CLASS TEST

S.No.: 01JPME_10062024

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

MECHANICAL ENGINEERING

Engineering Mechanics

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

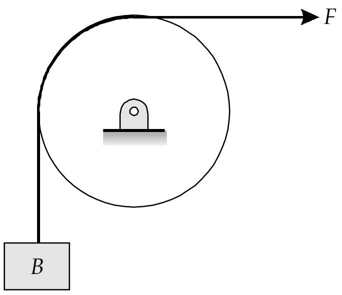
DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 A car starts from rest on a 700 meters long bridge. The coefficient of friction between the tyre and the road is 0.75. The minimum time taken to cross the bridge is

(a) 10.52 seconds (b) 11.94 seconds
(c) 15.08 seconds (d) 13.79 seconds

Q.2 The block B weighs 50 kg. The coefficient of static friction between the cable and the fixed pulley is 0.4. The minimum value of force F required to support the box is

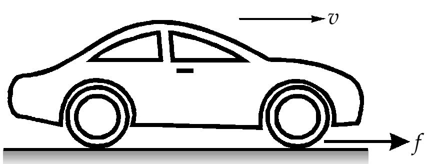


(a) 490.5 N (b) 261.7 N
(c) 113.7 N (d) 223.6 N

Q.3 A force $F = (10 + x)$ acts on a particle in the x-direction, where F is in Newton and x is in meter. The work done by this force during a displacement from $x = 0$ to $x = 3$ m is

(a) 3 J (b) 33 J
(c) 34.5 J (d) 17.25 J

Q.4 For a short period of time, the frictional driving force acting on the wheels of a 2500 kg car is $f = 600t^2$ N, where t is in seconds. If the car has a speed of 20 m/s at $t = 0$, then speed at t = 6s is

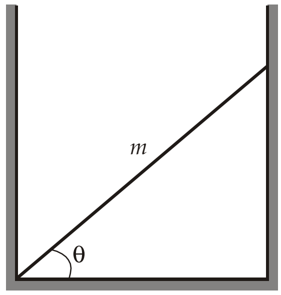


(a) 37.28 m/s (b) 2.72 m/s
(c) 17.28 m/s (d) 18.64 m/s

Q.5 Consider an object having a initial velocity of 8 m/s and is being accelerated at 2.4 m/s^2 for 10 seconds. The distance travelled by the object during the period of the acceleration is

(a) 100 m (b) 200 m
(c) 600 m (d) 400 m

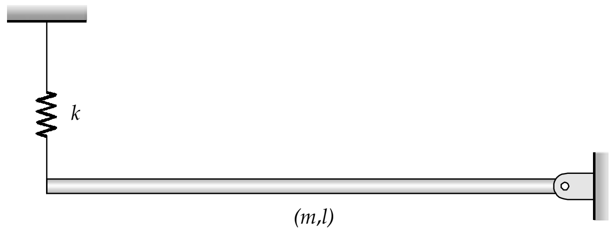
Q.6 A uniform stick of mass m is placed in a frictionless well as shown. The stick makes an angle θ with the horizontal. Then the force which the vertical wall exerts on the right end of stick is



(a) $\dfrac{mg}{2\cot\theta}$ (b) $\dfrac{mg}{2\tan\theta}$

(c) $\dfrac{mg}{2\cos\theta}$ (d) $\dfrac{mg}{2\sin\theta}$

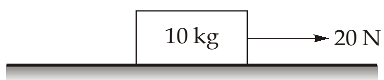
Q.7 For the system shown in figure, a rod of mass m and length l is hinged at one end and is horizontally supported by a spring of spring constant k on the other end. Neglecting the friction at hinge, the extension of the spring and force on the rod due to hinge respectively are



(a) $\dfrac{mg}{k}, \dfrac{mg}{2}$ (b) $\dfrac{mg}{2k}, \dfrac{mg}{4}$

(c) $\dfrac{mg}{k}, mg$ (d) $\dfrac{mg}{2k}, \dfrac{mg}{2}$

Q.8 A block of mass 10 kg is resting on a rough floor having static coefficient of friction 0.5. A force of 20 N is applied on the block. The friction force acting on the block due to the floor is
[Take g = 10 m/s^2]



(a) 50 N (b) 20 N
(c) 25 N (d) 100 N

Q.9 A constant force of 7.5 N accelerates a stationary particle of mass 30 grams through a displacement of 2.5 meters. The average power delivered during this displacement is approximately,

(a) 133 W (b) 166 W
(c) 266 W (d) 122 W

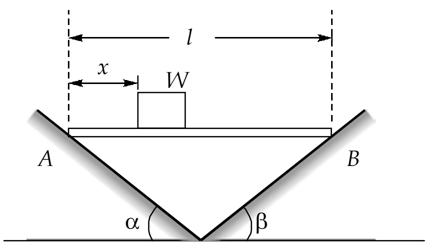
Q.10 **Statement (I):** Change in either the speed or the direction of motion of a moving particle results into acceleration.

Statement (II): For a particle moving in a circular motion in a plane with uniform speed, the acceleration is zero.

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is NOT the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

Q. No. 11 to Q. No. 30 carry 2 marks each

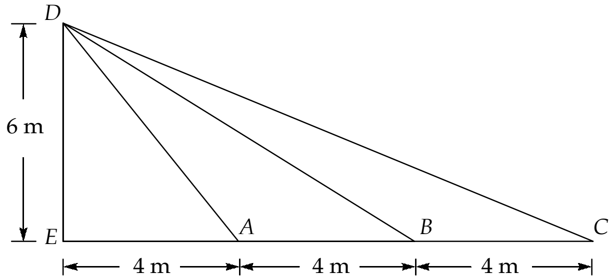
Q.11 A beam of length l and negligible weight is placed horizontally between two smooth inclined planes. A block with weight W rests upon the beam. At what distance 'x' the block must be placed in order to obtain equilibrium?



