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ENGINEERING MECHANICS

MECHANICAL ENGINEERING

Date of Test : 22/06/2024

ANSWER KEY

1.	(b)	7.	(b)	13.	(b)	19.	(d)	25.	(b)
2.	(b)	8.	(a)	14.	(c)	20.	(a)	26.	(d)
3.	(c)	9.	(b)	15.	(c)	21.	(c)	27.	(a)
4.	(c)	10.	(a)	16.	(a)	22.	(b)	28.	(b)
5.	(d)	11.	(b)	17.	(c)	23.	(a)	29.	(a)
6.	(d)	12.	(a)	18.	(d)	24.	(c)	30.	(b)

DETAILED EXPLANATIONS

1. (b)

As per given information,

$$h = 40 \text{ m}, u = 50 \text{ m/s}$$

Let the speed be 'v' when it strike to the ground

Apply law of conservation of energy

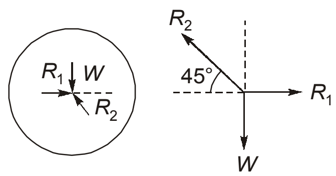
$$mgh + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

$$m \times 10 \times 40 + \frac{1}{2} \times m \times (50)^2 = \frac{1}{2} \times m \times v^2$$

$$400 + 1250 = \frac{v^2}{2}$$

$$v = 57.44 \text{ m/s}$$

2. (b)



$$R_2 \cos 45° = R_1$$

$$R_2 \sin 45° = W$$

$$\Rightarrow \quad R_2 = W\sqrt{2}$$

$$\therefore \quad R_1 = W\sqrt{2} \times \frac{1}{\sqrt{2}} = W$$

$$W = 50 \text{ N}$$

$$\therefore \quad R_1 = 50 \text{ N}$$

3. (c)

$$\text{Lagrangian, } L = T - V$$

$$= v^2\dot{u}^2 + 2\dot{v}^2 - u^2 + v^2$$

$$= v^2\left(1 + \dot{u}^2\right) + \left(2\dot{v}^2 - u^2\right)$$

The equation of motion, using langrangian (L) for $q = u$,

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{u}}\right) - \frac{\partial L}{\partial u} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left(2v^2\dot{u}\right) - (-2u) = 0$$

$$\Rightarrow \quad 2\left[v^2\ddot{u} + 2v\dot{v}\dot{u}\right] + 2u = 0$$

$$\Rightarrow \quad 2v^2\ddot{u} + 4v\dot{v}\dot{u} + 2u = 0$$

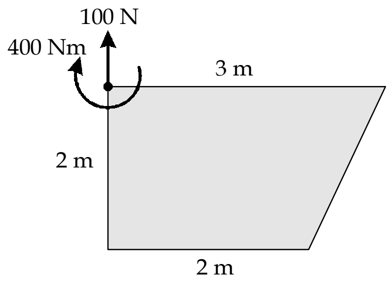
4. (c)

Force in member AH should be zero, as the AH is corner member with only two member connected to each other at 90°. Hence in both member AH and GH force is zero.

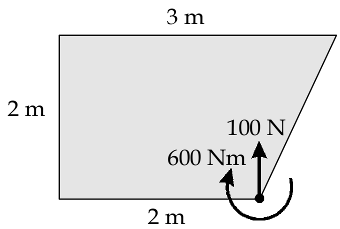
5. **(d)**

6. **(d)**

Force-couple system,



Equivalent force couple system,



7. **(b)**

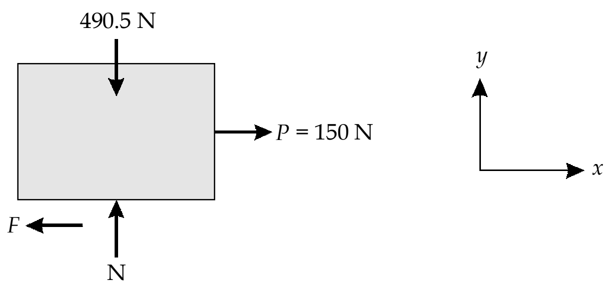
When body is at rest, for equilibrium,

$$N = 490.5 \text{ N}$$

Applied force, $P = 150$ N

Maximum static friction force,

$$F_{max} = \mu_s N$$
$$= 0.5\ (490.5)$$
$$= 245.25 \text{ N}$$



Because $P < F_{max}$, we conclude that the block is in static equilibrium and correct value of friction force is,

