CLASS TEST

S.No.: 02SK_HIJ_02072024

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ENGINEERING MATHEMATICS

EC-EE

Date of Test : 02/07/2024

ANSWER KEY

1.	(a)	7.	(a)	13.	(b)	19.	(a)	25.	(a)
2.	(a)	8.	(c)	14.	(b)	20.	(d)	26.	(b)
3.	(a)	9.	(b)	15.	(b)	21.	(c)	27.	(b)
4.	(a)	10.	(c)	16.	(a)	22.	(c)	28.	(a)
5.	(b)	11.	(c)	17.	(b)	23.	(b)	29.	(d)
6.	(b)	12.	(b)	18.	(d)	24.	(a)	30.	(a)

DETAILED EXPLANATIONS

1. **(a)**

Probability of defective item, $p = 0.2$

Probability of non-defective item,

$$q = 0.8$$

Probability that exactly 3 of the chosen items are defective,

$$= {}^{20}C_3 (p)^3 (q)^{17}$$

$$= \frac{20!}{17!3!}(0.2)^3(0.8)^{17} = 0.205$$

2. **(a)**

Given, $$y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + ... + \infty}}}$$

$$y = \sqrt{\cos x + y}$$

$$y^2 = \cos x + y$$

$$y^2 - y = \cos x$$

Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} - \frac{dy}{dx} = -\sin x$$

$$(1-2y)\frac{dy}{dx} = \sin x$$

3. **(a)**

$$\text{Ax} = 0 \text{ has non-trivial solution}$$

$$|A| = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ \alpha & 0 & 1 \end{vmatrix} = 0$$

$$\alpha(2-1) + 1(1-0) = 0$$

$$\alpha + 1 = 0$$

$$\alpha = -1$$

4. **(a)**

$$\iint \frac{1}{4}(F \cdot n)dA \;;\; \iint \frac{1}{4}(F \times dA)$$

By using Gauss divergence theorem

$$\Rightarrow \iiint \frac{1}{4}(\vec{\nabla} \cdot F)dV$$

$$\Rightarrow \iiint \frac{1}{4}\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)dV$$

$$\Rightarrow \iiint \frac{1}{4}(1+1+1)dV = \frac{3}{4} \times V_{sphere}$$

$$\Rightarrow \frac{3}{4} \times \frac{4}{3} \times \pi (1)^3 = \pi$$

5. (b)

For singular matrix,

$$|A| = 0$$

$$\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & -1 \\ b & b^2 & -1 \\ c & c^2 & -1 \end{vmatrix} = 0$$

$$abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$abc = 1$$

6. (b)

Putting, $z = x + iy$

and $b = b_1 + ib_2$

$$(b_1 - ib_2)(x + iy) + (b_1 + ib_2)(x - iy) = c$$

$$\Rightarrow \quad 2b_1 x + 2b_2 y = c$$

$\Rightarrow$ straight line

7. (a)

Conjugate of $1 + i$ with respect to $|z - 1| = 2$ is $1 - i$ as it is inside the circle.

8. (c)

Given equation is of the form $(D^2 + D)y = x^2 + 2x + 4$

$$\therefore \quad \text{P.I.} = \frac{1}{D(D+1)}(x^2 + 2x + 4)$$

$$= \frac{1}{1+D} \cdot \frac{1}{D}\left(x^2 + 2x + 4\right)$$

$$= \frac{1}{1+D}\left(\frac{x^3}{3} + x^2 + 4x\right)$$

$$= (1+D)^{-1}\left(\frac{x^3}{3} + x^2 + 4x\right)$$

$$= \left(1 - D + D^2 - D^3\right)\left(\frac{x^3}{3} + x^2 + 4x\right)$$

$$= \frac{x^3}{3} + x^2 + 4x - \left(x^2 + 2x + 4\right) + \left(2x + 2\right) - 2$$

$$= \frac{x^3}{3} + 4x - 4 + C$$

9. (b)

Given,

$$\frac{dy}{dx} = \sqrt{\frac{x}{y}}$$

$$\Rightarrow \int \sqrt{y}\,dy = \int \sqrt{x}\,dx + c$$

$$\Rightarrow \frac{2}{3}y^{3/2} = \frac{2}{3}x^{3/2} + c$$

$$y^{3/2} = x^{3/2} + c'$$

Given, $y(0) = 1$

$\Rightarrow$ $1 = c'$

$$y^{3/2} = x^{3/2} + 1$$

10. (c)

Comparing the given equation with general form of second order partial differential equation,

$$A = 1,$$

$$B = \frac{1}{2},$$

$$C = 0$$

$$\Rightarrow B^2 - 4AC = \frac{1}{4} > 0$$

$\therefore$ PDE is hyperbolic.

