} = E_{f2}$$



$$\because \quad P = \frac{VE_f}{X_S}\sin\delta$$

$$\therefore \quad P_1 = \frac{VE_f}{X_S}\sin\delta_1 \qquad \ldots\text{(i)}$$

$$\text{and} \quad P_2 = \frac{VE_{f2}}{X_S}\sin\delta_2 = \frac{VE_{f1}}{X_S}\sin\delta_2 \qquad \ldots\text{(ii)}$$

From (i) and (ii)

$$\frac{\sin\delta_2}{\sin\delta_1} = \frac{P_2}{P_1} = \frac{100}{80}$$

$$\Rightarrow \quad \delta_2 = \sin^{-1}[1.25\sin\delta_1]$$

17. (c)

$$V = 1.0\angle 0°\text{p.u.}$$

$$I_a = 1.0\angle 90°\text{p.u.} = j1.0 \text{ p.u.}$$

$$V_{\text{base}} = 33 \text{ kV}$$

$$E_f = V - I_a Z_s$$

$$= 1.0\angle 0° - (j1.0)\ (j0.5)$$

$$= 1 + 0.5$$

$$= 1.5 \text{ p.u.}$$

$$\therefore \quad E_{f(\text{actual})} = E_f \times 33 \text{ kV}$$

$$= 1.5 \times 33$$

$$= 49.5 \text{ kV}$$

18. (a)

No-load characteristic is the saturation curve plotted between E_b and field current with speed, n constant.

Regulation curves are the armature characteristics.

19. (a)

- Power transformer is used for the transmission purpose at heavy load, so its efficiency is greater than distribution transformer.
- Iron weight/Cu weight is less in power transformer.
- Power transformer is generally operated at full load, hence it is designed such that copper losses are minimal. However, a distribution transformer is operated at loads less than full load for most of the time. Hence it is designed such that the core losses are minimal.

20. (d)

- Size of single-phase induction motor is larger because of high magnetizing current due to larger air-gap.
- Copper losses are high due to single winding carrying all the current while in poly-phase windings share the current.
- Poor power factor is due to high magnetizing current in single phase induction motor.

Hence statement 1, 2 and 3 are correct.

21. (c)

$$P = 2$$

No. of parallel path for wave wound machine, $A = 2$

$$l = 15 \text{ cm} = 0.15 \text{ m}$$

$$d = 20 \text{ cm} = 0.2 \text{ m}$$

$$Z = 500 \text{ conductors}$$

$$I_a = 30 \text{ A}$$

$$B_{\text{avg}} = 0.5 \text{ T}$$

We have flux per pole $\phi = B_{av}\left(\frac{\pi dl}{P}\right) = 0.5\left(\frac{\pi \times 0.2 \times 0.15}{2}\right) = 0.024$ Wb

Electromagnetic torque, $T_e = \frac{60}{2\pi N} \times E_b I_a = \frac{60}{2\pi N} \times \frac{NP\phi Z}{60A} \times I_a$

$$= \frac{P\phi Z I_a}{2\pi A} = \frac{2 \times 0.024 \times 500 \times 30}{2\pi \times 2}$$

$$= 57.3 \text{ N-m}$$

22. (d)

$$V = 250 \text{ V}$$
$$R_a = 0.5\ \Omega$$
$$R_f = 125\ \Omega$$

Constant field current, $I_f = \frac{250}{125} = 2A$

At $N_1 = 800$ rpm, $I_{L1} = 20$ A

$$I_{a1} = 20 - 2 = 18 \text{ A}$$
$$E_{b1} = V - I_{a1}R_a = 250 - 18 \times 0.5 = 241 \text{ V}$$
$$T \propto \phi I_a$$

or $T \propto I_a$ $(\because \phi \text{ constant})$

For constant torque I_a constant

$\therefore$ $I_{a2} = I_{a1} = 18$ A

$$= 250 - 18(0.5 + 1.5) = 214 \text{ V}$$

Now, $\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$ $(\because E_b \propto N\phi \propto N)$

$$\frac{214}{241} = \frac{N_2}{800}$$

$$N_2 = 710.37 \text{ rpm}$$

23. (a)

Load saturation curve is drawn between terminal voltage versus field current.

