## CLASS TEST

## STRUCTURAL ANALYSIS

## CIVIL ENGINEERING

Date of Test : 16/05/2024

## ANSWER KEY

1. (a)
2. (b)
3. (a)
4. (b)
5. (c)
6. (c)
7. (a)
8. (a)
9. (c)
10. (a)
11. (c)
12. (a)
13. (d)
14. (b)
15. (c)
16. (b)
17. (b)
18. (a)
19. (d)
20. (b)
21. (b)
22. (c)
23. (d)
24. (d)
25. (b)
26. (c)
27. (d)
28. (a)
29. (d)
30. (d)

## DETAILED EXPLANATIONS

1. (a)

$$
\text { Number of members }=19
$$

Number of external reactions $=4$

$$
\text { Number of joints }=11
$$

$$
\therefore \quad\left(D_{s}\right)_{\text {Total }}=19+4-2 \times 11=1
$$

2. (c)

Taking moments about crown i.e. $C$ (from left),

$$
\begin{array}{lrl} 
& H_{A} \times 2 R & =V_{A} \times 2 R \\
\Rightarrow & H_{A} & =V_{A} \\
\text { Similarly, for BC, } & H_{B} & =V_{B} \\
\text { Now, as } & H_{A} & =H_{B} \text { and thus, } \\
\therefore & V_{A} & =V_{B}=\frac{W}{2}
\end{array}
$$

Now, inclination of $R_{A}, \tan \theta=\frac{V_{A}}{H_{B}}=1 \Rightarrow \theta=45^{\circ}$.
3. (c)

$$
\begin{aligned}
\text { Carry over factor } & =\frac{1}{2} \\
\qquad M_{A B} & =\frac{1}{2} \times 40=20 \mathrm{kNm}
\end{aligned}
$$

Now, moment about $A=0$;

$$
\begin{aligned}
V_{B} \times 5+40+20 & =0 \\
V_{B} & =-12 \mathrm{kN}=12 \mathrm{kN}(\downarrow)
\end{aligned}
$$

4. (b)

For stability, the moments on both the sides of support must balance each other.

$$
\begin{aligned}
\frac{w x^{2}}{2} & =\frac{3 w y^{2}}{2} \\
x & =\sqrt{3} y
\end{aligned}
$$

5. (b)

6. (c)
7. (b)

Internal static indeterminacy $=0$
External static indeterminacy $=9-3=6$

$$
\begin{aligned}
\therefore \quad D_{s} & =D_{s e}+D_{s i} \\
& =6+0 \\
& =6
\end{aligned}
$$

8. (a)


$$
H=\frac{4}{3} \frac{w R}{\pi} \quad \text { (for full loading) }
$$

$\therefore \quad$ For half-loading

$$
H=\frac{1}{2} \times \frac{4}{3} \frac{w R}{\pi}=\frac{2}{3} \frac{w R}{\pi}
$$

9. (a)

ILD for SF at $C$ is shown below


ILD for S.F. at C
Maximum positive shear force at $C$ will occur when udl cover the entire span $B C$.

$$
\begin{aligned}
\therefore \quad \text { Maximum S.F. } & =\frac{1}{2} \times 0.6 \times 6 \times 15 \\
& =27 \mathrm{kN}
\end{aligned}
$$

10. (b)

Due to sinking of support $A$,


Due to load $P$


Hence, reaction at $C$ decreases.
11. (c)
12. (d)

Elements in flexibility matrix can be positive or negative but the elements of leading diagonal must be positive since the displacement at any co-ordinate due to a unit force at that co-ordinate is always in the direction of unit force.
13. (a)

Taking hogging moments as negative and sagging moments as positive.
$\therefore$ Fixed end moment at A;

$$
\begin{aligned}
& M_{A}=\frac{-100 \times 2 \times 6^{2}}{8^{2}}+\frac{50 \times 8}{8}=-62.5 \mathrm{kNm} \\
& M_{B}=\frac{100 \times 6 \times 2^{2}}{8^{2}}-\frac{50 \times 8}{8}=-12.5 \mathrm{kNm}
\end{aligned}
$$



Taking moments about point $B$,
$-12.5-62.5-100 \times 6+50 \times 4+V_{A} \times 8=0$

$$
V_{A}=59.375 \mathrm{kN}
$$

14. (a)

Let $R_{A}$ and $R_{B}$ be the support reaction,

$$
\begin{aligned}
R_{A}+R_{B} & =5 \mathrm{~W} \\
\Sigma M_{A} & =0 \\
R_{B} \times 6 a & =W a+2 W \times 2 a+\mathrm{W} \times 3 a+\mathrm{W} \times 5 a \\
R_{B} & =\frac{13}{6} W(\uparrow) \\
R_{A} & =\frac{17}{6} W(\uparrow)
\end{aligned}
$$