11. (c)

Given, $PQRS = I$

$\Rightarrow$ $PQRSS^{-1} = I.S^{-1}$

$\Rightarrow$ $PQR = S^{-1}$

$\Rightarrow$ $PQRR^{-1} = S^{-1} R^{-1}$

$\Rightarrow$ $PQ = S^{-1} R^{-1}$

$\Rightarrow$ $SPQ = S.S^{-1} R^{-1} = R^{-1}$

$\Rightarrow$ $R^{-1} = SPQ$

12. (b)

For limit to exist, LHL = RHL

Now,

RHL $\lim_{x\to 0^+} f(x) = 4 - 6x = 4$

LHL $\lim_{x\to 0^-} f(x) = 3x + 4 = 4$

$\Rightarrow$ Limit exists

Now for continuity,

LHL = RHL = Functional value

At $x = 0$

$f(x) = -4 \neq$ RHL

$\Rightarrow$ Discontinous.

13. (b)

Auxiliary equations,

$$D^2 + 6D + 13 = 0$$

i.e. $m^2 + 6m + 13 = 0$

$\Rightarrow m = -3 \pm 2i$

$\Rightarrow$ Solution of *DE* is $\rightarrow \psi = e^{-3t}\,[c_1 \cos 2t + c_2 \sin 2t]$

Now, $\psi(0) = e^0\,[c_1.1 + 0] = 0$

$$c_1 = 0$$

So,

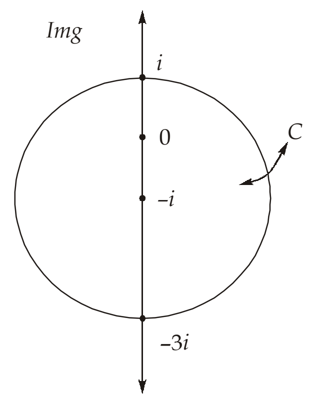
solution of *DE* is, $\psi = e^{-3t}\,[c_2 \sin 2t]$

$$\equiv e^{-3t}\,[c \sin 2t]$$

14. (b)

$$I = \oint_c \text{cosec } z dz$$

$$= \oint_c \frac{1}{\sin z} dz$$



Singular points, $z = 0, \pm \pi, \pm 2\pi$ ....

out of all singular points on $z = 0$ lies inside/on C

Now, $\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!}$.............

Now, $\oint_c \text{cosec } z dz = 2\pi i$ [sum of residue inside Contour]

Now, $\underset{z=0}{\text{Res}} \text{ cosec } z = \underset{z=0}{\text{Res}} \frac{1}{\sin z} = \text{coefficient of } \frac{1}{z} \text{ in} \left[\frac{1}{z - \frac{z^3}{3!} + \frac{z^5}{5!}}\right]$

$$= \text{coefficient of } z \text{ in} \left[\frac{1}{z}\left(1 - \frac{z^2}{3!} + \frac{z^4}{5!}....\right)^{-1}\right] \text{at } z = 0$$

$\Rightarrow \oint_c \text{cosec } z dz = 2\pi i\,(1)$

15. (b)

$$A = \frac{A+A^T}{2} + \frac{A-A^T}{2}$$

Where, $$B = \frac{A+A^T}{2} \text{ is symmetric matrix}$$

$$C = \frac{A-A^T}{2} \text{ is skew symmetric matrix}$$

Now, $$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \\ 2 & 1 & 5 \end{bmatrix}$$

$\Rightarrow$ $$B = \frac{A+A^T}{2} = \frac{1}{2}\begin{bmatrix} 2 & 5 & 5 \\ 5 & 8 & 2 \\ 5 & 2 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5/2 & 5/2 \\ 5/2 & 4 & 1 \\ 5/2 & 1 & 5 \end{bmatrix}$$

16. (a)

LHL $$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\pi \sin x}{x} \to \left[\frac{0}{0}\text{form}\right]$$

Applying L Hospital rule,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \pi \cos x = \pi \quad \text{...(i)}$$

Similarly,

RHL $$\lim_{x\to 0^+} f(x) = \pi$$

...(ii)

Also,

RHL $$f(0) = \frac{22}{7} \quad \text{...(iii)}$$

So, from (i), (ii) and (iii),

LHL = RHL ≠ functional value

$\Rightarrow$ $f(x)$ is not continuous at $x = 0$

Note : $\pi \neq \frac{22}{7}$. It is approximated to $\frac{22}{7}$.

