CLASS TEST

S.No.: 01JPECEE_10062024

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EC-EE

SIGNAL & SYSTEM

Duration : 1:00 hr. **Maximum Marks : 50**

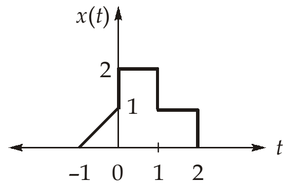
Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

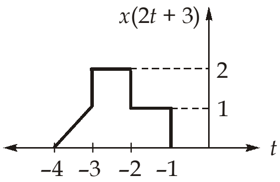
DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

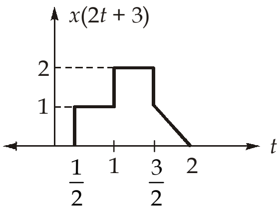
Q.No. 1 to Q.No. 10 carry 1 mark each

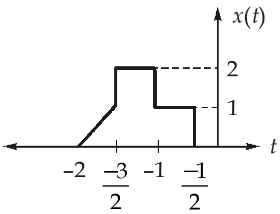
Q.1 For the signal $x(t)$ shown in figure below:

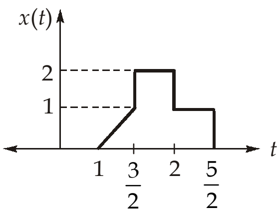


Which of the following signal represents $x(2t + 3)$.

(a)


(b)


(c)


(d)


Q.2 The Fourier transform of the signal $e^{-(2t-2)}u(t-1)$ is

(a) $\dfrac{e^{-j2\omega}}{2+j\omega}$ (b) $\dfrac{e^{-j\omega}}{2+j\omega}$

(c) $\dfrac{e^{-j2\omega}}{2+j2\omega}$ (d) $\dfrac{e^{j\omega}}{2-j\omega}$

Q.3 The energy of the signal

$$x(t)=\left(\frac{\sin 200\pi t}{\pi t}\right)*\left(\frac{\sin 100\pi t}{\pi t}\right)$$

is ________. (* denotes convolution)

(a) 100 (b) 50

(c) 200 (d) 400

Q.4 Let $X(z)$ be the z-transform of a discrete time signal $x(n)$, where $x(n)=\sum_{k=0}^{\infty} a^k\delta[n-5k]$. Then $X(z)$ is represented by

(a) $\dfrac{z}{z^5-a}$ (b) $\dfrac{z^5}{z-a}$

(c) $\dfrac{z^5}{z^5-a}$ (d) $\dfrac{z^4}{z^5-a}$

Q.5 The Nyquist sampling rate of the signal $x(t) = (1 + \cos 300\pi t)^2 (\sin 4000\pi t)^2$ is

(a) 4.3 kHz

(b) 4 kHz

(c) 8.6 kHz

(d) 8 kHz

Q.6 Let $x(t) = \dfrac{\sin(10\pi t)}{\pi t}$ be the continuous time signal, the condition on the sampling interval 'T_s' so that $x(t)$ is uniquely represented by the discrete-time sequence $x[n] = x[nT_s]$ is

(a) $T_s > \dfrac{1}{10}$

(b) $T_s < \dfrac{1}{10}$

(c) $T_s > 10\pi$

(d) $T_s < \dfrac{1}{5\pi}$

Q.7 Consider the sequence $x[n]=[-2-j5, \underset{\uparrow}{1-j}, 2]$.

The conjugate anti-symmetric part of the sequence is

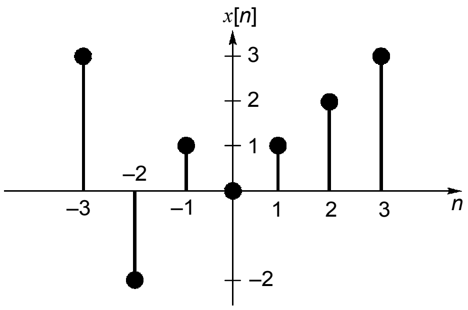
(a) $[2-j2.5, \underset{\uparrow}{j}, -2-j2.5]$

(b) $[2-j2.5, -\underset{\uparrow}{j}, -2-j2.5]$

(c) $[-2-j2.5, -\underset{\uparrow}{j}, 2-j2.5]$

(d) $[-2-j2.5, \underset{\uparrow}{j}, -2+j2.5]$

Q.8 Consider the sequence given below



the value of $|X(e^{j\pi})|$ is ________.

(a) 8 (b) 16
(c) 9 (d) 18

Q.9 Let $x(t)$ be a periodic signal with fundamental period T and Fourier series coefficients a_K. The Fourier series coefficient of $Re\{x(t)\}$ (Where Re (.) denotes the real part of the signal) is

(a) $\dfrac{a_K + a^*_{-K}}{2}$ (b) $\dfrac{a_K - a^*_{-K}}{2}$

(c) $\dfrac{a^*_K + a_{-K}}{2}$ (d) $\dfrac{a^*_K - a^*_K}{2}$

Q.10 The transfer function of a causal discrete time system is $H(z) = \dfrac{z}{z-0.2}$. Then the impulse response will be

(a) stable with ROC : $|z| < 0.2$
(b) unstable with ROC : $|z| > 0.2$
(c) stable with ROC : $|z| > 0.2$
(d) None of these

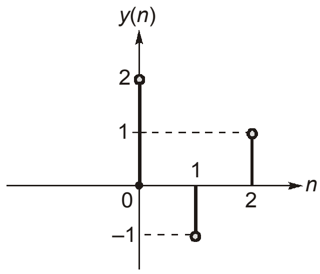
Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 Consider $y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-2k)$, if $y(t)$ $= Ae^{-t}$ for $0 \le t < 2$, then value of A is

