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# SIGNAL & SYSTEM

EC-EE

Date of Test : 20/05/2024

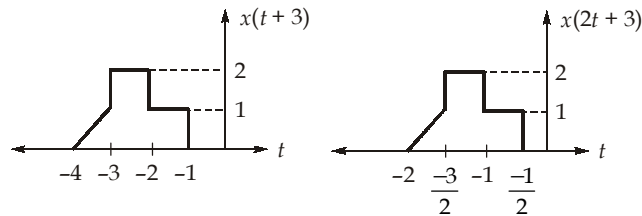
## ANSWER KEY >

- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (c) | 7. (c)  | 13. (b) | 19. (c) | 25. (d) |
| 2. (b) | 8. (a)  | 14. (b) | 20. (d) | 26. (b) |
| 3. (a) | 9. (a)  | 15. (d) | 21. (a) | 27. (b) |
| 4. (c) | 10. (c) | 16. (c) | 22. (d) | 28. (a) |
| 5. (c) | 11. (b) | 17. (c) | 23. (b) | 29. (c) |
| 6. (b) | 12. (b) | 18. (c) | 24. (a) | 30. (b) |

## DETAILED EXPLANATIONS

1. (c)

The signal  $x(2t + 3)$  can be obtained by first shifting  $x(t)$  to the left by 3 units and then scaling by 2 units.



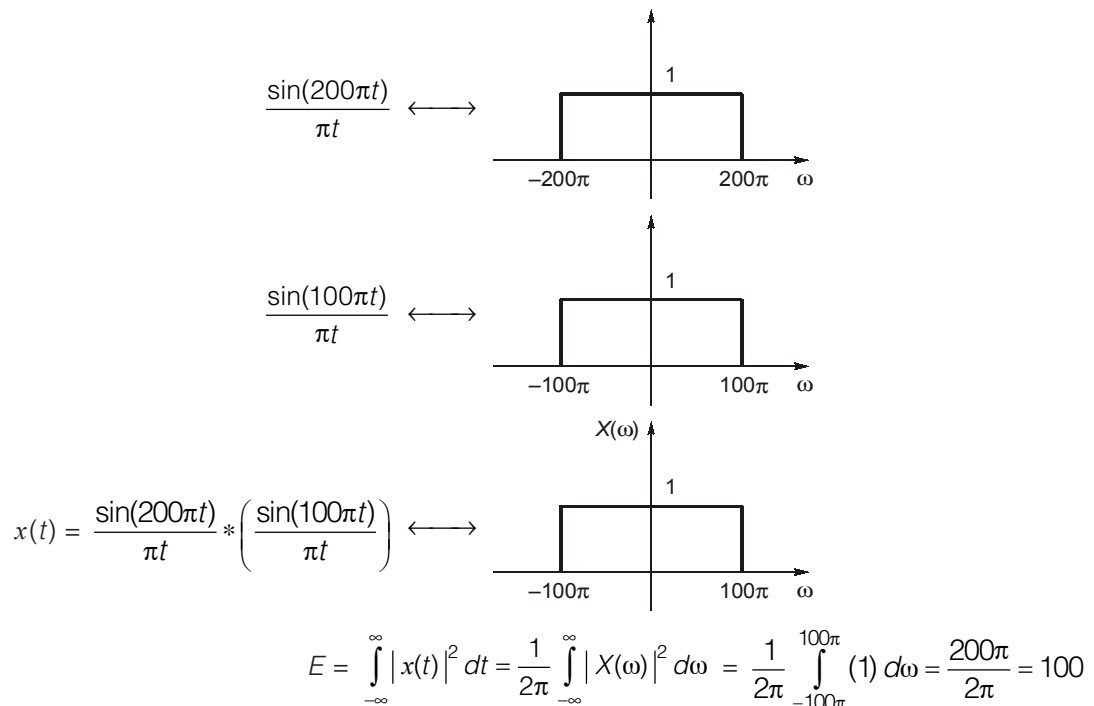
2. (b)

$$e^{-(2t-2)} u(t-1) = e^{-2(t-1)} u(t-1)$$

Now, 
$$e^{-2t} u(t) \leftrightarrow \frac{1}{2 + j\omega}$$

$$e^{-2(t-1)} u(t-1) \leftrightarrow \frac{e^{-j\omega}}{2 + j\omega}$$

3. (a)



4. (c)