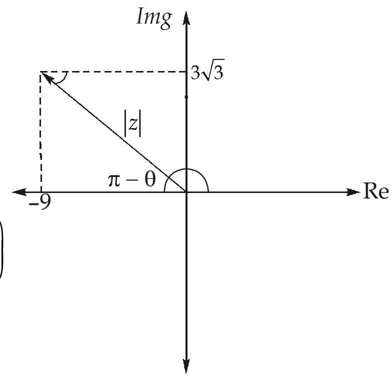
17. (b)

Since, $$P\left(\frac{X}{Y}\right) = \frac{P(X\cap Y)}{P(Y)}$$

$\Rightarrow$ $$P(X \cap Y) = P\left(\frac{X}{Y}\right).P(Y) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$$

18. (d)

$$|z| = \sqrt{9^2 + \left(3\sqrt{3}\right)^2}$$
$$= \sqrt{81 + 27}$$
$$= \sqrt{108}$$
$$= 6\sqrt{3}$$



$$\leq \quad \angle\theta = \tan^{-1}\left(\frac{+3\sqrt{3}}{-9}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$
$$= \pi - \frac{\pi}{6}$$
$$= \frac{5\pi}{6}$$

19. (a)

At $z = 0$ is a pole of order 2,

$$\Rightarrow \quad \underset{z\to 0}{\text{Res}} = \lim_{z\to 0}\left[\frac{1}{(2-1)!}\frac{d}{dz}\left[\frac{1-z}{1+z}\right]\right]$$
$$= \lim_{z\to 0} 1 \cdot \left[\frac{(1+z)(-1)-(1-z)(1)}{(1+z)^2}\right]$$
$$= \frac{-1-1}{1} = -2 \qquad \text{...(1)}$$
$$\Rightarrow \quad \underset{z\to -1}{\text{Res}} f(z) = \lim_{z\to -1}\frac{1-z}{z^2} = \frac{2}{1} = 2 \qquad \text{...(2)}$$

Sum of residue = – 2 + 2 = 0

20. (d)

We know that,

$$L^{-1}\left[\frac{1}{s^2}\right] = t$$

By shifting rule,

$$L^{-1}\left[\frac{1}{(s-1)^2}\right] = te^t$$

21. (c)

$$|Adj(A)| = |A|^{n-1} \qquad \text{.....(1)}$$

Where n is order of A,

Now,

$$|A| = 1\begin{vmatrix}5 & 1\\4 & 3\end{vmatrix} + 0 + 1.\begin{vmatrix}2 & 1\\5 & 1\end{vmatrix}$$
$$\Rightarrow \quad |A| = 11 - 3$$
$$\Rightarrow \quad |A| = 8$$

Using (1),

$$|Adj(A)| = 8^{3-1} = 8^2 = 64$$

22. **(c)**

Let,

$$y = (\cos(\cos(\cos......x)))$$

$$\Rightarrow \quad y = \cos y$$

As $\quad y - \cos y = f(y)$

$$\Rightarrow \quad f'(y) = 1 + \sin y$$

Using Newton – Raphson's method,

and initial guess value, $x_0 = 1$

$$\Rightarrow \quad f(x_0) = 1 - \cos 1 = 0.4597$$

Now First iteration

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1 - \frac{0.4597}{1 + \sin 1}$$

$$\Rightarrow \quad x_1 = 0.75036$$

Now, $\quad f(x_1) = 0.0189$

Second Iteration,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\Rightarrow \quad x_2 = 0.75036 - \frac{0.0189}{0.75036 + \sin(0.75036)}$$

$$\Rightarrow \quad x_2 = 0.7372$$

Now,

$$f(x_2) = 3.153 \times 10^{-3} \simeq 0$$

$$y = x_2 = 0.7372$$

$$\Rightarrow \quad I = \int_0^1 x\cos(\cos(\cos........x))\,dx = \int_0^1 xy dx = \int_0^1 0.7372.x dx$$