(a) $\dfrac{1}{1-e}$ (b) $\dfrac{1}{1-e^{-2}}$

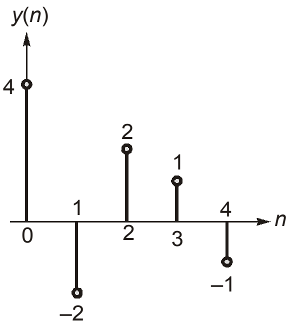
(c) $\dfrac{1}{1-e^{-3}}$ (d) $\dfrac{1-e}{1-e^{-2}}$

Q.12 Consider a discrete-time system which is both linear and time invariant. For an input $x(n) = \delta(n)$, the output $y(n)$ is given by,

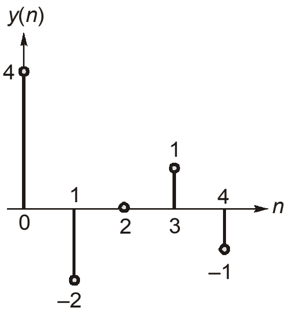


If $x[n] = 2\delta[n] - \delta[n-2]$, then the output $y(n)$ can be given by

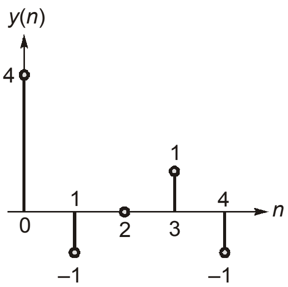
(a)



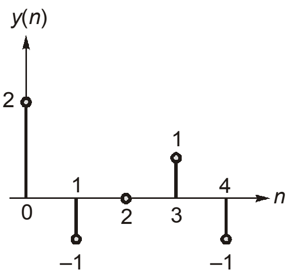
(b)



(c)



(d)



Q.13 If $f(t)$ is a real even continuous time signal, then its Fourier transform will be

(a) $\int_0^\infty f(t)\cos(2\omega t)\,dt$

(b) $2\int_0^\infty f(t)\cos(\omega t)\,dt$

(c) $\int_0^\infty f(t)\sin(\omega t)\,dt$

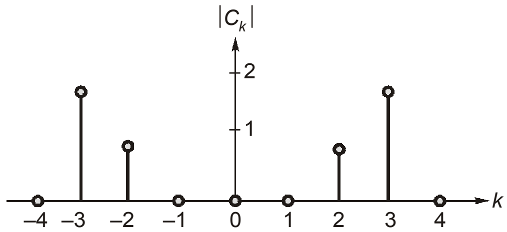
(d) $2\int_0^\infty f(t)\sin(\omega t)\,dt$

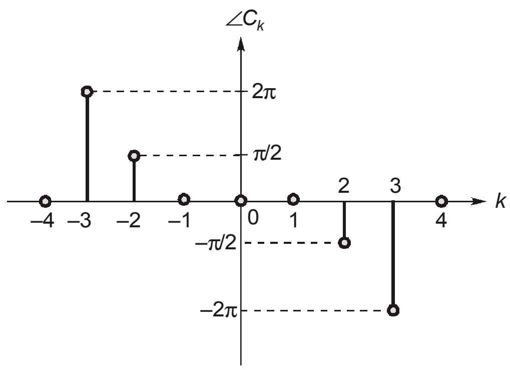
Q.14 The inverse Laplace transform of the signal $X(s) = \log(s+2) - \log(s+3)$. Then $x(t)$ is equal to
(Assume all initial conditions are zero).

(a) $\left(\dfrac{e^{3t}-e^{-2t}}{t}\right)u(t)$ (b) $\left(\dfrac{e^{-3t}-e^{-2t}}{t}\right)u(t)$

(c) $\left(\dfrac{e^{-3t}-e^{2t}}{t}\right)u(t)$ (d) $\left(\dfrac{e^{3t}-e^{2t}}{t}\right)u(t)$

Q.15 A time domain signal $x(t)$ is represented by the following exponential Fourier series spectra (magnitude and phase).





The corresponding signal $x(t)$ can be given by

(Assume that, $\omega_0 = \pi$)

(a) $4\cos 3\pi t - 2\sin 2\pi t$

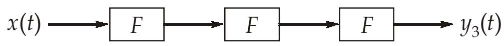
(b) $4\sin 3\pi t + 2\cos 2\pi t$

(c) $4\sin 3\pi t - 2\cos 2\pi t$

(d) $4\cos 3\pi t + 2\sin 2\pi t$

Q.16 Suppose that the system F takes the Fourier transform of the input, as shown in below figure.

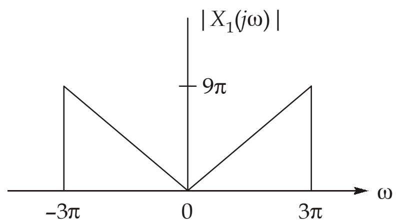
Then the output signal $y_3(t)$ of the system shown in the figure below is

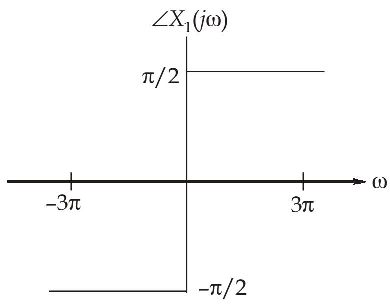


(a) $(2\pi)^3\,X(t)$ (b) $(2\pi)^3\,X(-t)$

(c) $(2\pi)^4\,X(t)$ (d) $(2\pi)^4\,X(-t)$

Q.17 A signal $x_1(t)$ whose fourier transform is $X_1(j\omega)$ has the following magnitude and phase response as shown below:





Then determine the signal $x(t)$.

