CLASS TEST

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 03/06/2024

ANSWER KEY

1. (a)	7. (d)	13. (a)	19. (c)	25. (c)
2. (a)	8. (b)	14. (d)	20. (b)	26. (a)
3. (d)	9. (d)	15. (d)	21. (c)	27. (d)
4. (d)	10. (b)	16. (c)	22. (b)	28. (d)
5. (a)	11. (a)	17. (c)	23. (a)	29. (a)
6. (d)	12. (b)	18. (a)	24. (b)	30. (a)

DETAILED EXPLANATIONS

1. (a)

$$P(-1 \le x \le 1) = \int_{-1}^{1} (0.1)dx$$
$$= 2 \times \frac{1}{10} = \frac{1}{5}$$

2. (a)

$$\lim_{x \to 0} \frac{\log x}{\cot x}$$
$$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\text{cosec}^2 x} = -\lim_{x \to 0} \frac{\sin^2 x}{x} \qquad \left(\text{from } \frac{\infty}{\infty}\right)$$
$$= -\lim_{x \to 0} \frac{2\sin x \cos x}{1} = 0 \qquad \left(\text{from } \frac{0}{0}\right)$$

3. (d)

$$u = \sin x$$
$$du = \cos x \, dx$$
$$x = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = 1$$
$$x = -\pi \Rightarrow u = \sin(-\pi) = 0$$
$$\int_{-\pi}^{\pi/2} \cos(x)\cos(\sin(x))dx = \int_0^1 \cos u \, du$$
$$= |\sin u|_0^1$$
$$= (\sin 1) - \sin(0) = \sin 1$$

4. (d)

$$\frac{e^x}{(1-e^x)}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

Integrating on both sides, we get,

$$-\ln(1 - e^x) + \ln(\tan y) = C_1$$
$$\ln\left(\frac{\tan y}{(1-e^x)}\right) = C_1$$
$$\frac{\tan y}{(1-e^x)} = e^{C_1} = C$$
$$\tan y = C(1 - e^x)$$

5. (a)

$$(D^2 + D)y = x^2 + 2x + 8$$

The particular integral is,

$$PI = \frac{x^2 + 2x + 8}{D(1+D)}$$

$$= \frac{1}{D}(1+D)^{-1}(x^2 + 2x + 8) = \frac{1}{D}(1 - D + D^2 - D^3 + \ldots)(x^2 + 2x + 8)$$

$$= \frac{1}{D}(x^2 + 2x + 8 - 2x - 2 + 2) = \frac{1}{D}(x^2 + 8) = \frac{x^3}{3} + 8x$$

6. (d)

Let,

$$U = V + W$$

$$V = \frac{x^2y^2z^2}{x+y+z}$$

$$V(tx, ty, tz) = \frac{t^6x^2y^2z^2}{t(x+y+z)} = t^5V(x,y,z)$$

$\therefore V$ is homogeneous function of degree 5,

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z} = 5V \quad \ldots \text{(i)} \quad \text{[From Euler's theorem]}$$

$$W(tx, ty, tz) = \log\left(\frac{t^2x^2y + t^2yz + t^2zx}{t^2x^2 + t^2y^2 + t^2z^2}\right) = t^0W(x,y,z)$$

$$\Rightarrow \quad x\frac{\partial W}{\partial x} + y\frac{\partial W}{\partial y} + z\frac{\partial W}{\partial z} = 0 \quad \ldots \text{(ii)}$$

Equation (i) and equation (ii),

$$x\left(\frac{\partial V}{\partial x} + \frac{\partial W}{\partial x}\right) + y\left(\frac{\partial V}{\partial y} + \frac{\partial W}{\partial y}\right) + z\left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial z}\right) = 5V$$

$$x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} + z\frac{\partial U}{\partial z} = 5\frac{x^2y^2z^2}{x+y+z}$$

7. (d)

$$y = \sqrt{a^x + y}$$

$$y^2 = a^x + y$$

$$2y\frac{dy}{dx} = a^x \ln a + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{a^x \ln a}{(2y-1)}$$

8. (b)

Let $$y = \lim_{x\to0^+} x^x$$

Taking natural log, $$\ln y = \lim_{x\to0^+} \ln\left(x^x\right) = \lim_{x\to0^+} x\ln x = \lim_{x\to0^+} \frac{\ln x}{1/x}$$

Using L' Hospital's rule, $$\ln y = \lim_{x\to0^+} \frac{1/x}{-1/x^2} = \lim_{x\to0^+} -x = 0$$

$\Rightarrow$ $$y = e^0 = 1$$

9. (d)

$$f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2}{2!}f''(3) \text{ ....}$$

