CLASS TEST

S.No.:02_CE_DEF_17062024

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

SOLID MECHANICS

CIVIL ENGINEERING

Date of Test : 17/06/2024

ANSWER KEY

1.	(a)	7.	(d)	13.	(a)	19.	(c)	25.	(a)
2.	(c)	8.	(c)	14.	(a)	20.	(a)	26.	(d)
3.	(d)	9.	(a)	15.	(b)	21.	(a)	27.	(b)
4.	(a)	10.	(b)	16.	(c)	22.	(a)	28.	(a)
5.	(c)	11.	(c)	17.	(c)	23.	(a)	29.	(a)
6.	(a)	12.	(c)	18.	(a)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

The length of column is very large as compared to its cross-sectional dimensions.

2. (c)

As per Rankine theory:

$$\frac{1}{P_R} = \frac{1}{P_{CS}} + \frac{1}{P_E}$$

$$P_R = \frac{P_{CS} \times P_C}{P_{CS} + P_E} = \frac{P_{CS}}{1 + \frac{P_{CS}}{P_E}}$$

$\because$ $$P_{SC} = \sigma_{CS} A$$

$$P_E = \frac{\pi^2 EI}{Le^2}$$

$$P_R = \frac{P_{CS}}{1 + \frac{\sigma_{CS} A}{\frac{\pi^2 EI}{Le^2}}} = \frac{P_{CS}}{1 + \frac{\sigma_{CS} A Le^2}{\pi^2 EI}}$$

$\therefore$ $$k = \sqrt{\frac{I}{A}}$$

$\Rightarrow$ $$k^2 = \frac{I}{A}$$

Hence $$P_R = \frac{P_{CS}}{1 + \frac{\sigma_{CS}}{\pi^2 E}\left(\frac{Le}{k}\right)^2}$$

i.e., option (c) is correct.

3. (d)

We know stress in rod, when load is applied suddenly,

$$\sigma = \frac{2P}{A} = \frac{2 \times 20 \times 10^3}{1000} = 40 \text{ N/mm}^2$$

$$\text{Volume of rod} = l.A. = 2.5 \times 10^3 \times 10^3$$

$$= 2.5 \times 10^6 \text{ mm}^3$$

Strain energy which can be absorbed by the rod,

$$U = \frac{\sigma^2}{2E} \cdot V = \frac{40^2 \times 2.5 \times 10^6}{2 \times 200 \times 10^3}$$

$$= 10 \times 10^3 \text{ N-mm}$$

$$= 10 \text{ N-m}$$

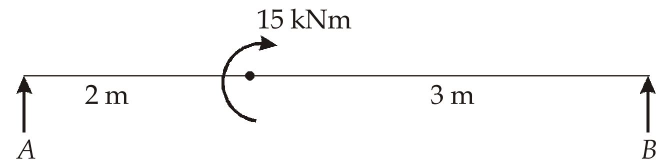
4. (a)

$$y_{max} = \frac{Wa\left(l^2 - a^2\right)^{3/2}}{9\sqrt{3}\,EIl}$$

This value of maximum deflection occurs at x from A

$$x = \sqrt{\frac{l^2 - a^2}{3}}$$

5. (c)



$$R_A + R_B = 0$$

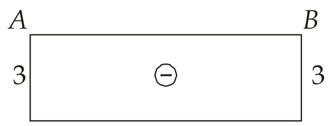
$$\Sigma M_A = 0$$

$$\Rightarrow \quad R_B \times 5 = 15$$

$$\Rightarrow \quad R_B = 3 \text{ kN}$$

$$R_A = -3 \text{ kN}$$

Now the for SFD will be as shon below.