Given,  $x(n) \xrightarrow{z} X(z)$

by the definition of z-transform,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=0}^{\infty} a^k \delta[n-5k] z^{-n}$$

The term  $\delta[n-5k]$  is equal 1 if  $n = 5k$  and equal to zero otherwise.

$$\therefore X(z) = \sum_{k=0}^{\infty} a^k z^{-5k} \quad [ \because n = 5k ]$$

$$= \frac{1}{1 - az^{-5}}$$

or 
$$X(z) = \frac{z^5}{z^5 - a}$$

5. (c)

$$(1 + \cos 300\pi t)^2 \rightarrow f_{1 \max} = 300 \text{ Hz}$$

$$(\sin 4000\pi t)^2 \rightarrow f_{2 \max} = 4000 \text{ Hz}$$

$$f_{\max} = f_{1 \max} + f_{2 \max} = 4300 \text{ Hz}$$

$$f_s = 2f_{\max} = 8.6 \text{ kHz}$$

6. (b)

Given, 
$$x(t) = \frac{\sin(10\pi t)}{\pi t}$$

Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 & ; \quad |\omega| \leq 10\pi \\ 0 & ; \quad |\omega| > 10\pi \end{cases}$$

or

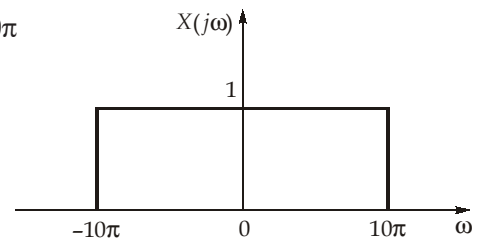
$\therefore$  The maximum frequency ' $\omega_m$ ' present in  $x(t)$  is  $\omega_m = 10\pi$

Hence we require,

$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$\therefore T_s < \frac{1}{10}$



7. (c)

Conjugate anti-symmetric part of  $x[n]$  is  $\frac{x[n] - x^*[-n]}{2}$ .

$$x^*[-n] = [2, 1 + j, -2 + j5]$$

$\therefore \frac{x[n] - x^*[-n]}{2} = \frac{[-2 - j5, 1 - j, 2] - [2, (1 + j), -2 + j5]}{2} = [-2 - j2.5, -j, 2 - j2.5]$

8. (a)

$$X(e^{j\omega}) = e^{j\omega} + e^{-j\omega} + 2(e^{2j\omega} - e^{-2j\omega}) + 3(e^{3j\omega} + e^{-3j\omega})$$

$$= 2\cos\omega + 4j\sin(2\omega) + 2\cos 3\omega = 2\cos(\pi) + 4j\sin(2\pi) + 6\cos(3\pi)$$

$$= -2 + 0 - 6 = -8$$

$$|X e^{j\pi}| = 8$$

9. (a)

$$\operatorname{Re} \{x(t)\} = \frac{x(t) + x^*(t)}{2}$$

∴ The Fourier coefficient of  $x^*(t)$  are

$$b_K = \frac{1}{T} \int_T x^*(t) e^{-jK \frac{2\pi}{T} t} dt$$

Taking conjugate on both sides

$$b_K^* = \frac{1}{T} \int_T x(t) e^{-j(-K) \frac{2\pi}{T} t} dt$$

$$\therefore a_{-K} = b_K^*$$

$$\therefore \text{Fourier series Coefficient of } \operatorname{Re} \{x(t)\} = \frac{a_K + a_{-K}^*}{2}$$

10. (c)

$$\text{Given, } H(z) = \frac{z}{z-0.2} = \frac{1}{1-0.2z^{-1}} \quad \text{ROC: } |z| > 0.2$$

Since the ROC:  $|z| > 0.2$ , which includes unit circle.

∴ The impulse response will be stable.

11. (b)

$$\text{Given, } y(t) = e^{-t} u(t) * \sum_{k=-\infty}^{\infty} \delta(t-2k)$$

$$= e^{-t} u(t) * (\dots + \delta(t+4) + \delta(t+2) + \delta(t) + \delta(t-2) + \delta(t-4) + \dots)$$

Using convolution property of impulse response,

$$\text{i.e., } x(t) * \delta(t-t_0) = x(t-t_0)$$

$$y(t) = \dots + e^{-(t+4)} u(t+4) + e^{-(t+2)} u(t+2) + e^{-t} u(t) + e^{-(t-2)} u(t-2) + e^{-(t-4)} u(t-4)$$

+ ...

