UPPSC-AE 2020

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Electrical Engineering

Power Systems

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY Publications

Power Systems

Contents

CHAPTER 1

Line Parameters and Performance of Transmission Lines

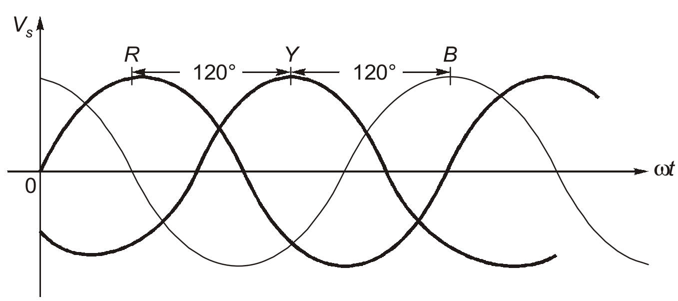
1.1 Introduction

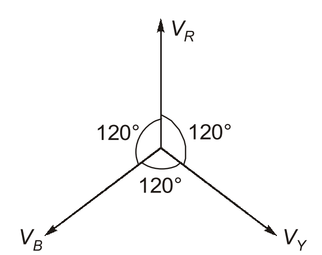
An "Electric power system" is a network of electrical components used to supply, transmit and use electric power. An example of an electric power system is the network that supplies a region's home and industry with power for sizable regions, this power system is called "the grid" and can be broadly divided into the generators that supply the power, the transmission system that carries the power from the generating stations to study the load centers and the distribution system that feeds the power to nearby homes and industries.

This chapter deals with fundamentals of power systems which includes polyphase AC circuits in order to study the characteristics of transmission lines. An electric transmission line can be represented by series combination of resistance, inductance and shunt combination of conductance and capacitance. This chapter thus deals with the series line parameters and shunt line parameters of a transmission line.

1.2 Three-Phase AC Systems

- A 3-ϕ circuit has an ac voltage generator (alternator) that produces three sinusoidal voltages that are identical except for a phase angle difference of 120° electrical.





Here, Phase sequence = RYB

$$V_R = V_m \sin\omega t = V\angle 0° \text{ Volt}$$

$$V_Y = V_m \sin(\omega t - 120°) = V\angle{-120°} \text{ Volt}$$

$$V_B = V_m \sin(\omega t - 240°) = V\angle{-240°} \text{ Volt} = V\angle{+120°} \text{ Volt}$$

NOTE: In general for a n-phase systems, phase difference is 360°/*n*.

1.2.1 Phase Sequence

- It is the order by which the phase voltages attains their peak value. The phase sequence may be positive, negative or zero (no particular sequence).
- RYB is a universally adopted phase sequence.

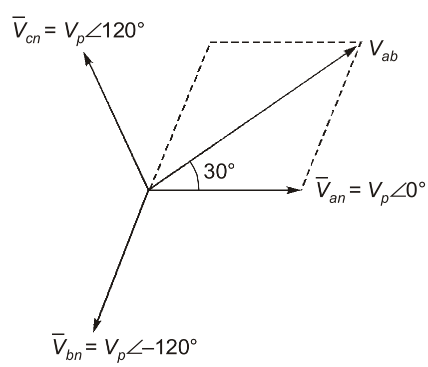
- For a 3-ϕ system phase sequence must be defined.
  (a) Positive phase sequence: i.e. RYB, YBR, BRY
  (b) Negative phase sequence: i.e. RBY
  (c) Zero phase sequence: No particular order of phase sequence
- For balanced 3-ϕ system: $I_R + I_Y + I_B = 0$
  For unbalanced 3-ϕ system: $I_R + I_Y + I_B \neq 0$

NOTE
- Phase voltage (V_p): It is the voltage between any one of the phase and neutral.
- Line voltage (V_L): Voltage measured between any two phases is known as line voltage.
- Phase current (I_P): The current flowing in any one phase is called phase current.
- Line current (I_L): The current flowing in the line is line current.