24. (c)

In closed slot,

1. Net air gap is low, so reluctance is less.
   So, magnetizing current is less.
   Hence power factor improved.
2. Reluctance is less so leakage reactance is more.
3. As leakage flux is more than useful flux hence starting torque will be less.

Hence statements 1, 2 and 4 are correct.

25. (c)

Field current, $I_f = \frac{250}{250\Omega} = 1A$

Full laod current, I_{Lfl} = 30 A

$I_{afl} = I_{Lfl} - I_f = 30 - 1 = 29\text{A}$

Under stalling condition,

$N = 0$

⇒ $E_b = 0$

∴ $E_b = V - I_{ast}(R_a + R_{ex})$

$0 = 250 - I_{ast}(0.5 + 5)$

⇒ $I_{ast} = \frac{250}{5.5} = 45.45A \text{ or } \frac{500}{11}A$

∴ $T \propto I_a$ (∵ ϕ constant for shunt motor)

⇒ $\frac{T_{st}}{T_{fl}} = \frac{I_{ast}}{I_{afl}} = \frac{500/11}{29} = \frac{500}{319}$

26. (a)

$i_1(t) = 5\sin(100\pi t)$ A

Area of the core, $A = 2 \times 10^{-3}$ m^2

Flux in the core, $\phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{N_1 \mu i_1(t) A}{2\pi r} = \frac{N_1(500\mu_0)(5\sin 100\pi t)\times 2\times 10^{-3}}{2\pi r}$

$$= \frac{250\times 500\times 4\pi\times 10^{-7}\times 5\sin(100\pi t)\times 2\times 10^{-3}}{2\pi\times 20\times 10^{-2}}$$

$= 1.25 \times 10^{-3}\sin(100\ \pi t)\text{Wb}$

∴ $e_2 = -N_2\frac{d\phi}{dt} = -N_2\frac{d}{dt}(1.25\times 10^{-3}\sin(100\pi t))$

$= -N_2 \times 1.25 \times 10^{-3} \times 100\pi\cos(100\pi t)$

$= -125\pi \times 10^{-3} N_2\cos(100\pi t)$

$e_2 = -125\pi \times 10^{-3} N_2\sin(100\pi t - 90°)$ volts

⇒ RMS Value, $E_2 = \frac{125\pi\times 10^{-3}N_2}{\sqrt{2}} = \frac{125\pi}{2\sqrt{2}}$

⇒ $N_2 = 500$

27. (d)

The MMF (armature ampere turn) method of voltage regulation for an alternator always gives a lower value than the actual voltage regulation. This is because the MMF method replaces the armature leakage reactance with an equivalent additional armature reaction MMF.

28. (a)

V-curves in synchronous machines are the plots of armature current versus field excitation at constant output power.

29. (d)

$$\text{Slip at no-load } (s_0) = 0.3\% = 0.003$$

$$\text{Synchronous speed, } N_s = \frac{120f}{P} = \frac{120\times 60}{4} = 1800 \text{ rpm}$$

$$\text{No-load speed, } N_0 = (1 - s_0)N_s = (1 - 0.003) \times 1800$$

$$= 0.997 \times 1800$$

$$= 1794.6 \text{ rpm}$$

$$\% \text{ speed regulation} = 4.74\%$$

$$\Rightarrow \quad \frac{N_0 - N_{fl}}{N_{fl}} \times 100 = 4.74\%$$

$$\frac{1794.6}{N_{fl}} - 1 = 0.0474$$

$$\frac{1794.6}{N_{fl}} = 1.0474$$

$$\Rightarrow \quad N_{fl} = \frac{1794.6}{10474} = 1713.39 \text{ rpm}$$

30. (a)

For constant V/f:

Starting torque :

$$T_{st} \propto \frac{V^2}{f^3} \propto \left(\frac{V}{f}\right)^2 \frac{1}{f}$$

Maximum torque :

$$T_{max} \propto \frac{V^2}{f^2} \propto \left(\frac{V}{f}\right)^2$$

As frequency increases, starting torque decreases and maximum torque remain constant.

