CLASS TEST

S.No.: 01SP_ABCDEFG_200424

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# STRENGTH OF MATERIAL

## MECHANICAL ENGINEERING

**Date of Test : 20/05/2024**

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (c) | 7. | (d) | 13. | (c) | 19. | (c) | 25. | (a) |
| 2. | (b) | 8. | (c) | 14. | (c) | 20. | (c) | 26. | (a) |
| 3. | (d) | 9. | (a) | 15. | (c) | 21. | (a) | 27. | (a) |
| 4. | (b) | 10. | (a) | 16. | (a) | 22. | (b) | 28. | (a) |
| 5. | (b) | 11. | (a) | 17. | (b) | 23. | (c) | 29. | (a) |
| 6. | (b) | 12. | (c) | 18. | (b) | 24. | (b) | 30. | (b) |

## DETAILED EXPLANATIONS

**1. (c)**

$$\text{Proof resilience} = \text{Total strain energy upto elastic limit}$$

$$= \frac{\tau^2}{2G} \times \text{Volume} = \frac{250^2}{2 \times (80000)} \times 50^3 = 48828.125 \text{ Nmm}$$

$$= 48.828 \text{ N-m}$$

$$\approx 49 \text{ N-m}$$

**2. (b)**

For equal area,

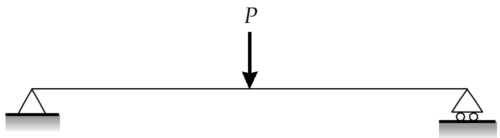
$$\frac{\pi}{4} d_1^2 = \frac{\pi}{4}\left(d_2^2 - d^2\right)$$

$$\Rightarrow \quad d^2 = d_2^2 - d_1^2$$

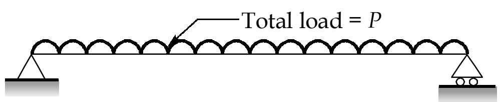
$$\frac{Z_2}{Z_1} = \frac{\left(\dfrac{d_2^4 - d^4}{d_2/2}\right)}{\left(\dfrac{d_1^4}{d_1/2}\right)} = \frac{\left(d_2^2 - d^2\right)\left(d_2^2 + d^2\right)}{d_2 d_1^3}$$

$$= \frac{\left(d_2^2 + d^2\right)}{d_1 d_2} = \frac{\left(d_2^2 + d_2^2 - d_1^2\right)}{d_1 d_2} = \frac{2d_2^2 - d_1^2}{d_1 d_2}$$

**3. (d)**




$$\theta_{\max 1} = \frac{PL^2}{16EI}$$




$$\theta_{\max 2} = \frac{PL^2}{24EI}$$

$$\% \text{ decrease in maximum slope} = \frac{\theta_{\max 1} - \theta_{\max 2}}{\theta_{\max 1}} \times 100 = \frac{\frac{1}{16} - \frac{1}{24}}{\frac{1}{16}} \times 100\% = 33.33\%$$

**4. (b)**

From Mohr circle,

$\sigma_1 = 100$ MPa, $\sigma_2 = -20$ MPa

$$\text{Absolute } \tau_{max} = \max\left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_1}{2}\right|, \left|\frac{\sigma_2}{2}\right|\right] = \max[60, 50, 10] = 60\,\text{MPa}$$

**5. (b)**

$$4\text{ mm} = \alpha L \Delta T - \frac{\sigma}{E}L$$

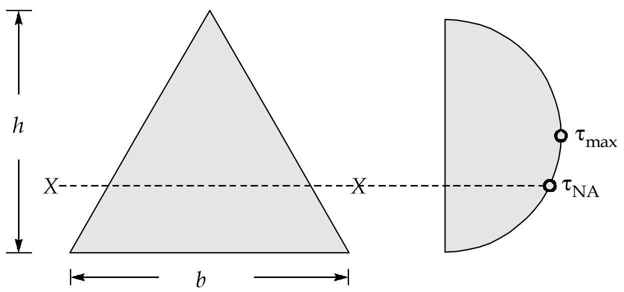
$$4 = \left(10 \times 10^{-6}\right) \times 10^4 \times \Delta T - \frac{20}{200 \times 10^3} \times 10^4$$

