CLASS TEST

S.No.: 01JP_CS_EM_260324

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COMPUTER SCIENCE & IT

Engineering Mathematics

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q. No. 1 to Q. No. 10 carry 1 mark each

Q.1 Which of the following represents the solution to the system of equation?

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

(a) 12, –4 (b) –12, –4
(c) –12, 4 (d) 12, 4

Q.2 The normal distribution $N(\mu, \sigma^2)$ with mean $\mu \in R$ and variance $\sigma^2 > 0$ has probability distribution function:

$$N(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right) \text{ for } x \in R$$

The difference of median and mean is ________.

(a) μ (b) σ
(c) $-\mu$ (d) 0

Q.3 A bag contains 15 defective items and 35 non defective items. If three items are selected at random without replacement, what will be the probability that all three items are defective?

(a) $\frac{1}{40}$ (b) $\frac{13}{560}$
(c) $\frac{15}{34}$ (d) $\frac{12}{499}$

Q.4 Which one of the following represents the eigen vectors of matrix $\begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix}$?

(a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Q.5 Find the limit?

$$\lim_{x\to\infty}\left[1+\frac{3}{2x}\right]^{5x}$$

(a) e^{15} (b) e^{3}
(c) $e^{15/2}$ (d) $e^{5/3}$

Q.6 Consider the following function:

$$f(x) = \begin{cases} -1.5x^2, & x \le -2 \\ 6x-5, & x > -2 \end{cases}$$

Which of the following is true at $x = -2$?
(a) Continuous but not differentiable
(b) Differentiable and continuous both
(c) Differentiable but not continuous
(d) neither continuous nor differentiable

Q.7 Consider a man is known to speak truth 3 out of 5 times, he throw a die and reports the number obtained is 2. What is the probability that the number obtained is actually 2?

(a) $\frac{13}{30}$ (b) $\frac{3}{13}$
(c) $\frac{1}{10}$ (d) None of the above

Q.8 Given that the determinant of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is –96, the determinant of the matrix $\begin{bmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{bmatrix}$ is

(a) 192 (b) 384
(c) –384 (d) –768

Q.9 The value of $\lim_{x\to 4}\frac{(2x)^{1/3}-2}{2x-8}$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$
(c) $\frac{1}{24}$ (d) $\frac{1}{12}$

Q.10 The maximum and minimum of the function $f(x) = x^3 - 6x^2 + 9x + 1$, $x \in [0, 5]$, attain at x = ________ respectively.

(a) 0 and 5 (b) 5 and 0
(c) 3 and 0 (d) –1 and –3

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 Consider X be a random variable with E(X) = 10 and Var(X) = 25. What is the positive value of a and b such that Y = aX – b has expectation 0 and variance 1?

(a) $a = 1, b = 2$ (b) $a = 0.2, b = 2$
(c) $a = 0.2, b = 1$ (d) $a = 0.2, b = 0.5$

Q.12 What is the standard deviation of a uniformly distributed variable between 0 and $\frac{1}{2}$?

(a) $\frac{1}{2\sqrt{12}}$ (b) $\frac{1}{\sqrt{12}}$
(c) $\frac{2}{\sqrt{12}}$ (d) $\frac{1}{\sqrt{6}}$

Q.13 For a given matrix $M = \begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$ where $i = \sqrt{-1}$, the inverse of matrix M is

(a) $\frac{1}{225}\begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$

(b) $\frac{1}{225}\begin{bmatrix} i & 12-9i \\ 12+9i & -i \end{bmatrix}$

(c) $\frac{1}{224}\begin{bmatrix} 12-9i & i \\ -i & 12+9i \end{bmatrix}$

(d) $\frac{1}{224}\begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$

Q.14 Consider 'A' is a set containing n elements. A subset 'P' of 'A' is chosen at random. The set 'A' is reconstructed by replacing the elements of 'A'. A subset 'Q' of 'A' is again chosen at random. What is the probability that 'P' and 'Q' have no common element?

(a) $(0.75)^n$ (b) $(0.85)^n$
(c) $(0.95)^n$ (d) None of these

Q.15 Consider the following function:

$$f(x) = \begin{cases} \dfrac{x-c}{1+c}, & \text{if } x \le 0 \\ x^2 + c, & \text{if } x > 0 \end{cases}$$

Which of the following value of c, for which function is continuous for every 'x'?