31. (b)

$$T_{max} = T_{st}$$

$$\Rightarrow \quad s_{max} = s_{st} = 1$$

Now, $$T_{fl} = \frac{1}{2}T_{max}$$

$$\Rightarrow \quad \frac{T}{T_{max}} = \frac{2s \cdot s_{max}}{s_{max}^2 + s^2}$$

$$\frac{T_{fl}}{T_{\max}} = \frac{2s_{fl} \cdot s_{\max}}{s_{\max}^2 + s_{fl}^2} = \frac{2 \times s_{fl} \times 1}{1^2 + s_{fl}^2}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{2s_{fl}}{1 + s_{fl}^2}$$

$$s_{fl}^2 - 4s_{fl} + 1 = 0$$

$$s_{fl} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$\Rightarrow \quad = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2}$$

$$\Rightarrow \quad = 2 \pm \frac{1}{2} 2\sqrt{3} = 2 \pm \sqrt{3}$$

$$= 3.7321 \text{ or } 0.268$$

$$\Rightarrow \quad s_{fl} = 0.268$$

32. **(c)**

$$R_2' = 0.9\ \Omega$$

$$s_{\max 1} = 0.3$$

$$P = 4$$

$$f = 50 \text{ Hz}$$

$$\Rightarrow \quad N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

At maximum slip, $s_{\max 1} = \dfrac{R_2'}{X_2'}$

$$0.3 = \frac{0.9}{X_2'}$$

$$\Rightarrow \quad X_2' = 3\ \Omega$$

Now, at new slip of $s_{\max 2} = 0.4$

$$s_{\max 2} = \frac{R_2' + R_{\text{ext}}}{X_2'}$$

$$0.4 = \frac{0.9 + R_{\text{ext}}}{3}$$

$$R_{\text{ext}} = 0.3\ \Omega$$

And, $N_r = (1 - s_{\max 2}) N_s = (1 - 0.4) \times 1500$

$$= 900 \text{ rpm}$$

33. **(c)**

With auto-transformer starting supply line current at starting

$$I_{st} = x^2 I_{sc}$$

But, $I_{st} = 2I_{fl}$

And, $I_{sc} = 4 \times I_{fl}$

$\Rightarrow$ $2I_{fl} = x^2(4I_{fl})$

$\Rightarrow$ $x = \frac{1}{\sqrt{2}} = 0.707$ or 70.7%

34. **(a)**

Given $I_{sc} = \frac{100\left(A^2+1\right)}{(B\times 3)} = \frac{100\left[\left(\sqrt{2}\right)^2+1\right]}{1\times 3} = 100\text{ A}$

With star-delta starting, starting line current,

$$I_{st} = \frac{1}{\sqrt{3}} I_{sc} = \frac{100}{\sqrt{3}} = 57.74\text{ A}$$

35. **(d)**

All the statements are correct.

36. **(c)**

Given: $\frac{s}{2-s} = 0.0101$

$\Rightarrow$ $\frac{s}{2-s} = \frac{101}{10,000}$

$\Rightarrow$ $10,000\text{ s} = 202\text{–}101\text{s}$

$\text{s} = 0.02$

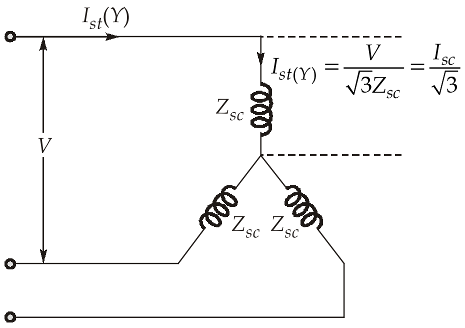
$$\text{Rotor speed} = (1-s)N_s = \frac{(1-0.02)\times 120\times 50}{4}$$