$$\Delta T = 50°\text{C}$$

Thus, rod is to be heated upto $\Delta T + 20 = 70°$C

**6. (b)**

In triangular section,




For triangular cross-section,

$$\tau_{max} = \frac{3}{2}\tau_{avg}$$

and,

$$\tau_{NA} = \frac{4}{3}\tau_{avg}$$

$\Rightarrow$

$$\tau_{NA} = \frac{8}{9}\tau_{max} = \frac{8}{9} \times 9 = 8\,\text{MPa}$$

**7. (d)**

$\in_A = \in_{0°} = \in_x = 530 \times 10^{-6}$

$\in_C = \in_{90°} = \in_y = -80 \times 10^{-6}$

$$(\in_n)_\theta = \left(\frac{\in_x + \in_y}{2}\right) + \left(\frac{\in_x - \in_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin\theta$$

$$(\in_n)_{\theta=45°} = \in_B = 225 \times 10^{-6} + 305 \times 10^{-6}\cos 90° + \frac{\gamma_{xy}}{2}(\sin 90°) = 420 \times 10^{-6}$$

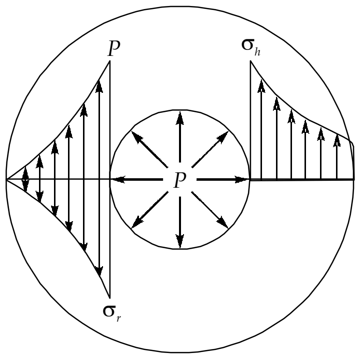
$\gamma_{xy} = 390 \times 10^{-6}$

$$\in_{1,2} = \frac{\in_x + \in_y}{2} \pm \sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{530-80}{2} \pm \sqrt{\left(\frac{530+80}{2}\right)^2 + \left(\frac{390}{2}\right)^2}\right] \times 10^{-6} = [225 \pm 362] \times 10^{-6}$$

$\therefore\ \in_{1,2} = 587 \times 10^{-6}, -137 \times 10^{-6}$

**8. (c)**




When thick cylinder is subjected to internal pressure 'p', hoop stress (tensile) develops which is maximum at inner radius and minimum at outer radius hyperbolic. The radial stress ($\sigma_r$) develope which is compressive in nature and has maximum magnitude at internal radius and varies hyperbolic to zero value at outer radius.

The longitudinal stress developed is tensile in nature and remain constant along the length of cylinder.

**9. (a)**

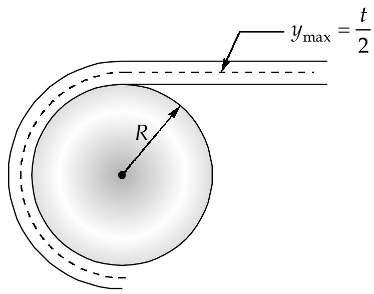
$$E = 300 \times 1000 \text{ N/mm}^2$$

$$y_{max} = \frac{2}{2} = 1 \text{ mm}$$

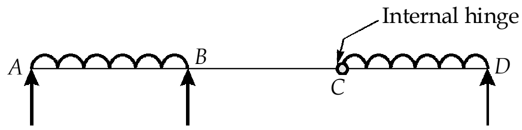
$$R = \frac{2000}{2} = 1000 \text{ mm}$$

$$(\sigma_b)_{max} = \frac{Ey_{max}}{R_1} = \frac{Ey_{max}}{R + \frac{t}{2}} \approx \frac{Ey_{max}}{R}$$

$$= \frac{300 \times 10^3}{1000} = 300 \text{ N/mm}^2$$




**10. (a)**




- At point C, there is an internal hinge, bending moment, BM = 0 and hence it will become a point of contraflexure.
- Between A and B, beam is subjected to uniformly distributed load, so the variation of BMD will of second degree. Similar BMD will be represented in between C and D.