(a) 2 (b) –2
(c) 0 (d) Both (b) and (c)

Q.16 Which of the following matrix is LU decomposible?

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix}$

Q.17 Consider the following table with data recorded over a month with 30 days:

		Weather: Sunny	Weather: Not sunny
Mood	Good	12	9
	Not Good	4	5

If Rahul recorded on each day, whether it was sunny or not sunny and whether Rahul's mood was good or not good. If given day is sunny, then what is the probability that on given day Rahul's mood is good?

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$
(c) $\frac{5}{16}$ (d) $\frac{16}{30}$

Q.18 The value of the integral given below is:

$$\int_{\pi/6}^{\pi/3} \frac{\operatorname{cosec}^2 x}{\cot^2 x}\, dx$$

(a) $\frac{2}{3}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\frac{3}{2}$ (d) $2\sqrt{3}$

Q.19 What is the value of $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$?

(a) abc (b) $\sqrt[2]{abc}$

(c) $\sqrt[3]{abc}$ (d) $(abc)^3$

Q.20 An artillery target may be either at point 1 with probability $\frac{8}{9}$ or at point 2 with probability $\frac{1}{9}$. We have 21 shells, each of which can be fired at point 1 or point 2. Each shell may hit the target, independently of other shells, with probability $\frac{1}{2}$. If 12 shells are fired at point 1 and 9 shells are fired at point 2, what is the probability that the target is hit?

(a) $\frac{8}{9}2^{12}+\frac{1}{9}2^{9}$

(b) $\frac{8}{9}\left(\frac{1}{2^{12}}\right)+\frac{1}{9}\left(\frac{1}{2^{9}}\right)$

(c) $\frac{8}{9}\left(1-\frac{1}{2^{12}}\right)+\frac{1}{9}\left(1-\frac{1}{2^{9}}\right)$

(d) None of these

Q.21 A matrix has eigen values –6 and –3, the corresponding eigen vectors are $\begin{bmatrix}3\\-6\end{bmatrix}$ and $\begin{bmatrix}3\\-3\end{bmatrix}$ respectively. The matrix is

(a) $\begin{bmatrix}-3 & 0\\0 & -6\end{bmatrix}$ (b) $\begin{bmatrix}3 & 3\\-3 & -6\end{bmatrix}$

(c) $\begin{bmatrix}0 & 3\\-6 & -9\end{bmatrix}$ (d) $\begin{bmatrix}3 & 6\\-6 & -12\end{bmatrix}$

Q.22 A function $y = 7x^2 + 12x$ is defined over an open interval $x = (1, 3)$. At least at one point is this interval, $\frac{dy}{dx}$ is exactly

(a) 26 (b) 40

(c) 62 (d) 54

Q.23 A random variable x has the following probability distribution.

x	0	1	2	3	4
$P(x)$	c	$2c$	$2c$	c^2	$5c^2$

The mean and variance of x is

(a) 1.638, 1.45 (b) 1.638, 1.204

(c) 1.204, 1.45 (d) 1.45, 1.638

Q.24 Consider function $f(x) = (x^2 - 4)^2$ where x is a real number. Which of the following is true about given function?

(a) Has only one minima

(b) Has only two minima

(c) Has three minima

(d) Has three maxima

Q.25 Assume A and B are matrix of size $n \times n$, which of the following is true?

(a) If A is invertible, the $ABA^{-1} = B$.

(b) If A is an indempotent non-singular matrix, then A must be the identity matrix.

(c) If the coefficient matrix A of the system $Ax = b$ is invertible, then the system has infinitely many solution.

(d) If $AB = B$ then B is identity matrix.

Q.26 Which one of the following represents the eigen vectors of matrix $\begin{bmatrix}4 & 6\\2 & 8\end{bmatrix}$?

(a) $\begin{bmatrix}3\\1\end{bmatrix}$ (b) $\begin{bmatrix}-3\\-1\end{bmatrix}$

(c) $\begin{bmatrix}1\\3\end{bmatrix}$ (d) $\begin{bmatrix}2\\2\end{bmatrix}$

Q.27 If a number x is selected from natural numbers 1, 2, 3, 4, ... 20. The probability that x follows $x+\frac{50}{x}>15$ is _______.