6. (a)

$$\Delta = \frac{wl^2}{2E}$$

$$= \frac{\left(89.2 \times 10^{-6}\right) \times \left(15 \times 10^3\right)^2}{2 \times \left(90 \times 10^3\right)} = 0.11 \text{ mm}$$

7. (d)

- Vertical drop at B shows point load on the loaded beam having magnitude equal to (14 + 4) = 18 kN.
- 1° shear force diagram between B and D represents. Uniformly distributed load. Whose magnitude is

$$\frac{(-4) - (-16)}{8} = \frac{12}{8} 1.5 \text{ kN/m}$$

or

$$\frac{9 - (+3)}{4} = \frac{6}{4} = 1.5 \text{ kN/m}$$

8. (c)

$$\text{Poisson's ratio, } \mu = \frac{3K - 2G}{6K + 2G}$$

$$= \frac{3 \times 6.93 \times 10^4 - 2 \times 2.65 \times 10^4}{6 \times 6.93 \times 10^4 + 2 \times 2.65 \times 10^4} = 0.33$$

9. (a)

Moment at E = 0,

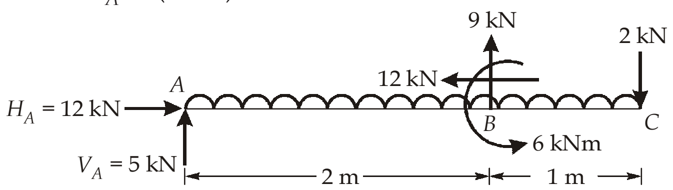
$$\Rightarrow \quad H_A \times 2 = \frac{4 \times 3^2}{2} + 2 \times 3,$$

$$\Rightarrow \quad H_A = 12 \text{ kN} = H_E$$

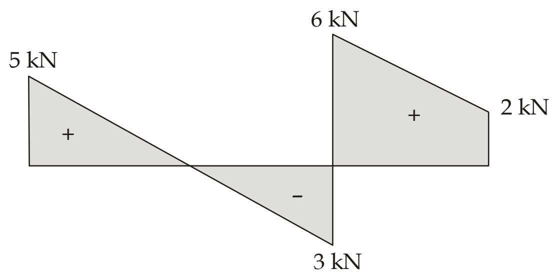
$$\tan\theta = \frac{V_E}{H_E} = \frac{1.5}{2} = \frac{3}{4};$$

$$\therefore \quad V_E = \frac{3}{4} H_E = 9 \text{ kN},$$

$$\therefore \quad V_A = (4 \times 3) + 2 - 9 = 5 \text{ kN}$$



SFD

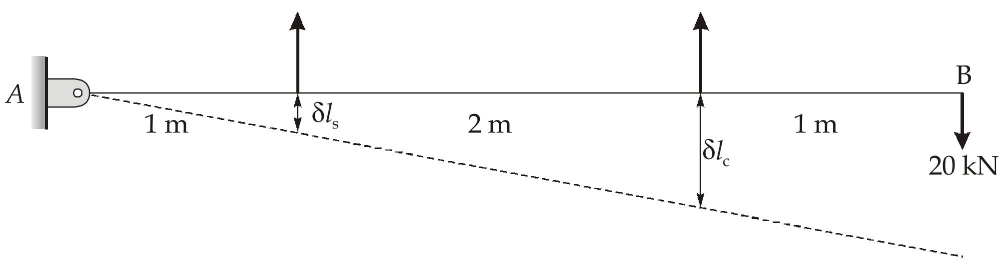


$\therefore$ Maximum S.F. = 6 kN

10. (b)

If a force acts on a body, then resistance to the deformation is known as stress.

11. (c)



Let P_s = Load shared by steel rod

and P_c = Load shared by copper rod

Now elongation of steel rod due to load P_s

$$\delta l_s = \frac{P_s l_s}{A_s E_s} = \frac{P_s \times 1 \times 10^3}{200 \times 200 \times 10^3} = 0.025 \times 10^{-3} P_s$$

and elongation of copper rod due to load P_C

$$\delta l_c = \frac{P_c l_c}{A_c E_c} = \frac{P_C \times 2 \times 10^3}{400 \times 100 \times 10^3} = 0.05 \times 10^{-3} P_c$$

From the geometry of elongation of the steel rod and copper rod

$$\frac{\delta l_C}{3} = \delta l_s$$

$$\Rightarrow \qquad \delta l_c = 3\delta l_s$$

$$\Rightarrow \qquad 0.05 \times 10^{-3} P_c = 3 \times 0.025 \times 10^{-3}\, P_s$$

$$\Rightarrow \qquad P_c = 1.5\, P_s$$

$$\frac{P_c}{P_s} = 1.5$$

12. (c)

Length (l) = 16 m, A = 4 mm^2

Weight of wire ABC, W = 20 N

E = 200 GPa

Deflection at B consists of deflection of wire AB due to self weight plus deflection due to weight of wire BC.