**11. (a)**

At section $A$,

Bending moment , $M$ = 400 × 1 = 400 N-m

Torsion, $(T)$ = 400 × 0.75 = 300 N-m

The principal stresses at top extremity of the vertical diameter, at the section A

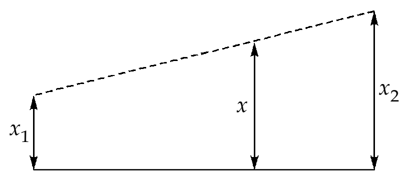
$$\sigma_{1,2} = \frac{16}{\pi d^3}\left(M \pm \sqrt{M^2 + T^2}\right)$$

$$= \frac{16}{\pi(100)^3}\left(400 \pm \sqrt{400^2 + 300^2}\right) \times 10^3$$

$$\sigma_1 = \frac{16}{\pi \times (1000)}(400 + 500) = 4.58\ \text{N/mm}^2$$

$$\sigma_2 = \frac{16}{\pi \times (1000)}(400 - 500) = -0.509\ \text{N/mm}^2$$

**12. (c)**




$$x_1 = \frac{F \cdot b}{(a+b)k_1}$$

$$x_2 = \frac{F \cdot a}{(a+b)k_2}$$

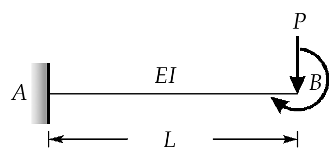
$$x = x_1 + \frac{(x_2 - x_1)}{(a+b)}a = \frac{x_1 b + x_2 a}{a+b}$$

$$= \left(\frac{F}{(a+b)^2}\right)\left(\frac{b^2}{k_1} + \frac{a^2}{k_2}\right)$$

$$k_{eq} = \frac{F}{x} = \left(\frac{(a+b)^2}{\frac{a^2}{k_2} + \frac{b^2}{k_1}}\right)$$

**13. (c)**

Since portion BC is rigid, BC will remain straight




$$\text{Deflection at } B = \frac{PL^3}{3EI} + \frac{PL^3}{2EI} = \frac{5}{6}\frac{PL^3}{EI}$$

$$\text{Slope at } B = \frac{PL^2}{2EI} + \frac{PL^2}{EI} = \frac{3}{2}\frac{PL^2}{EI}$$

Deflection at $C$ = Deflection at B + Slope at $B$ × $L$

$$= \frac{5}{6}\frac{PL^3}{EI} + \frac{3}{2}\frac{PL^2}{EI} \times L = \frac{7}{3}\frac{PL^3}{EI}$$

**14. (c)**

$$\frac{P_2}{P_1} = \frac{I_2}{L_2^2} \times \frac{L_1^2}{I_1} = \left(\frac{d_2}{d_1}\right)^4 \times \left(\frac{L_1}{L_2}\right)^2 = \frac{(0.8)^2}{(1.1)^2} = 0.338$$

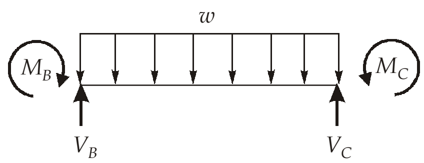
Percentage decrease = (1 – 0.338) × 100 = 66.148% $\simeq$ 66%

**15. (c)**

For sections $AB$ and $CD$, the beam may be modeled as


$M(x)$ is linear with respect to $x$.

For section $BC$, the beam is modeled as




$M(x)$ is parabolic, reaching a maximum near or at the center.

