CLASS TEST

S.No.: 01SKEC_ABCDEFG_160524

MADE EASY
India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

COMMUNICATIONS

ELECTRONICS ENGINEERING

Date of Test : 16/05/2024

ANSWER KEY

1.	(d)	7.	(a)	13.	(a)	19.	(b)	25.	(b)
2.	(c)	8.	(a)	14.	(a)	20.	(b)	26.	(b)
3.	(c)	9.	(b)	15.	(a)	21.	(c)	27.	(b)
4.	(d)	10.	(c)	16.	(a)	22.	(c)	28.	(a)
5.	(c)	11.	(c)	17.	(d)	23.	(c)	29.	(d)
6.	(c)	12.	(a)	18.	(a)	24.	(c)	30.	(a)

Detailed Explanations

1. (d)

For matched filter,

$$(\text{SNR})_{\max} = \frac{2E_s}{N_0}$$

$$E_s = \text{Energy of the signal } s(t)$$

$$= \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_0^2 (4)^2 dt = 32$$

So, $$(\text{SNR})_{\max} = \frac{2(32)}{N_0} = \frac{64}{N_0}$$

2. (c)

Both the given statements are correct.

3. (c)

For coherent BPSK,

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{(N_0/2)}}\right)$$

$$= Q\left(\sqrt{\frac{10\times10^{-6}}{1\times10^{-9}}}\right) = Q\left(\sqrt{10^4}\right) = Q(100)$$

4. (d)

Bit rate, $R_b = 100$ kbps

Chip rate, $R_c = 7.2$ Mcps

$$\text{Processing gain} = \frac{R_c}{R_b} = \frac{7.2\times1000}{100} = 72$$

5. (c)

The condition required to eliminate the slope-overload distortion is,

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{\max} = |2\pi f_m \sin(2\pi f_m t)|_{\max}$$

$$2f_s \geq 2\pi f_m$$

$$f_s \geq \pi f_m \approx 3.14 f_m$$

$$f_{s\,(\min)} = 3.14 f_m$$

6. (c)

The amplitude of uniformly distributed is equal to $\frac{1}{K}$.

$\therefore$ $$E[X^{K-1}] = \int_{-\infty}^{\infty} x^{K-1} f_x(x) dx$$

$$= \frac{1}{K}\int_0^k x^{K-1} dx = \frac{1}{K}\left|\frac{x^K}{K}\right|_0^K = (K)^{K-2}$$

7. (a)

$$H(X) = 0.9\log_2 \frac{1}{0.9} + 0.1\log_2 \frac{1}{0.1} = 0.469 \text{ bits/message}$$

$$\therefore \quad H(X^2) = 2H(X) = 2 \times 0.469 = 0.938 \text{ bits/message}$$

8. (a)

Since, $Y = 2X$

Thus, $\frac{dx}{dy} = \frac{1}{2}$

$$f_Y(y) = \frac{dx}{dy} f_X(x) = \frac{dx}{dy} f_X\left(\frac{y}{2}\right)$$

$$\therefore \quad K = \frac{1}{2}$$

9. (b)

Power spectral density is always a even function and only (b) satisfies this condition.

10. (c)

$$H = P\log_2 \frac{1}{P} + (1-P)\log_2 \frac{1}{1-P}$$

$$\frac{dH}{dp} = 0$$

Which gives as $P = \frac{1}{2}$

11. (c)

$$X(t) = 6e^{At}$$

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)] = E[6e^{At_1}\, 6e^{At_2}]$$

$$= 36\left[\frac{1}{2}\int_0^2 e^{A(t_1+t_2)} dA\right] = 18\left[\frac{e^{A(t_1+t_2)}}{t_1+t_2}\right]_0^2 = \frac{18}{t_1+t_2}[e^{2(t_1+t_2)} - 1]$$

12. (a)

The procedure are identical, let $\in = e$,

The above value should be non-negative, this will happen only for $\left(\frac{T-t_o}{T}\right)$ and also since x can have only two values A and 0, thus

Here, CDF $F_X(x|\varepsilon = e) = P\{X \le x | \varepsilon = e\}$

$$= \left[\frac{(T-t_o)}{T}u(x) + \frac{t_o}{T}u(x-A)\right]$$

Because 'x' can have only value of zero and A.