(a) $\dfrac{l}{1+\dfrac{\tan\beta}{\tan\alpha}}$ (b) $\dfrac{l}{1+\dfrac{\tan\alpha}{\tan\beta}}$

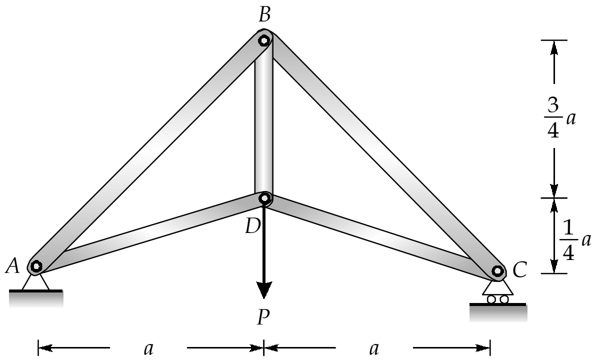
(c) $\dfrac{l}{1+\dfrac{\sin\beta}{\sin\alpha}}$ (d) $\dfrac{l}{1+\dfrac{\cos\beta}{\sin\alpha}}$

Q.12 The magnitudes of the forces exerted on the pillar at D by the cables A, B and C are equal. The magnitude of the total moment about E due to the forces exerted by the three cables at D is 2700 kN-m. The magnitude of force in cable A is



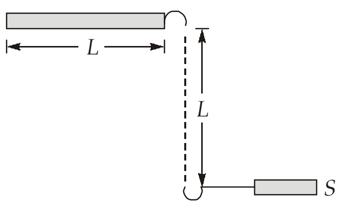
(a) 151.2 kN (b) 133.5 kN
(c) 190.1 kN (d) 200.9 kN

Q.13 Members AB and BC can each support a maximum compressive force of 800 Newtons, and members AD, DC and BD can support a maximum tensile force of 2000 N. If a = 6 m, the greatest load P that the truss can support is



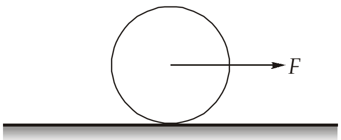
(a) 503.8 N (b) 848.5 N
(c) 647.9 N (d) 981.6 N

Q.14 The rod of mass m and length L = 1 m is released from rest without rotating. When it falls a distance L, the right end strikes the hook S, which provides a permanent connection. The angular velocity (ω) of the rod after it has rotated through 90° is (Treat the impact as non-impulsive).



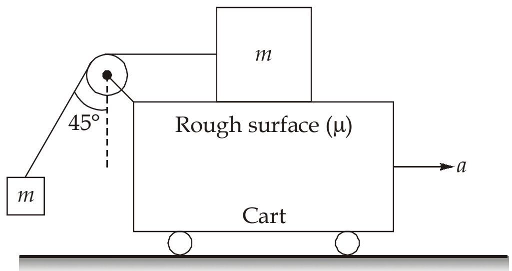
(a) 8.57 rad/s (b) 10.23 rad/s
(c) 18.12 rad/s (d) 15.21 rad/s

Q.15 A solid circular disc weighs 12 kg and radius 6 cm is initially stationary as shown in the figure. If a force F = 10 N is applied, then the linear acceleration of the disc is



(a) 0.28 m/s^2 (b) 1.80 m/s^2
(c) 0.55 m/s^2 (d) 0.60 m/s^2

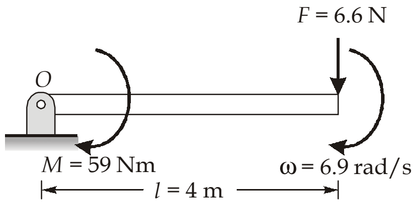
Q.16 The cart shown in figure is moving towards right with a constant acceleration 'a'. Two blocks of equal mass m are supported on the cart as shown. Given that when the block at top surface is just about to slide, other block is hanging at 45° from the vertical. For the system at the instant shown in the figure which one of the above statement is INCORRECT?



(a) The acceleration of cart is g m/s^2.
(b) The tension in the wire connecting blocks is $\sqrt{2}mg$.
(c) The coefficient of friction between the block and the rough surface is $(\sqrt{2}+1)$.
(d) The tension in the wire connecting blocks is $(\sqrt{2}+1)$mg.

Q.17 If acceleration of an object is given as a = –6s^{-3}, where 's' is the displacement. Motion starts with infinite displacement from rest, then what will be the velocity of the object at 6 m?
(a) 0.167 m/s (b) 2.3 m/s
(c) 0.408 m/s (d) 0.68 m/s

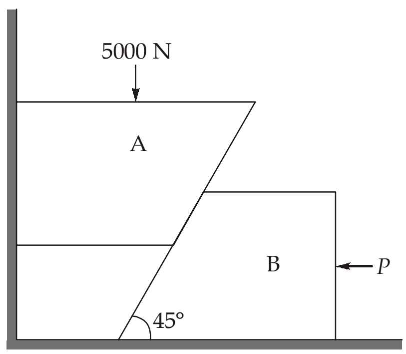
Q.18 At the instant shown in figure, 8.4 kg slender rod has an angular velocity of 6.9 rad/s and is subjected to force F = 6.6N and moment M = 59 N-m. If the rod length L = 4 m, what will be the angular velocity of the rod when it is rotated by 90° downwards?
(Assuming the force is always perpendicular to the axis of the rod. (g = 9.81 m/s^2))



(a) 7.807 rad/s (b) 7.493 rad/s
(c) 8.252 rad/s (d) 15.079 rad/s

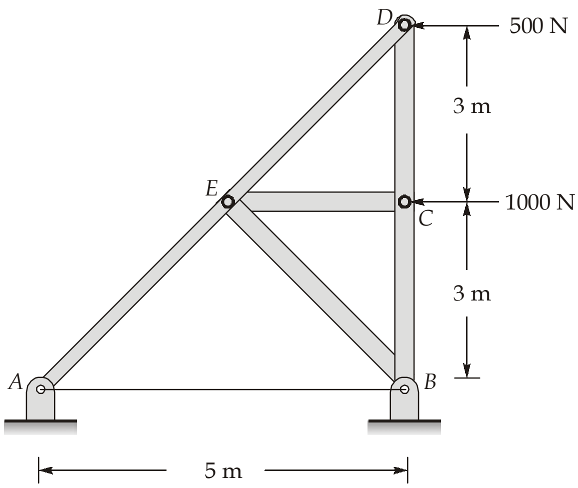
Q.19 A single threaded screw jack has a pitch of 12 mm and a mean diameter of 80 mm. The coefficient of static friction between the screw and the nut is 0.15 and that of kinetic friction is 0.10. The force P required to be applied at the end of a 600 mm long lever to just lift a weight of 25 kN is
(a) 663.34 N (b) 331.67 N
(c) 211.03 N (d) 422.06 N

Q.20 In the shown figure, the block A supports a weight of 5000 N and it is to be prevented from sliding down by applying a horizontal force P on the block B. If the coefficient of friction at all surfaces of contact is 0.2, the smallest force P required to maintain the equilibrium is [Neglect the weight of the blocks]



(a) 4117.62 N (b) 3661.52 N
(c) 2058.81 N (d) 1830.76 N

Q.21 For the following truss system as shown in figure, which one of the following statement is INCORRECT?