**16. (a)**

Given, a biaxial stress system,

$$\sigma_x = 100 \text{ MPa}, \sigma_y = 60 \text{ MPa}$$

For maximum obliquity of the resultant with the normal to a plane is given by

$$\tan\theta = \sqrt{\frac{\sigma_x}{\sigma_y}} = \sqrt{\frac{100}{60}} = 1.29$$

or $\theta = 52.24°$

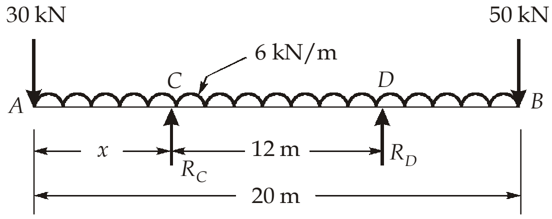
Direct stress, 
$$\sigma_\theta = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta$$
$$= 100\cos^2 52.24° + 60\sin^2 52.24°$$
$$= 100 \times 0.375 + 60 \times 0.625 = 37.5 + 37.5 = 75 \text{ MPa}$$

Shear stress, 
$$\tau_\theta = -\frac{1}{2}\left(\sigma_x - \sigma_y\right)\sin 2\theta$$
$$= -\frac{1}{2}(100 - 60)\sin 104.48° = -19.365 \text{ MPa}$$

Resultant stress, 
$$\sigma_r = \sqrt{\sigma_\theta^2 + \tau_\theta^2} = \sqrt{75^2 + 19.365^2} = 77.46 \text{ MPa}$$

**17.** **(b)**

Let the left support $C$ be at a distance $x$ meters from $A$.




Now, $R_C = R_D$ (Given)

$$\Sigma V = 0$$

$$R_C + R_D - 30 - 6 \times 20 - 50 = 0$$

$$\Rightarrow \quad 2R_C = 30 + 120 + 50$$

$$\Rightarrow \quad R_C = 100 \text{ kN}$$

$$\therefore \quad R_D = 100 \text{ kN}$$

$$\Sigma M_A = 0$$

$$100x + 100(12 + x) - 6 \times 20 \times 10 - 50 \times 20 = 0$$

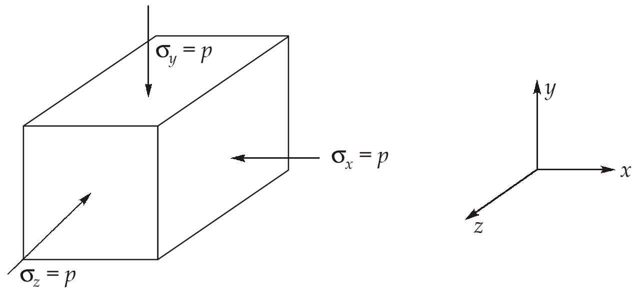
$$200x = 1000$$

$$x = 5 \text{ m}$$

**18.** **(b)**

Under water, the solid will be subjected to hydrostatic pressure (compressive) of equal magnitude on all sides as shown in figure. In the three principal directions, the strains will be

$$\varepsilon_x = \frac{\sigma}{E}(1-2\mu) = \varepsilon_y = \varepsilon_z$$




Therefore,

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{3p}{E}(1-2\mu)$$

$$\text{Change in volume, } \varepsilon_v = \frac{0.05}{100} = \frac{3p}{E}(1-2\mu)$$

$$p = \frac{0.05}{100} \times \frac{E}{3(1-2\mu)} = \frac{0.05 \times 200{,}000}{100 \times 3(1-2\times 0.3)} = 83.33 \text{ N/mm}^2 \quad \text{...(1)}$$

$$\text{Pressure at any depth, } p = wh = 10{,}080h \text{ N/m}^2 \quad \text{...(2)}$$

From eq. (1) and (2) we get,

$$10{,}080h = 83.33 \times 10^6$$

$$\Rightarrow \quad h = 8267 \text{ m}$$

**19. (c)**

Impact loading

$$\delta = \text{Max. instantaneous extension} = 1.25 \text{ mm}$$

$$W = Mg = 60 \times 9.81 = 588.6 \text{ N}$$

$$\therefore \quad \sigma = E\frac{\delta}{L} = \frac{2.05\times10^5\times1.25}{2.5\times10^3} = 102.5 \text{ N/mm}^2$$

