CLASS TEST

S.No. : 01 JP_ME_26032024

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MECHANICAL ENGINEERING

Engineering Mathematics

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 Consider the function defined by,

$$f(x) = \begin{cases} \dfrac{\sin(3p-1)x}{3x}; & x < 0 \\ \dfrac{\tan(3p+1)x}{2x}; & x > 0 \end{cases}$$

If this function is known to be differentiable at $x = 0$, then the value of p is

(a) $\frac{1}{3}$
(b) $\frac{4}{3}$
(c) $-\frac{1}{3}$ or $\frac{1}{3}$
(d) $-\frac{5}{3}$

Q.2 The order and degree of differential equation of family of curves $y = e^x (A\cos x + B\sin x)$, are respectively

(a) 1 and 1
(b) 2 and 1
(c) 2 and 2
(d) 1 and 2

Q.3 If $y(0) = 0$, then the solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = ax + by$ is

(a) $\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = \frac{a+b}{ab}$
(b) $\frac{e^{ax}}{a} + \frac{e^{bx}}{b} = a+b$
(c) $\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = \frac{1}{a+b}$
(d) $\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = \frac{1}{a} - \frac{1}{b}$

Q.4 A vector

$$\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (2xy - axz + 2z)\hat{k}$$

is known to be solenoidal. The value of "a" is

(a) 2
(b) –3
(c) –2
(d) Can't be determined

Q.5 Consider a function $\phi(x, y, z) = x^2yz + 4xz^2$. The greatest rate of increase of ϕ at point (1, –2, 1) is

(a) $\sqrt{37}$
(b) $\sqrt{39}$
(c) $\sqrt{27}$
(d) $\sqrt{35}$

Q.6 The solution to the system of equations is

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

(a) 12, –4
(b) –12, –4
(c) –12, 4
(d) 12, 4

Q.7 $\lim\limits_{x \to 0} \frac{\ln(1+5x)}{e^{7x}-1}$ is equal to

(a) 0
(b) $\frac{5}{7}$
(c) $\frac{3}{10}$
(d) 1

Q.8 A bag contains 15 defective items and 35 non defective items. If three items are selected at random without replacement, what will be the probability that all three items are defective?

(a) $\frac{1}{40}$
(b) $\frac{13}{560}$
(c) $\frac{15}{34}$
(d) $\frac{12}{499}$

Q.9 If $x = b(2 - \cos\theta)$ and $y = b(\sin\theta + \theta)$, then $\frac{dx}{dy}$ will be equal to

(a) $\tan\left(\frac{\theta}{2}\right)$
(b) $\cot\left(\frac{\theta}{2}\right)$
(c) $\sin\left(\frac{\theta}{2}\right)$
(d) $\cos\left(\frac{\theta}{2}\right)$

Q.10 A coin is tossed 8 times. What is the probability of getting tails exactly 6 times?

(a) $\frac{7}{16}$
(b) $\frac{7}{64}$
(c) $\frac{1}{8}$
(d) $\frac{7}{32}$

Q.No. 11 to Q.No. 30 carry 2 marks each

Q.11 A curve given by $x^2 + 4y^2 = 36$ is revolved around x axis. The volume of solid generated is

(a) 64π unit^3
(b) 72π unit^3
(c) 144π unit^3
(d) 48π unit^3

Q.12 Consider the differential equation given below:

$$\frac{dy}{dx} + yf'(x) = f(x)\cdot f'(x)$$

Here $f(x)$ is purely a function of x. The solution of the equation is

(a) $ye^{f(x)} = f(x)\,[e^{f(x)} + 1] + c$
(b) $ye^{f(x)} = e^{f(x)} + f(x) + c$
(c) $\log\,[y + f(x)] + f(x) = 0$
(d) $\log\,[1 + y - f(x)] + f(x) = c$

Q.13 The particular integral of the differential equation $D^2(D^2 + 4)y = 96x^2$ for $x = 2$ will be

(a) 8 (b) 5
(c) 9 (d) 2

Q.14 $f(z) = u + iv$ is an analytic function. If $u(x, y) = 2x(1 - y)$, then $v(x, y)$ will be