Now deflection of wire at B due to self weight of wire AB.

$$\delta l_1 = \frac{\frac{W}{2} \cdot \frac{l}{2}}{2AE} = \frac{10 \times 8 \times 10^3}{2 \times 4 \times 200 \times 10^3} = 0.05 \text{ mm}$$

and deflection of wire at B due to weight of wire BC

$$\delta l_2 = \frac{\frac{W}{2} \cdot \frac{l}{2}}{AE} = \frac{10 \times 8 \times 10^3}{4 \times 200 \times 10^3} = 0.1 \text{ mm}$$

Total deflection of wire at B = $\delta l_1 + \delta l_2$ = 0.05 + 0.1 = 0.15 mm

13. (a)

Strain energy stored in hollow shaft, $U = \frac{\tau_{max}^2}{4G}\left[\frac{D^2 + d^2}{D^2}\right]V$

$$= \frac{80^2}{4 \times 100 \times 10^3}\left[\frac{80^2 + 60^2}{80^2}\right] \times 10^6$$

$$= \frac{10000 \times 10^6}{4 \times 10^5} = 2.5 \times 10^4 \text{ N-mm}$$

$$= 25 \text{ N-m}$$

14. (a)

Direct longitudinal stress,

$$\sigma_x = \frac{90 \times 10^3}{30 \times 30} = 100 \text{ MPa} \qquad \text{(Compressive)}$$

$$\in_x = \frac{1}{E}\left[-\sigma_x + \nu\left(\sigma_y + \sigma_z\right)\right] \quad \text{...(i)}$$

$$\in_y = \in_z = \frac{1}{E}\left[-\sigma_y + \nu\left(\sigma_x + \sigma_z\right)\right] = 0 \quad \text{...(ii)}$$

As we know $\sigma_y = \sigma_z$

Also on solving equation (ii)

$$\sigma_y = \frac{\nu}{1-\nu}\sigma_x = \frac{0.25}{1-0.25}\sigma_x = \frac{\sigma_x}{3}$$

So,
$$\in_x = \frac{1}{E}\left[-\sigma_x + \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$= \frac{1}{E}\left[-\sigma_x + \nu \times 2\sigma_y\right]$$

$$= \frac{1}{E}\left[-\sigma_x + 0.25 \times 2 \times \frac{\sigma_x}{3}\right]$$

$$= \frac{1}{E}\left[-\sigma_x + \frac{0.5\sigma_x}{3}\right]$$

$$= \frac{1}{100 \times 10^3}\left[-100 + \frac{0.5 \times 100}{3}\right]$$

$$= \frac{1}{100 \times 10^3}\left[-\frac{250}{3}\right]$$

$$\delta l = l\in_x = \frac{1}{100 \times 10^3}\left[-\frac{250}{3}\right] \times 100$$

$= 0.083$ mm (Reduction in length)

15. (b)

We know deflection of spring,

$$\delta = \frac{64WR^3n}{Gd^4}$$

where, $W = 100$ N, $R = 25$ mm, $n = 12$, $G = 80$ GPa, $d = 5$ mm

So,
$$\delta = \frac{64 \times 100 \times (25)^3 \times 12}{80 \times 10^3 \times 5^4} = 24 \text{ mm}$$

16. (c)

We know that area of triangular beam section

$$A = \frac{\sqrt{3}}{4}a^2 \text{ for equilateral triangle}$$

$$= \frac{\sqrt{3}}{4}(100)^2 = 2500\sqrt{3} \text{ mm}^2$$

Average shear stress across the section

$$\tau_{avg} = \frac{F}{A} = \frac{13 \times 10^3}{2500\sqrt{3}} = 3 \text{ MPa}$$

So maximum shear stress for triangular section

$$\tau_{max} = 1.5\ \tau_{avg}$$
$$= 1.5 \times 3 = 4.5 \text{ MPa}$$

17. (c)