(a) $\dfrac{3}{\pi t}(\cos 3\pi t - \sin 3\pi t)$

(b) $\dfrac{3}{\pi t}(\pi t\cos 3\pi t - \sin 3\pi t)$

(c) $\dfrac{3}{\pi t^2}(3\pi t\cos 3\pi t - \sin 3\pi t)$

(d) $\dfrac{3}{\pi^2 t^2}(3t\cos 3\pi t - \sin 3\pi t)$

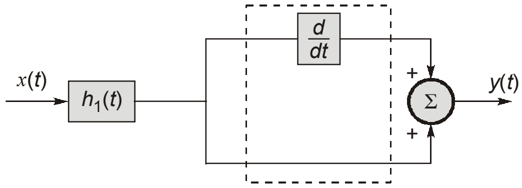
Q.18 The Laplace transform of a signal given below is

$$x(t) = \begin{cases} \cos(\pi t); & 0 < t < 1 \\ 0 \quad ; & \text{otherwise} \end{cases}$$

(a) $\dfrac{1-e^{-s}}{s^2+\pi^2}$ (b) $\dfrac{1-e^{-\pi s}}{s^2+\pi^2}$

(c) $\dfrac{s[1+e^{-s}]}{s^2+\pi^2}$ (d) $\dfrac{s[1-e^{-s}]}{s^2+\pi^2}$

Q.19 Consider the system shown below

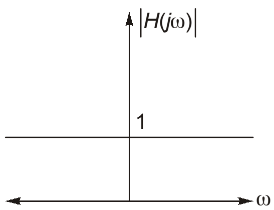


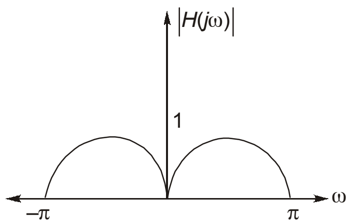
If $h_1(t) = e^{-t}\,u(t)$, then the the impulse response of the entire system is

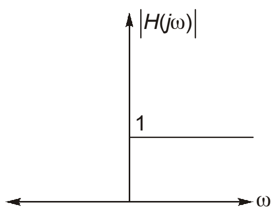
(a) $e^{-t}\,u(t)$ (b) $2e^{-t}\,u(t) + \delta(t)$

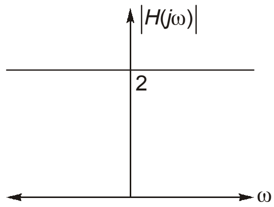
(c) $\delta\,(t)$ (d) None of these

Q.20 The magnitude plot of the Fourier transform of the signal $H(j\omega) = \dfrac{1+2e^{-j\omega}}{1+\dfrac{1}{2e^{-j\omega}}}$ is

(a)


(b)


(c)


(d)


Q.21 The discrete time signal $x[n]$ has Fourier transform of $X(e^{j\omega}) = \dfrac{\sin\frac{3\omega}{2}}{\sin\frac{\omega}{2}}$. Then the value of $\dfrac{1}{\pi}\left|\displaystyle\int_{-\pi}^{\pi}\frac{d}{d\omega}\left[X(e^{j\omega})\right]\right|^2 d\omega$ is

(a) 4 (b) 6

(c) 8 (d) 12

Q.22 An ideal phase shifter is defined by the frequency response

$$H(\omega) = \begin{cases} -2j\,; & \omega > 0 \\ 2j\,; & \omega < 0 \end{cases}$$

The impulse response of the above phase shifter is

(a) πt (b) $(\pi t)^{-1}$

(c) $\dfrac{(\pi t)^{-1}}{2}$ (d) $2(\pi t)^{-1}$

Q.23 The discrete Fourier transform [DFT] of a discrete sequence $x[n]$ is $X[k] = \{\underset{\uparrow}{6}, 7, 8, 9\}$. If the DFT of the sequence $g[n] = x[n-2]_{\text{mod N}}$ is $G[k]$, then $G[1]$ is

(a) 7 (b) –7

(c) 9 (d) –9

Q.24 Let $x[n]$ be a real periodic sequence with fundamental period N_0 and Fourier series coefficients $C_k = a_k + jb_k$, where a_k and b_k both are real. If N_0 is even, then $C_{N_0/2}$ is

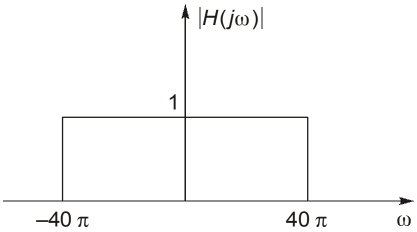
(a) purely real

(b) purely imaginary

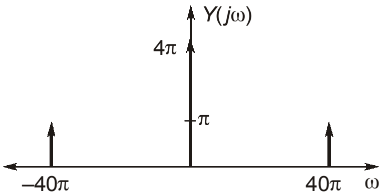
(c) complex

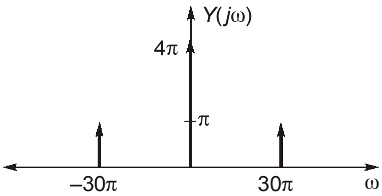
(d) none

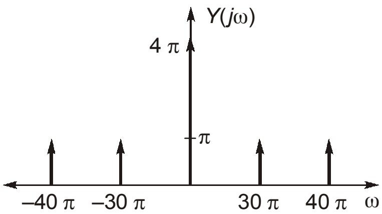
Q.25 A continuous time signal $x(t)$ is defined as $x(t) = 2 + \cos(50\pi t)$ is sampled with sampling interval $t_s = 0.025$ sec and passed through an ideal low pass filter whose frequency response is as shown in figure below

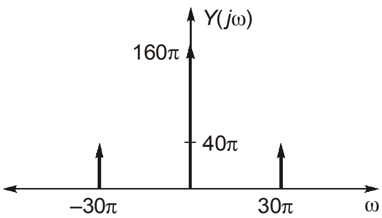


The spectrum of output will be

(a)


(b)


(c)


(d)


Q.26 The output $y[n]$ of a discrete time LTI system is found to be $2\left(\frac{1}{3}\right)^n u[n]$ when input $x[n]$ is $u[n]$. The output $y[n]$ is $\left[k_1\left(\frac{1}{2}\right)^n + k_2\left(\frac{1}{3}\right)^n\right]u[n]$ when input is $\left(\frac{1}{2}\right)^n u[n]$. The value of $k_1 + k_2$ is

(a) 1 (b) 2
(c) 3 (d) 4

Q.27 A real signal $x[n]$ with its Fourier transform $X(e^{j\omega})$ has following properties.