In the range  $0 \leq t < 2$ , we may write  $y(t)$  as,

$$y(t) = [\dots + e^{-(t+4)} u(t+4) + e^{-(t+2)} u(t+2) + e^{-t} u(t) + e^{-(t-2)} u(t-2) + e^{-(t-4)} u(t-4) + \dots] (u(t) - u(t-2))$$

$$= (e^{-t} + e^{-(t+2)} + e^{-(t+4)} + \dots); \quad 0 \leq t < 2$$

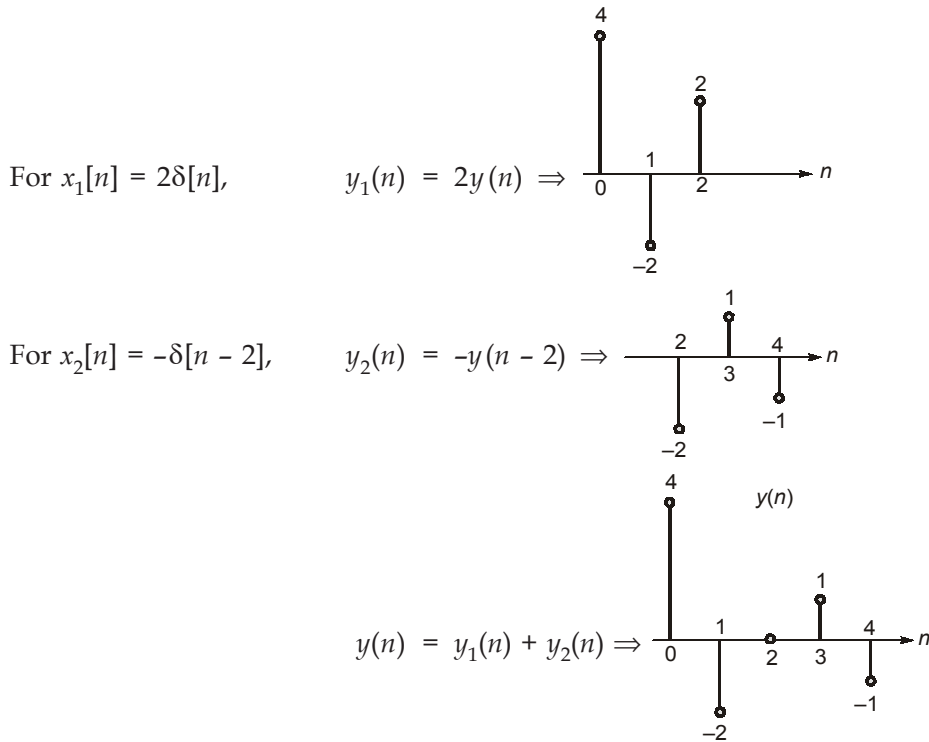
$$= e^{-t} (1 + e^{-2} + e^{-4} + \dots); \quad 0 \leq t < 2$$

$$= e^{-t} \left[ \frac{1}{1-e^{-2}} \right]; \quad 0 \leq t < 2$$

$$\therefore y(t) = A e^{-t} \text{ for } 0 \leq t < 2$$

$$\therefore A = \frac{1}{1-e^{-2}}$$

12. (b)



13. (b)

$$\begin{aligned}
 F(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt \\
 &= \int_{-\infty}^{\infty} [f(t)\cos\omega t - jf(t)\sin\omega t] dt \\
 &= \int_{-\infty}^{\infty} f(t)\cos\omega t dt - j \int_{-\infty}^{\infty} f(t)\sin\omega t dt
 \end{aligned}$$

$f(t) \Rightarrow$  even signal

$f(t)\cos\omega t \Rightarrow$  even signal

$f(t)\sin\omega t \Rightarrow$  odd signal

$$\int_{-\infty}^{\infty} f(t)\sin\omega t dt = 0$$

$$\int_{-\infty}^{\infty} f(t)\cos\omega t dt = 2 \int_0^{\infty} f(t)\cos\omega t dt$$

$$\therefore F(\omega) = 2 \int_0^{\infty} f(t)\cos\omega t dt$$

14. (b)

