CLASS TEST

S.No.: 03IGCE_DEF_04072024

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 04/07/2024

ANSWER KEY

1.	(a)	7.	(d)	13.	(d)	19.	(a)	25.	(c)
2.	(b)	8.	(a)	14.	(b)	20.	(c)	26.	(b)
3.	(b)	9.	(b)	15.	(a)	21.	(a)	27.	(a)
4.	(c)	10.	(d)	16.	(b)	22.	(b)	28.	(d)
5.	(a)	11.	(b)	17.	(c)	23.	(d)	29.	(c)
6.	(b)	12.	(d)	18.	(d)	24.	(d)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 2R_1$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or $$y = -4$$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

$\Rightarrow$ $$x = 12$$

$\therefore$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$

2. (b)

$$\lim_{x\to 0} \frac{\ln(1+5x)}{e^{7x}-1} \qquad \left(\frac{0}{0} \text{ indeterminate form}\right)$$

Applying L' Hospitals rule

$$\lim_{x\to 0} \frac{\ln(1+5x)}{e^{7x}-1} = \lim_{x\to 0} \frac{5}{(1+5x)7e^{7x}} = \frac{5}{7}$$

3. (b)

Probability of first item being defective,

$$P_1 = \frac{15}{50}$$

Probability of second item being defective,

$$P_2 = \frac{14}{49}$$

Probability of third item being defective,

$$P_3 = \frac{13}{48}$$

Probability that all three are defective,

$$P = P_1P_2P_3 = \frac{15}{50} \times \frac{14}{49} \times \frac{13}{48} = \frac{13}{560}$$

4. (c)

The characteristic equation $[A - \lambda I] = 0$

i.e. $$\begin{bmatrix} 4-\lambda & 6 \\ 2 & 8-\lambda \end{bmatrix} = 0$$

or $(4-\lambda)(8-\lambda)-12 = 0$

or $32-8\lambda-4\lambda+\lambda^2-12 = 0$

$\Rightarrow$ $\lambda^2-12\lambda+20 = 0$

$\Rightarrow$ $\lambda^2-10\lambda-2\lambda+20 = 0$

$\Rightarrow$ $(\lambda-10)(\lambda-2) = 0$

$\Rightarrow$ $\lambda = 10, 2$

Corresponding to $\lambda = 10$, we have

$$[A-\lambda I]x = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $-6x+6y = 0$

$\Rightarrow$ $x = y$

$2x-2y = 0$

$\Rightarrow$ $x = y$

i.e. eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = 2$, we have

$$[A-\lambda I]x = \begin{bmatrix} 2 & 6 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $2x+6y = 0$ i.e. eigen vector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

5. (a)

Given, $x = b(2-\cos\theta)$, $y = b(\sin\theta+\theta)$

$\therefore$ $$\frac{dx}{d\theta} = b\sin\theta,$$

$$\frac{dy}{d\theta} = b(\cos\theta+1)$$

$$\frac{dx}{dy} = \frac{dx/d\theta}{dy/d\theta} = \frac{b\sin\theta}{b(\cos\theta+1)}$$

$$= \frac{2b\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)}{b\times 2\cos^2\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$$

6. (b)

Inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\therefore$ $$\begin{bmatrix} 2 & 3 \\ 6 & 8 \end{bmatrix}^{-1} = \frac{1}{16-18}\begin{bmatrix} 8 & -3 \\ -6 & 2 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 8 & -3 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3/2 \\ 3 & -1 \end{bmatrix}$$

7. (d)

x	1	2	3	4	5	6
$y = f(x)$	1	8	27	64	125	216
y_n	y_0	y_1	y_2	y_3	y_4	y_5

$$I = \int_1^6 x^3 dx = \frac{h}{2}\left[(y_0 + y_5) + 2y_1 + 2y_2 + 2y_3 + 2y_4\right]$$

$$= \frac{1}{2}\left[(1 + 216) + 2(8 + 27 + 64 + 125)\right]$$

$$= \frac{1}{2}\left[217 + 448\right] = 332.5$$

8. (a)

$$f(x) = \lim_{x \to 0}\left[\frac{2x^3 + 3x^2}{4x^3 - 5x^2}\right]$$

Since this has $\frac{0}{0}$ form, limit can be found by repeated application of L'Hospitals rule.

$$f(x) = \lim_{x \to 0}\left[\frac{6x^2 + 6x}{12x^2 - 10x}\right]$$

$$= \lim_{x \to 0}\left[\frac{12x + 6}{24x - 10}\right] = \left[\frac{12 \times 0 + 6}{24 \times 0 - 10}\right] = \frac{-6}{10}$$

$$= \frac{-3}{5}$$

9. (b)

$$P(T) = 0.5$$

Probability of getting tails exactly 6 times is

$$8C_6(0.5)^6(0.5)^2 = \frac{7}{64}$$

10. (d)

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Where, $f(x) = x^2 - N$

$f'(x) = 2x$

Thus, $$x_{i+1} = x_i - \frac{x_i^2 - N}{2x_i} = x_i - \frac{x_i}{2} + \frac{N}{2x_i}$$

$$= \frac{x_i}{2} + \frac{N}{2x_i} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$$

11. **(b)**

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x-2) = 0$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^3 - 8) = 0$$

Also $f(2) = 0$

Thus $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

∴ f is continuous at $x = 2$

and $Lf'(2) = 1$ and $Rf'(2) = 12$

∴ f is not differentiable at $x = 2$.