(a) $x^2 + y^2 - 2y + c$ (b) $x^2 - y^2 + 2y + c$
(c) $2x^2 - y^2 + c$ (d) $x^2 + 2y + c$

Q.15 If X is a random variable with PDF given by,

$$f(x) = \begin{cases} kx & ; \quad 0 < x \le 2 \\ 2k & ; \quad 2 < x \le 4 \\ -kx + 6k & ; \quad 4 < x \le 6 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

The value of k and mean value of X are respectively

(a) $\frac{1}{8}$ and 3 (b) $\frac{1}{8}$ and $\frac{11}{6}$
(c) 3 and $\frac{13}{6}$ (d) $\frac{1}{6}$ and 3

Q.16 The value of integral $I = \int_0^{\pi/2} \sqrt{1 + \sec x}\, dx$ will be

(a) 2π (b) $\pi/2$
(c) $\pi/4$ (d) π

Q.17 For the differential equation $(2y - 3x)dx + xdy = 0$, the initial condition is zero i.e. $y = 0$ for $x = 0$. The value of y for $x = 2$ will be

(a) 1 (b) 2
(c) 3 (d) 0.5

Q.18 Four cards were drawn from a pack of 52 cards. The probability that they are a king, a queen, a jack and an ace.

(a) $\frac{512}{54145}$ (b) $\frac{64}{54145}$
(c) $\frac{256}{270725}$ (d) $\frac{64}{270725}$

Q.19 A parametric curve defined by $x = \sin\left(\frac{\pi k}{2}\right), y = \cos\left(\frac{\pi k}{2}\right)$ in the range $0 \le k \le 1$ is rotated about the y-axis by 360 degree. Area of the surface generated is

(a) 2π (b) π
(c) $\frac{\pi}{2}$ (d) 4π

Q.20 For the function $f(y) = y^2e^{-y}$, the maximum occurs when y is equal to

(a) 1 (b) 3
(c) 2 (d) 4

Q.21 The number of satellites launched worldwide in a month follows Poisson distribution with mean as 6.8. The probability of launch of less than 3 satellites during a randomly selected month is

(a) 0.034 (b) 0.34
(c) 0.068 (d) 0.0034

Q.22 The equation of the curve passing through the point $\left(0, \frac{\pi}{3}\right)$ satisfies the following differential equation is $\sin x \cos y dx + \cos x \sin y dy = 0$

(a) $\cos x \cos y = \frac{1}{2}$ (b) $\sin x \cos y = 0$
(c) $\cos x \cos y = \frac{\sqrt{3}}{2}$ (d) $\sin x \sin y = 0$

Q.23 The polynomial, $P(x) = x^5 + x + 2$, has

(a) All real roots
(b) 3 real and 2 complex roots
(c) 1 real and 4 complex roots
(d) None of these

Q.24 If $\int \sec^3\theta\, d\theta = a(\sec\theta \tan\theta) + b\, ln|\sec\theta + \tan\theta| + c$, then the value of $(a + b)$ is

(a) 1 (b) 2
(c) 3 (d) 4

Q.25 The value of $\int_C \bar{F}.\overline{dr}$, where $\bar{F} = x^2y^2\,\bar{i} + y\,\bar{j}$ and C is the curve $y^2 = 4x$ in the xy-plane from (0, 0) to (4, 4), is

(a) 66 (b) 132
(c) 264 (d) 528

Q.26 The function, $f(x) = 2x^3 - 3x^2 - 36x + 10$, has a local maximum value at 'x' equals to

(a) –2 (b) –1
(c) 3 (d) 4

Q.27 What is the length of the curve, $3x^2 = y^3$, between $y = 0$ and $y = 1$?

(a) $\frac{1}{18}\left(7\sqrt{7}-8\right)$ (b) $\frac{1}{9}\left(7\sqrt{7}-8\right)$

(c) $\frac{1}{3}\left(7\sqrt{7}-8\right)$ (d) None of these

Q.28 The area bounded by $y = x^3$ and $y = x$ in the third quadrant is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

Q.29 The solution of $(x+1)\frac{dy}{dx}+1=2e^{-y}$ is

(a) $(x + 1)(2 - e^y) = k$ (b) $(x + 1)(2 - e^{-y}) = k$
(c) $(x - 1)(2 - e^{-y}) = k$ (d) $(x + 1)(2 + e^y) = k$
where, k is a constant

Q.30 Find the solution of $\frac{d^2y}{dx^2} = y$ which passes through the origin and the point $\left(In\,2, \frac{3}{4}\right)$.