1.2.2 Types of Three-Phase Connections

There are basically two types of 3-phase connections:
(a) Star (Y) connection (b) Delta (Δ) connection

(a) Star (Y) connection:



Phase and Magnitude Relations between the Phase and Line Voltage of a Y-Connection

- The set of voltage V_{ab}, V_{bc} and V_{ca} are called the ***line voltages***, and the set of voltages V_{an}, V_{bn} and V_{cn} are referred as the ***phase voltages***.
- For a balanced system, each phase voltage has the same magnitude

$$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$$

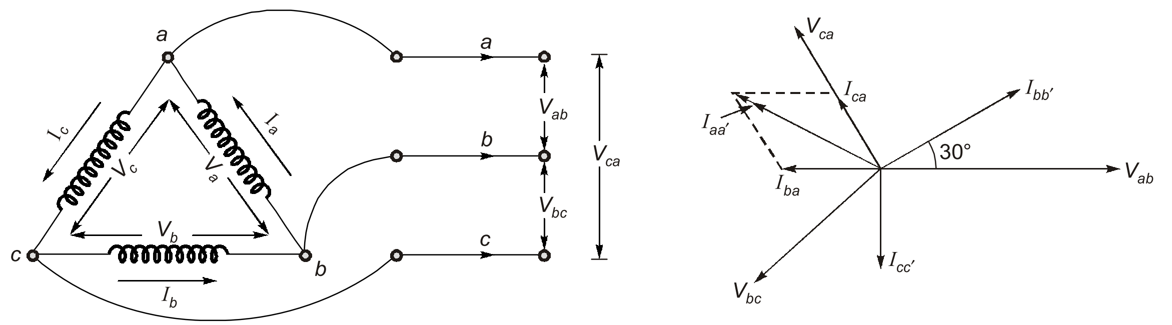
where V_p denotes the effective magnitude of the phase voltage

$$V_{ab} = \sqrt{3}\, V_p\angle 30°$$
$$V_{bc} = \sqrt{3}\, V_p\angle -90°$$
$$V_{ca} = \sqrt{3}\, V_p\angle 150°$$

NOTE
- The line voltages (V_L) constitute a balanced three-phase voltage system whose magnitudes are $\sqrt{3}$ times the phase voltages (V_p); $\boxed{V_L = \sqrt{3}\, V_p}$.
- The line current (I_L) and the phase current (I_p) have the same magnitude; $\boxed{I_L = I_p}$
- With star connected system, protection can be provided by connecting a protective device between neutral and earth for earth fault detection.

(b) Delta (Δ) connection:

For phase sequence (abc):



Relations between Phase and Line Currents in a Δ-Connection

NOTE

- The line and phase voltages have the same magnitude;

$$|V_L| = |V_p|$$

- A set of balanced three phase currents yields a corresponding set of balanced line currents that are $\sqrt{3}$ times the phase value;

$$I_L = \sqrt{3}\, I_p$$

Power Calculations

- Single phase power, $P_{1\text{-}\phi} = V_{Ph}\, I_{Ph} \cos\phi$
- Three phase power, $P_{3\text{-}\phi} = 3\, V_{Ph}\, I_{Ph} \cos\phi$
- Three phase power in star connection,

$$P_{3\text{-}\phi(Y)} = 3 \cdot \frac{V_L}{\sqrt{3}} \cdot I_L \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

- Three phase power in delta connection,

$$P_{3\text{-}\phi(\Delta)} = 3 \cdot V_L \cdot \frac{I_L}{\sqrt{3}} \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

- Three phase reactive power,

$$P_{(Y)} \text{ or } P(\Delta) = \sqrt{3}\, V_L I_L \sin\phi$$

- Total apparent power, $S = \sqrt{3}\, V_L I_L$

Example - 1.1 Three equal impedances ($R + jX$) connected in delta carry a balanced line current of I_L. The total active and reactive power drawn by these are

(a) $I_L^2 R$ and $I_L^2 X$ respectively (b) $3 I_L^2 R$ and $3 I_L^2 X$ respectively
(c) $I_L^2 R/3$ and $I_L^2 X/3$ respectively (d) $I_L^2 X$ and $I_L^2 R$ respectively

[UPPSC]

Solution: (a)