$$F = 150 \text{ N}$$

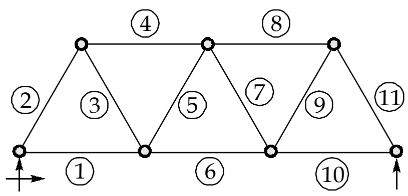
8. **(a)**

Torque,
$$T = mg \times \frac{L}{2}$$

$$I_0 = \frac{mL^2}{3}$$

$$\alpha = \frac{T}{I_0} = \frac{mgL}{2} \times \frac{3}{mL^2} = \frac{1.5g}{L}$$

9. **(b)**



Total number of members, $i = 11$

Number of reactions, $r = 3$

Total number of joints $= 2 \times j = 2 \times 7 = 14$

$$i + r = 2j$$

$$11 + 3 = 2 \times 7$$

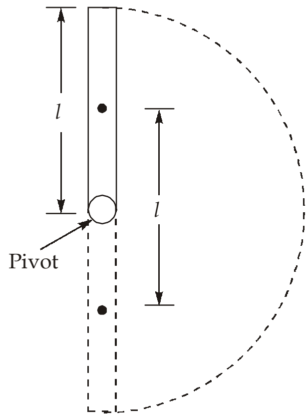
$$14 = 14$$

Therefore, the truss is stable and internally determinate.

10. (a)

As the rod reaches it lowest position, the center of mass is lowered by a distance l. Its gravitational potential energy is decreased by mgl.

Rotation occurs about the horizontal axis through the clamped end.



Moment of inertia, $I = \dfrac{ml^2}{3}$

Now, by work energy theorem;

Total work done = Change in kinetic energy

$$(\Delta W)_{mgl} = (KE)_f - (KE)_i$$

$$mgl = \frac{1}{2}I\omega^2 - 0$$

$$\frac{1}{2}I\omega^2 = (mgl)$$

$$\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 = (mgl)$$

$$\omega^2 = \frac{6g}{l}$$

$$\omega = \sqrt{\frac{6g}{l}}$$

Linear speed of the free end at given instant, $v = l\omega$

$$V = l \times \sqrt{\frac{6g}{l}}$$

$$V = \sqrt{6gl}$$

11. (b)

As per given information,

$m = 30$ kg; $\quad r = 0.2$ m

$\omega = 20$ rad/s; $\quad T = 5$ Nm

$F = 10$ N

$$I = \frac{1}{2}mr^2 = \frac{1}{2} \times 30 \times 0.2^2 = 0.6 \text{ kg.m}^2$$

Let the disk rotate an angle of θ rad.

From work energy principle

$$T \cdot \theta + F \times r \cdot \theta = \frac{1}{2} \times I \times \omega^2 \qquad [\because \text{ Workdone = change in energy}]$$

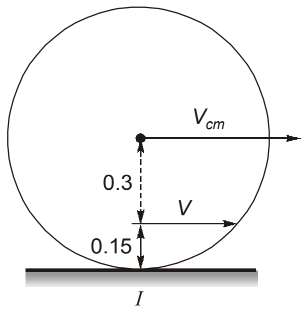
$$5\cdot\theta + 10 \times 0.2 \times \theta = \frac{1}{2}\times 0.6 \times (20)^2$$

$$7\cdot\theta = 120$$

$$\theta = 17.14 \text{ rad}$$

$$\text{Number of revolution} = \frac{\theta}{2\pi} = \frac{17.14}{2\pi} = 2.73 \text{ rev}$$

12. (a)



$$V_{cm} = 0.45\,\omega$$

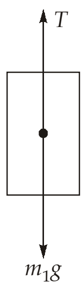
$$\omega = \frac{3}{0.45} \text{ rad/s}$$

$$V = 0.15\,\omega = \frac{0.15\times 3}{0.45}$$

$$V = 1 \text{ m/s}$$

13. (b)