For quadratic approximation, neglecting rest terms,

$$f(x) = e^{-3} + (x-3)\left(-e^{-3}\right) + \frac{(x-3)^2}{2}e^{-3}$$

$$f(x) = e^{-3}\left[1 + (3-x) + \frac{(x-3)^2}{2}\right]$$

$$f(x) = e^{-3}\left[1 + 3 - x + \frac{x^2 + 9 - 6x}{2}\right]$$

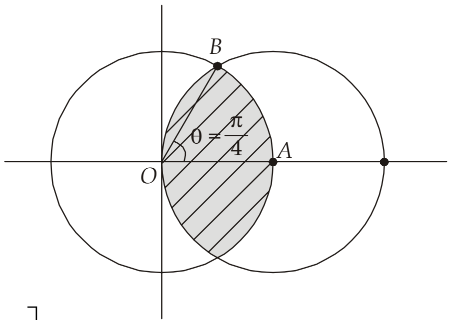
$$f(x) = \frac{1}{2}e^{-3}\left[8 - 2x + x^2 + 9 - 6x\right]$$

$$f(x) = \frac{1}{2}e^{-3}\left[x^2 - 8x + 17\right]$$

10. (b)

Given: $r = \sqrt{2}a$ and $r = 2a\cos\theta$

Point of intersection,



$$\sqrt{2}a = 2a\cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

$$\text{Area, } A = 2\left[\frac{1}{2}\int_0^{\pi/4} r^2 d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/2} r^2 d\theta\right]$$

$$A = \int_0^{\pi/4} \left(\sqrt{2}a\right)^2 d\theta + \int_{\pi/4}^{\pi/2} 4a^2\cos^2\theta d\theta$$

$$A = 2a^2 \times \frac{\pi}{4} + 2a^2\left(\frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2}\right)$$

$$A = a^2(\pi - 1)$$

11. (a)

$$f(x) = xe^{-x}$$
$$f(0) = 0$$
$$f'(x) = e^{-x} - xe^{-x}, \qquad f'(0) = 1$$
$$f''(x) = -e^{-x} - e^{-x} + xe^{-x}, \qquad f''(0) = -2$$
$$f'''(x) = 2e^{-x} + e^{-x} - xe^{-x}, \qquad f'''(0) = 3$$
$$\Rightarrow \quad xe^{-x} = x - 2\frac{x^2}{2!} + 3\frac{x^3}{3!}$$
$$\Rightarrow \quad xe^{-x} = x - x^2 + \frac{x^3}{2}$$

12. (b)

$$\int_0^4 f(t)\,dt = \int_0^1 \left(1 - 3t^2\right)dt + \int_1^4 2t\,dt = t - t^3\Big|_0^1 + t^2\Big|_1^4 = 15$$

13. (a)

$$\int_1^2 \frac{(x-1)^3}{x^2}dx = \int_1^2 \frac{x^3 - 1 + 3x - 3x^2}{x^2}dx = \int_1^2 \left(x - \frac{1}{x^2} + \frac{3}{x} - 3\right)dx$$
$$= \left[\frac{x^2}{2} + \frac{1}{x} + 3\ln x - 3x\right]_1^2$$
$$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - 1\right) + 3\ln 2 - 3(2-1) = 3\ln 2 - 1$$

14. (d)

$$y = \sqrt[3]{x} \Rightarrow x = y^3$$
$$y = \frac{x}{9} \Rightarrow x = 9y$$
$$\Rightarrow \quad 9y = y^3$$
$$y = \pm 3 \text{ (Point of intersection)}$$

For first quadrant, $y = 3$, $x = 27$

$$V = \int_0^3 A(y)\,dy = \int_0^3 \pi\left[(9y)^2 - \left(y^3\right)^2\right]dy$$
$$V = \pi\int_0^3 \left(81y^2 - y^6\right)dy = 416.57\pi \text{ unit}^3$$

15. (d)

It is an exact differential equation since

$$\frac{\partial M}{\partial y} = 12xy^2 = \frac{\partial N}{\partial x}$$
$$\Rightarrow \int M\,dx + \int (\text{Terms of N not containing } x)\,dy = C$$
$$\int \left(x^4 + 4xy^3\right)dx + \int y^4\,dy = C$$
$$\frac{1}{5}\left(x^5 + 10x^2y^3 + y^5\right) = C$$

16. (c)

$$\underset{z=1}{\text{Res}}\, f(z) = \lim_{z\to 1}(z-1)f(z) = \lim_{z\to 1}\frac{(z+6)(z-3)}{(z+2)} = \frac{7\times(-2)}{3} = -\frac{14}{3}$$

$$\underset{z=-2}{\text{Res}}\, f(z) = \lim_{z\to -2}(z+2)f(z) = \lim_{z\to -2}\frac{(z+6)(z-3)}{(z-1)} = \frac{4\times(-5)}{-3} = \frac{20}{3}$$

$$\text{Sum of residues} = \underset{z=1}{\text{Res}}\, f(z) + \underset{z=-2}{\text{Res}}\, f(z)$$

$$= -\frac{14}{3}+\frac{20}{3} = \frac{6}{3} = 2$$

17. (c)

$$e^{1/z} = 1+\frac{1}{z}+\frac{1}{2!z^2}+\frac{1}{3!z^3}+\ldots\ldots\ldots$$

So, $e^{1/z}$ has pole at $z = 0$ which lies inside 'C' so, using Cauchy's residue theorem,

$$\int_C e^{1/z}dz = 2\pi i\,[\Sigma\text{Res}(e^{1/z}\text{ at poles inside C})]$$

$$\underset{z\to 0}{\text{Res}}\, e^{1/z} = 1$$

$$\int_C e^{1/z}dx = 2\pi i\,[1] = 2\pi i$$

18. (a)