(a) $\frac{10}{20}$ (b) $\frac{14}{20}$

(c) $\frac{4}{20}$ (d) $\frac{15}{20}$

Q.28 Let $P = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$. If $A = PDP^{-1}$, then A^5 is

(a) $\begin{bmatrix} 8 & 12 \\ 0 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 32 & 0 \\ 0 & -32 \end{bmatrix}$

(c) $\begin{bmatrix} 32 & -192 \\ 0 & -32 \end{bmatrix}$ (d) $\begin{bmatrix} 32 & -96 \\ 0 & -32 \end{bmatrix}$

Q.29 Two matrixes A and B are given below:

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}, B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$$

If the rank of matrix A is N, then the rank of matrix B is

(a) $\frac{N}{2}$ (b) $N - 1$

(c) N (d) $2N$

Q.30 Consider the following system of equations:

$$\begin{aligned} 8x + 3y - 2z &= 8 \\ 2x + 3y + 5z &= 9 \\ 2x + 3y + \lambda z &= \mu \end{aligned}$$

The system of equations has no solution for values of λ and μ given by

(a) $\lambda = 5$ and $\mu \neq 9$

(b) $\lambda = 5$ and $\mu = 9$

(c) $\lambda \neq 5$ and $\mu = 9$

(d) $\lambda \neq 5$ and $\mu \neq 9$

■■■■

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ENGINEERING MATHEMATICS

COMPUTER SCIENCE & IT

Date of Test : 26/03/2024

ANSWER KEY

1.	(a)	7.	(b)	13.	(c)	19.	(c)	25.	(b)
2.	(d)	8.	(d)	14.	(a)	20.	(c)	26.	(d)
3.	(b)	9.	(d)	15.	(d)	21.	(c)	27.	(b)
4.	(c)	10.	(b)	16.	(b)	22.	(b)	28.	(c)
5.	(c)	11.	(b)	17.	(b)	23.	(a)	29.	(c)
6.	(d)	12.	(a)	18.	(b)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$R_2 + 2R_1$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or $$y = -4$$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

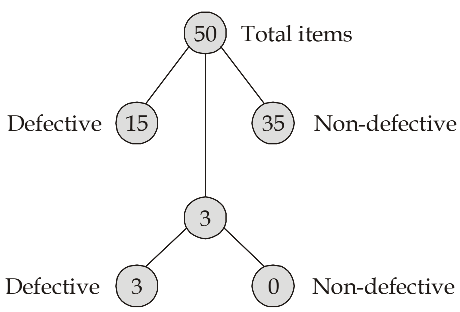
$\Rightarrow$ $$x = 12$$

$\therefore$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$

2. (d)

Mean, median and mode are all same (μ) for normal distribution.

3. (b)



$$\text{Required probability} = \frac{{}^{15}C_3 \times {}^{35}C_0}{{}^{50}C_3}$$

$$= \frac{15 \times 14 \times 13}{50 \times 49 \times 48} = \frac{13}{560}$$

4. (c)

The characteristic equation $|A - \lambda I| = 0$

i.e. $$\begin{vmatrix} 4-\lambda & 6 \\ 2 & 8-\lambda \end{vmatrix} = 0$$

or $(4 - \lambda)(8 - \lambda) - 12 = 0$

or $32 - 8\lambda - 4\lambda + \lambda^2 - 12 = 0$

$\Rightarrow$ $\lambda^2 - 12\lambda + 20 = 0$

$\Rightarrow$ $\lambda^2 - 10\lambda - 2\lambda + 20 = 0$

$\Rightarrow$ $(\lambda - 10)(\lambda - 2) = 0$

$\Rightarrow$ $\lambda = 10, 2$

Corresponding to $\lambda = 10$, we have

$$[A - \lambda I]x = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $-6x + 6y = 0$

$\Rightarrow$ $x = y$

$2x - 2y = 0$

$\Rightarrow$ $x = y$

i.e. eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = 2$, we have

$$[A - \lambda I]x = \begin{bmatrix} 2 & 6 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $2x + 6y = 0$ i.e. eigen vector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