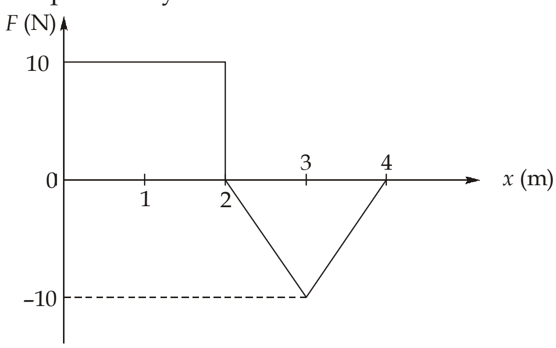


(a) The force in the member CD is 600 N and is tensile in nature.
(b) The force in the member DE is 780 N and is tensile in nature.
(c) The force in the member CE is 1000 N and is compressive in nature.
(d) The force in the member BC is 600 N and is tensile in nature.

Q.22 The mass of a helicopter is 10000 kg. If takes off vertically at time t = 0. The pilot advances the throttle so that the upward thrust of its engine (in kN) is given as a function of time (in seconds) by, $T = 200 + 2t^3$. The magnitude of the linear impulse due to the forces acting on the helicopter from t = 0 sec to t = 4 sec is [Use g = 10 m/s^2]

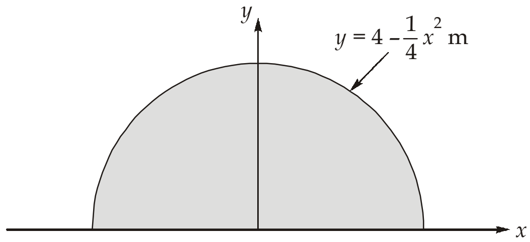
(a) 272 kN-s (b) 928 kN-s
(c) 528 kN-s (d) 384 kN-s

Q.23 The following graph shows the acceleration of a 5 kg particle as an applied force moves it from rest along the x-axis. The speed of the particle by the time it reaches at x = 4 m is



(a) zero (b) 2 m/s
(c) 8 m/s (d) 4 m/s

Q.24 A homogeneous thin plate of uniform thickness and 20 N weight is shown in figure. The moment of inertia of the plate about the y-axis is

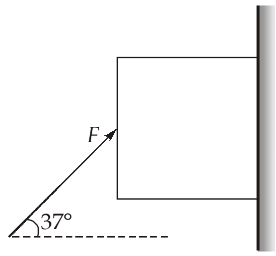


(a) 6.55 kg-m^2 (b) 3.20 kg-m^2
(c) 68.26 kg-m^2 (d) 21.33 kg-m^2

Q.25 A moment M is applied to a uniform disc I of mass 20 kg and radius 0.2 m which drives another uniform disc II of mass 40 kg and radius 0.3 m, without slip occurring between them. If the angular acceleration of disc I is 8.33 rad/s^2 the value of M is

(a) 3.33 Nm (b) 6.66 Nm
(c) 20 Nm (d) 10 Nm

Q.26 A 2 kg block is pushed against a wall by a force F = 80 N as shown in the figure. The coefficient of friction is 0.25. The magnitude of acceleration of the block is

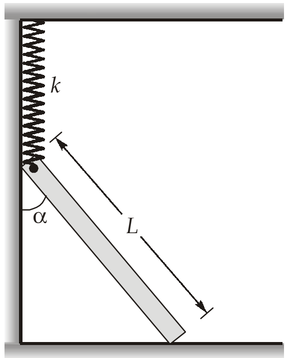


(a) 12 m/s^2 (b) 6 m/s^2
(c) 14 m/s^2 (d) 7 m/s^2

Q.27 A wheel of moment of inertia 1 kg-m^2 is rotating about a shaft at an angular speed of 100 rev/minute. A second wheel is set into rotation at 200 rev/minute and is coupled to the same shaft so that the both wheels finally rotate with a common angular speed of 130 rev/minute. The moment of inertia of the second wheel is

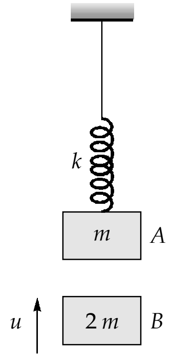
(a) 2.33 kg-m^2 (b) 0.70 kg-m^2
(c) 0.43 kg-m^2 (d) 4.33 kg-m^2

Q.28 The length of a 4 kg bar is 6 meters. The floor and wall are smooth. The spring is unstretched when the angle α = 0°. If the bar is in equilibrium at α = 30°, then the spring constant is



(a) 24.88 N/m (b) 3.85 N/m
(c) 3.33 N/m (d) 6.66 N/m

Q.29 Block *A* is hanging from a vertical spring and is at rest. Block *B* strikes the block *A* with velocity *u* and sticks to it. Then the value of *u* for which the spring just attains natural length is



(a) $\sqrt{\dfrac{60\, mg^2}{k}}$ (b) $\sqrt{\dfrac{15\, mg^2}{4\, k}}$

(c) $\sqrt{\dfrac{10\, mg^2}{k}}$ (d) $\sqrt{\dfrac{3\, mg^2}{k}}$

Q.30 A uniform disc of radius *R* and mass *m* lies in the *x*-*y* plane with its centre at origin. Its moment of inertia about *z*-axis is equal to its moment of inertia about the line $y = x + C$. The value of *C* is

(a) $\pm\dfrac{R}{2}$ (b) $\pm\dfrac{R}{\sqrt{2}}$

(c) $\pm\dfrac{R}{4}$ (d) $\pm R$

■■■■

CLASS TEST

S.No.: 01JPME_10062024

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

ENGINEERING MECHANICS

CIVIL ENGINEERING

Date of Test : 10/06/2024

ANSWER KEY

1.	(d)	7.	(d)	13.	(b)	19.	(b)	25.	(d)
2.	(b)	8.	(b)	14.	(a)	20.	(c)	26.	(b)
3.	(c)	9.	(a)	15.	(c)	21.	(b)	27.	(c)
4.	(a)	10.	(c)	16.	(d)	22.	(c)	28.	(a)
5.	(b)	11.	(a)	17.	(c)	23.	(b)	29.	(b)
6.	(b)	12.	(d)	18.	(a)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

1. (d)

Without slipping, maximum acceleration provided by friction is given as

$$a = \mu g = 0.75 \times 9.81 = 7.36 \text{ m/s}^2$$

Using second equation of kinematics as the car is starting from rest,

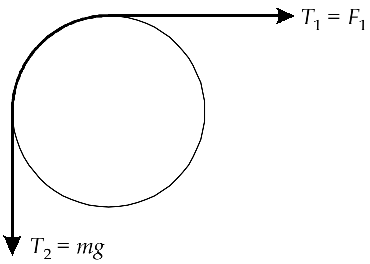
$$s = ut + \frac{1}{2}at^2$$

$$\therefore \quad s = \frac{1}{2}at^2 \qquad (u = 0)$$

$$\therefore \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 700}{7.36}} = \sqrt{100.217}$$

$$= 13.79 \text{ seconds}$$

2. (b)