Per phase impedance, $Z = R + jX$

Given, Line current $= I_L$

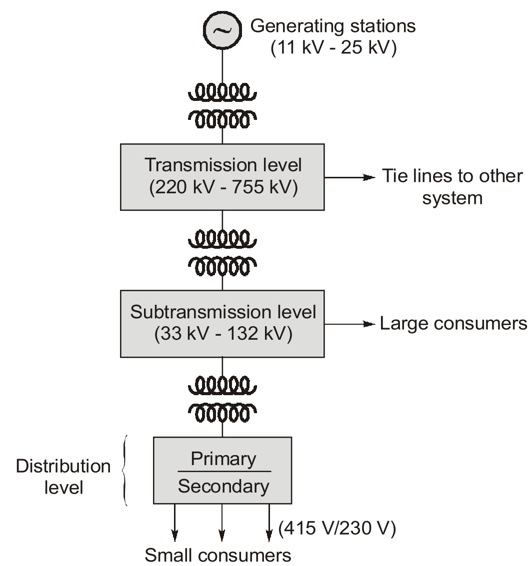
For delta connection, $I_P = \frac{I_L}{\sqrt{3}}$

Total active power, $P_T = 3(I_P)^2 R = 3\left(\frac{I_L^2}{3}\right)R = I_L^2 R$

Similarly total reactive power drawn will be $I^2 X$.

1.3 Electric Supply System

- The conveyance of electric power from a power station to consumer's premises is electric supply system.
- An electric supply system consists of three principles components i.e. power station, the transmission and the distribution system.
- Now-a-day, 3-phase 3-wire ac system is universally adopted for generation and transmission of electric power.
- However, distribution of electric power is done by 3-phase 4-wire ac system.
- Schematic diagram depicting power system structure.



(Typical AC power supply scheme)

Generating Station

- Station where the electric power is produced.
- The usual generation voltage is from (11-25 kV).
- Due to several advantages of high voltage transmission, the generation voltage is stepped up at the generating station.

Primary Transmission

- Generally the primary transmission is carried at 66 kV, 132 kV, 220 kV, 400 kV or 765 kV with help of 3-phase 3-wire overhead system from power plant after stepping up voltage to above mentioned voltage levels to receiving stations at the outskirts of city.

Secondary Transmission

- The primary transmission lines terminate at receiving station and here the voltage is reduced to 33 kV/66 kV for further distribution, by step down transformers.

Primary Distribution

- The secondary transmission line terminates at the substation (ss) where the voltage is reduced from 33 kV to 11 kV, 3-phase, 3-wire system.
- Large consumers are generally supplied power at 11 kV for further handling with their own substations.

Secondary Distribution

- The electric power from primary distribution line (11 kV) is delivered to distribution substations (DS).

- The substations near consumer step down the voltage to 415 V, 3-phase 4-wire for secondary distribution.
- Various voltage levels in power system:

Voltage Level	Voltage Lines
(a) Low voltage	230 V(1-ϕ), 400 V(3-ϕ)
(b) High voltage	11 kV, 33 kV
(c) Extra high voltage (EHV)	66 kV, 132 kV, 220 kV
(d) Modern EHV	400 kV
(e) Ultra high voltage	765 kV and above

NOTE

- The voltage at the point of commencement of supply shall not vary by more than
  - $\Rightarrow$ $\pm 6\%$ in the case of the low voltage (upto 250 V) and medium voltage (upto 650 V).
  - $\Rightarrow$ $+6\%$ to -9% in the case of high voltage (i.e. upto 33 kV).
  - $\Rightarrow$ $\pm 12\frac{1}{2}\%$ in the case of extra high voltage (i.e. exceeding 33 kV).
- An ideal transmission line has 0% regulation.

1.3.1 Effect of System Voltage on Transmission

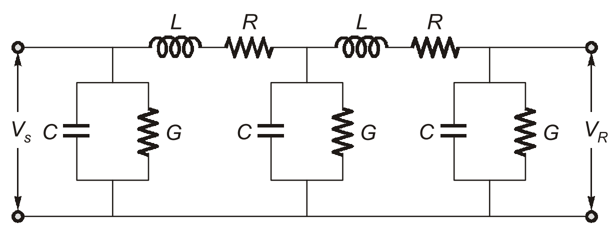
- Power loss in the line is inversely proportional to the system voltage and power factor both.
- Percentage voltage drop in resistance decreases with the increase in the system voltage.
- Weight of the conductor material for the line will decreases with the increase in supply voltage and power factor.
- Efficiency of transmission, increases with the increase of supply voltage and power factor.
- Higher supply voltages also enhances the system stability.
- The problems encountered with high voltages are the insulation of the equipment, corona, radio and television interference.
- The voltage level of a system is therefore governed by the amount of power to be transmitted and the length of the line.