5. (c)

$$\lim_{x\to\infty}\left[1+\frac{3}{2x}\right]^{5x}$$

Put limit $x \to \infty$

1^{∞} from create,

So, we know, for form 1^{∞}

$$\lim_{x\to\infty} f(x)^{g(x)} = e^{\left(\lim\limits_{x\to\infty}(f(x)-1)\cdot g(x)\right)}$$

Apply in given function:

$$= e^{\lim\limits_{x\to\infty}\left[1+\frac{3}{2x}-1\right]5x}$$

$$= e^{\lim\limits_{x\to\infty}\left[\frac{3}{2x}\right]5x}$$

$$= e^{15/2}$$

6. (d)

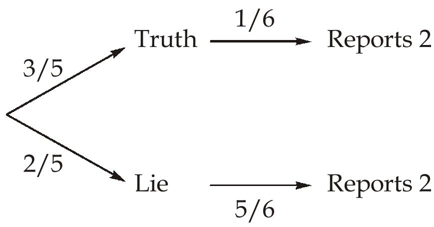
Check for continuous:

$$\begin{aligned} f(-2) &= -1.5 \times (-2)^2 = -6 \\ f(-2^+) &= 6(-2) - 5 = -17 \\ f(-2^-) &= -1.5 \times (-2)^2 = -6 \\ f(-2^-) &\neq f(-2^+) \end{aligned}$$

Function is not continuous, hence cannot be differentiable i.e. differentiable $\rightarrow$ continuous.

7. (b)

Applying Bayes Theorem:



So,

$$P(\text{spoke truth/reports 2}) = \frac{P(\text{spoke truth} \cap \text{reports 2})}{P(\text{reports 2})} = \frac{\frac{3}{5} \times \frac{1}{6}}{\frac{3}{5} \times \frac{1}{6} + \frac{2}{5} \times \frac{5}{6}} = \frac{3}{13}$$

8. (d)

$$D = -96 \text{ for the given matrix}$$

$$|A| = \begin{vmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{vmatrix} = 2^3 \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

(Taking 2 common from each row)

$$\therefore \quad \text{Det}(A) = (2)^3 \times D$$
$$= 8 \times (-96)$$
$$= -768$$

9. (d)

$$\lim_{x \to 4} \frac{(2x)^{1/3} - 2}{2x - 8}$$

Above form is $\left(\frac{0}{0}\right)$ by putting the value $x = 4$

Applying L' Hospital rule

$$= \lim_{x \to 4} \frac{\frac{1}{3}(2x)^{\left(\frac{1}{3}-1\right)} \times 2}{2} = \lim_{x \to 4} \frac{1}{3}(2x)^{\left(-\frac{2}{3}\right)}$$

$$= \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$$

10. (b)

$$f(x) = x^3 - 6x^2 + 9x + 1$$
$$f'(x) = 3x^2 - 12x + 9 = 0$$
$$x^2 - 4x + 3 = 0$$
$$x^2 - 3x - x + 3 = 0$$
$$x(x-3) - 1(x-3) = 0$$
$$(x-1)(x-3) = 0$$
$$x = 1, x = 3$$
$$f''(x) = 6x - 12$$

We need to check at all the extremum points i.e. 1, 3, 0, 5.

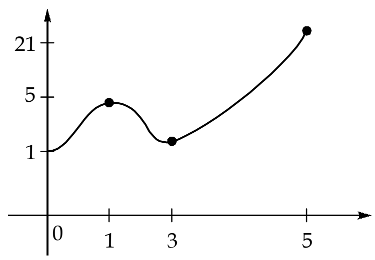
At 1, $f''(x) = -6 < 0$ (maximum)

At 3, $f''(x) = 6 > 0$ (minimum)

Taking into account all points:

$$f(0) = 1$$
$$f(1) = 5$$
$$f(3) = 1$$
$$f(5) = 21$$

Hence roughly graph can be drawn like:



Thus, maximum at 5 and minimum can be at 0 or 3.