The angle of contact between the cable and the round support is $\theta = \frac{\pi}{2}$ radians.

$$\therefore \quad T_2 = T_1 e^{\mu_s \theta}$$

$$T_2 = T_1 e^{0.4 \times \frac{\pi}{2}}$$

$$T_2 = T_1 e^{0.628} = 1.874 T_1$$

$$T_1 = \frac{T_2}{1.874} = \frac{50 \times 9.81}{1.874} = 261.74 \text{ N}$$

3. (c)

The work done in a small displacement dx is given as,

$$dw = \vec{F} \cdot \vec{dx} = F dx$$

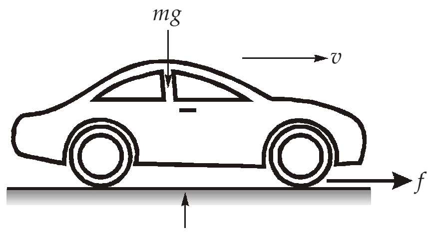
$$w = \int dw = \int_0^3 (10 + x) dx$$

$$w = 10[x]_0^3 + \frac{1}{2}\left[x^2\right]_0^3 = 10(3-0) + 0.5(9-0)$$

$$= 30 + 4.5 = 34.5 \text{ Joules}$$

4. (a)

Using impulse-momentum theorem,



F.B.D.

$$mu + \int F dt = mv$$

$$mu + \int_0^6 600t^2 dt = mv$$

$$2500 \times 20 + 600\left(\frac{t^3}{3}\right)_0^6 = 2500 \times v$$

$$50000 + 600 \times \frac{216}{3} = 2500 \times v$$

$$v = \frac{93200}{2500} = 37.28 \text{ m/s}$$

5. (b)

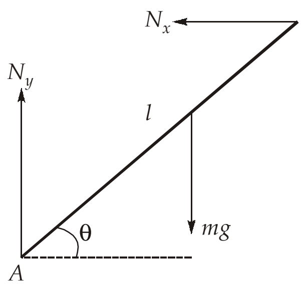
Given u = 8 m/s, a = 2.4 m/s^2, t = 10s

Using,
$$S = ut + \frac{1}{2}at^2$$

$$S = 8 \times 10 + \frac{1}{2} \times 2.4 \times 10^2$$

$$S = 80 + 120 = 200 \text{ m}$$

6. (b)



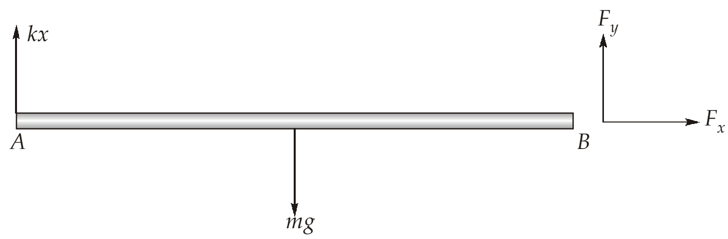
The free body diagram of the rod is shown. From the equilibrium of the rod and taking moment about the end A, we get,

$$mg \times \frac{l}{2}\cos\theta = N_x \times l \sin\theta$$

$$N_x = \frac{mg}{2\tan\theta}$$

7. (d)

Free body diagram of rod is given as



Taking moment about B.

$$mg \times \frac{l}{2} = kx \times l$$

$$\therefore \qquad x = \frac{mg}{2k}$$

Since, there is no external horizontal force on the rod, so $F_x = 0$ and $F_y + kx = mg$

$$F_y + \frac{mg}{2k} \times k = mg$$

$$F_y = \frac{mg}{2}$$

8. (b)

As the block is at rest, the net horizontal force acting on it should be zero. Therefore, friction force is equal to 20 N.

9. (a)

$$\text{Acceleration of particle, } a = \frac{F}{m} = \frac{7.5}{30 \times 10^{-3}} = 250 \text{ m/s}^2$$

Time taken to cover 2.5 meters distance is

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 2.5}{250}} = 0.141 \text{ seconds}$$

Velocity after this displacement,

$$v = \sqrt{2as} = \sqrt{2 \times 250 \times 2.5} = 35.35 \text{ m/s}$$

$$P_{av} = \frac{\text{Total work done}}{\text{Time}} = \frac{\text{Change in KE}}{\text{Time}}$$

$$= \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2} \times 30 \times 10^{-3} \times 35.35^2}{0.141}$$

$$= 133 \text{ Watts}$$

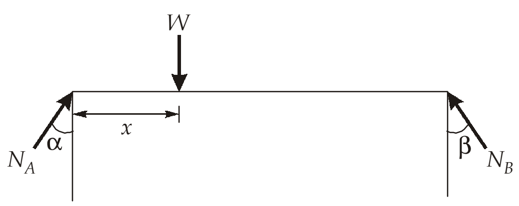
10. (c)

Acceleration is defined as the rate of change of velocity with respect to time. So, a change in either the speed or the direction of motion or both results into acceleration. Statement I is correct.

For a particle moving in circular motion with a constant speed, the direction of velocity is changing at every instant. Therefore, the particle is having an acceleration. So, statement II is false.

11. (a)

Free-body diagram of the beam is drawn as,



From equilibrium equation,

$$N_A \cos\alpha + N_B \cos\beta = W \qquad \text{...(i)}$$

$$N_A \sin\alpha = N_B \sin\beta \quad \text{...(ii)}$$

Moment about A is given by,

$$W \times x = N_B \cos\beta \times l \quad \text{...(iii)}$$

Solving these equations,

$$N_B = \frac{Wx}{l\cos\beta}$$

Putting this value of N_B in equation (ii),

$$N_A = \frac{Wx}{l\cos\beta}\frac{\sin\beta}{\sin\alpha} = \frac{Wx}{l}\frac{\tan\beta}{\sin\alpha}$$

Now, putting the values N_A and N_B in equation (i)

$$\frac{Wx}{l}\frac{\tan\beta}{\sin\alpha}\times\cos\alpha + \frac{Wx}{l\cos\beta}\times\cos\beta = W$$

$$\frac{Wx}{l}\frac{\tan\beta}{\tan\alpha} + \frac{Wx}{l} = W$$

$$\frac{x}{l}\left(\frac{\tan\beta}{\tan\alpha}+1\right) = 1$$

$$x = \frac{l}{1+\dfrac{\tan\beta}{\tan\alpha}}$$

12. (d)