(i) $x[n] = 0$ for $n > 0$

(ii) $x[0]$ is non negative

(iii) $\text{Im}\left(X(e^{j\omega})\right) = \sin(\omega) - \sin(2\omega)$

(iv) $\frac{1}{2}\int_{-\pi}^{\pi}\left|X\left(e^{j\omega}\right)\right|^2 d\omega = 3$

The signal $x[n]$ is

(a) $2\delta[n] - \delta[n+1] - \delta[n+3]$
(b) $\delta[n] + \delta[n+1] - \delta[n+2]$
(c) $\delta[n+1] + \delta[n] + \delta[n-3]$
(d) $3\delta[n] + 2\delta[n+1]$

Q.28 A discrete time system with input $x[n]$ and output $y[n]$ is described by the relation

$$Y(e^{j\omega}) = e^{-j\omega}X(e^{j\omega}) + \frac{dX(e^{j\omega})}{d\omega}$$

The response of the system for input $x[n] = \delta[n]$ is

(a) $\frac{\sin\pi(n-1)}{\pi(n-1)}$ (b) $\delta[n-1] + 2\delta[n-1]$

(c) $u[n-1] + \delta[n]$ (d) $r[n-1] + u[n]$

Q.29 Consider a causal LTI system with frequency response $H(j\omega) = \frac{1}{3 + j\omega}$ for a particular input $x(t)$, this system is observed to produce the output as $y(t) = e^{-3t}\,u(t) - e^{-4t}\,u(t)$. Then the input $x(t)$ is

(a) $e^{-3t}\,u(t)$ (b) $e^{-t}\,u(t)$
(c) $e^{-4t}\,u(t)$ (d) $e^{4t}\,u(t)$

Q.30 Consider the z-transform:

$$X(z) = \frac{1}{1 - 2.5z^{-1} + z^{-2}}$$, it is found to be $X(z)$ is causal signal. Then the value of $x[n]$ at $n = 0$ is

(a) 0 (b) 1
(c) 2 (d) 3

■■■■

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SIGNAL & SYSTEM

EC-EE

Date of Test : 10/06/2024

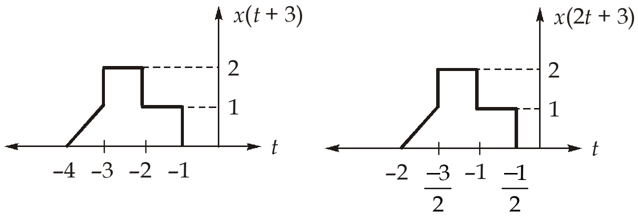
ANSWER KEY

1.	(c)	7.	(c)	13.	(b)	19.	(c)	25.	(d)
2.	(b)	8.	(a)	14.	(b)	20.	(d)	26.	(b)
3.	(a)	9.	(a)	15.	(d)	21.	(a)	27.	(b)
4.	(c)	10.	(c)	16.	(c)	22.	(d)	28.	(a)
5.	(c)	11.	(b)	17.	(c)	23.	(b)	29.	(c)
6.	(b)	12.	(b)	18.	(c)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

1. (c)

The signal $x(2t + 3)$ can be obtained by first shifting $x(t)$ to the left by 3 units and then scaling by 2 units.



2. (b)

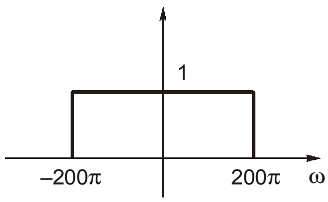
$$e^{-(2t-2)}\,u(t-1) = e^{-2(t-1)}\,u(t-1)$$

Now,
$$e^{-2t}\,u(t) \leftrightarrow \frac{1}{2+j\omega}$$

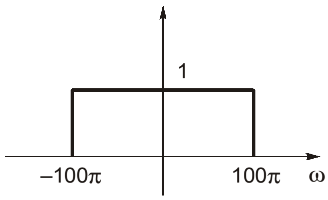
$$e^{-2(t-1)}\,u(t-1) \leftrightarrow \frac{e^{-j\omega}}{2+j\omega}$$

3. (a)

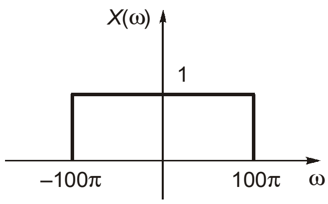
$$\frac{\sin(200\pi t)}{\pi t} \longleftrightarrow$$



$$\frac{\sin(100\pi t)}{\pi t} \longleftrightarrow$$



$$x(t) = \frac{\sin(200\pi t)}{\pi t} * \left(\frac{\sin(100\pi t)}{\pi t}\right) \longleftrightarrow$$



$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega = \frac{1}{2\pi}\int_{-100\pi}^{100\pi} (1)\,d\omega = \frac{200\pi}{2\pi} = 100$$

4. (c)

Given, $x(n) \overset{z}{\longleftrightarrow} X(z)$

by the definition of z-transform,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=0}^{\infty} a^k \delta[n-5k] z^{-n}$$

The term $\delta[n - 5k]$ is equal 1 if $n = 5k$ and equal to zero otherwise.