Thus, PDF $f_X(x|\varepsilon = e) = \left[\frac{(T-t_o)}{T}\right]\delta(x) + \frac{t_o}{T}\delta(x-A)$

13. (a)

Given, $y = \frac{1}{2}(x + |x|)$

when $x > 0$, $$y = \frac{1}{2}(x+|x|) = \frac{1}{2}(x+x) = \frac{2x}{2} = x, \; y > 0$$

and $y = x, \quad y > 0$

$$\therefore \quad F_Y(y) = P[X \le y | X \ge 0]$$

$$= \frac{P[X \le y, X \ge 0]}{P(X \ge 0)} = \frac{P(X \le y, X \ge 0)}{1 - P(X < 0)}$$

$$= \frac{P(0 \le X \le y)}{1 - P(X < 0)} = \frac{F_X(y) - F_X(0)}{1 - F_X(0)}$$

$$\therefore \quad f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{f_X(y)}{1 - F_X(0)}$$

14. (a)

$$\int_{-\infty}^{\infty} f_X(x) dx = 1$$

$$\int_{0}^{\infty} Cxe^{-x} dx = 1$$

$$C\left[x\left(\frac{e^{-x}}{-1}\right) - 1\left(\frac{e^{-x}}{1}\right)\right]_0^{\infty} = 1$$

$$C(0 + 1) = 1$$

$$\therefore \quad C = 1$$

15. (a)

The probability density function of the input variable r can be given by,

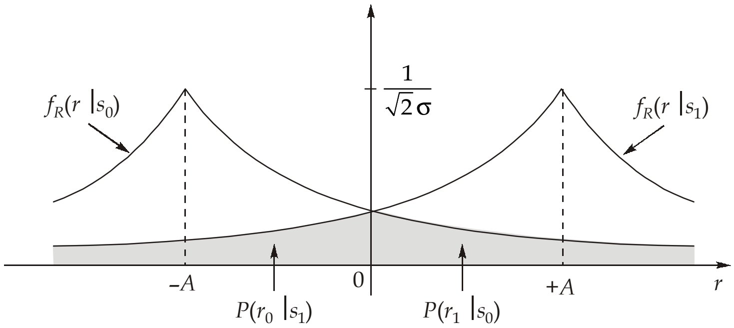
$$f_R(r|s_0) = \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|r+A|}{\sigma}} \quad ; \quad \text{when "0" is transmitted}$$

$$f_R(r|s_1) = \frac{1}{\sqrt{2}\sigma} e^{\frac{-\sqrt{2}|r-A|}{\sigma}} \quad ; \quad \text{when "1" is transmitted}$$

Given that, $r_{th} = 0$. So, the detector will make the decision in favour of logic-0 when $r < 0$ and in favour of logic-1 when $r > 0$.

The probability of error can be given by,

$$P_e = \begin{pmatrix} \text{Probability of decision} \\ \text{made in favour of 0} \\ \text{when 1 is transmitted} \end{pmatrix} P(s_1) + \begin{pmatrix} \text{Probability of decision} \\ \text{made in favour of 1} \\ \text{when 0 is transmitted} \end{pmatrix} P(s_0)$$



It is given that, $P(s_0) = P(s_1) = \frac{1}{2}$ and from the above graph it is clear that $P(r_0|s_1) = P(r_1|s_0)$.

So,
$$P_e = \frac{1}{2}P(r_0|s_1) + \frac{1}{2}P(r_1|s_0) = P(r_1|s_0) = P(r_0|s_1)$$

$$= \int_0^\infty f_R(r|s_0)\,dr = \frac{1}{\sqrt{2}\sigma}\int_0^\infty e^{-\frac{\sqrt{2}|r+A|}{\sigma}}\,dr$$

$$= \frac{e^{-\frac{\sqrt{2}A}{\sigma}}}{\sqrt{2}\sigma}\int_0^\infty e^{-\sqrt{2}r/\sigma}\,dr = \frac{e^{-\frac{\sqrt{2}A}{\sigma}}}{\sqrt{2}\sigma}\left[-\frac{e^{-\sqrt{2}r/\sigma}}{\sqrt{2}/\sigma}\right]_0^\infty$$