The angles between the pillar ED and three cables are

$$\alpha_A = \tan^{-1}\left(\frac{4}{6}\right) = 33.7°$$

$$\alpha_B = \tan^{-1}\left(\frac{8}{6}\right) = 53.1°$$

$$\alpha_C = \tan^{-1}\left(\frac{12}{6}\right) = 63.4°$$

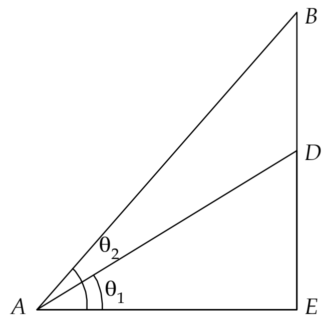
The vertical components of each force at point D exert no moment about E. Noting that $F_A = F_B = F_C$, the magnitude of the moment about E due to the horizontal components is

$$\Sigma M_E = F_A\left(\sin\alpha_A + \sin\alpha_B + \sin\alpha_C\right)\times 6 = 2700$$

$$F_A = \frac{2700}{6\times\left(\sin\alpha_A + \sin\alpha_B + \sin\alpha_C\right)} = \frac{2700}{6\times(0.55+0.8+0.89)}$$

$$F_A = 200.89 \text{ kN}$$

13. (b)



$$\tan\theta_1 = \frac{DE}{AE} = \frac{\frac{1}{4}a}{a} = \frac{1}{4} = 0.25$$

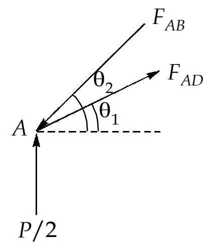
$$\theta_1 = \tan^{-1}(0.25) = 14.04°$$

$$\tan\theta_2 = \frac{BE}{AE} = \frac{a}{a} = 1$$

$$\theta_2 = \tan^{-1}(1) = 45°$$

Given: $F_{AB} = 800$ N (C)

Joint A:



$\Sigma F_x = 0;$

$$F_{AD}\cos\theta_1 = 800\cos\theta_2$$

$$F_{AD} = 800\frac{\cos 45°}{\cos 14.04°}$$

$$F_{AD} = 583.01 \text{ N} < 2000 \text{ N}$$

$\Sigma F_y = 0;$

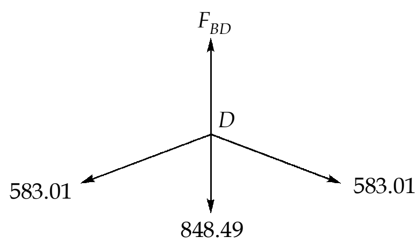
$$\frac{P}{2} + F_{AD}\sin\theta_1 = F_{AB}\sin\theta_2$$

$$P = (2)(800\sin 45° - 583.01\sin 14.04°)$$

$$P = 2(565.68 - 141.44)$$

$$P = 848.49 \text{ N}$$

Joint *D*,



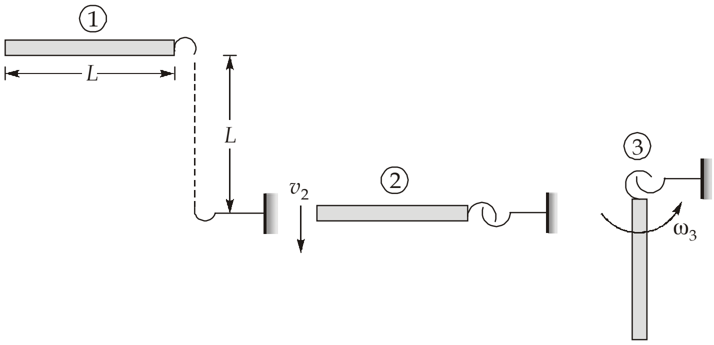
$\Sigma F_y = 0;$

$$F_{DB} = 848.49 + 2 \times 583.01 \times \sin 14.04$$
$$= 1131.29 \text{ N} < 2000 \text{ N}$$

Therefore, $$P_{max} = 848.49 \text{ N}$$

14. (a)

The three different situations of motion of rod is shown as :



Using energy conservation between (1) and (2),

$$U_1 + K_1 = U_2 + K_2$$

$$\Rightarrow \quad mgL + 0 = 0 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad v_2 = \sqrt{2gL}$$

From momentum conservation before and after striking the hook

$$\therefore \quad P_1 = P_2$$

$$\Rightarrow \quad mv_2r = I\omega_2$$

$$\Rightarrow \quad m\sqrt{2gL} \times \frac{L}{2} = \left(\frac{mL^2}{3}\right)\omega_2$$

$$\Rightarrow \quad \omega_2 = \frac{3}{2}\sqrt{\frac{2g}{L}}$$

Energy conservation between (2) and (3),

$$U_2 + K_2 = U_3 + K_3$$

$$0 + \frac{1}{2}\left(\frac{1}{3}ML^2\right) \times \frac{9}{4} \times \frac{2g}{L} = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega_3^2 - Mg\frac{L}{2}$$

$$\frac{3}{4}gL = \frac{1}{6}L^2\omega_3^2 - g\left(\frac{L}{2}\right)$$

$$\omega_3 = \sqrt{\frac{7.5g}{L}} = \sqrt{\frac{7.5 \times 9.81}{1}} = \sqrt{73.575}$$

$$\omega_3 = 8.57 \text{ rad/sec}$$

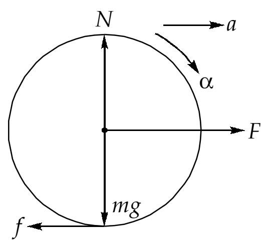
15. (c)

Free body diagram of the disc is given as

$$F - f = ma \quad \text{...(i)}$$

$$fR = I\alpha = \frac{1}{2}mR^2\alpha = \frac{1}{2}mR^2\frac{a}{R}$$

$$fR = \frac{mRa}{2}$$



$$f = \frac{ma}{2} \quad \text{...(ii)}$$

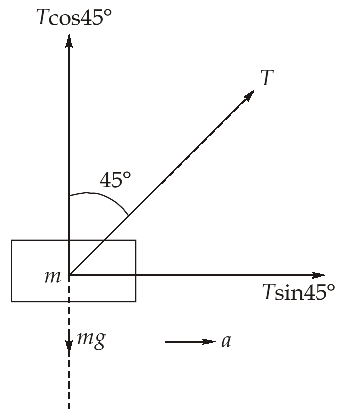
From equation (i) and (ii),

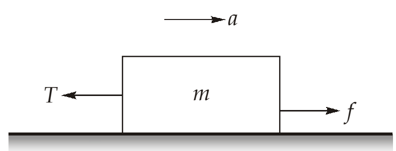
$$F = \frac{3ma}{2}$$

$$\Rightarrow \quad a = \frac{2F}{3m} = \frac{2\times 10}{3\times 12} = 0.55 \text{ m/s}^2$$

16. (d)

Free body diagram of the blocks are:





$$T\sin 45° = ma \quad \text{... (i)}$$

$$T\cos 45° = mg \quad \text{... (ii)}$$

Dividing equation (i) by (i),

$$\frac{T\sin 45°}{T\cos 45°} = \frac{ma}{mg}$$

$$\Rightarrow \quad a = g$$

From equation (ii), $T = \dfrac{mg}{\cos 45°} = \sqrt{2}\, mg$.