11. (b)

We know that, $E(X) = 10$

and $Var(X) = 25$

Now, $E(Y) = E(aX - b) = 0$

$$aE(X) - b = 0$$

$\Rightarrow$ $a(10) - b = 0$

$$10a - b = 0 \qquad \text{...(i)}$$

Given, $Var(Y) = 1$

$$Var(aX - b) = a^2\, Var(X) = 1$$

$\Rightarrow$ $25a^2 = 1$

i.e $a = \pm\frac{1}{5}$

$a = \frac{1}{5}$ (taking positive values only)

By putting value of 'a' in equation (i)

We get $b = 2$

12. **(a)**

For rectangular distribution

$$\text{Variance} = \frac{(b-a)^2}{12}$$

Here, $$a = 0, b = \frac{1}{2}$$

$$\therefore \quad \text{Variance} = \frac{\left(\frac{1}{2}-0\right)^2}{12} = \frac{\frac{1}{4}}{12} = \frac{1}{4\times 12}$$

$$\text{Then standard deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{\frac{1}{4\times 12}} = \frac{1}{2\sqrt{12}}$$

13. **(c)**

Given matrix is $M = \begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$

Determinant of $M = \begin{vmatrix} 12+9i & -i \\ i & 12-9i \end{vmatrix} = (12 + 9i)(12 - 9i) + i^2$

$$= (12^2 - 9^2 i^2) + i^2$$

$$= 225 - 1 = 224$$

$$\therefore \quad \text{Inverse of } M = M^{-1} = \frac{1}{|M|}(adjM)$$

$$= \frac{1}{224}\begin{bmatrix} 12-9i & i \\ -i & 12+9i \end{bmatrix}$$

14. **(a)**

Let, $\text{'A'} = \{a_1, a_2, a_3 \,........\, a_n\}$

There is an element a_1 of 'A' and two subsets 'P' and 'Q', then four possibilities

$$\left.\begin{array}{l} \text{(a) } a_1 \in P \text{ and } a_1 \in Q \\ \text{(b) } a_1 \in P \text{ and } a_1 \notin Q \\ \text{(c) } a_1 \notin P \text{ and } a_1 \in Q \\ \text{(d) } a_1 \notin P \text{ and } a_1 \notin Q \end{array}\right\} \text{4 choices}$$

Total number of ways selecting 'P' and 'Q' = 2^n

$\Rightarrow \quad 2^n \times 2^n = 4^n$ ways

$\Rightarrow \quad n(S) = 4^n$

Number of favorable elements = 3^n

$$P(E) = \frac{n(E)}{n(S)} = \frac{3^n}{4^n}$$

$$= (0.75)^n$$

15. (d)

function $f(x)$ is continuous for every $x \neq 0$ (since $\frac{x-c}{1+c}$ and $x^2 + c$ are polynomials, and polynomials are continuous).

$$f(0) = \frac{0-c}{1+c} = \frac{-c}{1+c}$$

$$\lim_{x\to 0^-} \frac{0-c}{1+c} = \frac{-c}{1+c}$$

$$\lim_{x\to 0^+} 0^2 + c = c$$

Since $f(x)$ is continuous for every x, hence continuous for $x = 0$.

$$\Rightarrow \quad f(0) = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\Rightarrow \quad \frac{-c}{1+c} = c$$

$$\Rightarrow \quad -c = c\,(1 + c)$$

$$c^2 + 2c = 0$$

$$c = -2 \text{ or } c = 0$$

So option (d) is correct answer

16. (b)

To check matrix is LU decomposible by checking if principal minors have non-zero determinants.

Check (a):

$$|A_1| = |\,1\,| = 1 \neq 0$$

Now $$|A_2| = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = 0$$

So option (a) is not LU decomposible.

Check (b):

$$\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \text{ here } |A_1| = 3,\ |A_2| = \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

So LU decomposible.

Check (c):

$$\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} \text{ here } |A_1| = 0$$

So not LU decomposible.

Check (d):

$$\begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix} \text{ here } |A_1| = 1 \neq 0 \text{ but}$$

$$|A_2| = \begin{vmatrix} 1 & -3 \\ -2 & 6 \end{vmatrix} = |6 - 6| = 0$$

So not LU decomposible.

17. (b)

Let, P(G) represent given day mood is good.
P(S) represent given day is sunny.