$$\Rightarrow \quad I = 0.7372.\frac{x^2}{2}\Big|_0^1$$

$$\Rightarrow \quad I = 0.7372.\frac{1}{2} = 0.3686 \simeq 0.369 \simeq 0.37$$

23. **(b)**

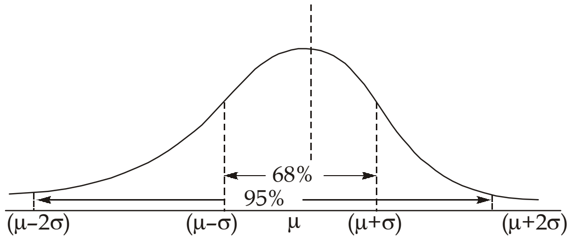
Given,

Mean, $\quad \mu = 1200$

Variance, $\quad \sigma^2 = 9 \times 10^4$

$\Rightarrow$ Standard deviation, $\sigma = \sqrt{9 \times 10^4} = 300$

Using Standard normal curve,



Probability of finding tigers between

$$(\mu - 2\sigma) \ \& \ (\mu + 2\sigma) = 0.95$$
$$\mu - 2\sigma = 1200 - 2 \times 300 = 600$$
$$\mu + 2\sigma = 1200 + 2 \times 300 = 1800$$

i.e. $P(600 \le X \le 1800) = 0.95$

$\Rightarrow P(X \le 600) + P(X \ge 1800) = 0.05$

Since normal curve is symmetric wrt mean value,

So, $P(X \le 600) = P(X \ge 1800)$

$\Rightarrow \quad 2P(X \ge 1800) = 0.05$

$\Rightarrow \quad P(X \ge 1800) = 0.025$

24. (a)

$$f_1(z) = z^3$$

if $z = x + iy$

$$z^2 = (x + iy)^2 = x^2 - y^2 + 2ixy$$
$$z^3 = (x^2 - y^2 + 2ixy)(x + iy)$$
$$= (x^3 - 3xy^2) + (3x^2y - y^3)i$$
$$u = x^3 - 3xy^2$$
$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$
$$\frac{\partial u}{\partial y} = -6xy$$
$$v = 3x^2y - y^3$$
$$\frac{\partial v}{\partial y} = 3x^2 - 3y^2$$
$$\frac{\partial v}{\partial x} = 6xy$$

$\because \quad \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$

and $\quad \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

$\therefore f_1(z) = z^3$ is analytic for all z-values

Now, $f_2(z) = \log z$

$$= \log(x + iy)$$
$$= \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{y}{x}$$
$$u = \frac{1}{2}\log(x^2 + y^2)$$

$$v = \tan^{-1}\frac{y}{x}$$

$$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = \frac{y}{x^2+y^2} = \frac{-\partial v}{\partial x}$$

$\because$ C–R equation are satisfied but the partial derivatives are not continuous at (0, 0)

$\Rightarrow f_2(z)$ is analytic everywhere except $z = 0$

$\Rightarrow$ Option (a) is correct.

25. **(a)**

Complete Solution CS

$$CS = CF + PI$$

Now Auxilliary equation

$$(D^2 + 4D + 6)y = 0$$

$$\Rightarrow \quad m^2 + 4m + 6 = 0$$

$$\Rightarrow \quad m = -2 \pm \sqrt{2}\,i$$

So $$C.F \rightarrow \left[c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x\right] \quad \text{....(1)}$$

Now,

$$PI \rightarrow \frac{3^x}{D^2+4D+6} = \frac{e^{xln3}}{D^2+4D+6}$$

$$\Rightarrow \quad P.I = \frac{e^{xln3}}{(ln3)^2 + 4.ln3 + 6} = \frac{e^{xln3}}{11.6} = \frac{3^x}{11.6}$$

$$\Rightarrow \quad C.S : y(x) = e^{-\sqrt{2}x}\left[c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x\right] + \frac{3^x}{11.6}$$

26. **(b)**

Let, P_1, P_2, P_3, P_4 be probability of selection in 1st, 2nd, 3rd & 4th attempt respectively,

Now,

$$P_1 = \frac{1}{24};\ P_2 = \frac{1}{24}[1+0.5]$$

$$P_2 = \frac{1}{24}\times\frac{3}{2}$$

$$P_3 = \frac{1}{24}\times\frac{3}{2}[1+0.5] = \frac{1}{24}\times\left(\frac{3}{2}\right)^2$$

$$P_4 = \frac{1}{24}\times\left(\frac{3}{2}\right)^3$$

Now let A_i be selection in i^{th} attempt & $\overline{A_i}$ be unsuccessful attempt,