Applying Newton's law equation for the block placed on the cart.

$$f - T = ma$$

$$\mu mg - T = ma$$

$$\mu mg = T + ma = \sqrt{2}\, mg + mg$$

$$\mu mg = mg\left(\sqrt{2}+1\right)$$

$$\mu = \sqrt{2}+1$$

17. (c)

$$v\frac{dv}{ds} = a$$

$$-6s^{-3} = v\frac{dv}{ds}$$

$$\left[a = \frac{dv}{dt}, dt = \frac{dS}{v} \quad a = \frac{dv}{\left(\frac{dS}{v}\right)} = v\left(\frac{dv}{dS}\right)\right]$$

$$\int_{\infty}^{6} -6s^{-3}ds = \int_{0}^{v} vdv$$

$$\left[-\frac{6}{-2}s^{-2}\right]_{\infty}^{6} = \frac{v^2}{2}$$

$$\left[\frac{3}{s^2}\right]_{\infty}^{6} = \frac{v^2}{2}$$

$$v^2 = \left[\frac{6}{s^2}\right]_{\infty}^{6}$$

$\Rightarrow$
$$v^2 = \frac{6}{6\times 6} = \frac{1}{6}$$

$$v = 0.408 \text{ m/s}$$

18. (a)

Given data: m = 8.4 kg, ω = 6.9 rad/s, F = 6.6 N, M = 59 Nm, L = 4 m, $\omega_{\theta = 90°}$ = ?

Moment of Inertia of rod about hinge O,

$$I_O = \frac{mL^2}{12} + m\times\left(\frac{L}{2}\right)^2 = \frac{mL^2}{3} = \frac{8.4\times 4\times 4}{3} = 44.8 \text{ kg m}^2.$$

By conservation of energy:

$$mgh_{cm} + (M + F \times L)\Delta\theta = \frac{1}{2}I_0\left(\omega_1^2 - \omega_0^2\right)$$

$$\text{For } \theta = 90°,\ h_{cm} = 2 \text{ m}$$

$$(8.4\times 9.81\times 2) + (59 + 6.6\times 4)\times\frac{\pi}{2} = \frac{1}{2}\times(44.8)\left[\omega_1^2 - 6.9^2\right]$$

$$\frac{298.954\times 2}{44.8} = \omega_1^2 - 6.9^2$$

$$\omega_1^2 = 60.9562$$

$$\omega_1 = 7.807 \text{ rad/s}$$

19. (b)

Given: Pitch (P) = 12 mm, Mean radius (r) = $\frac{80}{2} = 40\text{ mm}$, Coefficient of static friction (μ_s) = 0.15,

Coefficient of kinetic friction (μ_k) = 0.10, Lever length (a) = 600 mm, Weight to be lifted (W) = 25 kN.
Since, the screw is single threaded, lead (C) = Pitch(P) = 12 mm.
Determination of helix angle,

$$\tan\theta = \frac{L}{2\pi r} = \frac{12}{2\pi\times 40} = 0.0477$$

$$\theta = \tan^{-1}(0.0477) = 2.733°$$

Force required to just lift a weight of 25 kN.

$$\tan\phi_s = \mu_s$$
$$\phi_s = \tan^{-1}(\mu_s) = \tan^{-1}(0.15)$$
$$\phi_s = 8.53°$$
$$\phi_s + \theta = 8.53° + 2.733° = 11.263°$$
$$\tan(\phi_s + \theta) = \tan(11.263) = 0.199$$

Therefore, the force required to just raise the load is given as:

$$P = \frac{Wr}{a}\tan(\phi_s + \theta)$$
$$= \frac{25000 \times 0.04}{0.6} \times 0.199 = 331.67 \text{ N}$$

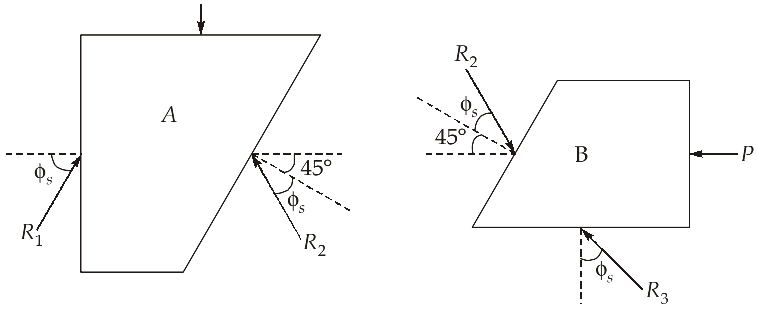
20. (c)

Coefficient of friction, $\mu_s = 0.2$.

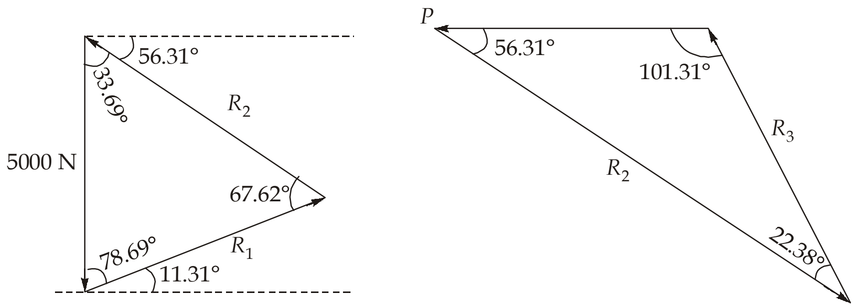
Here the force P is required to maintain the equilibrium. The direction of impending motion of the block A is downwards and that of block B is rightwards.