$$P_e = \frac{1}{2}e^{-\frac{\sqrt{2}A}{\sigma}}$$

The SNR at the input of the detector can be given by,

$$\text{SNR} = \frac{(\pm A)^2}{\sigma^2} = \left(\frac{A}{\sigma}\right)^2$$

So, the relation between P_e and SNR can be given by,

$$P_e = \frac{1}{2}e^{-\sqrt{2(\text{SNR})}}$$

To achieve a maximum error probability of 10^{-5},

$$\frac{1}{2}e^{-\sqrt{2(\text{SNR})}} \le 10^{-5}$$

$$-\sqrt{2(\text{SNR})} \le \ln(2 \times 10^{-5})$$

$$\text{SNR} \ge 58.534 \text{ (or) } 17.6741 \text{ dB}$$

So,
$$(\text{SNR})_{\min} = 58.538 \text{ (or) } 17.6741 \text{ dB}$$

16. (a)

$$E_1 = \frac{1}{4}\left[(0)^2 + (2A)^2 + (\sqrt{2}A)^2 + (\sqrt{2}A)^2\right] = 2A^2 \Rightarrow d_1 = \sqrt{E_1} = \sqrt{2}A$$

$$E_2 = \frac{1}{4}\left[(A)^2 + (A)^2 + (A)^2 + (A)^2\right] = A^2 \Rightarrow d_2 = \sqrt{E_2} = A$$

Since $d_1 > d_2$

$\therefore$ $p_1 < p_2$

So, $E_1 > E_2$

17. (d)

Maximum distance separable (MDS) codes satisfy the Singleton bound.

Singleton bound, $d_{min} = n - k + 1$

For (6, 1, 6) $\Rightarrow$ $n - k + 1 = 6$ and $d_{min} = 6$

For (6, 5, 2) $\Rightarrow$ $n - k + 1 = 2$ and $d_{min} = 2$

For (7, 6, 2) $\Rightarrow$ $n - k + 1 = 2$ and $d_{min} = 2$

For (7, 4, 3) $\Rightarrow$ $n - k + 1 = 4$ and $d_{min} = 3$

So, the code given in option (d) is not an MDS code.

18. (a)

The quantization level for region R_3 will be,

$$x_{q3} = \frac{1+5}{2} = 3$$

So, the quantization noise due to rounding off of the samples lie in the region R_3 alone is,

$$N_{Q3} = E\left[(X - x_{q3})^2\right] = \int_{<R_3>} (x-3)^2 f_X(x)\,dx$$

$$f_X(x) = \begin{cases} \frac{1}{5}; & -3 \le x \le 2 \\ 0; & \text{otherwise} \end{cases}$$

So, $$N_{Q3} = \frac{1}{5}\int_1^5 (x^2 - 6x + 9)dx = \frac{1}{5}\left[\frac{x^3}{3} - 3x^2 + 9x\right]_1^5 = \frac{36}{5} = 7.2$$

19. (b)

$$f_I = f_c + 2(IF)$$

$$IF = 10 \text{ MHz}$$

$$\left(\frac{C_{max}}{C_{min}}\right) = \left(\frac{f_{Lo_2}}{f_{Lo_1}}\right)^2$$

In order to avoid image frequency

$$f_{Lo_1} = f_{o_1} + IF = 88 + 10 = 98 \text{ MHz}$$

$$f_{Lo_2} = f_{o_2} + IF = 108 + 10 = 118 \text{ MHz}$$

$\therefore$ $$\frac{C_{max}}{C_{min}} = \left(\frac{118}{98}\right)^2 = 1.449 : 1$$

20. (b)

$$f_{Lo_1} = f_{c_1} + f_{IF} = 5 + 0.5 = 5.5 \text{ MHz}$$

$$f_{Lo_2} = f_{c_2} + f_{IF} = 10 + 0.5 = 10.5 \text{ MHz}$$

21. (c)

$$y(t) = 4x(t) + 10x^2(t)$$

$$\therefore \quad y(t) = 4[m(t)+\cos(\omega_c t)]+10[m(t)+\cos(\omega_c t)]^2$$

$$= 4m(t)+4\cos(\omega_c t)+10m^2(t)+\frac{10}{2}+\frac{10}{2}\cos(2\omega_c t)+20m(t)\cos(\omega_c t)$$

$$\therefore \quad y(t) = 4\cos(\omega_c t)+20m(t)\cos(\omega_c t)=4[1+5m(t)]\cos(\omega_c t)$$

Now, $\max\{|m(t)|\} = A_m$

$$\therefore \quad \mu = \max\{5|m(t)|\}$$

$$\mu = 5A_m$$

$$0.8 = 5A_m$$

$$A_c = 0.16$$

22. (c)

$$\frac{dm(t)}{dt} = \begin{cases} 1 & ; \quad 0 \le t \le t_1 \\ 0 & ; \quad t_1 \le t \le t_2 \\ 1 & ; \quad t_2 \le t \le t_3 \\ 0 & ; \quad t_3 \le t \le \infty \end{cases}$$

$\therefore$ The wave is PM wave.