Cylinder

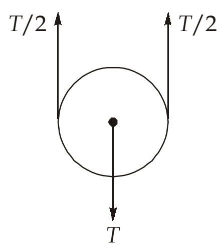


From Newton's first law,

$$m_1g - T = 0$$

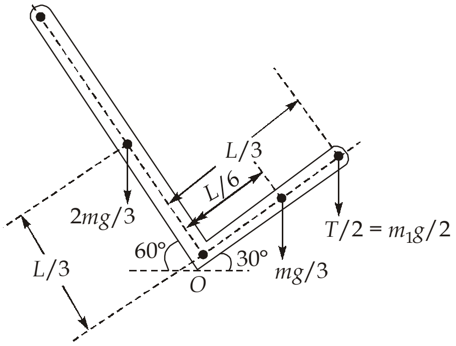
$$T = m_1g$$

Pulley



$$\frac{T}{2} = \frac{m_1g}{2}$$

To cause loss of contact at A, reaction at A will be zero.



$\Sigma M_o = 0$

$$\frac{2mg}{3}\times\frac{L}{3}\cos 60° - \frac{mg}{3}\times\frac{L}{6}\cos 30° - \frac{T}{2}\times\frac{L}{3}\cos 30° = 0$$

$$\Rightarrow \quad \frac{2mg}{9}gL\cos 60° = \frac{m}{18}gL\cos 30° + \frac{m_1 g}{2}\times\frac{L}{3}\cos 30°$$

$$\Rightarrow \quad \frac{2m}{9}\cos 60° = \frac{m}{18}\cos 30° + \frac{m_1}{6}\cos 30°$$

$$m_1 = 0.436 \text{ m}$$

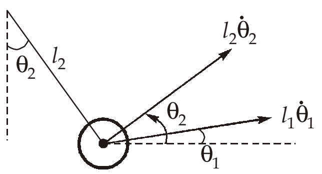
14. (c)

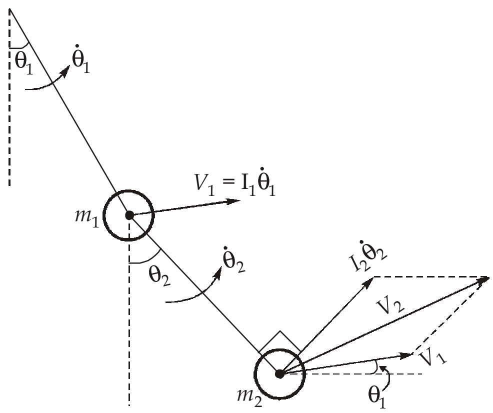
Let the general co-ordinates are $q_1 = \theta_1$ and $q_2 = \theta_2$, c [two degree of freedom so two general co-ordinates]

$$L = T - V$$

$$T = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

where $v_1^2 = (l_1\dot{\theta}_1)^2$ and V_2 is resultant of $l_1\dot{\theta}_1$ and $l_2\dot{\theta}_2$





By using parallelogram law,
Resultant velocity V_2

$$\Rightarrow \quad V_2^2 = l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1 l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_2 - \theta_1)$$

Here, $\theta_2 - \theta_1 \approx 0$

$\Rightarrow \quad \cos(\theta_2 - \theta_1) \approx 1$

So, total kinetic energy, $T = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\left[l_1^2\dot{\theta}_1^2 + l_2\dot{\theta}_2^2 + 2l_1 l_2\dot{\theta}_1\dot{\theta}_2\right]$ ...(i)

Potential energy change due to θ_1 and θ_2.

$$\Rightarrow \quad V = m_1 g l_1(1 - \cos\theta_1) + m_2 g[l_1(1 - \cos\theta_1) + l_2(1 - \cos\theta_2)]$$

$$\Rightarrow \quad V = m_1 g l_1 \frac{\theta_1^2}{2} + \frac{m_2 g}{2}[l_1\theta_1^2 + l_2\theta_2^2] \qquad \text{...(ii)}$$