The free body-diagrams of the block are:

[Angle of friction: $\phi_s = \tan^{-1}\mu$, $\phi_s = \tan^{-1}(0.2)$, $\phi_s = 11.31°$]



Making force triangles for A and B



Applying Lami's theorem for block A

$$\frac{5000}{\sin(67.62°)} = \frac{R_1}{\sin(33.69°)} = \frac{R_2}{\sin(78.69°)}$$

$$\therefore \quad R_2 = 5000 \times \frac{\sin(78.69°)}{\sin(67.62°)} = 5302.27 \text{ N}$$

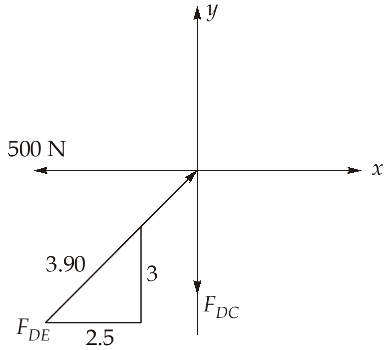
From Lami's theorem for block B

$$\frac{P}{\sin(22.38°)} = \frac{R_2}{\sin(101.31°)} = \frac{R_3}{\sin(56.31°)}$$

$$\therefore \quad P = R_2 \times \frac{\sin(22.38°)}{\sin(101.31°)}$$

$$P = 5302.27 \times \frac{\sin(22.38°)}{\sin(101.31°)} = 2058.81 \text{ N}$$

21. **(b)**



Beginning by analyzing the equilibrium of joint D.

$\Sigma F_x = 0$

$$F_{DE}\cos\theta - 500 = 0$$

$$F_{DE} = \frac{500}{\cos\theta} = \frac{500}{\left(\frac{2.5}{3.90}\right)} = 780 \text{ N}$$

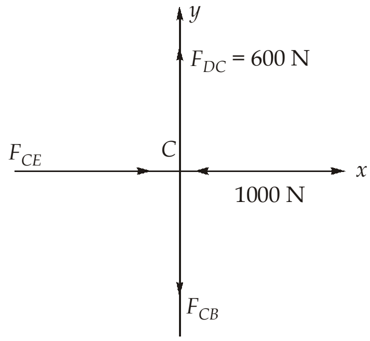
F_{DE} is compressive in nature.

$\Sigma F_y = 0$

$$F_{DC} = F_{DE}\sin\theta$$

$$F_{DC} = 780 \times \frac{3}{3.90} = 600 \text{ N}$$

F_{DC} is tensile in nature.



Free-body diagram of the joint C,

$\Sigma F_x = 0$

$$F_{CE} = 1000 \text{ N}$$

F_{CE} is compressive in nature,

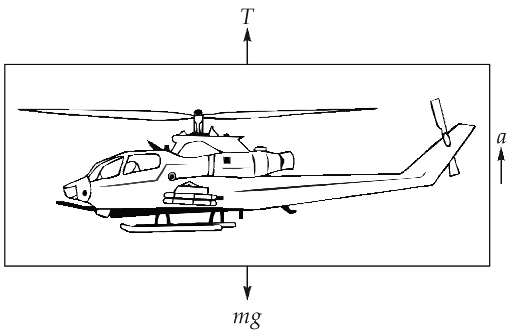
$\Sigma F_y = 0,$ $\quad 600 - F_{CB} = 0$

$$F_{CB} = 600 \text{ N}$$

F_{CB} is tensile in nature.

22. **(c)**

Free-body diagram of the helicopter is given by:



Net force on the helicopter is given as,

$$F_{\text{net}} = T - mg = (200 + 2t^3 - 100) \text{ kN}$$

Impulse of the net force is given as

$$I = \int_0^4 F_{net}\,dt = \int_0^4 \left(200 + 2t^3 - 100\right)dt$$

$$= \int_0^4 \left(2t^3 + 100\right)dt = 2\left[\frac{t^4}{4}\right]_0^4 + 100[t]_0^4$$

$$= \frac{2}{4}\left[4^4 - 0\right] + 100[4 - 0]$$

$$= 128 + 400 = 528 \text{ kN-s}$$

23. (b)

The area under the force displacement curve will give the net work done by the force on the particle.

$$W_{net} = 10\times 2 - \frac{1}{2}\times 10\times 2 = 20 - 10 = 10\text{ J}$$

Using work energy theorem,

$$W_{net} = \text{Change in kinetic energy}$$

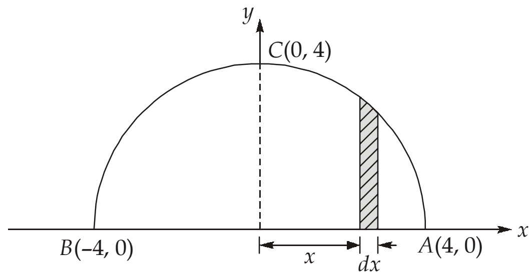
$$10 = (KE)_f - (KE)_i$$

$$10 = \frac{1}{2}Mv^2 - 0$$

$$v = \sqrt{\frac{20}{M}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2\text{ m/s}$$

24. (a)

The co-ordinates of the plate on the axis are:



Let ρ be the area density of the plate,

$$\rho = \frac{m}{A} = \frac{W}{gA}$$

The mass of an element ydx at a distance x from the y-axis is,

$$dm = \frac{W}{gA}ydx$$

Using the formula for moment of inertia,

$$I_{y\text{-axis}} = \int r^2 dm = \int x^2 \frac{W}{gA} ydx$$

$$= \frac{W}{gA}\int_{-4}^{4} x^2\left(4 - \frac{x^2}{4}\right)dx = \frac{W}{gA}\int_{-4}^{4}\left(4x^2 - \frac{x^4}{4}\right)dx$$

$$= \frac{W}{gA}\left[\frac{4x^3}{3} - \frac{x^5}{20}\right]_{-4}^{4} = \frac{W}{gA}(68.26) \quad \text{... (i)}$$

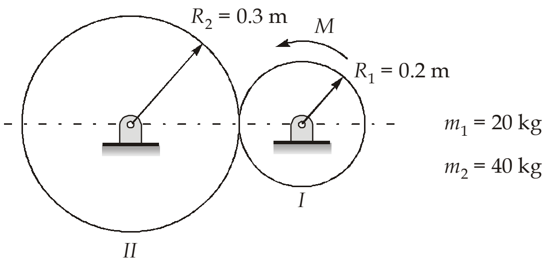
Area of the plate is $\quad A = \int\left(4 - \frac{x^2}{4}\right)dx = \left[4x - \frac{x^3}{12}\right]_{-4}^{4}$

$$A = 21.33\text{ m}^2$$

Putting this value of area in equation (i)

$$I_{y\text{-axis}} = \frac{W}{g\times 21.33}\times(68.26) = \frac{20}{(9.81\times 21.33)}\times(68.26) = 6.5458\text{ kg-m}^2 \simeq 6.55\text{ kg-m}^2$$

25. (d)



$$\text{Moment of inertia, } I_1 = \frac{m_1R_1^2}{2} = \frac{20\times 0.2^2}{2} = 0.4 \text{ kgm}^2$$

$$I_2 = \frac{m_2R_2^2}{2} = \frac{40\times 0.3^2}{2} = 1.8 \text{ kgm}^2$$

A force of friction F acts between disc I and II which drives disc II.