Pass section line $I-I$ through member $M_{1} L_{2}$ and $M_{1}-U_{3}$ dividing the truss into sub assembles.
Consider equilibrium of joint $M_{1}$

$$
\Sigma F_{x}=F_{M_{1}-U_{3}} \times \cos 45^{\circ}+F_{M_{1}-L_{2}} \cos 45^{\circ}=0
$$

$$
\begin{aligned}
F_{M_{1}-U_{3}} & =-F_{M_{1}-L_{2}} \\
\Sigma F_{y} & =\frac{17 W}{6}-W+F_{M_{1}-U_{3}} \sin 45^{\circ}-F_{M_{1}-L_{2}} \sin 45^{\circ}=0 \\
\Rightarrow \quad \frac{11 W}{6}+\frac{2}{\sqrt{2}} \times F_{M_{1}-U_{3}} & =0 \quad\left[\because F_{M_{1}-U_{3}}-F_{M_{1}-L_{2}}\right] \\
F_{M_{1}-U_{3}} & =\frac{11 \sqrt{2}}{12} W \text { (compression) }
\end{aligned}
$$

15. (d)

Moment distribution method is also known as stiffness method because the structure is analysed using relative stiffness of members at a joint.
16. (a)

Using Maxwell theorem,

$$
\begin{aligned}
300 \times \Delta_{A} & =100 \times \Delta_{B}+200 \times \Delta_{C} \\
\Delta_{A} & =\frac{100 \times 10}{300}+\frac{200 \times 15}{300}=\frac{40}{3} \mathrm{~mm}
\end{aligned}
$$

17. (d)


ILD for B.M. at any section $C$
Let ' $x$ ' be the length of UDL on 8 m portion of beam,
For Max. BM at C,

$$
\begin{aligned}
\frac{x}{8} & =\frac{6-x}{24-8} \\
x & =2 \mathrm{~m} \\
M_{\max } & =\frac{1}{2} \times\left(4+\frac{16}{3}\right) \times 2 \times 5+\frac{1}{2} \times\left(\frac{16}{3}+4\right) \times 4 \times 5 \\
& =140 \mathrm{kNm}
\end{aligned}
$$

18. (a)

$$
\begin{aligned}
& \text { Vertical reaction; } V=\frac{w l}{2}=\frac{15 \times 150}{2}=1125 \mathrm{kN} \\
& \text { Horizontal reaction; } H=\frac{w l^{2}}{8 h}=\frac{15 \times 150^{2}}{8 \times 10}=4218.75 \mathrm{kN} \\
& \therefore \quad \text { Maximum tension }=T_{\max }=\sqrt{V^{2}+H^{2}}=\sqrt{1125^{2}+4218.75^{2}}=4366.17 \mathrm{kN}
\end{aligned}
$$

CE

$$
\begin{array}{rlrl} 
& & \text { Minimum tension } & =T_{\min }=\mathrm{H}=4218.75 \mathrm{kN} \\
\therefore \quad & T_{\max }-T_{\min } & =147.42 \mathrm{kN}
\end{array}
$$

19. (b)

No rigid body motion is possible in figure (1) but in figure (2), rigid body motion is possible as shown below.

20. (c)

$$
\begin{aligned}
& \text { Let the reaction at the roller }(R) \text { be redundant } \\
& \qquad \begin{aligned}
\left(\Delta_{B}\right)_{A B} & =\left(\Delta_{B}\right)_{B C} \\
\Rightarrow \quad \frac{5 \times(4)^{4}}{8 E I}-\frac{R(4)^{3}}{3 E I} & =\frac{5(2)^{4}}{8 E I}+\frac{R(2)^{3}}{3 E I} \\
\Rightarrow \quad & =6.25 \mathrm{kN} \\
\therefore \quad \text { Moment at } A, M_{A} & =R \times 4-5 \times 4 \times 2=6.25 \times 4-5 \times 4 \times 2 \\
& =25-40 \\
& =-15 \mathrm{kN}-\mathrm{m}
\end{aligned}
\end{aligned}
$$

Therefore, the magnitude of moment at $\mathrm{A}=15 \mathrm{kN}-\mathrm{m}$.
21. (b)

$$
\begin{aligned}
& H=\frac{120}{\pi} \times \sin ^{2} 30^{\circ}+\frac{100}{\pi}+\frac{180}{\pi} \times \sin ^{2} 60^{\circ} \\
& H=84.35 \mathrm{kN}
\end{aligned}
$$

22. (d)