23. (c)

According to MAP criterion

$$\frac{f_X(r|s_1)}{f_X(r|s_o)} \underset{H_1}{\overset{H_0}{\lessgtr}} \frac{P_o}{P_1}$$

Now, $$f_X(r \mid s_1) = \frac{1}{\sqrt{\pi N_o}}\exp -\frac{(x-\mu)^2}{N_o}$$

and $$f_X(r \mid s_1) = \frac{1}{\sqrt{\pi N_o}}\exp -\frac{(x-1)^2}{N_o}$$

$$f_X(r \mid s_0) = \frac{1}{\sqrt{\pi N_o}}\exp -\frac{(x+1)^2}{N_o}$$

$\therefore$ ACC to MAP criterion

$$\frac{\exp -\frac{(t_o-1)^2}{N_o}}{\exp -\frac{(t_o+1)}{2}} = \frac{P_0}{P_1} = 2$$

$$4t_o = 2\ln 2$$

$$t_o = \frac{1}{2}\ln 2 = 0.346$$

24. (c)

$$P_e = 1 - P_c$$

Now, since we are using ML criterion, and the input symbols are equally likely, then we can directly choose the output based on the maximum value of the transmission probabilities.

$$P_c = P(y_1)\,P(y_1 \mid x_1) + P(x_3)\cdot P(y_2 \mid x_3) + P(x_1)\cdot P(y_3 \mid x_1)$$

$$= \frac{1}{3}[0.5+0.5+0.4] = \frac{7}{15}$$

$$\therefore \quad P_e = 1-\frac{7}{15} = \frac{8}{15} = 0.533$$

25. (b)

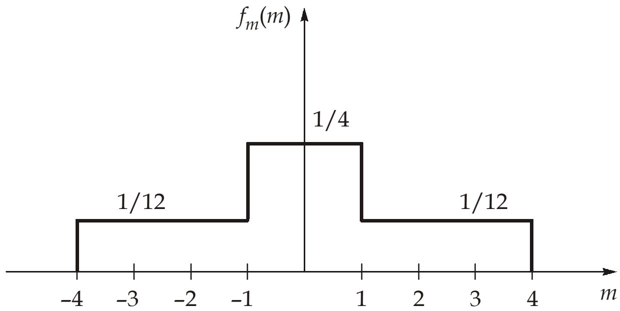
Probability of error in BPSK system with a phase mismatch of ϕ_e is,

$$P_e = Q\left[\sqrt{\frac{2E_b\cos^2\phi_e}{N_0}}\right] = Q\left[\sqrt{\frac{2E_b(\cos 30°)^2}{N_0}}\right]$$

$$P_e = Q\left[\sqrt{\frac{3E_b}{2N_0}}\right] = Q\left[\sqrt{\frac{(1.5)E_b}{N_0}}\right]$$

26. (b)

To create an optimum quantize of 3-bits all the message symbols should be equiprobable thus, the area under each quantized value must be same



Thus, for a 3-bit quantizer we need 8 levels. From the graph we can observe that the quantization level can be chosen as $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$ and $\pm\frac{7}{2}$.

Thus, signal power, $$\sigma_m^2 = \int_{-4}^{4} m^2 f_m(m)\,dm$$

$$= 2\left[\left[\frac{1}{4}\frac{m^3}{3}\right]_0^1 + \left[\frac{1}{12}\cdot\frac{m^3}{3}\right]_1^4\right] = \frac{11}{3} \text{ Watts}$$

Quantized noise power,

$$\sigma_q^2 = 2\left[\int_0^1\left(m-\frac{1}{2}\right)^2\times\frac{1}{4}dm + \int_1^2\left(m-\frac{3}{2}\right)^2\times\frac{1}{12}dm + \int_2^3\left(m-\frac{5}{2}\right)^2\times\frac{1}{12}dm + \int_3^4\left(m-\frac{7}{2}\right)^2\times\frac{1}{12}dm\right]$$