From bending equation, $\frac{f}{y} = \frac{f_{max}}{y_{max}}$

$\therefore$ $$f = \frac{f_{max}}{y_{max}} \times y$$

$\therefore$ Force on shaded area $= \frac{f_{max}}{y_{max}} \times \Sigma Ay$

$$= \frac{f_{max}}{y_{max}}(A\bar{y})$$

[where A is shaded area, $\bar{y}$ = distance of centroid of shaded area from N.A.]

$$= \frac{90}{12} \times \left[\frac{15}{2} \times 12\right] \times \frac{2}{3} \times 12$$

$$= 5400 \text{ kg}$$

18. (a)

$$\tau_{max} = \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$$

$$= \left[\frac{16}{\pi(100)^3}\sqrt{(8)^2 + (6)^2}\right] \times 10^6$$

$$= \frac{16}{\pi} \times \frac{10 \times 10^6}{10^6} = 50.93 \text{ MPa}$$

19. (c)

Shear center for channel section,

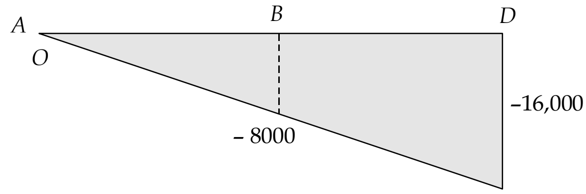
$$e = \frac{3b^2}{6b + h} = \frac{b}{2 + \frac{h}{3b}}$$

Now, $$\frac{h}{3b} = \frac{150}{3 \times 50} = 1$$

$$e = \frac{50}{2 + 1} = \frac{50}{3} = 16.67 \text{ mm}$$

20. (a)

Bending moment diagram (N.m)



Since the cross section of the beam is not constant, the maximum stress occurs either at the section just to the left of B (M_B = –8000 Nm) or at the section at D (M_D = –16,000 Nm)

Section Modulus (z) at the two sections

$$Z_{AB} = \frac{bh_{AB}^2}{6} = \frac{50 \times 100^2}{6} = 83{,}333.3 \text{ mm}^3$$

$$Z_{BD} = \frac{bh_{BD}^2}{6} = \frac{50 \times 150^2}{6} = 187{,}500 \text{ mm}^3$$

Maximum bending stress,

$$(\sigma_B)_{max} = \frac{|M_B|}{Z_{AB}} = \frac{8000 \times 10^3}{83{,}333.33} = 96 \text{ MPa}$$

$$(\sigma_D)_{max} = \frac{|M_D|}{Z_{BD}} = \frac{16{,}000 \times 10^3}{187{,}500} = 85.3 \text{ MPa}$$

The maximum bending stress in the beam is

$$\sigma_{max} = 96 \text{ MPa}$$

21. (a)

The maximum shear stress may occur at the neutral axis (where Q is largest) or at level a-a in the lower fin (where the with of the cross section is smaller than at the neutral axis)

Shear stress at neutral axis:

Area above the neutral axis is used.

$$Q = A'\bar{y}' = (60 \times 219) \times \frac{219}{2} = 1438830 \text{ mm}^3$$

Shear stress,
$$\tau_1 = \frac{VQ}{Ib} = \frac{72 \times 10^3 \times 1438830}{(440 \times 10^6) \times 60}$$

$$\tau_1 = 3.92 \text{ MPa}$$

Shear stress at a-a:

Area below the line a-a is easier to use.

$$Q = A'\bar{y}' = (36 \times 225) \times \left(267 - \frac{225}{2}\right) = 1251450 \text{ mm}^3$$

Shear stress, $$\tau_2 = \frac{VQ}{Ib} = \frac{72\times10^3\times1251450}{440\times10^6\times36}$$

$$\tau_2 = 5.69 \text{ MPa}$$

The maximum shear stress is the larger of above two values occurring at *a*-*a*.

