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STRUCTURAL ANALYSIS

CIVIL ENGINEERING

Date of Test: 16/05/2024

ANSWER KEY >

1.	(a)	7.	(b)	13.	(a)	19.	(b)	25.	(c)
2.	(c)	8.	(a)	14.	(a)	20.	(c)	26.	(a)
3.	(c)	9.	(a)	15.	(d)	21.	(b)	27.	(c)
4.	(b)	10.	(b)	16.	(a)	22.	(d)	28.	(b)
5.	(b)	11.	(c)	17.	(d)	23.	(d)	29.	(b)
6.	(c)	12.	(d)	18.	(a)	24.	(d)	30.	(d)

DETAILED EXPLANATIONS

1. (a)

Number of members = 19

Number of external reactions = 4

Number of joints = 11

$$D_s$$
_{Total} = 19 + 4 - 2 × 11 = 1

2. (c)

Taking moments about crown i.e. C (from left),

$$H_A \times 2R = V_A \times 2R$$

 \Rightarrow $H_A = V_A$
Similarly, for BC, $H_B = V_B$
Now, as $H_A = H_B$ and thus,

$$\therefore V_A = V_B = \frac{W}{2}$$

Now, inclination of R_A , $\tan \theta = \frac{V_A}{H_B} = 1 \implies \theta = 45^\circ$.

3. (c)

Carry over factor =
$$\frac{1}{2}$$

$$M_{AB} = \frac{1}{2} \times 40 = 20 \text{ kNm}$$

Now, moment about A = 0;

$$V_B \times 5 + 40 + 20 = 0$$

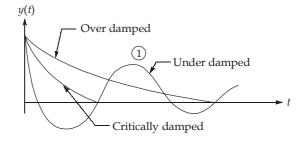
 $V_B = -12 \text{ kN} = 12 \text{ kN } (\downarrow)$

4. (b)

For stability, the moments on both the sides of support must balance each other.

$$\frac{wx^2}{2} = \frac{3wy^2}{2}$$
$$x = \sqrt{3}y$$

5. (b)



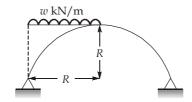
6. (c)

Internal static indeterminacy = 0

External static indeterminacy = 9 - 3 = 6

$$D_s = D_{se} + D_{si}$$
$$= 6 + 0$$
$$= 6$$

8. (a)



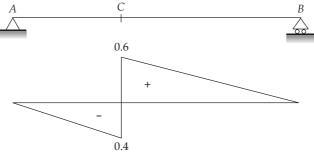
$$H = \frac{4}{3} \frac{wR}{\pi}$$
 (for full loading)

:. For half-loading

$$H = \frac{1}{2} \times \frac{4}{3} \frac{wR}{\pi} = \frac{2}{3} \frac{wR}{\pi}$$

9. (a)

ILD for SF at C is shown below



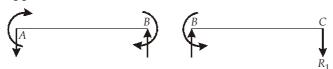
ILD for S.F. at C

Maximum positive shear force at C will occur when udl cover the entire span BC.

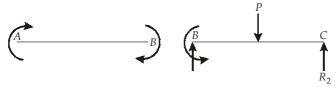
$$\therefore \qquad \text{Maximum S.F.} = \frac{1}{2} \times 0.6 \times 6 \times 15$$
$$= 27 \text{ kN}$$

10. (b)

Due to sinking of support *A*,



Due to load P



$$\therefore \qquad \qquad R_C = R_1(\downarrow) + R_2(\uparrow) = R_2 - R_1 < R_2$$

Hence, reaction at *C* decreases.

11. (c)

12. (d)

Elements in flexibility matrix can be positive or negative but the elements of leading diagonal must be positive since the displacement at any co-ordinate due to a unit force at that co-ordinate is always in the direction of unit force.

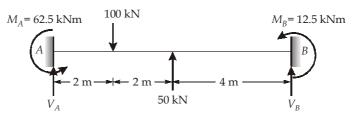
13. (a)

Taking hogging moments as negative and sagging moments as positive.

∴Fixed end moment at A;

$$M_A = \frac{-100 \times 2 \times 6^2}{8^2} + \frac{50 \times 8}{8} = -62.5 \text{ kNm}$$

 $M_B = \frac{100 \times 6 \times 2^2}{8^2} - \frac{50 \times 8}{8} = -12.5 \text{ kNm}$



Taking moments about point B,

$$-12.5 - 62.5 - 100 \times 6 + 50 \times 4 + V_A \times 8 = 0$$

 $V_A = 59.375 \text{ kN}$

14. (a)