1.4 Introduction to Transmission Lines

- The performance of a transmission line is governed by it's four parameters namely series resistance R and inductance L, shunt capacitance C and conductance G.
- The resistance R is due to the fact that every conductor offers opposition to the flow of current.
- The inductance L is due to the fact that the current carrying conductor is surrounded by the magnetic lines of force. The capacitance of the line is due to the fact that the voltage which is always at lower potential than the conductor and the air between them forms a dielectric medium.
- The shunt conductance is mainly due to flow of leakage currents over the surface of the insulators.
- The line resistance causes voltage drop (IR volts) and power loss (I^2R watts) in the line, line inductance causes voltage drop ($2\pi fLI$ volts), line capacitance produces a current called "charging current".
- The performance of the transmission line can be determined by following two parameters:

(a) Percentage efficiency
(b) Percentage regulation

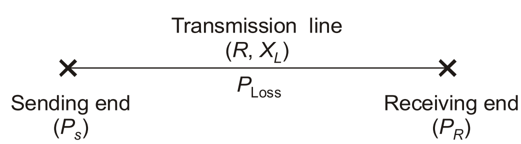
- Resistance, R is due to the fact that every conductor offers opposition to the flow of current.
- Inductance, L is due to the fact that the current carrying conductor has surrounding magnetic lines of force.
- Capacitance, C results due to potential difference between the conductors and the air serving as a dielectric medium between them.
- Shunt conductance G, which is due to leakage over line insulators is almost neglected in overhead transmission lines.



(A transmission line model)

- The line resistance causes voltage drop (IR) and power loss (I^2R) in the transmission line, line inductance causes voltage drop ($IX_L = \omega LI$), line capacitance produces a charging current (I_C).

(a) Efficiency of a Transmission Line



- Let a transmission line with R and X_L as series parameters and P_S, P_R respectively are sending and receiving end powers. Losses in the line is $P_{Loss} = P_S - P_R$
- Efficiency of a transmission line is defined as the ratio of power delivered at the receiving end to the power sent from the sending end.
- Percentage efficiency of transmission line is given by,

$$\%\eta = \frac{P_R}{P_S} \times 100 = \frac{P_S - P_{Loss}}{P_S} \times 100 = \frac{P_R}{P_R + P_{Loss}} \times 100$$

NOTE
- Higher the efficiency of transmission line, better is the performance of the transmission line.
- An ideal transmission line has 100% efficiency (i.e. zero losses) which is not possible practically.

(b) Voltage Regulation of a Transmission Line

- It is defined as the rise in voltage at the receiving end expressed as percentage of full load voltage, when full load at a specified power factor is thrown off i.e.

$$\%\text{regulation} = \frac{V_{RNL} - V_{RFL}}{V_{RFL}} \times 100$$

where, V_{RNL} = Magnitude of no-load receiving end voltage
V_{RFL} = Magnitude of full load receiving end voltage

- This is basically a voltage drop in the line due to resistance and inductance of the line.

NOTE

- An ideal transmission line has 0% regulation.
- Lower the voltage regulation, better is the performance of transmission line.

ABCD Parameters:

- Sending end quantities V_s, I_s can be represented in terms of receiving end quantities V_r, I_r by the following equations,

- $V_s = AV_R + BI_R$
  $I_s = CV_R + DI_R$
  where A, B, C, D are the parameters of transmission on line.
- In matrix form, $$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$
- Calculation of A, B, C, D:

$$A = \left.\frac{V_s}{V_r}\right|_{I_r=0} \qquad C = \left.\frac{I_s}{V_r}\right|_{I_r=0}$$

$$B = \left.\frac{V_s}{I_r}\right|_{V_r=0} \qquad D = \left.\frac{I_s}{I_r}\right|_{V_r=0}$$

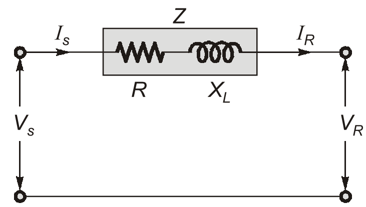
1.4.1 Classification of Transmission Lines

- Transmission lines are classified based on three criterion and they are:
  (a) Length of transmission line
  (b) Operating voltage
  (c) Effect of capacitance
- On the basis of length of transmission line, transmission lines are classified as:
  (i) Short transmission line
  (ii) Medium transmission line
  (iii) Long transmission line