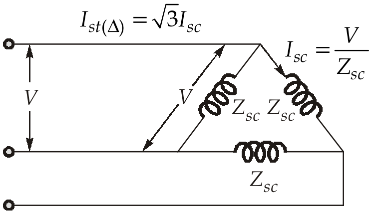
$$= 1470\text{ rpm}$$

37. **(c)**

When star-delta starting method is employed the starting line current is $\frac{1}{3}$ times the Direct-ON-line (DOL) starting line current.

$$I_{st(y)} = \frac{1}{3} I_{st(\Delta)}$$

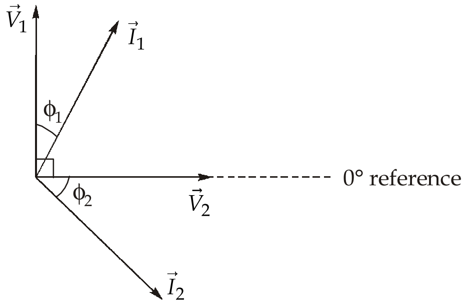




38. **(d)**

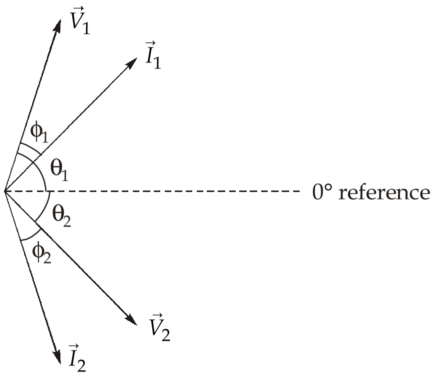
From Scott connection, a balanced 2-ϕ load (like furnace etc) is supplied power from a balanced 3-ϕ supply.

The balanced 2-ϕ load side phasors are represented as



The respective phase displacement between the two phase voltages must be 90°,

Henc, it can be redrawn as



where, $\theta_1 + \theta_2 = 90°$

The transformation ratio of the main transformer is $\frac{2}{\sqrt{3}}$ times the transformation ratio of the teaser transformer,

i.e. $a_m = \frac{2}{\sqrt{3}} a_T$

or $$a_T = \frac{\sqrt{3}}{2}a_m$$

where $$a_M = \frac{N_1}{N_2}$$

39. (b)

Voltage regulation, V.R. is given by

$$R_{pu}\cos\phi + X_{pu}\sin\phi = V.R.$$

$\therefore$ $\%R(0.8) + \%X(0.6) = 4\%$ $(\cos\phi = 0.8)$

and $\%R(0.6) + \%X(0.8) = 4.4\%$ $(\cos\phi = 0.6)$

on solving, $\%R = 2$

and $\%X = 4$

For maximum regulation,

$$\cos\phi = \cos\theta$$

$$= \frac{\%R}{\%Z} = \frac{2}{\sqrt{2^2+4^2}} = \frac{2}{\sqrt{20}} \simeq 0.45$$

40. (a)

If field excitation is such that $E_f \cos\phi > V$ the motor is said to be over-excited and it draws leading current.

41. (c)

We know that,

$$T \propto V^2$$

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^2$$

$$T_2 = \left(\frac{300}{400}\right)^2 \times 160$$

$$T_2 = \frac{9}{16}\times 160 = 90 \text{ N-m}$$

43. (c)

Power equation for salient pole machine is

$$P = \underbrace{\frac{VE_f}{X_d}\sin\delta}_{\text{Electromagnetic Power}} + \underbrace{\frac{V^2}{2}\left[\frac{1}{X_q}-\frac{1}{X_d}\right]\sin 2\delta}_{\text{Reluctance Power}}$$

Reluctance power still exists even if the field excitation is reduced to zero.