So,

$$P_{\text{selection}} = A_1 + \overline{A_1}A_2 + \overline{A_1}\,\overline{A_2}\,A_3 + \overline{A_1}\,\overline{A_2}\,\overline{A_3}\,A_4$$

$$= \frac{1}{24} + \frac{23}{24}\times\frac{1}{24}\times\frac{3}{2} + \frac{23}{24}\left(1-\frac{3}{48}\right)\times\frac{1}{24}\times\left(\frac{3}{2}\right)^2 + \frac{23}{24}\left(1-\frac{3}{48}\right)$$

$$\left(1-\frac{9}{96}\right).\frac{1}{24}\times\left(\frac{3}{2}\right)^3 = 0.3$$

27. (b)

To get ABC their are two ways,

i) $(AB)C$

Now, Number of multiplications in $AB = 2 \times 3 \times 4 = 24$

Now, $ABC = (AB)_{2\times4}\, C_{4\times2}$

Number of multiplication for $(AB)C = 2 \times 4 \times 2 = 16$

$\Rightarrow$ Total multiplication $= 24 + 16 = 40$

ii) $A(BC)$

Number of multiplication operations in $BC = 3 \times 4 \times 2 = 24$

Now,

$$ABC = A_{2\times3}\,(BC)_{3\times2}$$

Number of multiplication for A(BC) = $2 \times 3 \times 2 = 12$

$\Rightarrow$ Total multiplication $= 24 + 12 = 36$

$\Rightarrow$ Minimum Number $= 36$

28. (a)

$$\vec{\nabla}\phi = \left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right)\left[3x^2y - 4yz^2 + 6z^2x\right]$$

$$\Rightarrow \quad \vec{\nabla}\phi = (6xy + 6z^2)\,\hat{i} + (3x^2 - 4z^2)\,\hat{j} + (-8yz + 12\,zx)\,\hat{k}$$

Now at (1, 1, 1)

$$\vec{\nabla}\phi = 12\hat{i} - \hat{j} + 4\hat{k} \quad .....(1)$$

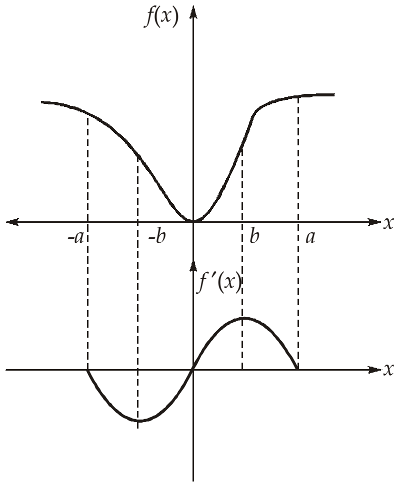
Also direction of line is, $\hat{A} = \dfrac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{17}}$ .....(2)

$\Rightarrow$ Directional derivative using (1) & (2)

$$\vec{\nabla}\phi.\hat{A} = \left(12\hat{i} - \hat{j} + 4\hat{k}\right)\left(\frac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{17}}\right)$$

$$= \frac{24 - 2 + 12}{\sqrt{17}} = \frac{34}{\sqrt{17}} = 2\sqrt{17}$$

29. (d)



i) From (–∞, a), $f(x)$ is constant $\Rightarrow f'(x) = 0$

ii) From (–a, –b), $f(x)$ decrases and also rate of decrement increases $\Rightarrow f'(x)$ increases with negative value.

iii) From (–b, 0), $f(x)$ decrease but rate of decrement decrease $\Rightarrow f'(x)$ decrease but remains negative.

from (2, 0), $f(x)$ increases with rate of increment increases

So

$f'(x)$ decreases but remains positive.

30. (a)

For given PDE,

$$\sin .dx = \cos y.dy = \tan z\ dz$$

$$\Rightarrow \quad \sin x\ dx = \cos y\ dy$$

$$\Rightarrow \quad \int \sin x\ dx = \int \cos y\ dy$$

$$\Rightarrow \quad -\cos x = \sin y + a$$

$$\Rightarrow \quad \sin y + \cos x = -a \qquad \text{...(i)}$$

& also,

$$\int \sin x\ dx = \int \tan z dz$$

$$\Rightarrow \quad -\cos x = \log \sec z + b$$

$$\Rightarrow \quad \log \cos z - \cos x = b \qquad \text{...(ii)}$$

from (i) and (ii),

$\psi(\sin y + \cos x, \log \cos z - \cos x) = 0$ is required solution

■■■■