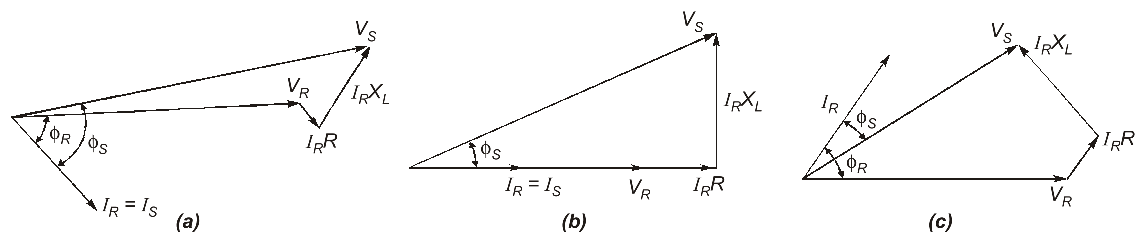
Transmission lines	Length of transmission lines	Operating voltage	Effect of capacitance(C)
Short transmission line	(0 – 80) km	(0 – 20) kV	Neglected
Medium transmission line	(80 – 160) km	(20 – 100) kV	C is lumped or concentrated
Long transmission line	(> 160) km	(> 100) kV	C is distributed

(A) Short Transmission Line

- **Equivalent circuit:**



- **Phasor diagram:**



(Phasor diagrams of a short transmission line for same V_R and I_R
(a) Lagging p.f. (0.7 lag), (b) unity p.f. and (c) leading p.f.

- **Sending end parameters:**

$$I_S = I_R$$
$$V_S = V_R + I_R Z$$

where, Z is zl, the total series impedance of the line (l being the total length of the line).

- **In matrix form:**

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

where, $ABCD$ parameters are $A = D$, $B = Z$, $C = 0$

NOTE

- $AD - BC = 1$, hence short line is also a reciprocal network.
- For a short transmission line, conductance and capacitance are neglected.
- For short line, line parameters are assumed to be lumped.
- In general, product of physical length (l) and frequency (f) for short transmission line:

$$\boxed{fl < 4000 \text{ Hz km}}$$

- In general, product of F and l is greater than 12000 Hz km for long transmission line.
- % Voltage regulation = $\dfrac{|V_s| - |V_R|}{|V_R|} \times 100$

  where V_s is sending end voltage or no load receiving end voltage and V_R is full load rated receiving end voltage.

Further, using phasor diagram (a) and (c),

$$V_s = V_R + |I_R| R\cos\phi_R + |I_R| X_L \sin\phi_R \quad \text{...(For lagging p.f.)}$$

$$V_s = V_R + |I_R| R\cos\phi_R - |I_R| X_L \sin\phi_R \quad \text{...(For leading p.f.)}$$

For lagging p.f.: $\%\text{V.R.} = \dfrac{|I_a|R\cos\phi_R + |I_a|X_L\sin\phi_R}{|V_a|} \times 100$

For leading p.f.: $\%\text{V.R.} = \dfrac{|I_a|R\cos\phi_R - |I_R|X_L\sin\phi_R}{|V_R|} \times 100$

Condition for zero voltage regulation:

- This is possible at leading p.f. load.
- For zero VR: $I_R R\cos\phi_R - IX_L\sin\phi_R = 0$

$$\Rightarrow \quad \boxed{\tan\phi_R = \frac{R}{X_L}}$$

or, $$\phi = \tan^{-1}\left(\frac{R}{X_L}\right)$$

Condition for maximum voltage regulation:

- Maximum regulation occurs at lagging power factor load.

  i.e. $\dfrac{d}{d\phi_R}$[Voltage regulation] = 0

$$\Rightarrow \quad \boxed{\tan\phi_R = \frac{X_L}{R}}$$

or, $$\phi_R = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Condition for negative voltage regulation:

- This occurs at leading p.f.
- Condition is, $\tan\phi_R > \dfrac{R}{X_L}$ (leading)
- Zero regulation and maximum regulation coincides when θ = 0.45°. i.e. cos(45°) = 0.707 (lag)
- Power factor has a great influence on voltage regulation, for lagging power factor the regulation is positive and for leading power factors may even be negative.