(a) $y = \frac{1}{2}e^x - e^{-x}$ (b) $y = \frac{1}{2}\left(e^x + e^{-x}\right)$

(c) $y = \frac{1}{2}\left(e^x - e^{-x}\right)$ (d) $y = \frac{1}{2}e^x + e^{-x}$

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ENGINEERING MATHEMATICS

MECHANICAL ENGINEERING

Date of Test : 26/03/2024

ANSWER KEY

1.	(d)	7.	(b)	13.	(a)	19.	(a)	25.	(c)
2.	(b)	8.	(b)	14.	(b)	20.	(c)	26.	(a)
3.	(a)	9.	(a)	15.	(a)	21.	(a)	27.	(b)
4.	(a)	10.	(b)	16.	(d)	22.	(a)	28.	(b)
5.	(a)	11.	(b)	17.	(b)	23.	(c)	29.	(a)
6.	(a)	12.	(d)	18.	(c)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

1. (d)

For function to be differentiable i.e. continuous $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$$f(0^-) = \lim_{x\to 0^-} \frac{\sin(3p-1)x}{3x} \times \frac{(3p-1)}{(3p-1)}$$

$$= \lim_{x\to 0^-} \frac{\sin(3p-1)x}{(3p-1)x} \times \frac{(3p-1)}{3} = \frac{(3p-1)}{3}$$

$$f(0^+) = \lim_{x\to 0^+} \frac{\tan(3p+1)x}{2x} \times \frac{(3p+1)}{(3p+1)}$$

$$= \lim_{x\to 0^+} \frac{\tan(3p+1)x}{(3p+1)x} \times \frac{3p+1}{2} = \frac{3p+1}{2}$$

For function to be continuous,

$$\frac{3p-1}{3} = \frac{3p+1}{2}$$

By solving, we get, $p = -\frac{5}{3}$

2. (b)

We have $y = e^x (A\cos x + B\sin x)$

$$y' = e^x (A\cos x + B\sin x) + e^x (-A\sin x + B\cos x)$$

$$= y + e^x [-A\sin x + B\cos x]$$

$$y'' = y' + e^x (-A\sin x + B\cos x) + e^x (-A\cos x - B\sin x)$$

$$= y' + y' - y - y$$

$$= 2y' - 2y$$

$\Rightarrow$ Order $= 2$

Degree $= 1$

3. (a)

$$\frac{dy}{dx} = e^{ax} \times e^{by}$$

$$\frac{dy}{e^{by}} = e^{ax} \times dx$$

$$\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$$

$y(0) = 0$

$\Rightarrow$ $c = -\left[\frac{1}{b} + \frac{1}{a}\right] = -\left[\frac{a+b}{ab}\right]$

4. (a)

$$\nabla \cdot \vec{F} = 0 \qquad \text{[For solenoidal vector]}$$

$$\frac{\partial(y^2 - z^2 + 3yz - 2x)}{\partial x} + \frac{\partial(3xz + 2xy)}{\partial y} + \frac{\partial(2xy - axz + 2z)}{\partial z} = 0$$

$$-2 + 2x - ax + 2 = 0$$

From here, $a = 2$

5. (a)

Greatest rate of increase of ϕ is magnitude of directional derivative at that point.

$$\nabla\phi = (2xyz + 4z^2)\hat{i} + x^2z\hat{j} + (x^2y + 8xz)\hat{k}$$

$$\nabla\phi\big|_{(1,-2,1)} = \hat{j} + 6\hat{k}$$

$$\text{Greatest rate of increase} = \sqrt{1^2 + 6^2} = \sqrt{37} = 6.08$$

6. (a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 2R_1$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or $y = -4$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

$\Rightarrow$ $x = 12$

$\therefore$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$

7. (b)

$$\lim_{x \to 0} \frac{\ln(1 + 5x)}{e^{7x} - 1} \qquad \left(\frac{0}{0} \text{ indeterminate form}\right)$$