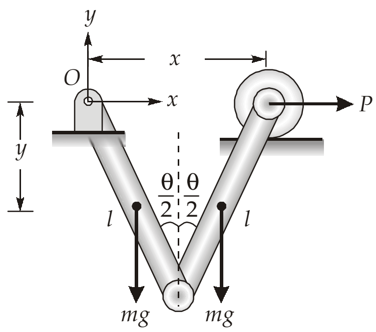
$$\left[\because\ 1 - \cos\theta = 2\sin^2\frac{\theta}{2} \approx \frac{\theta^2}{2}\right]$$

From equation (i) and (ii)

$$\text{Lagrangian, } L = T - V$$

$$= \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\left[l_2^2\dot{\theta}_2^2 + l_1^2\dot{\theta}_1^2 + 2l_1 l_2\dot{\theta}_1\dot{\theta}_2\right] - \left[m_1 g l_1\frac{\theta_1^2}{2} + \frac{m_2 g}{2}\left(l_1\theta_1^2 + l_2\theta_2^2\right)\right]$$

15. (c)



$$x = 2l\sin\frac{\theta}{2}$$

$$\partial x = l\cos\frac{\theta}{2}\partial\theta$$

$$y = -\frac{l}{2}\cos\frac{\theta}{2}$$

$$\partial y = +\frac{l}{4}\sin\frac{\theta}{2}\partial\theta$$

$$+P(\partial x) + (-2mg)\cdot\partial y = 0$$

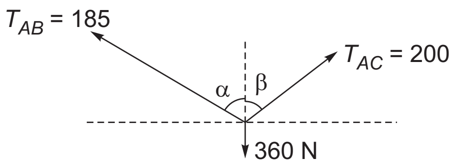
$$P\left(l\cos\frac{\theta}{2}\partial\theta\right) - 2mg\left(\frac{l}{4}\sin\frac{\theta}{2}\partial\theta\right) = 0$$

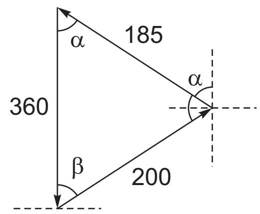
$$Pl\cos\frac{\theta}{2}\partial\theta = 2mg \times \frac{l}{4}\sin\frac{\theta}{2}\partial\theta$$

$$\tan\frac{\theta}{2} = \frac{2P}{mg}$$

$$\theta = 2\tan^{-1}\left(\frac{2P}{mg}\right)$$

16. (a)





Applying cosine rule

$$185^2 = 360^2 + 200^2 - 2 \times 360 \times 200 \cos\beta$$
$$\beta = 19.93°$$
$$200^2 = 360^2 + 185^2 - 2 \times 360 \times 185 \cos\alpha$$
$$\alpha = 21.62°$$

17. (c)

Moment about the point '*c*'

$$\text{(Vector method), } M_c = \vec{r} \times \vec{F}$$

$$\text{Force vector, } \vec{F} = 500 \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$

$$= 500\left(\frac{2\hat{i} - 4\hat{j} + 3\hat{k}}{\sqrt{(2)^2 + (-4)^2 + (3)^2}}\right)$$

$$= 92.847\left(2\hat{i} - 4\hat{j} + 3\hat{k}\right)$$

$$\text{Position vector, } r_{CA} = -2\hat{i} - 0\hat{j} + 0\hat{k}$$

$$M_C = \vec{r}_{CA} \times \vec{F}$$

$$= \left(-2\hat{i}\right)[92.847\left(2\hat{i} - 4\hat{j} + 3\hat{k}\right)]$$

$$M_C = 92.847\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 0 \\ 2 & -4 & 3 \end{vmatrix}$$