Efficiency: It is given by

$$\%\eta = \frac{\text{Power received at receiving end}}{\text{Power received at receiving end + losses}} \times 100$$

$$\%\eta = \frac{P}{P + 3I_r^2 R} \times 100$$

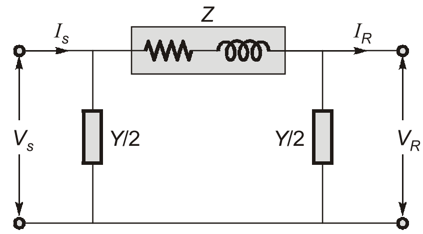
where, P = Power received at receiving end
R = Resistance per phase of the line

(B) Medium Transmission Line

- In medium transmission lines, the shunt admittance (usually capacitance) is not neglected.
- Basically these lines can be modelled into two basic circuits:
  (i) Nominal π-circuit (ii) Nominal-T circuit

(i) Nominal π-circuit:

Here the total shunt admittance is divided into two equal parts placed at the sending and the receiving ends of the line.



In matrix form:

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ Y\left[1+\frac{YZ}{4}\right] & 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

ABCD parameters,

$$A = 1+\frac{YZ}{2}, \qquad B = Z$$

$$C = Y\left[1+\frac{YZ}{4}\right], \qquad D = 1+\frac{YZ}{2}$$

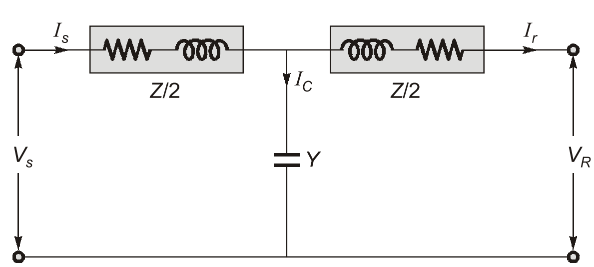
NOTE ▶

- $A = D = \left(1+\frac{YZ}{2}\right)$ therefore, a nominal-π method equivalent circuit of a medium transmission line is symmetrical.
- $AD - BC = \left(1+\frac{YZ}{2}\right)\left(1+\frac{YZ}{2}\right) - YZ\left(1+\frac{YZ}{4}\right) = 1$

  Therefore, it is a reciprocal network.

(ii) Nominal T-circuit:

Shunt admittance is assumed to be concentrated in the middle while the series impedance is split into two equal part.



In matrix form:

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z\left[1+\frac{YZ}{4}\right] \\ Y & 1+\frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

A, *B*, *C*, *D* parameters,

$$A = D = 1 + \frac{YZ}{2}, \quad C = Y$$

$$B = Z\left[1 + \frac{YZ}{4}\right]$$

NOTE

- Here,

$$A = D = \left(1 + \frac{YZ}{2}\right) \text{ and } AD - BC = \left(1 + \frac{YZ}{2}\right)^2 - YZ\left(1 + \frac{YZ}{4}\right) = 1.$$

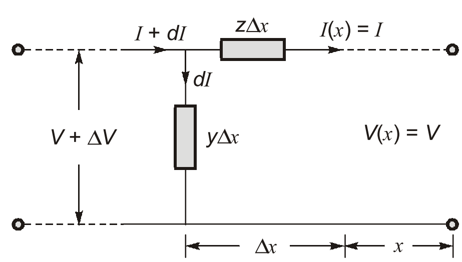
  Therefore, nominal-T representation of a medium transmission line is both symmetrical and reciprocal.
- Nominal-T and nominal-π with the obtained *ABCD* constants are not equivalent to each other which can be verified by applying star-delta transformation to either one.

(C) Long Transmission Line

- In long transmission line, the parameters of the lines are not lumped, but rather are distributed uniformly throughout the length of the line.
- Let's consider a differential element of length (Δx) at a distance (x) from the receiving end.

  ($z\Delta x$) and ($y\Delta x$) respectively are the series impedance and shunt impedance of the differential element section and

$$V(x) = \text{voltage at location } x$$
$$I(x) = \text{current at location } x$$



(Incremental length of transmission line)

Here, z = series impedance percent length = $r + j\omega l$
where r and l are series resistance and inductance per unit length.
y = shunt admittance per unit length = $g + j\omega c$
where g and c are shunt conductance and capacitance to neutral per unit length

- In matrix form:

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \frac{1}{Z_c} \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

- *ABCD* parameters are:

$$A = D = \cosh \gamma l, \qquad B = Z_c \sinh \gamma l$$

$$C = \frac{1}{Z_c} \sinh \gamma l$$

where, $\chi = \sqrt{yz}$ = propagation constant of transmission line

$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\omega l}{g + j\omega c}}$ = characteristics impedance of line

l = total length of long transmission line

NOTE

- Propagation Constant:

  Propagation constant is given by $\gamma = \sqrt{yz} = (\alpha + j\beta)$.