$$\therefore \quad X(z) = \sum_{k=0}^{\infty} a^k z^{-5k} \quad [\because n = 5k]$$

$$= \frac{1}{1 - az^{-5}}$$

or

$$X(z) = \frac{z^5}{z^5 - a}$$

5. (c)

$$(1 + \cos 300\pi t)^2 \rightarrow f_{1\,max} = 300 \text{ Hz}$$
$$(\sin 4000\pi t)^2 \rightarrow f_{2\,max} = 4000 \text{ Hz}$$
$$f_{max} = f_{1\,max} + f_{2\,max} = 4300 \text{ Hz}$$
$$f_s = 2f_{max} = 8.6 \text{ kHz}$$

6. (b)

Given, $$x(t) = \frac{\sin(10\pi t)}{\pi t}$$

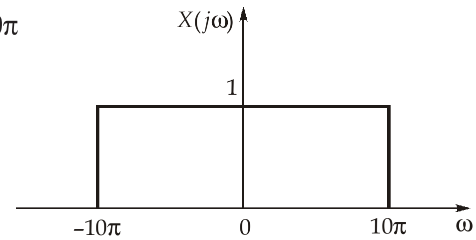
Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 \; ; & |\omega| \le 10\pi \\ 0 \; ; & |\omega| > 10\pi \end{cases}$$

or

$\therefore$ The maximum frequency 'ω_m' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,



$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$$\therefore \quad T_s < \frac{1}{10}$$

7. (c)

Conjugate anti-symmetric part of $x[n]$ is $\frac{x[n] - x^*[-n]}{2}$.

$$x^*[-n] = [2, 1 + j, -2 + j5]$$

$$\therefore \quad \frac{x[n] - x^*[-n]}{2} = \frac{[-2 - j5, 1 - j, 2] - [2, (1 + j), -2 + j5]}{2} = [-2 - j2.5, -\underset{\uparrow}{j}, 2 - j2.5]$$

8. (a)

$$X(e^{j\omega}) = e^{j\omega} + e^{-j\omega} + 2(e^{2j\omega} - e^{-2j\omega}) + 3(e^{3j\omega} + e^{-3j\omega})$$
$$= 2\cos\omega + 4j\sin(2\omega) + 2\cos 3\omega = 2\cos(\pi) + 4j\sin(2\pi) + 6\cos(3\pi)$$
$$= -2 + 0 - 6 = -8$$
$$|Xe^{j\pi}| = 8$$

9. (a)

$$Re\{x(t)\} = \frac{x(t)+x^*(t)}{2}$$

$\therefore$The Fourier coefficient of $x^*(t)$ are

$$b_K = \frac{1}{T}\int_T x^*(t)e^{-jK\frac{2\pi}{T}t}\,dt$$

Taking conjugate on both sides

$$b_K^* = \frac{1}{T}\int_T x(t)e^{-j(-K)\frac{2\pi}{T}t}\,dt$$

$\therefore$ $$a_{-K} = b_K^*$$

$\therefore$ Fourier series Coefficient of $Re\{x(t)\} = \dfrac{a_K + a_{-K}^*}{2}$

10. (c)

Given, $$H(z) = \frac{z}{z-0.2} = \frac{1}{1-0.2z^{-1}} \quad \text{ROC}: |z| > 0.2$$

Since the ROC : $|z| > 0.2$, which includes unit circle.

$\therefore$ The impulse response will be stable.

11. (b)

Given, $$y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty}\delta(t-2k)$$

$$= e^{-t}u(t) * (.....+\delta(t+4)+\delta(t+2)+\delta(t)+\delta(t-2)+\delta(t-4)+.....)$$

Using convolution property of impulse response,

i.e., $$x(t) * \delta(t-t_0) = x(t-t_0)$$

$$y(t) = ...+e^{-(t+4)}u(t+4)+e^{-(t+2)}u(t+2)+e^{-t}u(t)+e^{-(t-2)}u(t-2)+e^{-(t-4)}u(t-4)$$
+ ...

In the range $0 \le t < 2$, we may write $y(t)$ as,

$$y(t) = [...+e^{-(t+4)}u(t+4)+e^{-(t+2)}u(t+2)+e^{-t}u(t)+e^{-(t-2)}u(t-2)+e^{-(t-4)}u(t-4)+...]\,(u(t)-u(t-2))$$

$$= \left(e^{-t}+e^{-(t+2)}+e^{-(t+4)}+......\right);\quad 0 \le t < 2$$

$$= e^{-t}\left(1+e^{-2}+e^{-4}+......\right);\quad 0 \le t < 2$$

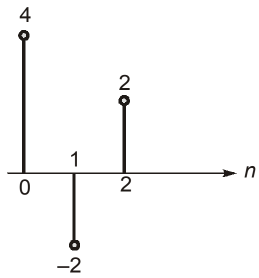
$$= e^{-t}\left[\frac{1}{1-e^{-2}}\right];\quad 0 \le t < 2$$

$\because$ $$y(t) = Ae^{-t} \text{ for } 0 \le t < 2$$

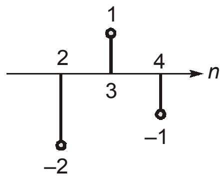
$\therefore$ $$A = \frac{1}{1-e^{-2}}$$

12. **(b)**

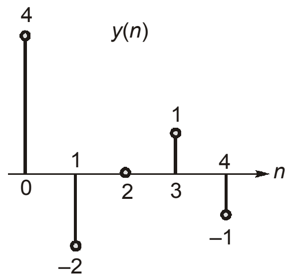
For $x_1[n] = 2\delta[n]$, $y_1(n) = 2y(n) \Rightarrow$



For $x_2[n] = -\delta[n-2]$, $y_2(n) = -y(n-2) \Rightarrow$



$y(n) = y_1(n) + y_2(n) \Rightarrow$



13. **(b)**

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} [f(t)\cos\omega t - jf(t)\sin\omega t]\,dt$$

$$= \int_{-\infty}^{\infty} f(t)\cos\omega t\,dt - j\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt$$