$$= 92.847\left(6\hat{j} + 8\hat{k}\right)$$

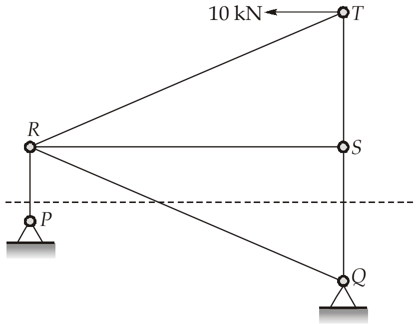
$$= 557.086\hat{j} + 742.776\hat{k}$$

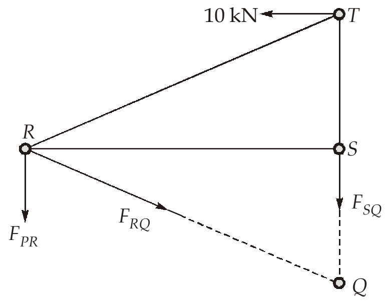
$$\text{Magnitude, } M_C = \sqrt{(557.086)^2 + (742.776)^2}$$

$$M_C = 928.47 \text{ Nm}$$

18. **(d)**

From method of section,





Taking moment about Q,

$$10 \times 6 + F_{PR} \times 4 = 0$$

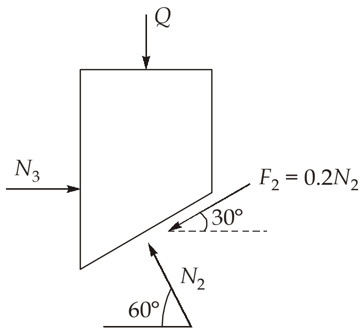
$$F_{PR} = -\frac{60}{4} = -15 \text{ kN}$$

$$= 15 \text{ kN (Compressive)}$$

19. **(d)**

From Newton's first law,

$\Sigma F_y = 0$



$$N_2 \sin 60° - 0.2N_2 \sin 30° - Q = 0$$

$$Q = 0.766\ N_2$$

$\Sigma F_X = 0$

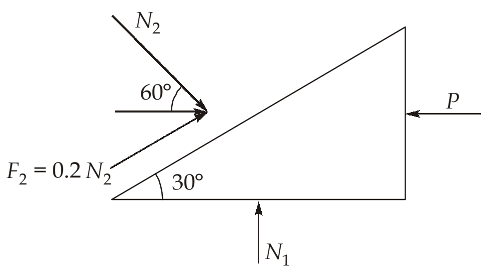
$$N_2 \cos 60° + 0.2N_2 \cos 30° - P = 0$$

$$P = 0.673\ N_2$$

$$\frac{P}{Q} = \frac{0.673}{0.766}$$

$$P = 0.878Q \simeq 0.9Q$$

$$\alpha = 0.9$$



20. (a)

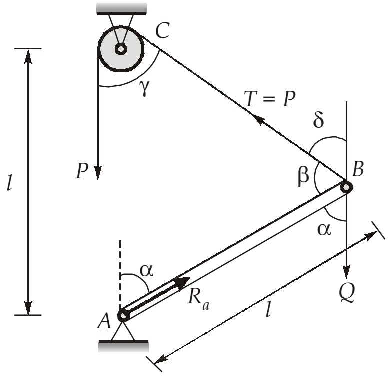
There are three forces acting on the bar AB; pull Q at B, tension in string T and reaction at point A i.e. R_a.

For isosceles triangle ABC,

$$\beta = \gamma = \left(\frac{\pi - \alpha}{2}\right) = 90° - \left(\frac{\alpha}{2}\right)$$

If there is no friction on pulley, tension in string BC will be P.

Taking moment about point A,



$$(P\cos\delta) \times (l\sin\alpha) + (P\sin\delta)(l\cos\alpha) = Ql\sin\alpha$$