Hence, statement II is incorrect.

45. (b)

- Due to large air gap in 3-ϕ I.M., it requires more exciting/no load current.
- Testing of large power transformer in loaded condition is not feasible because it carries large current and voltage. So, it is very difficult to arrange such high rating voltmeter, ammeter and wattmeter.

Section B : Control Systems-1 + Engineering Mathematics-1

46. (a)

$$t_s = \frac{3}{\xi\omega_n} = 1.5$$

$$\xi\omega_n = 2 \qquad \text{...(i)}$$

Peak time,
$$t_p = \frac{\pi}{\omega_d} = 0.2$$

$$\omega_d = 15.708 \text{ rad/sec} \approx 15.7 \text{ rad/sec}$$

But
$$\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{\omega_n^2 - (\xi\omega_n)^2}$$

$$15.7 = \sqrt{\omega_n^2 - 2^2}$$

$$(15.7)^2 = \omega_n{}^2 - 4$$

$$\omega_n = \sqrt{4+15.7^2} = 15.82 \text{ rad/sec}$$

47. (b)

Closed loop T.F.
$$T(s) = \frac{\dfrac{10}{s(s+9)}}{1+\dfrac{10}{s(s+9)}\times 0.3s} = \frac{10}{s(s+12)}$$

Now, the open loop transfer function of equivalent unity negative feedback system is

$$= \frac{10}{s(s+2)-10} = \frac{10}{(s^2+12s-10)} = G(s)H(s)$$

Velocity error constant,
$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$= \lim_{s\to 0} s\cdot\left[\frac{10}{s^2+12s-10}\right]$$

$$= \lim_{s\to 0}\frac{10s}{s^2+12s-10} = \frac{10\times 0}{0^2+12\times 0-10} = 0$$

$\therefore$ Steady state error for unity ramp input will be

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

48. (a)

$$G(s) = \frac{\dfrac{25}{s(s+4)}}{1+\dfrac{25}{s(s+4)}\times\alpha s} = \frac{25}{s(s+[25\alpha+4])}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + (25\alpha + 4)s + 25$$

$$\Rightarrow \quad \omega_n^2 = 25$$

$$\Rightarrow \quad \omega_n = 5 \text{ rad/s}$$

$$2 \times \xi \times \omega_n = 25\alpha + 4$$

$$2 \times 0.8 \times 5 = 25\alpha + 4$$

$$\alpha = 0.16$$

$$= \frac{4}{25}$$

49. (b)

The rise time, t_r is the time required for the response to rise from 10% to 90% of its final value.

50. (c)

The characteristic equation is

$$s^2 + \frac{s}{K} + \frac{6}{K} = 0$$

$$\omega_n^2 = \frac{6}{K}$$

$$\omega_n = \sqrt{\frac{6}{K}}\,;$$

and

$$2\xi\omega_n = \frac{1}{K}$$

$$2\times\frac{1}{2}\times\sqrt{\frac{6}{K}} = \frac{1}{K}$$

$$\Rightarrow \quad K = \frac{1}{6}$$

51. (a)

$$1 + G(s)H(s) = 0$$

$$\Rightarrow \quad (s^3 + bs^2 + 3s + 2) + K(s + 3) = 0$$

$$\Rightarrow \quad s^3 + bs^2 + (3 + K)s + (3K + 2) = 0$$

$$\begin{array}{c|cc} s^3 & 1 & (3+K) \\ s^2 & b & 3K+2 \\ s^1 & \dfrac{b(3+K)-(3K+2)}{b} & \\ s^0 & 3K+2 & \end{array}$$