$$\tau_{max} = 5.69 \text{ MPa}$$

22. (a)

$$J = \frac{\pi}{32}\left(d_0^4 - d_i^4\right)$$

$$T = \frac{G\theta}{L}\times J$$

$$= \frac{8\times10^4}{250}\times\left(0.1\times\frac{\pi}{180}\right)\left[\frac{\pi}{32}\times\left(150^4 - 75^4\right)\right] \text{ Nmm}$$

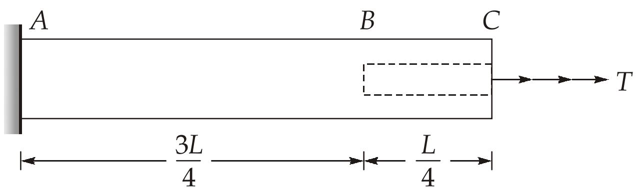
$$= 26.02 \text{ kNm}$$

Now, $P = Tw,$

where, $$w = \frac{2\pi N}{60}$$

$\therefore$ $$P = \frac{26.02\times2\pi\times500}{60} = 1362.40 \text{ kW}$$

23. (a)



$$\theta_{AC} = \theta_{AB} + \theta_{BC}$$

$$\theta_C = \frac{T_{AB}\times L_{AB}}{GJ_{AB}} + \frac{T_{BC}\times L_{BC}}{GJ_{BC}} \quad (\because \theta_A = 0)$$

$$J_{BC} = \frac{\pi}{32}\left[D^4 - \left(\frac{D}{2}\right)^4\right]$$

$$J_{BC} = \frac{\pi}{32}D^4\left[1 - \frac{1}{16}\right]$$

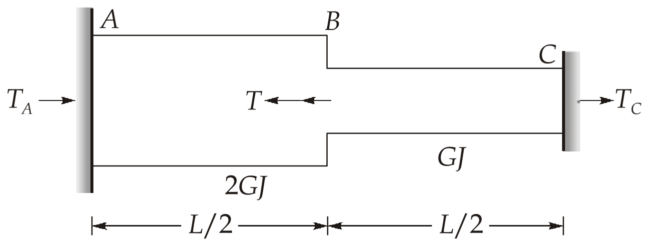
$$J_{BC} = \frac{15}{16}J_{AB} = \frac{15}{16}J$$

$\therefore$ $$\theta_C = \frac{T\times\frac{3L}{4}}{GJ} + \frac{T\times\frac{L}{4}}{G\times\frac{15}{16}J}$$

$$= \frac{3}{4}\frac{TL}{GJ} + \frac{4}{15}\frac{TL}{GJ}$$

$$= \frac{45TL + 16TL}{60GJ} = \frac{61}{60}\frac{TL}{GJ}$$

24. (c)



$$T = T_A + T_C \quad \text{...(i)}$$

$$\theta_{AB} + \theta_{BC} = 0$$

$$\frac{T_{AB}L_{AB}}{(GJ)_{AB}} + \frac{T_{BC}L_{BC}}{(GJ)_{BC}} = 0$$

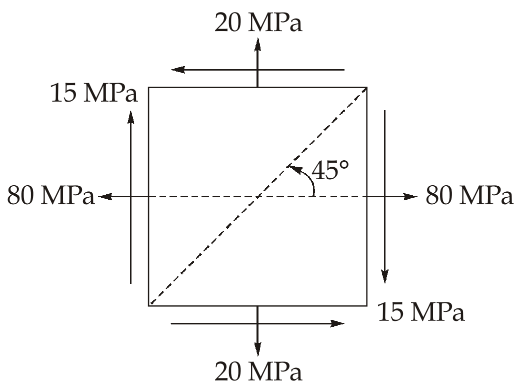
$$\frac{(-T_A)\frac{L}{2}}{2GJ} + \frac{(T - T_A)\frac{L}{2}}{GJ} = 0$$

$$\frac{-T_A}{2} + (T - T_A) = 0$$

$$T - \frac{3}{2}T_A = 0$$

$$T_A = \frac{2}{3}T$$

25. (a)