$$Pl\sin(\alpha + \delta) = Ql\sin\alpha$$

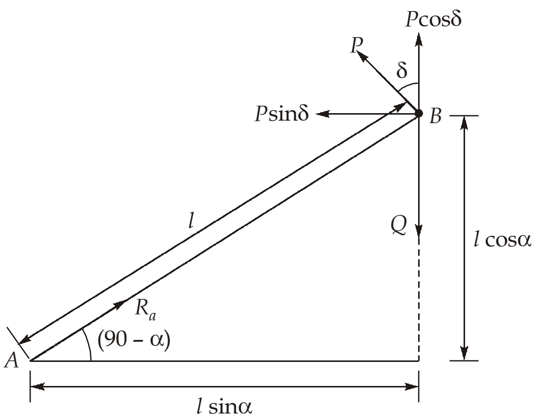
$$P\sin(180° - \beta) = Q\sin\alpha$$

$$P\sin\left[180 - 90 + \frac{\alpha}{2}\right] = Q\sin\alpha$$

$$P\cos\frac{\alpha}{2} = 2Q\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$\left(\cos\frac{\alpha}{2}\right)\left[P - 2Q\sin\frac{\alpha}{2}\right] = 0$$

or $$\sin\frac{\alpha}{2} = \frac{P}{2Q}$$



$$\alpha = 2\sin^{-1}\left(\frac{P}{2Q}\right) = 2\sin^{-1}\left(\frac{900}{2 \times 2200}\right) = 23.6057°$$

$$\alpha = 23.6057 \times \left(\frac{\pi}{180}\right) = 0.412 \text{ radian}$$

21. (c)

Given: M = 2000 kg, v_1 = 2 m/s, v_2 = 0, for drum m = 50 kg, k = 0.7 m, R = 0.75 m, h = 0.5 m

$$\Delta kE \text{ of mass} = \frac{1}{2}M\left(v_1^2 - v_2^2\right) = \frac{1}{2} \times 2000 \times \left(2^2 - 0^2\right) = 4000 \text{ J}$$

$$\Delta kE \text{ of drum} = \frac{1}{2}mk^2\left(\omega_1^2 - \omega_2^2\right) = \frac{1}{2} \times 50 \times 0.7^2 \times \left[\left(\frac{2}{0.75}\right)^2 - 0^2\right] = 87.11 \text{ J}$$

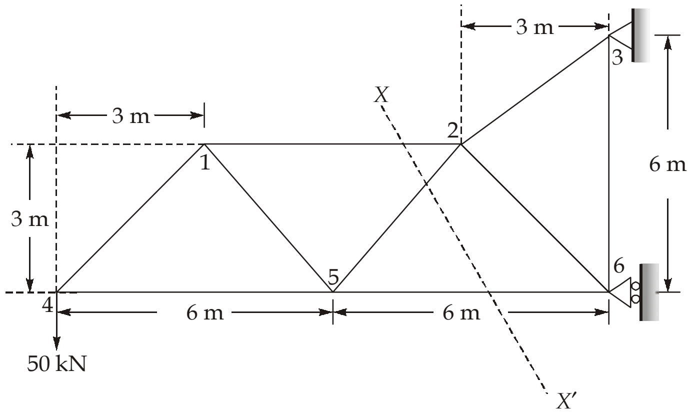
$$\Delta kE \text{ of the mass} = mgh = 2000 \times 9.81 \times 0.5 = 9810 \text{ J}$$

the total energy absorbed by the break is given by

$$E = \Delta kE \text{ of mass} + \Delta kE \text{ of drum} + \Delta PE \text{ of mass}$$
$$= 4000 + 87.11 + 9810$$
$$E = 13897.11 \text{ J} \simeq 13897 \text{ J}$$

22. (b)

Cutting section about, 1-2, 2-5 and 5-6:



Taking moment of left part about 5.

$$F_{1\text{-}2} = \frac{50 \times 6}{3} = 100 \text{ kN(T)}$$

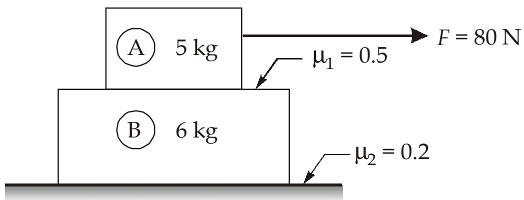
Cutting section through 1-2, 1-5 and 4-5 and balancing vertical forces for left part only,