Required probability is

$$P = \frac{4}{52}\times\frac{2}{51}+\frac{4}{52}\times\frac{2}{51} = \frac{4}{663}$$

19. (c)

$$P(0) + P(1) + P(2) + P(3) = 1$$

$$k + 2k + 3k + 4k = 1$$

$$k = 0.1$$

$$P(x < 2) = P(0) + P(1) = k + 2k = 0.3$$

$$P(x \le 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 0.6$$

$$P(x < 2) + P(x \le 2) = 0.9$$

20. (b)

Let,

$$f(x) = x^2 - 15$$

$$f'(x) = 2x$$

First iteration:

$$f(x_0) = f(3.5) = 3.5^2 - 15 = -2.75$$

$$f'(x_0) = f'(3.5) = 7$$

$$x_1 = 3.5-\frac{2.75}{7} = 3.8929$$

Second iteration:

$$f(x_1) = 0.1543$$

$$f'(x_1) = 7.7857$$

$$x_2 = 3.8929-\frac{0.1543}{7.7857} = 3.873$$

21. (c)

22. (b)

$$I = \frac{h}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2\right]$$

$$I = \frac{0.1}{3}[(1 + 0.8604) + 4(0.9975 + 0.9776) + 2 \times 0.9900]$$

$$I = 0.39136$$

23. (a)

$$L(\sin 2t) = \frac{2}{s^2+4}$$

$$L(t^2 \sin 2t) = \frac{d^2}{ds^2} \times \frac{2}{s^2+4} = \frac{12s^2 - 16}{(s^2+4)^3}$$

$$= \frac{-2d}{ds} \times \frac{1}{s^2+4} \times 2s = -4\frac{d}{ds} \times \frac{s}{(s^2+4)^2}$$

$$= -4\left[\frac{(s^2+4)^2 - 2(2s)(s^2+4)}{(s^2+4)^4}\right] = -4\left[\frac{s^2+4-3s^2}{(s^2+4)^3}\right]$$

$$= \frac{12s^2-16}{(s^2+4)^3}$$

24. (b)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x-1) = 0$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 - 1) = 0$$

Also $f(1) = 0$

Thus $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow$ f is continuous at $x = 1$

And Lf′(1) = 2, Rf′(1) = 1

$\Rightarrow f$ is not differentiable at $x = 1$

25. (c)

$$\int_0^5 f(x) \cdot dx = \frac{h}{2}\left[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)\right]$$

$$= \frac{1}{2}\left[(2 + 0.077) + 2(1 + 0.4 + 0.2 + 0.1176)\right]$$

$$\simeq 2.75$$

26. (a)

$[A]$ = diag [1×6, 2×1, 7×2] + diag [2, 3, 4]

$[A]$ = diag [6+2, 2+3, 14+4]

$$|A| = \begin{vmatrix} 8 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 18 \end{vmatrix} = 8 \times 5 \times 18 = 720$$

27. **(d)**

Three second order principal sub-matrix,

$$\begin{bmatrix}1 & 3\\5 & 11\end{bmatrix}, \begin{bmatrix}1 & 2\\7 & 6\end{bmatrix}, \begin{bmatrix}11 & 2\\3 & 6\end{bmatrix}$$

Three first order principal sub-matrix,
[1], [11], [6]

28. **(d)**

$$\text{Rank (AB)} \leq \min[\text{Rank (A), Rank (B)}]$$
$$\text{Rank (AB)} \leq 3$$

∵ We don't know the dimension of A and B, we cannot predict the exact rank of AB but its maximum rank will be 3.

29. **(a)**

$$[A] = \begin{bmatrix}0 & 2 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 2\\2 & 0 & 0 & 0\end{bmatrix}$$

$$[A - \lambda I] = \begin{bmatrix}-\lambda & 2 & 0 & 0\\0 & -\lambda & 2 & 0\\0 & 0 & -\lambda & 2\\2 & 0 & 0 & -\lambda\end{bmatrix} = 0$$

$$[A - \lambda I] = -\lambda\begin{vmatrix}-\lambda & 2 & 0\\0 & -\lambda & 2\\0 & 0 & -\lambda\end{vmatrix} - 2\begin{vmatrix}0 & 2 & 0\\0 & -\lambda & 2\\2 & 0 & -\lambda\end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^2(\lambda^2) - 2(-2)\begin{vmatrix}0 & 2\\2 & -\lambda\end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^4 - 16 = 0$$

$$(\lambda^2 - 4)(\lambda^2 + 4) = 0$$

$$(\lambda - 2)(\lambda + 2)(\lambda - 2i)(\lambda + 2i) = 0$$

$$\text{Eigen values} = 2, -2, 2i, -2i$$

30. **(a)**

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5}$$

$$\tan^{-1}(0.6) = 0.6 - \frac{0.6^3}{3} + \frac{0.6^5}{5}$$

$$\tan^{-1}(0.6) = 0.5435$$

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