Let R_A and R_B be the support reaction,

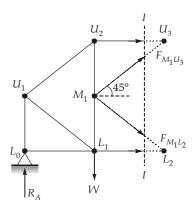
$$R_A + R_B = 5 W$$

$$\Sigma M_A = 0$$

$$R_B \times 6a = Wa + 2W \times 2a + W \times 3a + W \times 5a$$

$$R_B = \frac{13}{6}W(\uparrow)$$

$$R_A = \frac{17}{6}W(\uparrow)$$



Pass section line I – I through member M_1L_2 and M_1 – U_3 dividing the truss into sub assembles. Consider equilibrium of joint M_1

$$\Sigma F_x = F_{M_1 - U_3} \times \cos 45^\circ + F_{M_1 - L_2} \cos 45^\circ = 0$$

$$F_{M_1 - U_3} \ = \ -F_{M_1 - L_2}$$

$$\Sigma F_y = \frac{17W}{6} - W + F_{M_1 - U_3} \sin 45^\circ - F_{M_1 - L_2} \sin 45^\circ = 0$$

$$\Rightarrow \frac{11W}{6} + \frac{2}{\sqrt{2}} \times F_{M_1 - U_3} = 0$$

$$[\because F_{M_1-U_3}-F_{M_1-L_2}]$$

$$F_{M_1-U_3} = \frac{11\sqrt{2}}{12}W$$
 (compression)

15. (d)

> Moment distribution method is also known as stiffness method because the structure is analysed using relative stiffness of members at a joint.

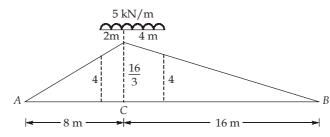
16. (a)

Using Maxwell theorem,

$$300 \times \Delta_A = 100 \times \Delta_B + 200 \times \Delta_C$$

$$\Delta_A = \frac{100 \times 10}{300} + \frac{200 \times 15}{300} = \frac{40}{3}$$
 mm

17. (d)



ILD for B.M. at any section C

Let 'x' be the length of UDL on 8 m portion of beam,

For Max. BM at C,
$$\frac{x}{8} = \frac{6-x}{24-8}$$

$$x = 2 \text{ m}$$

$$M_{\text{max}} = \frac{1}{2} \times \left(4 + \frac{16}{3}\right) \times 2 \times 5 + \frac{1}{2} \times \left(\frac{16}{3} + 4\right) \times 4 \times 5$$
$$= 140 \text{ kNm}$$

18. (a)

Vertical reaction;
$$V = \frac{wl}{2} = \frac{15 \times 150}{2} = 1125 \text{ kN}$$

Horizontal reaction;
$$H = \frac{wl^2}{8h} = \frac{15 \times 150^2}{8 \times 10} = 4218.75 \text{ kN}$$

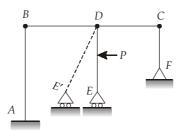
$$T_{\text{max}} = \sqrt{V^2 + H^2} = \sqrt{1125^2 + 4218.75^2} = 4366.17 \text{ kN}$$



Minimum tension =
$$T_{\text{min}}$$
 = H = 4218.75 kN
 T_{max} - T_{min} = 147.42 kN

19. (b)

No rigid body motion is possible in figure (1) but in figure (2), rigid body motion is possible as shown below.



20. (c)

Let the reaction at the roller (R) be redundant

$$\therefore \qquad (\Delta_B)_{AB} = (\Delta_B)_{BC}$$

$$\Rightarrow \frac{5 \times (4)^4}{8EI} - \frac{R(4)^3}{3EI} = \frac{5(2)^4}{8EI} + \frac{R(2)^3}{3EI}$$

$$\Rightarrow$$
 $R = 6.25 \text{ kN}$

.. Moment at
$$A$$
, $M_A = R \times 4 - 5 \times 4 \times 2 = 6.25 \times 4 - 5 \times 4 \times 2$
= $25 - 40$
= -15 kN-m

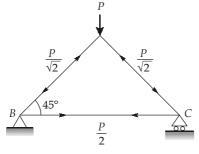
Therefore, the magnitude of moment at A = 15 kN-m.



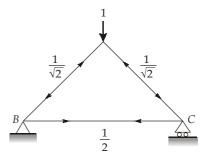
$$H = \frac{120}{\pi} \times \sin^2 30^\circ + \frac{100}{\pi} + \frac{180}{\pi} \times \sin^2 60^\circ$$

$$H = 84.35 \text{ kN}$$

22. (d)



P-forces



K-system of forces

(All other remaining members will have zero force)

Deflection at
$$A$$
,
$$\Delta_A = \Sigma \frac{PKL}{AE} = \frac{\frac{P}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{L}{\sqrt{2}}}{AE} \times 2 + \frac{\frac{P}{2} \times \frac{1}{2} \times L}{AE}$$

$$= \frac{PL}{AE} \left(\frac{1}{2\sqrt{2}} \times 2 + \frac{1}{4} \right) = 0.96 \frac{PL}{AE} = k \frac{PL}{AE}$$