$\Sigma F_V = 0$

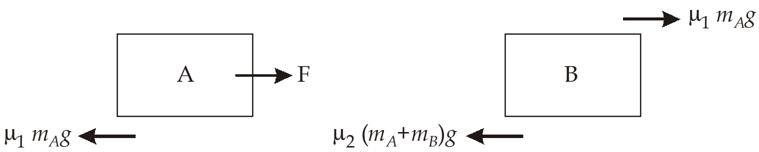
$$\Rightarrow \quad F_{1\text{-}5} \times \cos 45° + 50 = 0$$
$$F_{1\text{-}5}\cos 45° = -50$$
$$\frac{F_{1-5}}{\sqrt{2}} = -50 \Rightarrow F_{1\text{-}5} = 50\sqrt{2} \text{ kN(C)}$$

23. (a)

According to question:



FBD of (A) and (B)



Equation of motion for A,

$$F - \mu_1 m_A g = m_A a_A$$
$$80 - 0.5(5)(9.81) = (5)a_A$$
$$a_A = 11.095 \text{ m/s}^2$$

So, $V_A = U_A + a_A t = 0 + 11.095\ (0.1)$
$= 1.1095$ m/s

Now, equation of motion for B,

$$\mu_1 m_A g - \mu_2 (m_A + m_B) g = m_B a_B$$

$0.5(5)(9.81) - 0.2(5 + 6)9.81 = 6(a_B)$

$a_B = 0.4905$ m/s^2

So, $V_B = U_B + a_B t$
$= 0 + 0.4905(0.1) = 0.04905$ m/s

∴ Relative velocity of A with respect to B = $V_A - V_B$
$= 1.1095 - 0.04905$
$= 1.06045$ m/s

24. (c)

$I = 2000 \times 0.25^2 = 125$ kg-m^2

for retardation, $\omega = \omega_0 + \alpha t$

$\omega = 0$

$$\omega_0 = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60}$$

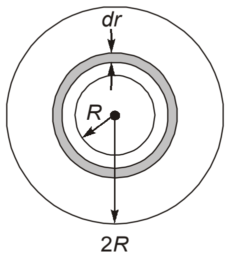
$t = 10$ min $= 600$ sec

$$\alpha = \frac{2\pi \times 3000}{60 \times 600} = 0.5236 \text{ rad/s}$$

So, average frictional torque,

$I\alpha = 65.44$ Nm

25. (b)



∴ I = Moment of inertia of disc

Total mass m is contained in area $3\pi R^2$

Let dm be the mass in area $2\pi r\, dr$

$$\therefore \quad dm = \frac{m \times 2\pi r dr}{3\pi R^2}$$

$$I = \int_R^{2R} dm\, r^2$$

$$\therefore \quad I = \frac{5}{2} m R^2$$

$$\text{K.E.} = (\text{K.E.})_{translation} + (\text{K.E.})_{rotation} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

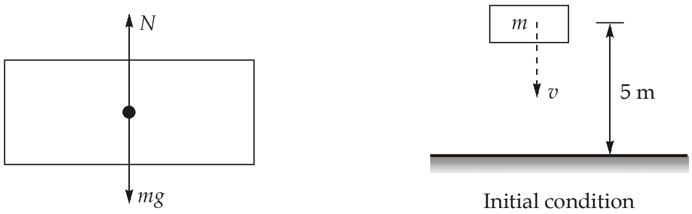
But $V = 2R\omega$

$$\therefore \quad \text{K.E.} = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{5R^2}{2}\times\frac{V^2}{4R^2} = \frac{1}{2}mv^2 + \frac{5}{16}mv^2$$

$$\text{K.E.} = \frac{8mv^2 + 5mv^2}{16} = \frac{13}{16}mv^2 = \frac{13}{16}\times 5\times(2)^2$$

$$\therefore \quad \text{K.E.} = 16.25 \text{ J}$$

26. (d)



Initial condition

Velocity when block reaches the ground = $\sqrt{2gh}$

$$= \sqrt{2\times 10\times 5} = 10 \text{ m/s}$$

By momentum conservation:

$(F) \times dt$ = Momentum just after striking the ground – momentum just before striking the ground