  Here, α is called *"attenuation constant"* and is measured in nepers per unit length. β is called *"Phase constant"* and is measured in radians per unit length.
- Here, $A = D$ and $AD - BC = 1$ therefore, a long transmission line is both symmetrical and reciprocal.

Characteristic Impedance (Z_c)

- If a line is terminated in it's characteristic impedance, receiving end voltage V_R is equal to $I_R Z_C$ and there is no reflection of either voltage or current wave. A line terminated in its characteristic impedance is called "flat line" or an infinite line.
- Value of Z_c is 400 Ω for single overhead line and Z_c is 200 Ω for two circuits in parallel and Z_c is 40 Ω for underground cable.

Surge Impedance (Z_s)

- It is reserved for the special case of lossless line.
- For a lossless line, $r = 0$, $g = 0$
- $Z_s = \sqrt{\frac{L}{C}}$ = surge impedance of the line.
- It is also called as natural impedance.

NOTE: Surge impedance is independent of length of the line.

Surge Impedance Loading (SIL)

- It is the power transmitted when the line is terminated through a purely resistive load equal to surge impedance.
- SIL is calculated as: $SIL = \frac{|V_L|^2}{Z_s}$

  where, $|V_L|$ = line to line voltage at load

  Z_s = surge impedance of the line
- The permissible loading of a transmission line may be expressed as a fraction of its *SIL*, and *SIL* provides a comparison of load carrying capabilities of lines.

Z_c and Z_s in Terms of Line Parameters

- **Characteristic impedance:**

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + jX_L}{G + jB_C}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Here, G = real part of admittance (Y) = conductance

and B_C = Imaginary part of admittance (Y) = capacitive susceptance.

- **Surge impedance:** Case of ideal transmission line ($R = 0$, $G = 0$)

Hence, $$Z_s = \sqrt{\frac{L}{C}}$$

NOTE

- Cables have low value of surge impedance because cables have relatively large capacitance and low inductance of cables.
  During flat line voltage profile, magnitude of voltage remains same at all points on the transmission line, but phase may be different.
- For distortion less line: $$\frac{R}{L} = \frac{G}{C}$$
  When loading is equal to SIL, flat voltage profile is obtained and SIL is also called as ideal loading capability of the line.
- The characteristic impedance of the line is equal to the geometric mean of the open-circuit and short-circuit impedances i.e., $Z_c = \sqrt{Z_{O.C.} \cdot Z_{S.C.}}$.

Example - 1.2 In a short transmission line, resistance and inductance are found to be equal and regulation appears to be zero, then the load will be

(a) Unity power factor (b) Zero power factor
(c) 0.707 lagging (d) 0.707 leading [UPPSC]

Solution: (d)

For zero voltage regulation, power factor has to be leading when $\tan\phi = \frac{X}{R}$

If $R = X$ (as given)

$\tan \phi = 1$

i.e. $\phi = 45°$

So, pf $= \cos 45°$

$= 0.707$ (leading)

Example - 1.3 A single load of 100 kVA is delivered at 2000 V over a transmission line having $R = 1.4\ \Omega$ and $X = 0.8\ \Omega$. The voltage at the sending end, when the power factor of the load is unity, will be

(a) 1.68 kV (b) 2.98 kV
(c) 2.07 kV (d) 2.84 kV [UPPSC]

Solution: (c)

Given, $R = 1.4\ \Omega$

$X = 0.8\ \Omega$

$\cos \phi = 1$

$V_R = 2$ kV

$$I_R = \frac{P_R}{V_R \cos\phi} = \frac{100 \times 10^3}{1 \times 2000} \; 50 \text{ A}$$