		Weather		
		Sunny	Not sunny	
Mood	Good	12	9	21
	Not Good	4	5	9
		16	14	30

So, $$P(G|S) = \frac{P(G \cap S)}{P(S)}$$

$$P(G \cap S) = \frac{12}{30}$$

$$P(S) = \frac{16}{30}$$

So, $$P(G|S) = \frac{\frac{12}{30}}{\frac{16}{30}} = \frac{12}{16} = \frac{3}{4}$$

18. (b)

Consider, $$u = \cot x$$

$$\frac{du}{dx} = -\text{cosec}^2 x$$

$$du = -\text{cosec}^2 x\, dx$$

$$-du = \text{cosec}^2 x\, dx$$

Now new limits:

$$x = \frac{\pi}{6} \to u = \cot\frac{\pi}{6} = \sqrt{3}$$

$$x = \frac{\pi}{3} \to u = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

Substitute new limits and $\text{cosec}^2 x\, dx$

$$\int_{\sqrt{3}}^{1/\sqrt{3}} \frac{-du}{u^2} = \left[\frac{u^{-2+1}}{-2+1}\right]_{\sqrt{3}}^{1/\sqrt{3}} = \left[u^{-1}\right]_{\sqrt{3}}^{1/\sqrt{3}} = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

19. (c)

$$\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(\frac{3 + a^x + b^x + c^x - 3}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1 + \frac{a^x + b^x + c^x - 3}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1 + \frac{(a^x - 1) + (b^x - 1) + (c^x - 1)}{3}\right)^{1/x}$$

We know that:

$$\lim_{x\to 0}(1+\lambda x)^{1/x} = e^{\lambda}$$

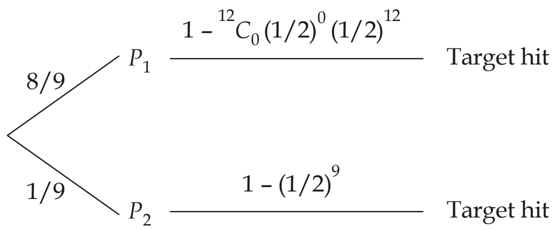
$$= e^{\lim_{x\to 0}\frac{(a^x-1)}{3x}+\frac{(b^x-1)}{3x}+\frac{(c^x-1)}{3x}} = e^{\lim_{x\to 0}\frac{1}{3}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right)}$$

$$= e^{1/3\ (\log a + \log b + \log c)} \qquad \left[\because \lim_{x\to 0}\frac{a^x-1}{x} = \log a\right]$$

$$= e^{1/3 \log (abc)} = e^{\log (abc)^{1/3}} = (abc)^{1/3}$$

$$= \sqrt[3]{abc}$$

20. (c)



$$P(\text{Target hit}) = \frac{8}{9}\left(1-\frac{1}{2^{12}}\right)+\frac{1}{9}\left(1-\frac{1}{2^{9}}\right)$$

So option (c) is correct answer.

21. (c)

$$AX = \lambda X$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -6 \end{bmatrix} = (-6)\begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

$$3a - 6b = -18 \qquad \text{... (i)}$$

$$3c - 6d = 36 \qquad \text{... (ii)}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -3 \end{bmatrix} = (-3)\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$3a - 3b = -9 \qquad \text{... (iii)}$$

$$3c - 3d = 9 \qquad \text{... (iv)}$$

From equation (i) and (iii), $a = 0$ and $b = 3$.
From equation (ii) and (iv), $c = -6$ and $d = -9$.

$$\therefore \qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -6 & -9 \end{bmatrix}$$

22. (b)

$$y = 7x^2 + 12x$$

Using Lagrange's mean value theorem:

At $x = 1, y = 7 + 12 = 19$

$x = 3, y = 63 + 36 = 99$

$$f'(x) = \frac{f(b)-f(a)}{b-a}$$
$$= \frac{99-19}{3-1} = 40$$

So option (b) is correct answer.

23. (a)

Since, $\sum_{x=0}^{4} P(x) = 1$

$$c + 2c + 2c + c^2 + 5c^2 = 1$$
$$6c^2 + 5c - 1 = 0$$
$$c = \frac{1}{6}, -1$$

Since $P(x) \geq 0$, the possible value of

$$c = \frac{1}{6}$$

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{5}{36}$
$xP(x)$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{3}{36}$	$\frac{20}{36}$

$$\text{Mean} = \sum_{x=0}^{4} xP(x) = 0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{36} + \frac{20}{36} = \frac{59}{36} = 1.638$$

$$\text{Variance} = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= \left[0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + 9\left(\frac{1}{36}\right) + 16\left(\frac{5}{36}\right) - \left(\frac{59}{36}\right)^2\right] = 1.45$$