$P$-forces


K-system of forces
(All other remaining members will have zero force)
Deflection at $A, \quad \Delta_{A}=\Sigma \frac{P K L}{A E}=\frac{\frac{P}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{L}{\sqrt{2}}}{A E} \times 2+\frac{\frac{P}{2} \times \frac{1}{2} \times L}{A E}$

$$
\begin{aligned}
& =\frac{P L}{A E}\left(\frac{1}{2 \sqrt{2}} \times 2+\frac{1}{4}\right)=0.96 \frac{P L}{A E}=k \frac{P L}{A E} \\
\Rightarrow \quad k & =0.96
\end{aligned}
$$

23. (d)

$$
U=\int_{0}^{L} \frac{M_{x}^{2} d x}{2 E I}=\int_{0}^{\frac{L}{2}} \frac{(P x)^{2} d x}{2 E I}=\frac{P^{2}}{2 E I}\left(\frac{x^{3}}{3}\right)_{0}^{L / 2}=\frac{P^{2} L^{3}}{48 E I}
$$

24. (d)

Apply a unit load at joint $E$


$$
\begin{equation*}
\delta_{E}=\Sigma K(L \alpha \Delta T) \tag{1}
\end{equation*}
$$

At joint A;

$$
\Sigma F_{y}=0
$$

$\Rightarrow \quad 0.5-K_{A B} \sin 60^{\circ}=0$
$\therefore \quad K_{A B}=\frac{1}{\sqrt{3}} \quad$ (compressive)
Also;

$$
K_{A B}=K_{C D} \quad \text { (Due to symmetry) }
$$

From (1)


$$
\therefore \quad \delta_{E}=\left(\left(\frac{1}{\sqrt{3}} \times 2 \times 12 \times 10^{-6} \times 20\right) \times 2 \mathrm{~m}\right)=0.55 \mathrm{~mm}
$$

25. (c)

By method of sections,

$\Sigma M_{G}=0 ; \Rightarrow F_{A B} \cos 45^{\circ} \times 2+F_{A B} \sin 45^{\circ} \times 2+40 \times 2+80 \times 6=0$
$\Rightarrow \quad \sqrt{2} F_{A B}+F_{A B} \sqrt{2}+80+480=0$
$\Rightarrow \quad F_{A B}=-197.989 \mathrm{kN} \simeq 197.99 \mathrm{kN} \simeq 198 \mathrm{kN}$ (compressive)
26. (a)

ILD for bending moment at $D$ is shown below

CE

$\therefore \quad$ Magnitude of maximum ordinate $=\frac{4 \times 4}{8}=2$
27. (c)

Stiffness of beam is given by,

$$
\begin{aligned}
K_{b} & =\frac{3 E I}{L^{3}} \\
& =\frac{3 \times\left(24000 \times 10^{6}\right) \times\left(1.2 \times 10^{-4}\right)}{(3.0)^{3}} \\
& =320 \times 10^{3} \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

Since both the springs will have the same displacement if the mass is displaced, and thus the springs are in parallel. Therefore,

$$
\begin{aligned}
K_{e} & =K_{b}+2 \mathrm{~K} \\
& =320 \times 10^{3}+2\left(40 \times 10^{3}\right)=400 \times 10^{3} \mathrm{~N} / \mathrm{m} \\
\text { Also, } \quad m & =10 \mathrm{kN} \\
\Rightarrow \quad m & =\frac{10000}{10}=1000 \mathrm{~kg}
\end{aligned}
$$

So,
Natural frequency,

$$
\begin{aligned}
\omega_{n} & =\sqrt{\frac{k_{e}}{m}}=\sqrt{\frac{400 \times 10^{3}}{1000}} \\
& =20 \mathrm{rad} / \mathrm{sec}
\end{aligned}
$$

28. (b)


$$
\begin{aligned}
j & =(S+1)(B+1)=\text { Number of joints } \\
m & =S(B+1)+B S=\text { Number of members } \\
r_{e} & =3(B+1) \\
D_{k} & =3 j-r_{e}-m \\
\therefore \quad D_{k} & =3(S+1)(B+1)-3(B+1)-S(B+1)-B S \\
& =S(B+2)
\end{aligned}
$$

29. (b)


$$
\begin{aligned}
\frac{12 E I \delta}{l^{3}}+\frac{12 E I \delta}{l^{3}} & =100 \\
\Rightarrow \quad \frac{24 E I \delta}{l^{3}} & =100 \\
M_{A} & =\frac{6 E I \delta}{l^{2}} \\
& =\frac{100 \times l}{4}=\frac{100 \times 5}{4}=125 \mathrm{kNm}
\end{aligned}
$$

30. (d)

$$
L=l+\frac{2}{3}\left[\frac{h^{2}}{l_{1}}+\frac{y_{c}^{2}}{l_{2}}\right]=50+\frac{2}{3}\left[\frac{9^{2}}{30}+\frac{4^{2}}{20}\right]=52.33 \mathrm{~m}
$$