Sending end voltage, $V_S = V_R + I_R \, . \, Z$

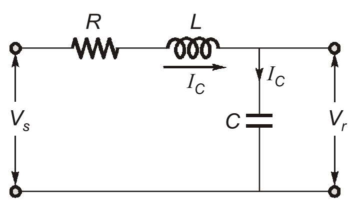
Here, $V_S = 2000 + (50\angle 0°)(1.4 + j0.8)$

$V_S = 2070.38 \angle 1.1° \text{ V}$

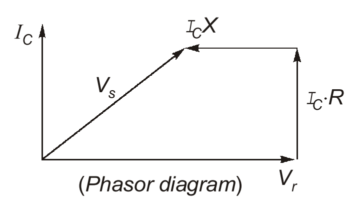
Magnitude, $|V_S| = 2070.38 \text{ V} \simeq 2.07 \text{ kV}$

1.5 Ferranti Effect

- When a medium or long transmission line operates under no load or light load condition, the receiving end voltage is greater than the sending end voltage. This is known as Ferranti effect.
- This effect is experienced only in medium and long transmission lines operating under no load or light load condition.
- This effect is due to voltage across line inductance due to charging current.
- Due to negligible capacitance and hence negligible charging current it does not occur in short transmission line.



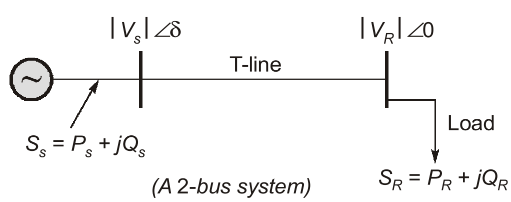
(*Line under NL condition*)



(*Phasor diagram*)

1.6 Power Flow through a Transmission Line

- Let, consider a single transmission line, two-bus system as shown:



(A 2-bus system)

- Let, δ = torque angle or load angle

$V_R = |V_R|\angle 0°$ = receiving end voltage

$V_s = |V_s|\angle\delta$ = sending end voltage

ABCD parameters, $A = D = |A|$

$B = |B|\angle\beta$

- We know that, $V_s = AV_R + BI_R$

$I_s = CV_R + DI_R$

Hence, $$I_R = \frac{V_s}{B} - \frac{A}{B} \cdot V_R$$

- Complex power per phase at receiving end,

$$S_R = V_R \cdot I_R^* = P_R + jQ_R$$

or,
$$S_R = \frac{|V_S||V_R|}{|B|}\angle(\beta-\delta) - \left|\frac{A}{B}\right||V_R|^2\angle(\beta-\alpha)$$

where, S_R = Complex power per phase at the receiving end
P_R = Active power per phase at the receiving end
and Q_R = Reactive power per phase at the receiving end

Now at receiving end:

Real power,
$$P_R = \frac{|V_s||V_R|}{|B|}\cos(\beta-\delta) - \frac{|A||V_R|^2}{|B|}\cos(\beta-\alpha)$$

Reactive power,
$$Q_R = \frac{|V_s||V_R|}{|B|}\sin(\beta-\delta) - \frac{|A||V_R|^2}{|B|}\sin(\beta-\alpha)$$

$$I_s = \frac{D}{B}V_s - \frac{1}{B}V_R \qquad [\because AD - BC = 1]$$

NOTE

- Maximum active power is delivered at the receiving end when $\beta = \delta$

$$\therefore \quad P_{max} = P|_{\beta=\delta} = \frac{|V_S||V_R|}{|B|} - \frac{|A||V_R|^2}{|B|}\cos(\beta-\alpha)$$

- When, $\beta = \delta$

$$Q_R = -\frac{|A||V_R|^2}{|B|}\sin(\beta-\alpha)$$

Therefore, for maximum active power at the receiving end, a leading VAR/KVAR is being delivered for all values of V_S.

1.6.1 Power Flow Equation for Short Transmission Line

For a short transmission line,
$A = D = 1\angle 0°$, $B = Z = |Z|\angle\theta$ and $C = 0$ (No admittance as C is neglected)

Here,
$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \text{Impedance angle}$$

Hence, $\alpha = 0$,
$$|A| = |D| = 1$$

Therefore, power flow equations be

$$P_R = \frac{|V_S||V_R|}{|Z|}\cos(\theta-\delta) - \frac{|V_R|^2}{|Z|}\cos\theta$$

$$Q_R = \frac{|V_S||V_R|}{|Z|}\sin(\theta-\delta) - \frac{|V_R|^2}{|Z|}\sin\theta$$