$f(t) \Rightarrow$ even signal
$f(t)\ \cos\omega t \Rightarrow$ even signal
$f(t)\ \sin\omega t \Rightarrow$ odd signal

$$\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt = 0$$

$$\int_{-\infty}^{\infty} f(t)\cos\omega t\,dt = 2\int_{0}^{\infty} f(t)\cos\omega t\,dt$$

$$\therefore \quad F(\omega) = 2\int_{0}^{\infty} f(t)\cos\omega t\,dt$$

14. **(b)**

Given, $X(s) = \log(s+2) - \log(s+3)$

Differentiating both the sides with respect to s

$$\frac{d}{ds}X(s) = \frac{1}{s+2} - \frac{1}{s+3} \quad \text{...(i)}$$

From the properties of Laplace transform, we know that,

$$tx(t) \longleftrightarrow -\frac{d}{ds}X(s)$$

Thus equation (i) can be written as,

$$-tx(t) = [e^{-2t} - e^{-3t}]u(t)$$

or,
$$x(t) = \left[\frac{e^{-3t} - e^{-2t}}{t}\right]u(t)$$

15. (d)

$$C_k = j\delta(k+2) - j\delta(k-2) + 2\delta(k+3) + 2\delta(k-3)$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} C_k e^{jk\pi t}$$

$$= je^{-j2\pi t} - je^{j2\pi t} + 2e^{-j3\pi t} + 2e^{j3\pi t}$$

$$= 4\cos(3\pi t) + 2\sin(2\pi t)$$

16. (c)

Given that,

Let,
$$y_1(t) = 2\pi X(-\omega)\big|_{\omega = t}$$

We have,
$$y_1(t) = 2\pi \int_{u=-\infty}^{\infty} x(u)e^{jut}du$$

Similarly, let $y_2(t)$ be the output due to passing $x(t)$ through 'F' twice.

$$y_2(t) = 2\pi \int_{v=-\infty}^{\infty} 2\pi \int_{u=-\infty}^{\infty} x(u)e^{juv}du\, e^{jtv}dv$$

$$= (2\pi)^2 \int_{u=-\infty}^{\infty} x(u) \int_{v=-\infty}^{\infty} e^{j(t+u)v}dv\, du$$

$$= (2\pi)^2 \int_{u=-\infty}^{\infty} x(u)(2\pi)\delta(t+u)du$$

$$= (2\pi)^3 X(-t)$$

Finally, let $y_3(t)$ be the output due to passing $x(t)$ through F three times

$$y_3(t) = 2\pi \int_{u=-\infty}^{\infty} (2\pi)^3 x(-u)e^{jtu}du$$

$$= (2\pi)^4 \int_{-\infty}^{\infty} e^{-jtu}x(u)du = (2\pi)^4 X(t)$$

17. (c)

The fourier transform of $x(t)$ can be written

$$X_1(j\omega) = |X_1(j\omega)| e^{j\angle X_1(j\omega)}$$

Let, $$X_{1a}(j\omega) = \begin{cases} 1; & |\omega| < 3\pi \\ 0; & \text{otherwise} \end{cases}$$

Note that, given $X_1(j\omega)$ is "$3jw$" times $X_{1a}(j\omega)$

$\therefore$ $$X_1(j\omega) = \begin{cases} 3j\omega; & |\omega| < 3\pi \\ 0; & \text{otherwise} \end{cases}$$

Since, at $$\omega = 3\pi,\ |X_1(j\omega)| = 9\pi \text{ and } \angle X_1(j\omega) = \frac{\pi}{2}$$

$$\omega = -3\pi,\ |X_1(j\omega)| = 9\pi \text{ and } \angle X_1(j\omega) = -\frac{\pi}{2}$$

By taking inverse fourier transform,

Thus, $$x_1(t) = 3\frac{d}{dt}x_{1a}(t)$$ [By using differential property]

also we can express $$x_{1a}(t) = \frac{\sin 3\pi t}{\pi t}$$

Thus, $$x_1(t) = 3\frac{d}{dt}\left[\frac{\sin 3\pi t}{\pi t}\right]$$

$$= \frac{3}{\pi} \times \frac{1}{t^2}[3\pi t \cos 3\pi t - \sin 3\pi t]$$

$\therefore$ $$x_1(t) = \frac{3}{\pi t^2}[3\pi t \cos 3\pi t - \sin 3\pi t]$$

18. (c)

We know that the Laplace transform of

$$\cos(at)u(t) = \frac{s}{s^2 + a^2}$$

$\therefore$ $$\cos(\pi t)u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function $x(t)$ can be written as,

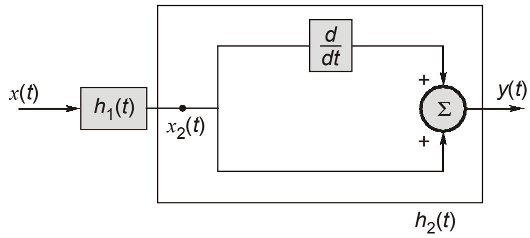
$$x(t) = \cos(\pi t)u(t) - \cos \pi t\, u(t-1)$$
$$= \cos(\pi t)u(t) - \cos \pi(t - 1 + 1)\, u(t-1)$$
$$= \cos(\pi t)u(t) - \cos[\pi(t-1) + \pi]\, u(t-1)$$
$$= \cos \pi\, t u(t) - [\cos \pi(t-1)(-1) - 0]\, u(t-1)$$
$$= \cos \pi\, t u(t) + \cos \pi(t-1)\, u(t-1)$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2 + \pi^2} + \frac{s e^{-s}}{s^2 + \pi^2}$$ [$\because x(t - t_0) = X(s) \cdot e^{-st_0}$, by shifting property]

$$X(s) = \frac{s[1 + e^{-s}]}{s^2 + \pi^2}$$

19. (c)