24. (b)

$$f(x) = (x^2 - 4)^2$$
$$f'(x) = 2(x^2 - 4) \times 2x$$
$$= 4x(x^2 - 4) = 0$$
$$x = 0, x = 2 \text{ and } x = -2 \text{ are the stationary points}$$
$$f''(x) = 4[x(2x) + (x^2 - 4) \times 1]$$
$$= 4[(2x^2 + (x^2 - 4] = 4[3x^2 - 4]$$
$$= 12x^2 - 16$$
$$f''(0) = -16 < 0 \quad \text{(So maxima at } x = 0)$$
$$f''(2) = (12)2^2 - 16 = 32 > 0 \quad \text{(So minima at } x = 2)$$
$$f''(-2) = 12(-2)^2 - 16 = 32 > 0 \quad \text{(So minima at } x = -2)$$

$\therefore$ There is only one maxima and only two minima for this function.

25. (b)

- $ABA^{-1} = B$ given,

$\Rightarrow$ $AB = BA$ since matrix multiplication is not commutative. So false even if A is invertible.

- A is idempotent, so $A^2 = A$, since A is non-singular, so it is invertible i.e. A^{-1} exist.

$$I = A^{-1} . A = A^{-1} . A^2 = IA = A$$

So A must be identity matrix. So true.

- If coefficient matrix A is invertible for $Ax = b$ then $x = A^{-1}$ unique solution exist. So false
- If B is zero matrix, then also $AB = B$ = zero matrix. So false

26. (d)

The characteristic equation is $|A - \lambda I| = 0$

i.e.,
$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & 8-\lambda \end{vmatrix} = 0$$
$$(4-\lambda)(8-\lambda) - 10 = 0$$
$$\lambda^2 - 12\lambda + 20 = 0$$
$$(\lambda - 10)(\lambda - 2) = 0$$
$$\lambda = 10, 2$$

Corresponding to $\lambda = 10$, we have
$$[A - \lambda I]X = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

which gives
$$\left.\begin{matrix} -6a + 6b = 0 \\ 2a - 2b = 0 \end{matrix}\right\} a = b$$

i.e., eigen vector can be the answer and is present in one of the option (d). Similarly $\lambda = 2$ also have eigen vectors i.e. not mentioned in any options.

27. (b)

$$x + \frac{50}{x} > 15$$
$$\Rightarrow \quad x^2 - 15x + 50 > 0$$
$$x^2 - 5x - 10x + 50 > 0$$
$$(x-5)(x-10) > 0$$

Cases :

(i) $x > 5$ and $x > 10 \Rightarrow x > 10$

(ii) $x < 5$ and $x < 10 \Rightarrow x < 5$

So, $x < 5 = \{1, 2, 3, 4\}$

$x > 10 = \{11, 12, 13, \ldots.., 20\}$

So, total favourable cases : 4 + 10 = 14

$$\text{The required probability} = \frac{14}{20}$$

28. (c)

$$P = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$
$$P^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$
$$A = PDP^{-1} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -6 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix}$$
$$A^2 = A \times A = \begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$
$$A^4 = A^2 \times A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

$$A^5 = A^4 \times A = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 32 & -192 \\ 0 & -32 \end{bmatrix}$$

29. (c)

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix} = AA^t$$

There are three cases for the rank of A.

Case I : rank $(A) = 0$

$\Rightarrow A$ is null. So, $B = AA^1$ also has to be null and hence rank (B) is also equal to 0. Therefore in this case rank (A) = rank (B).

Case II : rank $(A) = 2$

So, A has to be non-singular, i.e., $|A| \neq 0$. Therefore, $|B| = |A|^2$ is also $\neq 0$. So, rank $(B) = 2$. Therefore, in this case also rank (A) = rank (B).

Therefore, in all three cases rank (A) = rank (B). So, rank of A is N, then the rank of matrix B is also N.

30. (a)

Augmented matrix:

$$[A \mid B] = \left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$R_2 \leftarrow 4R_2 - R_1$

$$\left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 0 & 9 & 22 & 28 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

If $\lambda = 5$ and $\mu \neq 9$, then system has no solution because Rank$[A \mid B] \neq$ Rank [4].

■■■■