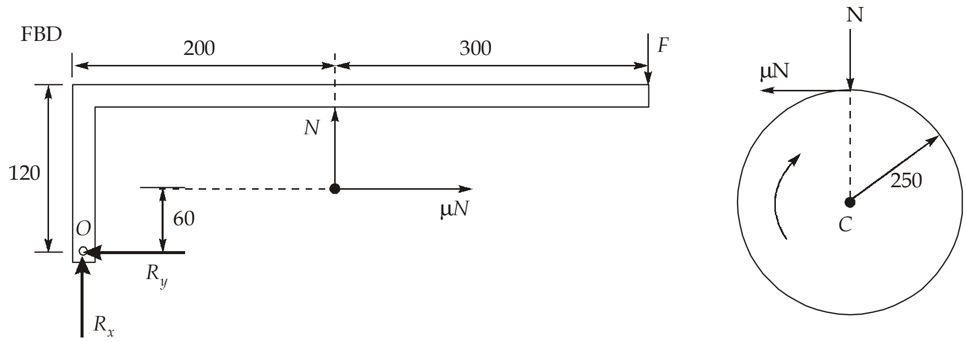
$$(N - mg)\times dt = m\times 0 - (-m\times 10)$$

$$(N - mg) = \frac{m\times 10}{dt}$$

$$N = \frac{10\times 10}{(1/10)} + 10\times 10$$

Force of interaction, $N = 1100$ N

27. (a)



For the drum, about C

$$\mu N \times 0.250 = 30$$

$$N = 400 \text{ N}$$

For the link,

$\Sigma T_{net\_O} = 0$: $\mu N \times 60 + F \times 500 = N \times 200$

$$F = \frac{N\times 200 - \mu N\times 60}{500}$$

$$F = \frac{400\times 200 - 0.3\times 400\times 60}{500}$$

$$F = 145.6 \text{ N}$$

28. (b)

$$mg(\sin\theta + \mu\cos\theta) = 3mg(\sin\theta - \mu\cos\theta)$$

$$(\sin 45° + \mu\cos 45°) = 3(\sin 45 - \mu\cos 45°)$$

$$\mu = 0.5$$

29. (a)

Taking all as a system

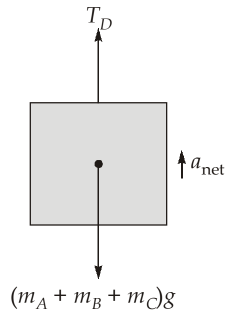
$$F_{net} = (m_A + m_B + m_C)\, a_{net}$$

$$a_{Net} = \frac{m_C a_C + m_B \times 0 + m_A a_A}{m_A + m_B + m_C}$$

$$a_C = +3 \text{ m/s}^2$$

$$a_A = -5 \text{ m/s}^2$$

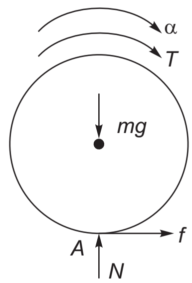
$$\Rightarrow \quad a_{net} = -\frac{7}{6}\text{m/s}^2$$



$$\Rightarrow \quad T_D - 30 \times g = (m_A + m_B + m_C)a_{net} = -30 \times \frac{7}{6}$$

$$\Rightarrow \quad T_D = 265 \text{ N}$$

30. (b)



$$I_A = I_{com} + md^2$$

$$= \frac{2}{5}\text{mR}^2 + \text{mR}^2$$

$$= \frac{7}{5}\text{mR}^2$$

for pure rolling, $a_{com} = \alpha R$

$$\Sigma T_A = I_A\,\alpha$$

$$T = \frac{7}{5}\text{mR}^2\,\alpha$$

$$T = \frac{7}{5}\text{mR a}_{com}$$

$$a_{com} = \frac{5T}{7\text{ mR}} \quad \text{...(i)}$$

$$\Sigma F_x = \text{m a}_{com}$$

$$f = \text{m}\cdot\frac{5T}{7\text{ mR}}$$

From equation (i)

$$= \frac{5T}{7\text{R}} = \frac{5 \times 14}{7 \times 0.5}$$

$$f = 20 \text{ N}$$

■■■■