Given,  $X(s) = \log(s + 2) - \log(s + 3)$

Differentiating both the sides with respect to  $s$

$$\frac{d}{ds} X(s) = \frac{1}{s+2} - \frac{1}{s+3} \quad \dots(i)$$

From the properties of Laplace transform, we know that,

$$tx(t) \longleftrightarrow -\frac{d}{ds} X(s)$$

Thus equation (i) can be written as,

$$-tx(t) = [e^{-2t} - e^{-3t}]u(t)$$

or,

$$x(t) = \left[ \frac{e^{-3t} - e^{-2t}}{t} \right] u(t)$$

15. (d)

$$C_k = j\delta(k+2) - j\delta(k-2) + 2\delta(k+3) + 2\delta(k-3)$$

$$\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} C_k e^{jk\pi t} \\ &= je^{-j2\pi t} - je^{j2\pi t} + 2e^{-j3\pi t} + 2e^{j3\pi t} \\ &= 4\cos(3\pi t) + 2\sin(2\pi t) \end{aligned}$$

16. (c)

Given that,

$$\text{Let, } y_1(t) = 2\pi X(-\omega) \Big|_{\omega=t}$$

$$\text{We have, } y_1(t) = 2\pi \int_{u=-\infty}^{\infty} x(u) e^{jut} du$$

Similarly, let  $y_2(t)$  be the output due to passing  $x(t)$  through 'F' twice.

$$\begin{aligned} y_2(t) &= 2\pi \int_{v=-\infty}^{\infty} 2\pi \int_{u=-\infty}^{\infty} x(u) e^{juv} du e^{jt v} dv \\ &= (2\pi)^2 \int_{u=-\infty}^{\infty} x(u) \int_{v=-\infty}^{\infty} e^{j(t+u)v} dv du \\ &= (2\pi)^2 \int_{u=-\infty}^{\infty} x(u) (2\pi) \delta(t+u) du \\ &= (2\pi)^3 X(-t) \end{aligned}$$

Finally, let  $y_3(t)$  be the output due to passing  $x(t)$  through F three times

$$\begin{aligned} y_3(t) &= 2\pi \int_{u=-\infty}^{\infty} (2\pi)^3 x(-u) e^{jtu} du \\ &= (2\pi)^4 \int_{-\infty}^{\infty} e^{-jtu} x(u) du = (2\pi)^4 X(t) \end{aligned}$$

17. (c)

The fourier transform of  $x(t)$  can be written

$$X_1(j\omega) = |X_1(j\omega)| e^{j\angle X_1(j\omega)}$$

Let, 
$$X_{1a}(j\omega) = \begin{cases} 1; & |\omega| < 3\pi \\ 0; & \text{otherwise} \end{cases}$$

Note that, given  $X_1(j\omega)$  is "3j\omega" times  $X_{1a}(j\omega)$

$$\therefore X_1(j\omega) = \begin{cases} 3j\omega; & |\omega| < 3\pi \\ 0; & \text{otherwise} \end{cases}$$

Since, at 
$$\omega = 3\pi, |X_1(j\omega)| = 9\pi \text{ and } \angle X_1(j\omega) = \frac{\pi}{2}$$

$$\omega = -3\pi, |X_1(j\omega)| = 9\pi \text{ and } \angle X_1(j\omega) = -\frac{\pi}{2}$$

By taking inverse fourier transform,

Thus, 
$$x_1(t) = 3 \frac{d}{dt} x_{1a}(t) \quad \text{[By using differential property]}$$

also we can express 
$$x_{1a}(t) = \frac{\sin 3\pi t}{\pi t}$$

Thus, 
$$\begin{aligned} x_1(t) &= 3 \frac{d}{dt} \left[ \frac{\sin 3\pi t}{\pi t} \right] \\ &= \frac{3}{\pi} \times \frac{1}{t^2} [3\pi t \cos 3\pi t - \sin 3\pi t] \end{aligned}$$

$$\therefore x_1(t) = \frac{3}{\pi t^2} [3\pi t \cos 3\pi t - \sin 3\pi t]$$

18. (c)

We know that the Laplace transform of

$$\cos(at)u(t) = \frac{s}{s^2 + a^2}$$

$$\therefore \cos(\pi t)u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function  $x(t)$  can be written as,