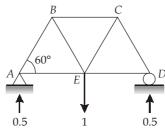
$$\Rightarrow$$
 $k = 0.96$

23. (d)

$$U = \int_{0}^{L} \frac{M_{x}^{2} dx}{2EI} = \int_{0}^{\frac{L}{2}} \frac{(Px)^{2} dx}{2EI} = \frac{P^{2}}{2EI} \left(\frac{x^{3}}{3}\right)_{0}^{L/2} = \frac{P^{2}L^{3}}{48EI}$$

24. (d)

Apply a unit load at joint E



$$\delta_E = \Sigma K(L\alpha \Delta T) \qquad ...(1)$$

At joint A;

$$\Sigma F_y = 0$$

$$\delta_E = \Sigma K(L\alpha \Delta T)$$
At joint A;
$$\Sigma F_y = 0$$

$$\Rightarrow 0.5 - K_{AB} \sin 60^\circ = 0$$

$$\therefore K_{AB} = \frac{1}{\sqrt{3}} \text{(compressive)}$$

Also;
$$K_{AB} = K_{CD}$$
 (Due to symmetry)

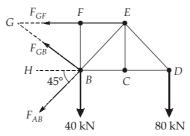
From (1)

$$A = \begin{cases} k_{AB} \\ 60^{\circ} \\ 0.5 \text{ kN} \end{cases}$$

$$\delta_E = \left(\left(\frac{1}{\sqrt{3}} \times 2 \times 12 \times 10^{-6} \times 20 \right) \times 2 \text{ m} \right) = 0.55 \text{ mm}$$

25. (c)

By method of sections,



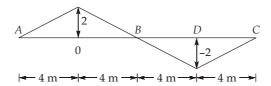
$$\Sigma M_G = 0$$
; $\Rightarrow F_{AB} \cos 45^\circ \times 2 + F_{AB} \sin 45^\circ \times 2 + 40 \times 2 + 80 \times 6 = 0$

$$\Rightarrow \quad \sqrt{2} \, F_{AB} + F_{AB} \sqrt{2} + 80 + 480 \, = 0$$

$$\Rightarrow$$
 F_{AB} = -197.989 kN \simeq 197.99 kN \simeq 198 kN (compressive)

26.

ILD for bending moment at *D* is shown below



$$\therefore \text{ Magnitude of maximum ordinate} = \frac{4 \times 4}{8} = 2$$

27. (c)

Stiffness of beam is given by,

$$K_b = \frac{3EI}{L^3}$$

$$= \frac{3 \times (24000 \times 10^6) \times (1.2 \times 10^{-4})}{(3.0)^3}$$

$$= 320 \times 10^3 \text{ N/m}$$

Since both the springs will have the same displacement if the mass is displaced, and thus the springs are in parallel. Therefore,

$$K_e = K_b + 2 K$$

$$= 320 \times 10^3 + 2 (40 \times 10^3) = 400 \times 10^3 \text{ N/m}$$
Also,
$$m = 10 \text{ kN}$$

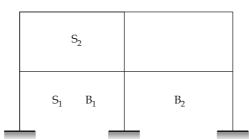
$$\Rightarrow \qquad m = \frac{10000}{10} = 1000 \text{ kg}$$

So,

Natural frequency,

$$\omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{400 \times 10^3}{1000}}$$
$$= 20 \text{ rad/sec}$$

28. (b)

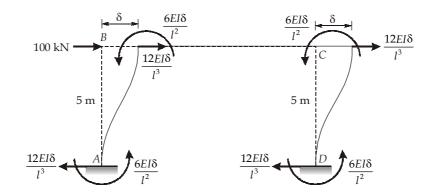


$$j = (S + 1) (B + 1) = \text{Number of joints}$$

 $m = S(B + 1) + BS = \text{Number of members}$
 $r_e = 3(B + 1)$
 $D_k = 3j - r_e - m$
 $D_k = 3(S + 1) (B + 1) - 3(B + 1) - S(B + 1) - BS$
 $= S(B + 2)$

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29. (b)



$$\frac{12EI\delta}{l^3} + \frac{12EI\delta}{l^3} = 100$$

$$\Rightarrow \frac{24EI\delta}{l^3} = 100$$

$$M_A = \frac{6EI\delta}{l^2}$$

$$= \frac{100 \times l}{4} = \frac{100 \times 5}{4} = 125 \text{ kNm}$$

30. (d)
$$L = l + \frac{2}{3} \left[\frac{h^2}{l_1} + \frac{y_c^2}{l_2} \right] = 50 + \frac{2}{3} \left[\frac{9^2}{30} + \frac{4^2}{20} \right] = 52.33 \text{ m}$$