$$= 2\left[\frac{1}{4}\int_{-1/2}^{1/2}\lambda^2 d\lambda + \frac{1}{12}\int_{-1/2}^{1/2}\lambda^2 d\lambda + \frac{1}{12}\int_{-1/2}^{1/2}\lambda^2 d\lambda + \frac{1}{2}\int_{-1/2}^{1/2}\lambda^2 d\lambda + \frac{1}{12}\int_{-1/2}^{1/2}\lambda^2 d\lambda\right]$$

$$= \int_{-1/2}^{1/2} \lambda^2 d\lambda = 2\int_{0}^{1/2} \lambda^2 d\lambda = \frac{1}{12}$$

$$(\text{SNR})_9 = \frac{\sigma_m^2}{\sigma_n^2} = \frac{11/3}{1/12} = 44$$

$$\therefore \quad (\text{SNR})_{9\,(\text{dB})} = 10\log_{10}(44) = 16.43 \text{ dB}$$

27. (b)

The angle of the modulated signal $s(t)$ can be given as,

$$\theta(t) = 2\pi f_c t + 4\sin(4000\pi t) + 3\cos(4000\pi t)$$

The instantaneous frequency of the modulated signal can be given as,

$$f_i = \frac{1}{2\pi}\frac{d[\theta(t)]}{dt}$$

$$f_i = f_c + \frac{1}{2\pi}[4\times 4000\pi\cos 4000\pi t + 3\times 4000\pi[-\sin 4000\pi t]]$$

$$= f_c + [8000\cos(4000\pi t) - 6000\sin(4000\pi t)]$$

$$= f_c + 2000 \times 5[\cos(4000\pi t + \alpha)] \text{ where } \alpha = \tan^{-1}\left(\frac{3}{4}\right)$$

$$f_{i(\max)} = f_c + 2000 \times 5$$

$$= 100 \text{ kHz} + 10 \text{ kHz}$$

$$f_{i(\max)} = 110 \text{ kHz}$$

28. (a)

The phase deviation of the modulated signal is

$$\phi(t) = 4\sin(1500\pi t) + 3\cos(1500\pi t) \text{ rad}$$

$$= \sqrt{4^2 + 3^2}\cos(1500\pi t - \alpha)$$

$$= 5\cos(1500\pi t - \alpha) \text{ rad where } \alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

$\therefore$ Maximum phase deviation of the signal $s(t)$ is,

$$\Delta\phi_{\max} = |\phi(t)|_{\max} = 5 \text{ radians}$$

29. (d)

$$P(X \le 0, Y \le 1) = \int_{x=-\infty}^{0}\int_{y=-\infty}^{1} f(x,y)\cdot dy\,dx$$

$$P(X \le 0, Y \le 1) = \frac{1}{2}\int_{-\infty}^{0} e^x \cdot dx \cdot \int_{-\infty}^{1} e^{-|y|}dy$$

$$P(X \le 0, Y \le 1) = \frac{1}{2}\left[e^x\right]_{-\infty}^{0}\left[\int_{-\infty}^{0} e^y\,dy + \int_{0}^{1} e^{-y}\,dy\right]$$

$$P(X \le 0, Y \le 1) = \frac{1}{2}\left[(e^y)_{-\infty}^{0} - (e^{-y})_0^1\right]$$

$$P(X \le 0, Y \le 1) = \frac{1}{2}\left[(1-0)-(e^{-1}-1)\right]$$

$$P(X \le 0, Y \le 1) = \frac{2-e^{-1}}{2}$$

30. (a)

$$S = E[x^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$S = \int_{-2}^{0} x^2\left(\frac{x}{4}+\frac{1}{2}\right)dx + \int_{0}^{2} x^2\left(-\frac{x}{4}+\frac{1}{2}\right)dx$$

$$S = \left[\frac{x^4}{16}+\frac{x^3}{6}\right]_{-2}^{0} + \left[-\frac{x^4}{16}+\frac{x^3}{6}\right]_{0}^{2} = -\frac{16}{16}+\frac{8}{6}-\frac{16}{16}+\frac{8}{6}=\frac{2}{3}$$

For a uniform quantizer, $N_q = \frac{\Delta^2}{12} = \left(\frac{2\times 2}{2^5}\right)^2 \times \frac{1}{2} = \frac{1}{768}$

Hence,

$$(\text{SQNR})_{\text{dB}} = 10\log_{10}\left(\frac{S}{N_q}\right)$$

$$(\text{SQNR})_{\text{dB}} = 10\log_{10}\left(\frac{2\times 768}{3}\right) = 27.1\text{ dB}$$

■■■■