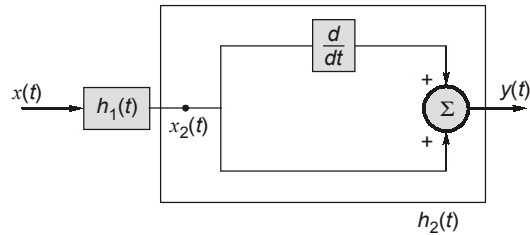
$$\begin{aligned} x(t) &= \cos(\pi t)u(t) - \cos \pi t u(t - 1) \\ &= \cos(\pi t)u(t) - \cos \pi(t - 1 + 1) u(t - 1) \\ &= \cos(\pi t)u(t) - \cos [\pi(t - 1) + \pi] u(t - 1) \\ &= \cos \pi t u(t) - [\cos \pi(t - 1) (-1) - 0] u(t - 1) \\ &= \cos \pi t u(t) + \cos \pi(t - 1) u(t - 1) \end{aligned}$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2 + \pi^2} + \frac{s e^{-s}}{s^2 + \pi^2} \quad [\because x(t - t_0) = X(s) \cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1 + e^{-s}]}{s^2 + \pi^2}$$

19. (c)



Let

$$x_2(t) = \delta(t)$$

$$h_2(t) = \left( \delta(t) + \frac{d}{dt} \delta(t) \right)$$

$$h_1(t) = e^{-t} u(t)$$

$$h(t) = e^{-t} u(t) * \left( \delta(t) + \frac{d}{dt} \delta(t) \right)$$

$$h(t) = e^{-t} u(t) * \delta(t) + e^{-t} u(t) * \frac{d}{dt} \delta(t)$$

$$= e^{-t} u(t) + \frac{d}{dt} (e^{-t} u(t)) * \delta(t)$$

$$= e^{-t} u(t) + \frac{d}{dt} (e^{-t} u(t))$$

$$= e^{-t} u(t) - e^{-t} u(t) + e^{-t} \delta(t)$$

$$h(t) = \delta(t) \quad \because e^{-t} \delta(t) = e^0 \delta(t) = \delta(t)$$

20. (d)

$$H(j\omega) = \frac{1 + 2e^{-j\omega}}{1 + \frac{1}{2e^{-j\omega}}} = \frac{1 + 2e^{-j\omega}}{2e^{-j\omega} + 1} \cdot 2e^{-j\omega}$$

$$|H(j\omega)| = 2$$

21. (a)

Given,

$$X(e^{j\omega}) = \frac{\sin \frac{3\omega}{2}}{\sin \frac{\omega}{2}} = \frac{e^{j3\omega/2} - e^{-j3\omega/2}}{e^{j\omega/2} - e^{-j\omega/2}} = \frac{e^{j3\omega/2} [1 - e^{-j3\omega}]}{e^{j\omega/2} [1 - e^{-j\omega}]}$$

$$= e^{j\omega} \left[ \frac{1 - e^{-j3\omega}}{1 - e^{-j\omega}} \right]$$

$$X(e^{j\omega}) = \frac{e^{j\omega}}{1 - e^{-j\omega}} - \frac{e^{-j2\omega}}{1 - e^{-j\omega}}$$

by taking inverse DTFT,

$$\begin{aligned} x[n] &= u[n+1] - u[n-2] \\ &= \begin{cases} 1; & -1 \leq n < 2 \\ 0; & \text{otherwise} \end{cases} \end{aligned}$$



From parseval's theorem,

$$\sum_{n=-\infty}^{\infty} |nx[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \left[ \frac{d}{d\omega} X(e^{j\omega}) \right] \right|^2 d\omega$$

$$\begin{aligned} \therefore \frac{1}{\pi} \int_{-\pi}^{\pi} \left| \left[ \frac{d}{d\omega} X(e^{j\omega}) \right] \right|^2 d\omega &= 2 \sum_{n=-\infty}^{\infty} |nx[n]|^2 d\omega \\ &= 2 \sum_{n=-1}^1 |n|^2 = 2[1+0+1] = 4 \end{aligned}$$

$$\frac{1}{\pi} \int_{-\pi}^{\pi} \left| \left[ \frac{d}{d\omega} X(e^{j\omega}) \right] \right|^2 d\omega = 4$$

22. (d)