For the system to be marginally stable (oscillatory)

$$b(3+K)-(3K+2) = 0$$

$$\Rightarrow \quad b(3+K) = (3K+2) \qquad \text{...(i)}$$

For oscillatory system at 2 rad/sec, auxiliary equation is

$$bs^2+(3K+2) = 0$$

$$\Rightarrow \quad b(j2)^2+(3K+2) = 0$$

$$\Rightarrow \quad -4b+3K+2 = 0$$

$$(3K+2) = 4b \qquad \text{...(ii)}$$

From (i) and (ii),

$$b(3+K) = 4b$$

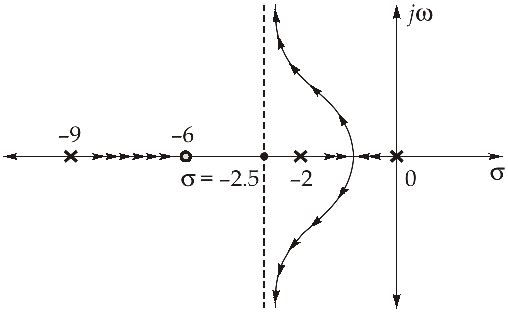
$$3+K = 4$$

$$\Rightarrow \quad K = 1$$

$$\therefore \quad b = \frac{3\times1+2}{4} = \frac{5}{4}$$

52. **(d)**

Root locus plot (approximated) is shown as



$$\text{Real part of poles} = 0, -2, -9$$

$$\text{Real part of zero} = -6$$

$$\text{No. of poles, } P = 3$$

$$\text{and number of zeros,} \quad Z = 1$$

$$P-Z = 3-1 = 2$$

$$\therefore \quad \text{Centroid, } \sigma = \frac{\Sigma\text{real\{Poles\}} - \Sigma\text{real\{Zeros\}}}{P-Z}$$

$$= \frac{0+(-2)+(-9)-(-6)}{2} = -2.5$$

$$\text{Now,} \quad 1+\frac{K(s+6)}{s(s+2)(s+9)} = 0$$

$$\Rightarrow \quad K = \frac{-s(s+2)(s+9)}{(s+6)} = -\frac{s^3+11s^2+18s}{(s+6)}$$

$$\frac{dK}{ds} = -\left[\frac{(3s^2+22s+18)(s+6)-(s^3+11s^2+18s)\times 1}{(s+6)^2}\right]$$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow \quad (3s^2 + 22s + 18)(s + 6) - (s^3 + 11s^2 + 18s) = 0$$

$$2s^3 + 29s^2 + 132s + 108 = 0$$

Solving this, we get $s = -1.04, -6.73 \pm j2.59$

Valid break away point is –1.04.

53. **(a)**

The sensitivity of the open loop system due to disturbance in the feedback path (feedback path not connected to the control or reference input for comparison) is always 'nil'

54. **(a)**

Given, differential equation is

$$\frac{dP(t)}{dt} - \frac{1}{2}P(t) = -200 \text{ is a linear differential equation}$$

Here, $W(t) = -\frac{1}{2},\ Q(t) = -200$

$$\text{I.F.} = e^{\int -\left(\frac{1}{2}\right)dt} = e^{-\frac{t}{2}}$$

Hence, solution is

$$P(t)\cdot\ (\text{I.F.}) = \int Q(t)\cdot(I.F.)\,dt$$

$$P(t)\cdot e^{-\frac{t}{2}} = 400\, e^{-\frac{t}{2}} + K$$

$$P(t) = 400 + K\, e^{\frac{t}{2}}$$

If $P(0) = 100$, then $K = -300$

$$P(t) = 400 - 300e^{t/2}$$

55. **(a)**

Every square matrix satisfies its characteristics equation,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & -2 & 4-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)\{(1 - \lambda)(4 - \lambda) + 2\} = 0$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$A^3 - 6A^2 + 11A - 6I = 0 \qquad \text{...(i)}$$