$$F \times R_2 = I_2\alpha_2 \quad \text{...(1)}$$

$$R_1\alpha_1 = R_2\alpha_2$$

$$\Rightarrow \quad 0.2 \times 8.33 = 0.3 \times \alpha_2$$

$$\alpha_2 = 5.55 \text{ m/s}^2$$

Put α_2 value in equation (1),

We get $\quad F = 33.32$ N

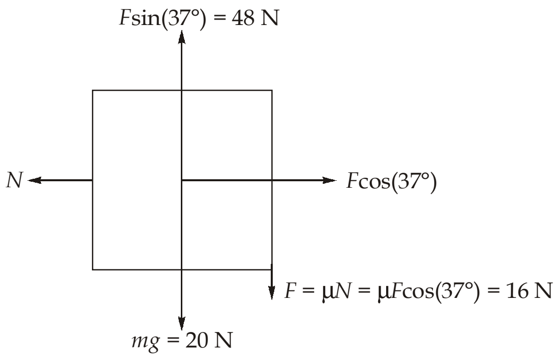
$$M - FR_1 = I_1\alpha_1$$

$$\Rightarrow \quad M - 33.32 \times 0.2 = 0.4 \times 8.33$$

$$M = 9.996 \simeq 10 \text{ Nm}$$

26. (b)

Free-body diagram of the block is given as:



As the upward force [Fsin(37°) = 48 N] is greater than the total downward force (20 + 16 = 36 N) hence, it has an upward acceleration,

$$F_{\text{net, } y} = ma$$

$$[48 - (20 + 16)] = 2a$$

$$48 - 36 = 2a$$

$$a = \frac{12}{2} = 6 \text{ m/s}^2$$

27. (c)

Given: N_1 = 100 rev/min, N_2 = 200 rev/min, N = 130 rev/min, I_1 = 1 kg-m^2.

Since the external torque acting on the two wheels system is zero, the angular momentum will be conserved.

$$L_i = L_f$$
$$I_1N_1 + I_2N_2 = (I_1 + I_2)N$$
$$1 \times 100 + I_2 \times 200 = (I_1 + I_2) \times 130$$
$$100 + 200I_2 = 130 + 130I_2$$
$$70I_2 = 30$$
$$I_2 = 0.4286 \text{ kg-m}^2 \simeq 0.43 \text{ kg-m}^2$$

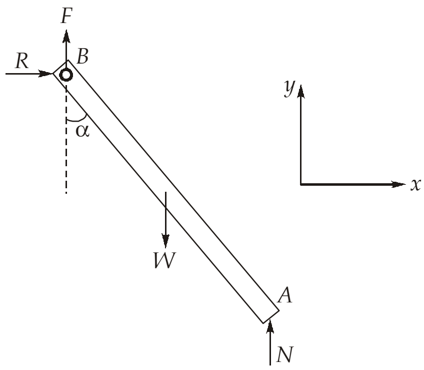
28. (a)

$$\text{Mass of bar, } m = 4 \text{ kg}$$
$$\text{Length of bar, } L = 6 \text{ m}$$

The elongation in the spring,

$$x = L - L\cos\alpha \qquad \text{... (i)}$$

Free-body diagram of the bar is given as:



From equilibrium equations,

$\Sigma F_x = 0,$ $\quad R = 0$

$\Sigma F_y = 0,$ $\quad F + N = W$

$$\Sigma M_A = 0,\ W\left(\frac{L}{2}\sin\alpha\right) - R(L\cos\alpha) - F(L\sin\alpha) = 0$$

as $R = 0$

$$W\left(\frac{L}{2}\sin\alpha\right) = F(L\sin\alpha)$$

$$\therefore \qquad F = \frac{W}{2} = \frac{4\times 10}{2} = 20\text{ N}$$

$\therefore$ Putting this value of F in equation (i),

$$F = k(L)\ (1 - \cos\alpha)$$
$$k = \frac{F}{L(1-\cos\alpha)} = \frac{20}{6(1-\cos 30°)}$$
$$k = 24.88 \text{ N/m}$$

29. (b)

The initial extension of the spring,

$$x_0 = \frac{mg}{k}$$

Using conservation of linear momentum to find the combined speed of A and B.

$$P_i = P_t$$
$$2\,m \times u + 0 = (3\,m) \times v$$

$$\Rightarrow \qquad v = \frac{2mu}{3m} = \frac{2u}{3}$$

For the spring to just attain its natural length, the combined blocks must rise by $\frac{mg}{k}$. Therefore, using conservation of mechanical energy:

$$E_i = E_f$$

$$\Rightarrow \frac{1}{2}\times 3m\left(\frac{2u}{3}\right)^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = (3mg)\left(\frac{mg}{k}\right)$$

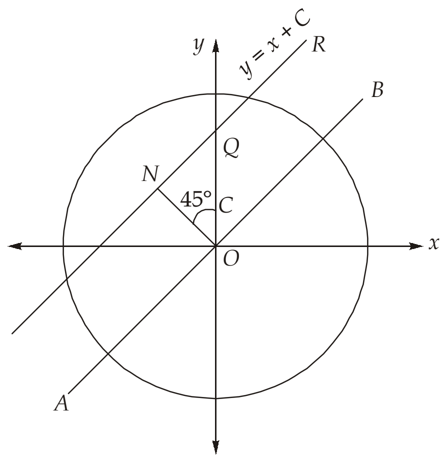
$$\Rightarrow \qquad \frac{2mu^2}{3} + \frac{m^2g^2}{2k} = \frac{3m^2g^2}{k}$$

$$\Rightarrow \qquad \frac{2mu^2}{3} = \left(\frac{m^2g^2}{k}\right)\left(\frac{5}{2}\right)$$

$$\Rightarrow \qquad u^2 = \frac{15mg^2}{4k}$$

$$\Rightarrow \qquad u = \sqrt{\frac{15mg^2}{4k}}$$

30. (b)



$$I_{PQR} = I_{AOB} + m(ON)^2$$

$$I_{PQR} = \frac{mR^2}{4} + m\left(\frac{C}{\sqrt{2}}\right)^2 \qquad \left[ON : C\cos(45°) = \frac{C}{\sqrt{2}}\right]$$

$$= \frac{mR^2}{4} + \frac{mC^2}{2}$$

$$I_z = I_{PQR}$$

$$\frac{mR^2}{2} = \frac{mR^2}{4} + \frac{mC^2}{2}$$

$$\frac{mR^2}{4} = \frac{mC^2}{2}$$

$$C = \pm\frac{R}{\sqrt{2}}$$

■■■■