By redrawing the given frequency response, we get,

We can write  $H(\omega) = -j2\text{sgn}(\omega)$

We know that,

For

$$\text{sgn}(t) \xleftrightarrow{\text{FT}} \frac{2}{j\omega}$$

By duality property

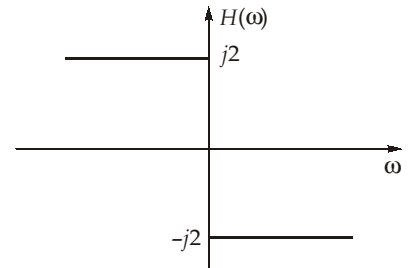
$$\frac{2}{jt} \xleftrightarrow{\text{FT}} 2\pi \text{sgn}(-\omega)$$

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} -2\pi \text{sgn}(\omega)$$

$$\frac{2}{\pi t} \xleftrightarrow{\text{FT}} -j2\text{sgn}(\omega)$$

or

$$= 2(\pi t)^{-1}$$



23. (b)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

$$g[n] = x[n-2]_{\text{mod } N}$$

$$G[k] = e^{-j\frac{2\pi}{N}(2)k} X[k]$$

$$G[1] = e^{-j\frac{2\pi}{4}(2)1} X[1] = e^{-j\pi} X[1]$$

$$G[1] = -X[1] = -7$$

24. (a)

By the definition of Fourier series,

We can write  $C_{N_0/2}$  for  $N_0$  is even,

$$C_{N_0/2} = \frac{1}{N_0} \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{N_0}{2}\right)\left(\frac{2\pi}{N_0}\right)n}$$

$$= \frac{1}{N_0} \sum_{n=0}^{N-1} x[n] e^{-j\pi n} = \frac{1}{N_0} \sum_{n=0}^{N-1} (-1)^n x[n] = \text{real}$$

25. (d)

Given,  $x(t) = 2 + \cos(50\pi t)$

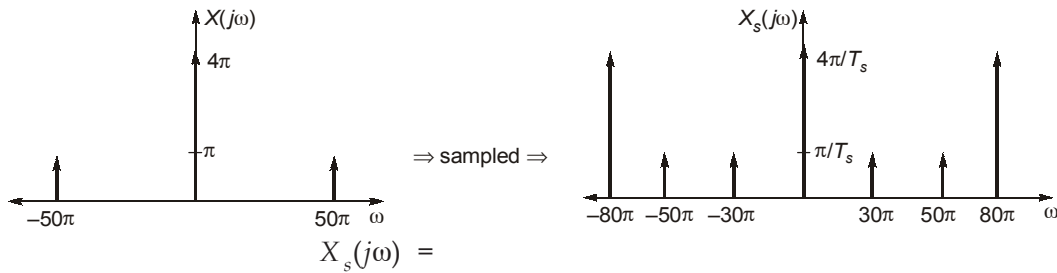
Frequency of signal  $\omega_{\text{sig}} = 50\pi$   
 $T_s = 0.025 \text{ sec}$

$\therefore$  sampling frequency  $\omega_s = \frac{2\pi}{T_s} = 80\pi \text{ rad/sec}$

then,  $X(j\omega) = 4\pi\delta(\omega) + \pi[\delta(\omega + 50\pi) + \delta(\omega - 50\pi)]$

Let the sampled signal be represented as  $X_s(j\omega)$ , where  $X_s(j\omega)$  is given as

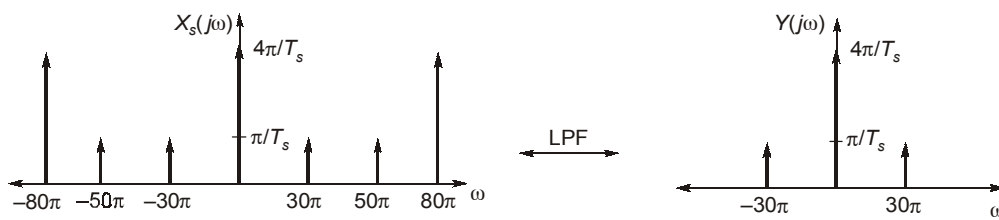
$$X_s(j\omega) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} X(j(\omega - m\omega_s))$$



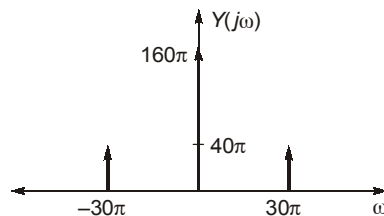
$$X_s(j\omega) = 40 \sum_{m=-\infty}^{\infty} [4\pi\delta(\omega - 80\pi m) + \pi\delta(\omega - 50\pi - 80\pi m) - \pi\delta(\omega + 50\pi - 80\pi m)]$$

now, the sampled input  $X_s(j\omega)$  is passed through a low passed filter having cut-off frequency at  $\omega = 40\pi$ .