Given, $6A^{-1} = A^2 + cA + dI$, multiplying both sides by A, we get

$$6I = A^3 + cA^2 + dA$$

$$A^3 + cA^2 + dA - 6I = 0 \qquad \text{...(ii)}$$

On comparing equation (i) and (ii), we get

$$c = -6 \text{ and } d = 11$$

56. (d)

Given differential equation is

$$y(1+xy)dx = xdy$$

$$ydx + xy^2dx = xdy$$

$$\frac{xdy - ydx}{y^2} = xdx$$

$$-d\left(\frac{x}{y}\right) = xdx$$

On integrating both sides, we get

$$\frac{-x}{y} = \frac{x^2}{2} + C \qquad \text{...(i)}$$

$\because$ It passes through (1, –1).

$\therefore$
$$1 = \frac{1}{2} + C$$

$$C = \frac{1}{2}$$

Now, from equation (i)

$$-\frac{x}{y} = \frac{x^2}{2} + \frac{1}{2}$$

$$x^2 + 1 = \frac{-2x}{y}$$

$$y = \frac{-2x}{x^2 + 1}$$

$$f\left(-\frac{1}{2}\right) = \frac{4}{5}$$

57. (c)

A square matrix M is invertible, if $|M| \neq 0$.

Let
$$M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

(a) Given, $$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix} \Rightarrow a = b = c = \alpha$$

$$M = \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \end{bmatrix}$$

$\Rightarrow$ $|M| = 0$

$\Rightarrow$ M is non-invertible.

(b) Given, $[b, c] = [a, b]$

$$a = b = c = \alpha$$

Again, $|M| = 0$

M is non-invertible.

(c) Given, $$M = \begin{bmatrix} \alpha & 0 \\ 0 & c \end{bmatrix}$$

$\Rightarrow$ $|M| = \alpha c \neq 0$

$\Rightarrow$ M is invertible.

(d) Given, $$M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$|M| = ac - b^2$$

ac is equal to square of the off diagonal element of M then $|M| = 0$

$\therefore$ M is invertible.

58. (d)

Since, $f(x)$ is continuous at $x = 0$.

$\therefore$ $f(0) = \text{LHL}$

$$\alpha = \lim_{h \to 0} \frac{1 - \cos 4h}{h^2}$$

$$= \lim_{h \to 0} \frac{2\sin^2 2h}{h^2} \times \frac{4}{4} = 8 \lim_{h \to 0} \left(\frac{\sin 2h}{2h} \right)^2 = 8 \times 1$$

$$\alpha = 8$$

59. (a)

The wave equation in one spatial dimension can be written as follows:

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$$

This equation is typically described as having only one spatial dimension x, because the only other independent variable is the time t.

60. (c)

Given, $$A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$$

Eigen value of the matrix,

$$\lambda = 1, 4$$

By properties of eigen values, for matrix $4A^{-1} + 3A + 2I$

(i) First eigen value $= 4 \times (1)^{-1} + 3 \times 1 + 2 = 9$

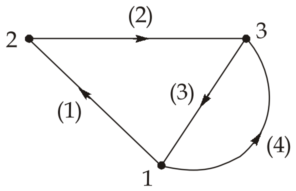
(ii) Second eigen value $= 4 \times (4)^{-1} + 3 \times 4 + 2 = 15$

Section C : Electrical Circuits-2 + Digital Electronics-2

61. (b)

To draw the graph,

Replace all resistors, inductors and capacitors by line segments, replace voltage source by short circuit and current source by an open circuit,



Complete incidence matrix (A_a)

$$A_a = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & -1 & 1 \\ 2 & -1 & 1 & 0 & 0 \\ 3 & 0 & -1 & 1 & -1 \end{array}$$

The reduced incidence matrix A is obtained by eliminating the last row from matrix A_a