Applying L' Hospitals rule

$$\lim_{x \to 0} \frac{\ln(1 + 5x)}{e^{7x} - 1} = \lim_{x \to 0} \frac{5}{(1 + 5x)7e^{7x}} = \frac{5}{7}$$

8. (b)

Probability of first item being defective,

$$P_1 = \frac{15}{50}$$

Probability of second item being defective,

$$P_2 = \frac{14}{49}$$

Probability of third item being defective,

$$P_3 = \frac{13}{48}$$

Probability that all three are defective,

$$P = P_1P_2P_3 = \frac{15}{50}\times\frac{14}{49}\times\frac{13}{48} = \frac{13}{560}$$

9. (a)

Given, $x = b(2 - \cos\theta)$, $y = b(\sin\theta + \theta)$

$$\therefore \quad \frac{dx}{d\theta} = b\sin\theta,$$

$$\frac{dy}{d\theta} = b(\cos\theta + 1)$$

$$\frac{dx}{dy} = \frac{dx/d\theta}{dy/d\theta} = \frac{b\sin\theta}{b(\cos\theta+1)}$$

$$= \frac{2b\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)}{b\times 2\cos^2\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$$

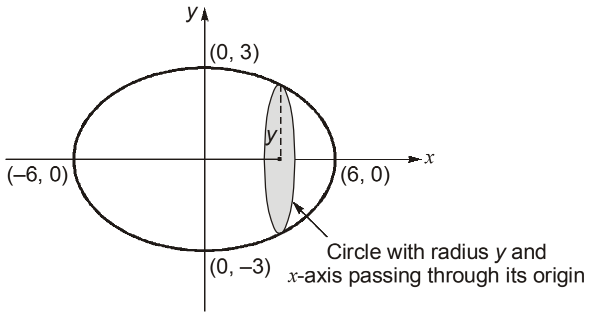
10. (b)

$$P(T) = 0.5$$

Probability of getting tails exactly 6 times is

$$8C_6(0.5)^6(0.5)^2 = \frac{7}{64}$$

11. (b)



$$\text{Volume generated} = \int_{-6}^{6}\pi y^2 dx = \int_{-6}^{6}\pi\left(\frac{36-x^2}{4}\right)dx$$

$$= \frac{\pi\times 2}{4}\int_0^6 (36 - x^2)dx = \frac{\pi}{2}\left[36x - \frac{x^3}{3}\right]_0^6$$

$$= 72\pi \text{ unit}^3$$

12. (d)

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF \cdot f(x) \cdot f'(x) dx$$

$$y \times e^{f(x)} = \int e^{f(x)} \cdot f(x) \cdot f'(x) dx$$

Let $f(x) = t$

$$f'(x)\ dx = dt$$

$$y \times e^t = \int e^t \cdot t dt$$

$$y.e^t = t.e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y + 1 - t) = -t + c'$$

$$\log\ [y + 1 - f(x)] + f(x) = c'$$

13. (a)

For particular integral,

$$PI = \frac{96x^2}{D^2(D^2+4)} = 96 \frac{1}{4D^2\left(1+\frac{D^2}{4}\right)} x^2 = \frac{96}{4}\left[\frac{\left(1-\frac{D^2}{4}\right)x^2}{D^2}\right]$$

$$= 24 \frac{\left(x^2 - \frac{1}{2}\right)}{D^2}$$

$$PI = 24\left[\frac{x^4}{4 \times 3} - \frac{x^2}{4}\right] = 2x^2(x^2 - 3)$$

$$PI\big|_{x=2} = 2 \times 2^2(4-3) = 8$$

14. (b)

$$u(x, y) = 2x(1 - y)$$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$$

$$dv = (2x)dx + 2(1 - y)dy$$

$$v = x^2 + 2y - y^2 + c$$

15. (a)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx + 6k)dx = 1$$

$$\left.\frac{kx^2}{2}\right|_0^2 + 2kx|_2^4 + \left.\left(\frac{-kx^2}{2} + 6kx\right)\right|_4^6 = 1$$