Therefore the output  $Y(j\omega)$  will contain only the components which are less than  $\omega = 40\pi$ .



Now by putting  $T_s = 0.025$ , we will get



26. (b)

$$X(z) = \frac{z}{z-1} \quad |z| > 1$$

$$Y(z) = \frac{2z}{z-\frac{1}{3}} \quad |z| > \frac{1}{3}$$

$$H(z) = \frac{2(z-1)}{z-\frac{1}{3}} \quad |z| > \frac{1}{3}$$

$$X'(z) = \frac{z}{z-\frac{1}{2}} \quad |z| > \frac{1}{2}$$

$$Y'(z) = H(z) \cdot X'(z)$$

$$= \frac{2z(z-1)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} \quad |z| > \frac{1}{2}$$

Taking inverse z transform

$$y[n] = \left[ -6\left(\frac{1}{2}\right)^n + 8\left(\frac{1}{3}\right)^n \right] u[n]$$

$$k_1 = -6, \quad k_2 = 8$$

so,

$$k_1 + k_2 = 2$$

27. (b)

If  $x[n]$  is real

$$\text{odd}[x[n]] \xrightarrow{FT} jIm[X(e^{j\omega})]$$

$$\begin{aligned} \therefore \text{odd}[x[n]] &= F^{-1} \left[ \frac{1}{2} (e^{j\omega} - e^{-j\omega} - e^{2j\omega} + e^{-2j\omega}) \right] \\ &= \frac{1}{2} [\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]] \end{aligned}$$

$$\therefore \text{odd}[x[n]] = \frac{x[n] - x[-n]}{2}$$

Since,

$$x[n] = 0 \text{ for } n > 0$$

$$x[n] = 2 \text{ odd}[x[n]]$$

$$= \delta[n+1] - \delta[n+2] \text{ for } n < 0$$

using Parseval's theorem

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$\begin{aligned}(x[0])^2 - 2 &= 3 \\ x[0] &= \pm 1 \\ \therefore x[0] &> 0 \\ \therefore x[n] &= \delta[n] + \delta[n+1] - \delta[n+2]\end{aligned}$$

28. (a)

$$\begin{aligned}x[n] &= \delta[n] \\ X(e^{j\omega}) &= 1\end{aligned}$$

$$\frac{dX(e^{j\omega})}{d\omega} = 0$$

$$\therefore Y(e^{j\omega}) = e^{-j\omega} X(e^{j\omega})$$

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j\omega} \cdot e^{j\omega n} d\omega = \frac{\sin \pi(n-1)}{\pi(n-1)}$$

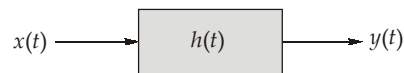
29. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3+j\omega}$$

and output,

$$y(t) = e^{-3t} u(t) - e^{-4t} u(t)$$



$$\text{We know that, } H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$$\therefore X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

By inverse Fourier transform of  $X(j\omega)$ , we have,

$$x(t) = e^{-4t} u(t)$$

30. (b)

$$\text{Given, } X(z) = \frac{1}{1-2.5z^{-1}+z^{-2}} = \frac{1}{(z^{-1}-2)\left(z^{-1}-\frac{1}{2}\right)}$$

$$\frac{1}{(z^{-1}-2)\left(z^{-1}-\frac{1}{2}\right)} = \frac{A}{z^{-1}-2} + \frac{B}{z^{-1}-\frac{1}{2}}$$

$$\therefore A = \frac{1}{2-\frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

$$B = \frac{1}{\frac{1}{2}-2} = -\frac{2}{3}$$

$$\begin{aligned} \therefore X(z) &= \frac{\frac{2}{3}}{z^{-1}-2} + \frac{-\frac{2}{3}}{z^{-1}-\frac{1}{2}} \\ &= \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{4}{3}}{1-2z^{-1}} \end{aligned}$$

Given  $X(z)$  is a causal system, the ROC is right of the right most pole.

$$\therefore |z| > 2$$

hence,

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{4}{3}(2)^n u[n]$$

$$\therefore x(0) = -\frac{1}{3} + \frac{4}{3} = \frac{3}{3} = 1$$

■■■■