Now potential energy lost by weight = Strain energy stored in bar

$$\therefore \quad W(h+\delta) = \frac{\sigma^2}{2E}\times V$$

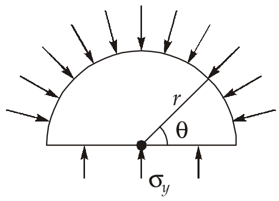
$$\text{or} \quad 588.6(h+1.25) = \frac{(102.5)^2}{2\times2.05\times10^5}\left[\frac{\pi}{4}(40)^2\times2500\right]$$

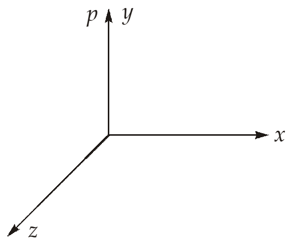
$$\text{or} \quad h+1.25 = 136.78$$

$$\Rightarrow \quad h = 136.78 - 1.25 = 135.53 \text{ mm}$$

**20. (c)**

$$\sigma_y dz(2r) = \int_0^{\pi} p(rd\theta)\,dz\sin\theta$$




$$2\sigma_y = p\int_0^{\pi}\sin\theta d\theta = p[-\cos\theta]_0^{\pi}$$

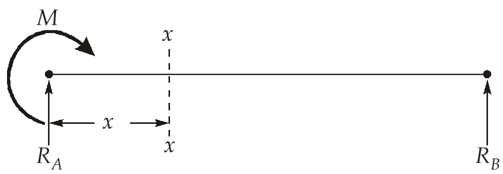
$$\sigma_y = p$$

Due to symmetry, $\sigma_x = \sigma_y = p$,

There will be no shear stress.

$$(\sigma_x, \sigma_y, \tau_{xy}) = (-p, -p, 0)$$

**21. (a)**




$$R_A = -\frac{M}{L} \qquad R_B = \frac{M}{L}$$

At any section, $$M_x = M - R_A x = M - \frac{M\times x}{L}$$

$$\theta_A = \frac{\partial U}{\partial M} = \int\frac{M}{EI}\frac{\partial M_x}{\partial M}dx \quad \text{(Modified castigliano's theorem)}$$

and, $$\frac{\partial M_x}{\partial M} = \left(1-\frac{x}{L}\right)$$

Hence, $$\theta_A = \int_0^L \frac{M\left(1-\frac{x}{L}\right)\left(1-\frac{x}{L}\right)}{EI}dx = \frac{ML}{3EI}$$

**22.** **(b)**

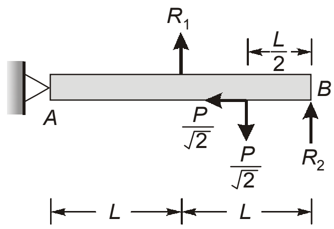
$$\tau_{max}\text{ (in plane)} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \sqrt{\left(\frac{25-(-50)}{2}\right)^2+30^2} = \sqrt{37.5^2+30^2}$$

$$\tau_{max} = 48.02\text{ MPa} \simeq 48\text{ MPa}$$

$$\sigma_{avg.} = \frac{\sigma_x+\sigma_y}{2} = \frac{25+(-50)}{2} = -12.5\text{ MPa}$$