Let

$$x_2(t) = \delta(t)$$

$$h_2(t) = \left(\delta(t) + \frac{d}{dt}\delta(t)\right)$$

$$h_1(t) = e^{-t}u(t)$$

$$h(t) = e^{-t}u(t) * \left(\delta(t) + \frac{d}{dt}\delta(t)\right)$$

$$h(t) = e^{-t}u(t) * \delta(t) + e^{-t}u(t) * \frac{d}{dt}\delta(t)$$

$$= e^{-t}u(t) + \frac{d}{dt}\left(e^{-t}u(t)\right) * \delta(t)$$

$$= e^{-t}u(t) + \frac{d}{dt}\left(e^{-t}u(t)\right)$$

$$= e^{-t}u(t) - e^{-t}u(t) + e^{-t}\delta(t)$$

$$h(t) = \delta(t) \qquad \because \; e^{-t}\delta(t) = e^{0}\delta(t) = \delta(t)$$

20. (d)

$$H(j\omega) = \frac{1+2e^{-j\omega}}{1+\dfrac{1}{2e^{-j\omega}}} = \frac{1+2e^{-j\omega}}{2e^{-j\omega}+1}\cdot 2e^{-j\omega}$$

$$|H(j\omega)| = 2$$

21. (a)

Given,

$$X(e^{j\omega}) = \frac{\sin\dfrac{3\omega}{2}}{\sin\dfrac{\omega}{2}} = \frac{e^{j3\omega/2}-e^{-j3\omega/2}}{e^{j\omega/2}-e^{-j\omega/2}} = \frac{e^{j3\omega/2}\left[1-e^{-j3\omega}\right]}{e^{j\omega/2}\left[1-e^{-j\omega}\right]}$$

$$= e^{j\omega}\left[\frac{1-e^{-j3\omega}}{1-e^{-j\omega}}\right]$$

$$X(e^{j\omega}) = \frac{e^{j\omega}}{1-e^{-j\omega}} - \frac{e^{-j2\omega}}{1-e^{-j\omega}}$$

by taking inverse DTFT,

$$x[n] = u[n+1] - u[n-2]$$

$$= \begin{cases} 1; & -1 \le n < 2 \\ 0; & \text{otherwise} \end{cases}$$

From parseval's theorem,

$$\sum_{n=-\infty}^{\infty} |nx[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\left[\frac{d}{d\omega}X(e^{j\omega})\right]\right|^2 d\omega$$

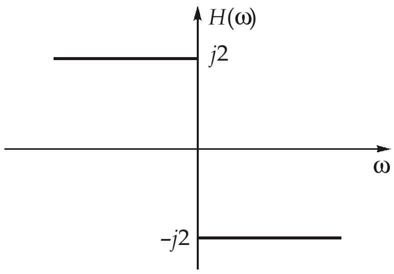
$$\therefore \quad \frac{1}{\pi}\int_{-\pi}^{\pi}\left|\left[\frac{d}{d\omega}X(e^{j\omega})\right]\right|^2 d\omega = 2\sum_{n=-\infty}^{\infty} |nx[n]|^2 d\omega$$

$$= 2\sum_{n=-1}^{1} |n|^2 = 2[1+0+1] = 4$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\left|\left[\frac{d}{d\omega}X(e^{j\omega})\right]\right|^2 d\omega = 4$$

22. (d)

By redrawing the given frequency response, we get,



We can write $H(\omega) = -j2\,\text{sgn}(\omega)$

We know that,

For $\quad \text{sgn}(t) \xleftrightarrow{\text{FT}} \dfrac{2}{j\omega}$

By duality property

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} 2\pi\,\text{sgn}(-\omega)$$

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} -2\pi\,\text{sgn}(\omega)$$

$$\frac{2}{\pi t} \xleftrightarrow{\text{FT}} -j2\,\text{sgn}(\omega)$$

or $\quad = 2(\pi t)^{-1}$

23. (b)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nK}$$

$$g[n] = x[n-2]_{\text{mod N}}$$

$$G[k] = e^{-j\frac{2\pi}{N}(2)k}X[k]$$

$$G[1] = e^{-j\frac{2\pi}{4}(2)1}X[1] = e^{-j\pi}X[1]$$

$$G[1] = -X[1] = -7$$

24. (a)

By the definition of Fourier series,

We can write $C_{N_0/2}$ for N_0 is even,

$$C_{N_0/2} = \frac{1}{N_0}\sum_{n=0}^{N-1} x[n]\,e^{-j\left(\frac{N_0}{2}\right)\left(\frac{2\pi}{N_0}\right)n}$$

$$= \frac{1}{N_0}\sum_{n=0}^{N-1} x[n]\, e^{-j\pi n} = \frac{1}{N_0}\sum_{n=0}^{N-1} (-1)^n x[n] = \text{real}$$

25. (d)

Given, $x(t) = 2 + \cos(50\pi t)$

Frequency of signal $\omega_{sig} = 50\pi$

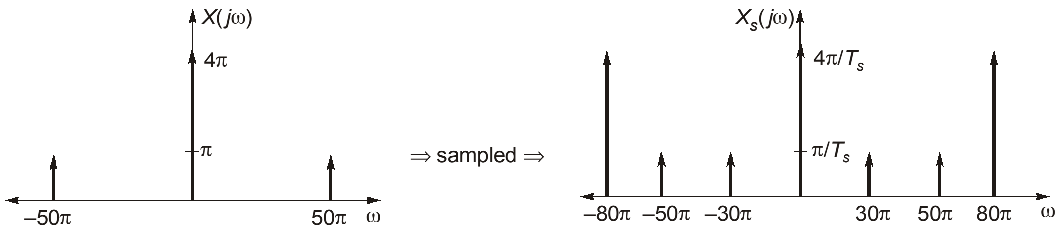
$T_s = 0.025$ sec

$\therefore$ sampling frequency $\omega_s = \frac{2\pi}{T_s} = 80\,\pi$ rad/sec

then, $X(j\omega) = 4\,\pi\delta(\omega) + \pi[\delta(\omega + 50\pi) + \delta(\omega - 50\pi)]$