$$\frac{k}{2}(2^2 - 0) + 2k(4-2) - \frac{k}{2}(6^2 - 4^2) + 6k(6-4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \quad \Rightarrow k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{1}{8}x^2 dx + \int_2^4 \frac{1}{4}x dx + \int_4^6 \left(-\frac{1}{8}x^2 + \frac{3}{4}x\right)dx$$

$$= \left.\frac{1}{8}\frac{x^3}{3}\right|_0^2 + \left.\frac{1}{4}\frac{x^2}{2}\right|_2^4 - \left.\frac{1}{8}\frac{x^3}{3}\right|_4^6 + \left.\frac{3}{4}\frac{x^2}{2}\right|_4^6$$

$$= \frac{1}{3} + \frac{3}{2} - \frac{19}{3} + \frac{15}{2} = 3$$

16. (d)

$$I = \int_0^{\pi/2} \sqrt{1+\sec x}\, dx = \int_0^{\pi/2} \sqrt{1+\frac{1}{\cos x}}\, dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{1+\cos x}}{\sqrt{\cos x}}\, dx = \int_0^{\pi/2} \frac{\sqrt{2}\cos(x/2)}{\sqrt{1-2\sin^2(x/2)}}\, dx$$

Let $\sin\frac{x}{2} = t,$ $\left\{\begin{array}{ll} x = 0, & t = 0 \\ x = \frac{\pi}{2} & t = \frac{1}{\sqrt{2}} \end{array}\right.$

$$\frac{1}{2}\cos\frac{x}{2}dx = dt$$

$$I = \int_0^{1/\sqrt{2}} \frac{2\sqrt{2}dt}{\sqrt{1-2t^2}}$$

$$= 2\sin^{-1}\left(\sqrt{2}t\right)\Big|_0^{1/\sqrt{2}} = 2\sin^{-1}\left(\sqrt{2}\times\frac{1}{\sqrt{2}}\right) - 2\sin^{-1}(0)$$

$$= 2\times\frac{\pi}{2} = \pi = 3.14$$

17. (b)

$$(2y - 3x)dx + xdy = 0$$

$$\frac{dy}{dx} + \frac{2}{x}y = 3$$

$$IF = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$$

$$y.x^2 = 3\int x^2 dx = x^3 + c$$

For $x = 0, y = 0$

$\Rightarrow$ $0 = 0 + c$

$\Rightarrow$ $c = 0$

For $x = 2,$ $y \times 2^2 = 2^3$

$$y = 2$$

18. (c)

$$\frac{4C_1 \cdot 4C_1 \cdot 4C_1 \cdot 4C_1}{52C_4} = \frac{4\times4\times4\times4}{(52\times51\times50\times49)/(4\times3\times2\times1)}$$

$$= \frac{4\times4\times4\times4\times3\times2}{52\times51\times50\times49} = \frac{256}{270725}$$

19. (a)

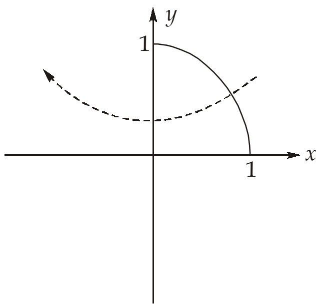
$x = \sin\left(\frac{\pi k}{2}\right), y = \cos\left(\frac{\pi k}{2}\right)$

Just by seeing, we can know that it represents a circle in $x - y$ plane, given by

$$x^2 + y^2 = 1$$

Given $0 \le k \le 1$, which gives $0 \le x \le 1$; $0 \le y \le 1$

or $$0 \le \frac{\pi k}{2} \le \frac{\pi}{2}$$



So we get a quarter circle, when this is rotated with respect to y-axis by 360 degree, it creates a hemisphere of radius 1.