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix}$$

The number of possible trees = $|AA^T|$

$$AA^T = \begin{bmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

$$|AA^T| = \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} = 6 - 1 = 5$$

62. (d)

$$L_1 + L_2 + 2M = 0.6 \qquad \text{...(i)}$$

$$L_1 + L_2 - 2M = 0.1 \qquad \text{...(ii)}$$

Also it is given that,

$$L_1 = 0.2 \text{ H}$$

$$\therefore \quad L_2 + 2M = 0.4 \qquad \text{...(iii)}$$

$$L_2 - 2M = -0.1 \qquad \text{...(iv)}$$

Solving (iii) and (iv),

$$L_2 = 0.15 \text{ H},$$

$$M = 0.125 \text{ H}$$

∴ Coefficient of coupling,

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.125}{\sqrt{0.2 \times 0.15}} = 0.721$$

63. (b)

The transfer function of a simple RC low-pass filter representing the relationship between its input and output signals is $H(s) = \frac{1}{1 + RCs}$.

64. (a)

The incidence matrix

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \uparrow \\ \text{Nodes} \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \leftarrow \text{Branches} \end{array} \\ \begin{pmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix} \\ \\ \\ \end{array}$$

65. (b)

For Z-parameter,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad \text{...(i)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \qquad \text{...(ii)}$$

For h-parameter, $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$

From equation (i), putting $I_1 = 0$, we get

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

For Y parameter, $Y_{22} = \frac{Z_{11}}{\Delta Z}$

For T-parameter, $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$

From equation (ii), $C = \dfrac{1}{Z_{21}}$

For g-parameter, $g_{22} = \left.\dfrac{V_2}{I_2}\right|_{V_1=0} = \dfrac{\Delta Z}{Z_{11}}$

66. **(c)**

The impedance matrix is given by,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & -j\omega M \\ -j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Here $L_1 = 3$ H, $L_2 = 8$ H, and $M = 2$ H

From figure (B), $L_{eq} = L_1 + L_2 + 2M = 3 + 8 + 4 = 15$ H

67. **(c)**

Every cutset has an even number of branches in common with every loop.

68. **(c)**

In an incidence matrix, incoming branches at any node are taken as negative and outgoing branches are taken as positive.

$$\therefore \qquad A_a = \begin{array}{c} \text{Nodes/} \\ \text{Branches} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

69. **(a)**

$$\log_2 126 = 6.98;$$

$$N = 7 \text{ D-FFs (greatest integer)}$$

70. **(b)**

- Sequential circuits store part information, unlike combinational circuit that depend only on present inputs. Therefore, statement-1 is correct.
- Sequenctial circuits use feedback loops to retain state information. Hence statement-2 is also correct.
- Sequential circuits depend on both past and present inputs. Thus, statement-3 is wrong.
- Due to feedback paths, sequential circuits often exhibit cyclic behaviour. Therefore, statement-4 is true.

71. **(c)**

Present state 1000

$$\text{Clock pulse } 92 = (16 \times 5) + 12$$

After 92 clock pulses, the state of MOD-16 up counter will be

(a) 12 states up the present state or (b) 4 states down the present state

1001, 1010, 1011, 1100, 1101, 1110, 1111, 0000, 0001, 0010, 0011, 0100

72. **(c)**

-

Type of *N*-bit ADC	**Maximum conversion time**
• Successive approximation	N clock cycles
• Counter digital ramp type	$2^N - 1$ clock cycles

- Flash converter is the fast converter. It uses no clock signal. It uses multiple comparators to increase speed of operation.
- Dual slope is a integrating type converter.

73. **(c)**

For transmission parameter,

$$A = D \quad \Rightarrow \quad \text{for Symmetry}$$

$$AD - BC = 1 \quad \Rightarrow \quad \text{for Reciprocity}$$

These conditions remain same for inverse transmission parameters too.

74. **(d)**

$$\because \qquad Z = [Y]^{-1}$$

$\therefore$ If Y parameters are reciprocal, then their equivalent Z-parameters are also reciprocal.

75. **(c)**

ECL transistor never goes into saturation which means fast operation. It continously remains in active region thus there is continous flow of current and high power losses.

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