σ_x = 80 MPa, σ_y = 20 MPa, τ_{xy} = –15 MPa, θ = 45°

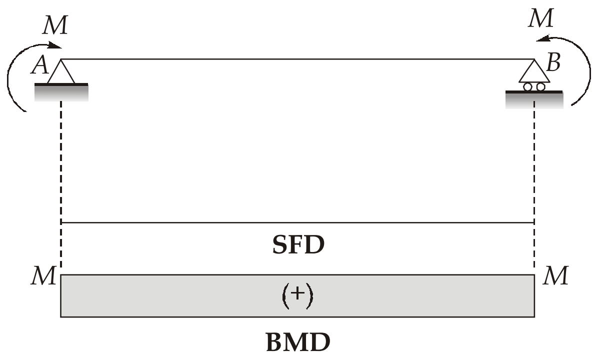
$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

$$= 80\cos^2 45° + 20\sin^2 45° + 2(-15)\cos 45° \sin 45°$$

$$= 80 \times \frac{1}{2} + 20 \times \frac{1}{2} - 2 \times 15 \times \frac{1}{2}$$

$$= 40 + 10 - 15 = 35 \text{ MPa}$$

26. (d)



Since, bending moment is constant.

Also, cross-section is constant throughout the length.

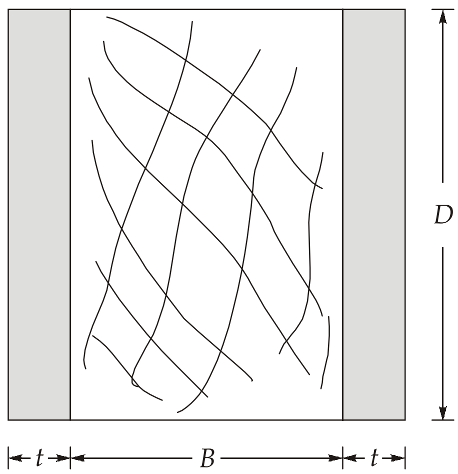
From bending formula, $\frac{\sigma}{y} = \underbrace{\frac{M}{I} = \frac{E}{R}}$

$$R = \frac{EI}{M}$$

$$R = \text{Constant}$$

Hence, deflection curve will be in the shape of a circular arc.

27. (b)



$$\text{MOR} = M_w + M_s$$

$$M = \sigma_w \times \frac{BD^2}{6} + m\sigma_w \times \frac{2tD^2}{6} \left(\because \frac{\sigma_s}{\sigma_w} = \frac{E_s}{E_w} = m \right)$$

$$M = \frac{\sigma D^2}{6}[B + 2mt]$$

28. (a)

Total deflection at free end, $\delta = \dfrac{wL^4}{8EI} = \dfrac{100(3)^4 \times 10^9}{8 \times 5 \times 10^{10}} = 20.25$ mm

Reaction at roller support is produced due to the deflection resisted i.e., (20.25 – 3) mm = 17.25 mm

$$\Rightarrow \quad 17.25 = \frac{P \times (3)^3 \times 10^9}{3EI}$$

$$P = 95.83 \text{ N}$$

29. (a)

$$\sigma_0 = \frac{3W_0L}{2nbt^2}$$

$$L = \frac{190 \times 2 \times 15 \times 75 \times 5^2}{3 \times 5 \times 10^3} = 712.5 \text{ mm}$$

Radius of curvature, $R = \dfrac{L^2}{8\delta_0} = \dfrac{(712.5)^2}{8 \times 20} = 3172.85$ mm

$$R = 3.17 \text{ m}$$

30. (a)

$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_{CI} = \delta_{steel} = 0.8 \text{ mm}$$

$$\delta_{CI} = \frac{P_{CI} \times 2000 \times 1000}{\frac{\pi}{4}\left(60^2 - 50^2\right) \times 10^5} = 0.8 \text{ mm}$$

$$\Rightarrow \quad P_{CI} = 11\pi \text{ kN}$$

$$\delta_{steel} = \frac{P_{steel} \times 2000 \times 1000}{\frac{\pi}{4} \times 50^2 \times 2 \times 10^5} = 0.8 \text{ mm}$$

$$\Rightarrow \quad P_{steel} = 50\pi \text{ kN}$$

$$\therefore \quad P = P_{CI} + P_{steel} = (11\pi + 50\pi) \text{ kN}$$

$$= 191.64 \text{ kN}$$

■■■■