Surface area of hemisphere,

$$A_S = 2\pi r^2$$
$$= 2\pi(1)^2 = 2\pi$$

20. (c)

$$f(y) = y^2 e^{-y}$$
$$f'(y) = y^2\left(-e^{-y}\right) + e^{-y}\times 2y$$
$$= e^{-y}\left(2y - y^2\right)$$

Putting $f'(y) = 0$

$$e^{-y}\left(2y - y^2\right) = 0$$

$$e^{-y}y(2-y) = 0$$

$y = 0$ or $y = 2$ are the stationary points

Now, $$f''(y) = e^{-y}(2-2y) + \left(2y - y^2\right)\left(-e^{-y}\right)$$

$$= e^{-y}\left(2 - 2y - 2y + y^2\right)$$

$$= e^{-y}\left(y^2 - 4y + 2\right)$$

At $y = 0$, $$f''(0) = e^{-0}(0 - 0 + 2) = 2$$

Since $f''(0) = 2$ is > 0 at $y = 0$ we have a minima

$$\text{Now, at } y = 2f''(2) = e^{-2}\left(2^2 - 4 \times 2 + 2\right)$$
$$= e^{-2}(4 - 8 + 2)$$
$$= -2e^{-2} < 0$$

$\therefore$ At $y = 2$ we have a maxima.

21. (a)

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$
$$P(x < 3) = P(0) + P(1) + P(2)$$
$$= \frac{\mu^0 e^{-\mu}}{0!} + \frac{\mu^1 e^{-\mu}}{1!} + \frac{\mu^2 e^{-\mu}}{2!}$$
$$= \frac{1}{e^{\mu}} + \frac{\mu}{e^{\mu}} + \frac{\mu^2}{2e^{\mu}}$$

As $\mu(\text{mean}) = 6.8$

$$\therefore \quad P(x < 3) = \frac{1 + 6.8 + \left(\frac{6.8^2}{2}\right)}{e^{6.8}} = \frac{30.92}{897.85} \simeq 0.034$$

22. (a)

$$\sin x \cos y dx + \cos x \sin y dy = 0$$

Divide by $\cos x \cos y$, we get ,

$$\tan x \, dx + \tan y dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$
$$\log \frac{1}{\cos x \cos y} = C_1$$
$$\cos x \cos y = C$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$
$$\frac{1}{2} = C$$

$\Rightarrow$ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

23. (c)

$$P(x) = x^5 + x + 2$$

It has a real root at $x = -1$

$$\Rightarrow \quad P(x) = (x^4 - x^3 + x^2 - x + 2)(x + 1)$$

Now, $x^4 - x^3 + x^2 + x + 2$ will give other 4 roots

To find roots,

$$\Rightarrow \quad x^4 - x^3 + x^2 - x + 2 = 0$$

$\Rightarrow \quad x^3(x-1) + x(x-1) + 2 = 0$

$\Rightarrow \quad x(x^2+1)(x-1) + 2 = 0$

In the above expression, $x^2 + 1$ is always positive. So, either 'x' or '$x - 1$' should be negative in order to satisfy the equation.

For $x > 1$, both (x) and $(x-1)$ are positive and,

For $x < 0$, both (x) and $(x-1)$ are negative

$\therefore$ x should lie within 0 and 1 in order to have real roots.

As $x \in (0, 1)$

$\Rightarrow \quad |x| < 1$

$\Rightarrow \quad |x^2+1| < 2$, $|x| < 1$ and $|x-1| < 1$

$\therefore$ The product of these three will be less than 2 and hence, no real value of 'x' can satisfy the equation

$$x^4 - x^3 + x^2 - x + 2 = 0$$

$\therefore$ The equation will have four imaginary roots apart from one real roots.

24. (a)

$$I = \int \sec^3\theta d\theta = \int \sec\theta.\sec^2\theta d\theta$$

$$= \sec\theta\int\sec^2\theta d\theta - \int\tan\theta(\sec\theta\tan\theta)d\theta$$

$$= \sec\theta\tan\theta - \int\tan^2\theta\sec\theta d\theta$$

$$\Rightarrow \quad I = \sec\theta\tan\theta - \int(\sec^2\theta - 1)\sec\theta d\theta$$

$$= \sec\theta\tan\theta - \int\sec^3\theta d\theta + \int\sec\theta d\theta$$

$$\Rightarrow \quad I = \sec\theta\tan\theta - I + ln|\sec\theta + \tan\theta| + c$$

$$\Rightarrow \quad I = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}ln|\sec\theta + \tan\theta| + c$$

$$\therefore \quad a + b = \frac{1}{2} + \frac{1}{2} = 1$$

25. (c)

$$\int_C \bar{F}.\overline{dr} = \int_C x^2y^2dx + y.dy$$

For curve C, $\quad y^2 = 4x$

and $\quad 2y\,dy = 4\,dx$

$$\Rightarrow \quad \int_C \bar{F}.\overline{dr} = \int_0^4 x^2(4x)dx + 2dx$$

$$= \int_0^4 (4x^3 + 2)dx = 264$$

26. (a)

To obtain maximum value of $f(x)$, first $f'(x)$ should be equated to zero.