Let the sampled signal be represented as $X_s(j\omega)$, where $X_s(j\omega)$ is given as

$$X_s(j\omega) = \frac{1}{T_s}\sum_{m=-\infty}^{\infty} X(j(\omega - n\omega_s))$$

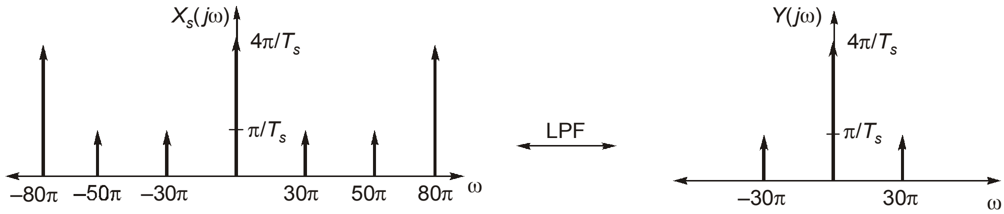


$$X_s(j\omega) =$$

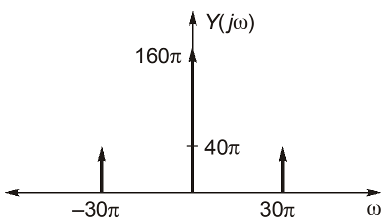
$$40\sum_{m=-\infty}^{\infty} [4\pi\,\delta(\omega - 80\pi)] + \pi\,\delta(\omega - 50\pi - 80\pi n) - \pi\,\delta(\omega + 50\pi - 80\pi n)]$$

now, the sampled input $X_s(j\omega)$ is passed through a low passed filter having cut-off frequency at $\omega = 40\pi$.

Therefore the output $Y(j\omega)$ will contain only the components which are less than $\omega = 40\,\pi$.



Now by putting $T_s = 0.025$, we will get



26. (b)

$$X(z) = \frac{z}{z-1} \qquad |z| > 1$$

$$Y(z) = \frac{2z}{z-\frac{1}{3}} \qquad |z| > \frac{1}{3}$$

$$H(z) = \frac{2(z-1)}{z-\frac{1}{3}} \qquad |z| > \frac{1}{3}$$

$$X'(z) = \frac{z}{z-\frac{1}{2}} \qquad |z| > \frac{1}{2}$$

$$Y'(z) = H(z) \cdot X'(z)$$

$$= \frac{2z(z-1)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} \qquad |z| > \frac{1}{2}$$

Taking inverse z transform

$$y[n] = \left[-6\left(\frac{1}{2}\right)^n + 8\left(\frac{1}{3}\right)^n\right]u[n]$$

$k_1 = -6, \quad k_2 = 8$

so, $\qquad k_1 + k_2 = 2$

27. (b)

If $x[n]$ is real

$$\text{odd}[x[n]] \xrightarrow{FT} jIm[X(e^{j\omega})]$$

$$\therefore \qquad \text{odd}[x[n]] = F^{-1}\left[\frac{1}{2}\left(e^{j\omega} - e^{-j\omega} - e^{2j\omega} + e^{-2j\omega}\right)\right]$$

$$= \frac{1}{2}\left[\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]\right]$$

$$\because \qquad \text{odd } [x[n]] = \frac{x[n] - x[-n]}{2}$$

Since, $\qquad x[n] = 0$ for $n > 0$

$x[n] = 2\ \text{odd}[x[n]]$

$= \delta[n+1] - \delta[n+2]$ for $n < 0$

using parshevel's theorem

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|X\left(e^{j\omega}\right)\right|^2 d\omega = \sum_{n=-\infty}^{\infty}|x[n]|^2$$

$$(x[0])^2 - 2 = 3$$

$$x[0] = \pm 1$$

$$\because \quad x[0] > 0$$

$$\therefore \quad x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$$

28. (a)

$$x[n] = \delta[n]$$

$$X(e^{j\omega}) = 1$$

$$\frac{dX(e^{j\omega})}{d\omega} = 0$$

$$\therefore \quad Y(e^{j\omega}) = e^{-j\omega}\, X(e^{j\omega})$$

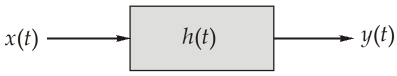
$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega}.e^{j\omega n}\, d\omega = \frac{\sin \pi(n-1)}{\pi(n-1)}$$

29. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3+j\omega}$$

and output, $\quad y(t) = e^{-3t}\, u(t) - e^{-4t}\, u(t)$



We know that, $\quad H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$$\therefore \quad X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

By inverse Fourier transform of $X(j\omega)$, we have,

$$x(t) = e^{-4t}\, u(t)$$

30. (b)

Given, $\quad X(z) = \dfrac{1}{1-2.5z^{-1}+z^{-2}} = \dfrac{1}{(z^{-1}-2)\left(z^{-1}-\frac{1}{2}\right)}$

$$\frac{1}{(z^{-1}-2)\left(z^{-1}-\frac{1}{2}\right)} = \frac{A}{z^{-1}-2} + \frac{B}{z^{-1}-\frac{1}{2}}$$

$$\therefore \quad A = \frac{1}{2-\frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

$$B = \frac{1}{\frac{1}{2}-2} = -\frac{2}{3}$$

$$\therefore \qquad X(z) = \frac{\frac{2}{3}}{z^{-1}-2} + \frac{-\frac{2}{3}}{z^{-1}-\frac{1}{2}}$$

$$= \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{4}{3}}{1-2z^{-1}}$$

Given $X(z)$ is a causal system, the ROC is right of the right most pole.

$\therefore\ |z| > 2$

hence, $$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{4}{3}(2)^n u[n]$$

$$\therefore \qquad x(0) = -\frac{1}{3}+\frac{4}{3} = \frac{3}{3} = 1$$

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