**23.** **(c)**

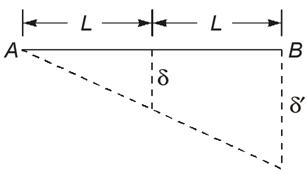
Let forces in spring (1) and (2) be $R_1$ and $R_2$




$$\Sigma M_A = 0$$

$$\Rightarrow \quad R_1\times L + R_2\times 2L = \frac{P}{\sqrt{2}}\times\frac{3L}{2}$$

$$\Rightarrow \quad R_1 + 2R_2 = \frac{3P}{2\sqrt{2}} \quad \text{...(i)}$$




From similar triangle, $$\frac{\delta}{L} = \frac{\delta'}{2L}$$

$$\Rightarrow \quad 2\delta = \delta' \quad \text{...(ii)}$$

Also, $\delta' = \dfrac{R_2}{K}$, $\delta = \dfrac{R_1}{2K}$

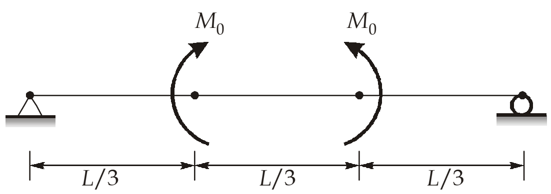
Putting in equation (ii)

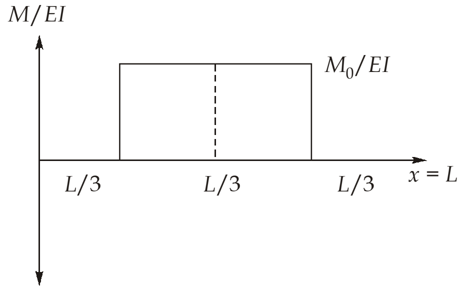
$$R_1 = R_2$$

$$\Rightarrow \quad 3R_1 = \frac{3P}{2\sqrt{2}}$$

$$\Rightarrow \quad R_1 = R_2 = \frac{P}{2\sqrt{2}}$$

**24. (b)**




Maximum deflection occurs at $x = \dfrac{L}{2}$

$$\delta_{max} = \text{Moment of area of } \frac{M}{EI} \text{ diagram between } x = 0 \text{ and } x = \frac{L}{2}$$

$$= \left(\frac{M_0}{EI} \times \frac{L}{6}\right) \times \frac{5}{12}L \quad \text{(Moment is calculated about } x = 0\text{)}$$

$$= \frac{5}{72}\frac{M_0 L^2}{EI}$$

**25. (a)**

Deflection of cantilever beam at free end.

1. Due to uniform loading, $w$

$$\Delta_1 = \frac{wL^4}{8EI}$$

2. Due to a point load, $P$

$$\Delta_2 = \frac{PL^3}{3EI}$$

Here $P$ is the spring force ($F_s$)

Net deflection due to superposition, $s$ is

$$s = \Delta_1 - \Delta_2$$

$$\frac{F_s}{k} = \frac{wL^4}{8EI} - \frac{F_s L^3}{3EI}$$

$$F_s = \frac{3kwL^4}{24EI + 8kL^3}$$

**26. (a)**

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

$$\frac{dM}{dx} = F$$

$$\frac{dF}{dx} = -q$$

From above three relations, we get

$$v'''' = -\frac{q(x)}{EI} \text{ or } \frac{d^4v}{dx^4} = -\frac{q(x)}{EI}$$

**27. (a)**

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Since $A$, $E$, $L$ are same.

$$P_1 : P_2 : P_3 = I_1 : I_2 : I_3$$

1. Circle

$$I = \frac{\pi d^4}{64} = \frac{A^2}{4\pi}$$

2. Square

$$I = \frac{a^4}{12} = \frac{A^2}{12}$$

3. Equilateral triangle

$$I = \frac{\sqrt{3}b^4}{96}$$

or $$I = \frac{A^2\sqrt{3}}{18}$$

$$P_1 : P_2 : P_3 = I_1 : I_2 : I_3 = 1 : \frac{\pi}{3} : \frac{2\pi\sqrt{3}}{9}$$

(Since all 3 cross-sections are symmetric, every centroidal axis has the same moment of Inertia).