$\Rightarrow \quad f'(x) = 6x^2 - 6x - 36 = 0$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\therefore \quad f'(x) = 0 \quad$ at $x = 3$ and -2

Now, $\quad f''(x) = 12x - 6$

$f''(3) = 30 > 0$

at $x = 3$, there is local minima

and $\quad f''(2) = -30 < 0$

$\therefore$ at $x = -2$, a local maxima is observed.

27. (b)

$$\text{Length of curve} = \int_0^1 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

Curve: $$3x^2 = y^3$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{\sqrt{3y}}{2}$$

$$\therefore \quad \text{Length} = \int_0^1 \sqrt{1+\frac{3y}{4}}\, dy$$

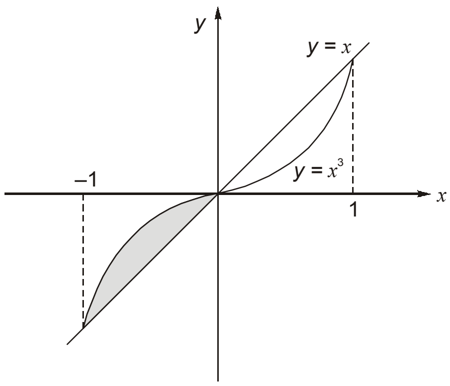
$$= \frac{1}{2}\int_0^1 \sqrt{4+3y}\, dy$$

$$= \frac{1}{2}\left[\frac{(4+3y)^{3/2}}{\frac{3}{2}\times 3}\right]_0^1$$

$$= \frac{1}{9}\left(7\sqrt{7}-8\right)$$

28. (b)

Point of inter-section of the two curves are $x = 0, 1, -1$



$$\text{Area} = \int_{-1}^{0}\left(x^3 - x\right)dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} = \frac{0-(-1)^4}{4} - \frac{0-(-1)^2}{2} = \frac{1}{4}$$

29. (a)

$$(x+1)\frac{dy}{dx}+1 = 2e^{-y}$$

$$\Rightarrow \quad (x+1)\frac{dy}{dx} = (2e^{-y}-1)$$

$$\Rightarrow \quad \frac{dy}{(2e^{-y}-1)} = \frac{dx}{x+1}$$

$$\Rightarrow \quad \frac{e^{y}dy}{2-e^{y}} = \frac{dx}{x+1}$$

$$\Rightarrow \quad -\log(2-e^{y}) = \log(x+1)+c$$

$$\Rightarrow \quad (x+1)(2-e^{y}) = k$$

30. (c)

$$\frac{d^2y}{dx^2} = y$$

$$\Rightarrow \quad D^2y = y \qquad (\therefore d/dx = D)$$

$$(D^2-1)y = 0$$

$$D^2-1 = 0$$

$$D = \pm 1$$

$$y = C_1 e^{x} + C_2 e^{-x}$$

Given point passes through origin

$$\Rightarrow \quad 0 = C_1 + C_2$$

$$C_1 = -C_2 \qquad \text{...(i)}$$

Also, point passes through (ln 2, 3/4)

$$\Rightarrow \quad \frac{3}{4} = C_1 e^{\ln 2} + C_2 e^{-\ln 2}$$

$$\frac{3}{4} = 2C_1 + \frac{C_2}{2}$$

$$\Rightarrow \quad C_2 + 4C_1 = 1.5 \qquad \text{...(ii)}$$

From (i) $C_1 = -C_2$, putting in (ii), we get

$$\Rightarrow \quad -3C_2 = 1.5$$

$$C_2 = -0.5$$

$$\therefore \quad C_1 = 0.5$$

$$\Rightarrow \quad y = 0.5(e^{x}-e^{-x})$$

$$y = \frac{e^{x}-e^{-x}}{2}$$

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