**28. (a)**

For a rectangular strain rosette

$$\varepsilon_x = \varepsilon_{0°} = 400 \times 10^{-6},\ \varepsilon_y = \varepsilon_{90°} = -100 \times 10^{-6}$$

and $$\gamma_{xy} = 2\varepsilon_{45°} - \varepsilon_x - \varepsilon_y$$

$$\gamma_{xy} = 2 \times 200 \times 10^{-6} - 400 \times 10^{-6} + 100 \times 10^{-6} = 100 \times 10^{-6}$$

Principal strains, $$\varepsilon_1, \varepsilon_2 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$= \frac{10^{-6}}{2}\left[(400 - 100) \pm \sqrt{(400+100)^2 + 100^2}\right]$$

$$= 404.95 \times 10^{-6} \text{ and } -104.95 \times 10^{-6}$$

Principal stresses

$$\sigma_1 = \frac{E(\mu\varepsilon_2 + \varepsilon_1)}{1-\mu^2} = \frac{210000(-0.3 \times 104.95 + 404.95) \times 10^{-6}}{1 - 0.3^2} = 86.2 \text{ MPa}$$

$$\sigma_2 = \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1-\mu^2} = \frac{210000(0.3 \times 404.95 - 104.95) \times 10^{-6}}{1 - 0.3^2} = 3.82 \text{ MPa}$$

**29. (a)**

Strain energy due to torsion,

$$U = \frac{T^2L}{2GJ}$$

Strain energy of portion $AB$,

$$U_{ab} = \frac{T_A^2\left(\frac{L}{2}\right)}{2GJ}$$

Strain energy for portion $BC$,

$$U_{bc} = \frac{(T_A - T_B)^2\left(\frac{L}{2}\right)}{2GJ} = 0$$

Total strain energy

$$U = U_{ab} + U_{bc}$$

$$U = \frac{100^2}{2\times 80\times 10^9\times 80000\times 10^{-12}}\left(\frac{1.5}{2}\right) = 0.5859 \text{ N-m}$$

or $\quad U = 585.9$ N-mm

**30. (b)**

$\Sigma F_x = 0,$

$\Rightarrow \quad H_A = P$

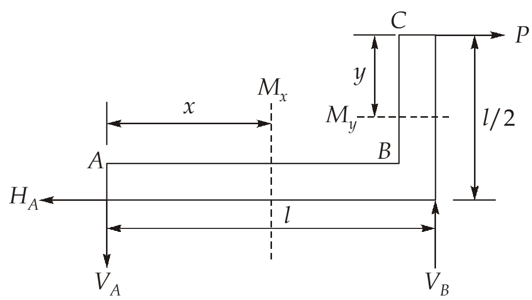
$\Sigma M_A = 0$

$\Rightarrow \quad V_B \times l = P \times \frac{l}{2}$

$$V_B = \frac{P}{2}$$

$\Sigma F_y = 0$

$\Rightarrow \quad V_A = V_B = \frac{P}{2}$




Strain energy stored by the bracket,

$$U = U_{AB} + U_{BC}$$

$$= \int_0^l \frac{M_x^2 dx}{2EI} + \int_0^{l/2} \frac{M_y^2 dy}{2EI} = \int_0^l \frac{\left(-\frac{Px}{2}\right)^2 dx}{2EI} + \int_0^{l/2} \frac{(Py)^2 dy}{2EI}$$

$$= \frac{P^2}{8EI}\left[\frac{x^3}{3}\right]_0^l + \frac{P^2}{2EI}\left[\frac{y^3}{3}\right]_0^{l/2}$$

$$= \frac{P^2}{8EI}\times\frac{l^3}{3} + \frac{P^2(l/2)^3}{6EI} = \frac{P^2l^3}{24EI} + \frac{P^2l^3}{48EI}$$

$$U = \frac{P^2 l^3}{16EI}$$

Horizontal deflection at $C$,

$$\delta_C = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\left(\frac{P^2 l^3}{16EI}\right)$$

$$\delta_C = \frac{2Pl^3}{16EI}$$

$$\delta_C = \frac{Pl^3}{8EI}$$

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