12. **(d)**

$D = -96$ for the given matrix

$$|A| = \begin{vmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{vmatrix} = 2^3 \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

(Taking 2 common from each row)

∴ $\text{Det}(A) = (2)^3 \times D$

$= 8 \times (-96) = -768$

13. **(d)**

$2x - 8 = 2h$ (say)

⇒ $2x = 8 + 2h$

⇒ $x = 4 + h$

∴ $$\lim_{h \to 0} \frac{(8+2h)^{1/3} - 2}{2h}$$

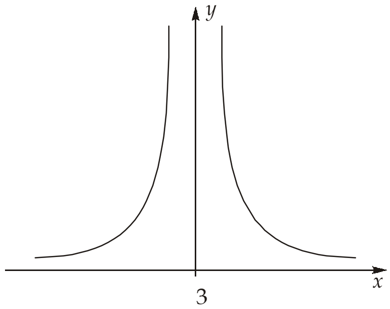
Above form is $\left(\frac{0}{0}\right)$ by putting the value $h = 0$

Applying L' Hospital rule $$\lim_{h \to 0} \frac{\frac{1}{3}(8+2h)^{\left(\frac{1}{3}-1\right)} \times 2}{2}$$

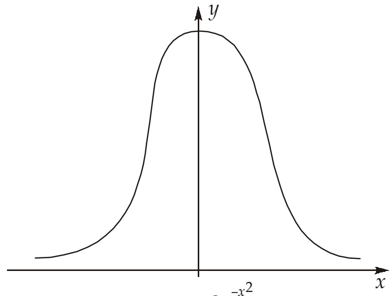
$$= \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$$

14. (b)

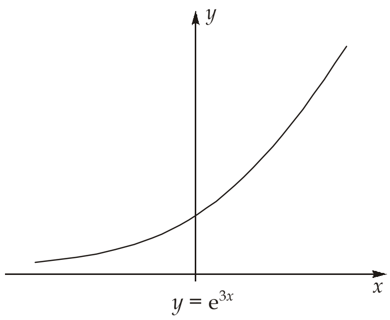
From the graphs below, we can see that only $2e^{-x^2}$ is strictly bounded.



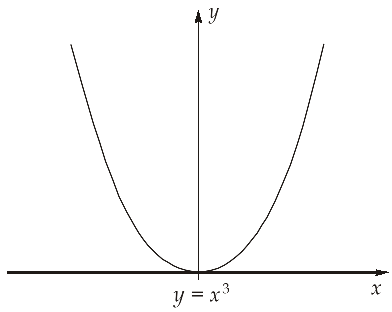
$y = \dfrac{3}{2x^2}$



$y = 2e^{-x^2}$



$y = e^{3x}$



$y = x^3$

15. (a)

Given that the partial differential equation is parabolic

$$\therefore \quad b^2 - 4ac = 0$$
$$b^2 - 4(4)(4) = 0$$
$$b^2 - 64 = 0$$
$$b^2 = 64$$

16. (b)

$$P[X > 1] = \int_1^\infty f(x)\,dx = \int_1^\infty e^{-2x}\,dx = \left.\frac{-e^{-2x}}{2}\right|_1^\infty$$

$$= -\left(\frac{e^{-2\infty}}{2} - \frac{e^{-2}}{2}\right) = \frac{e^{-2}}{2} = 0.067$$

17. (c)

$$\frac{dy}{dx} = 0.75y^2 \qquad (y = 1 \text{ at } x = 0)$$

Iterative equation by backward (implicit) Euler's method for above equation would be

$$y_{k+1} = y_k + h_f(x_{k+1}, y_{k+1})$$
$$y_{k+1} = y_k + h \times 0.75\, y^2_{k+1}$$
$$\Rightarrow \quad 0.75\, hy^2_{k+1} - y_{k+1} + y_k = 0$$

Putting $k = 0$ in above equation

$$0.75hy_1^2 - y_1 + y_0 = 0$$

Since $y_0 = 1$ and $h = 1$

$$0.75y_1^2 - y_1 + 1 = 0$$

$$\Rightarrow \quad y_1 = \frac{1 \pm \sqrt{1^2 - 3}}{2 \times 0.75} = \frac{2}{3}(1 \pm i\sqrt{2})$$

18. (d)

$$\frac{dx}{dt} = 9x - 11y$$

$$\frac{dy}{dt} = 7x + 13y$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & -11 \\ 7 & 13 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9x & -11y \\ 7x & +13y \end{bmatrix}$$

19. (a)

The volume of a solid generated by revolution about the x-axis, of the area bounded by curve $y = f(x)$, the x-axis and the ordinates $x = a$, $y = b$ is

Volume $\int_a^b \pi y^2 dx$

Here, $a = 2$, $b = 3$ and $y = 2\sqrt{x} \Rightarrow y^2 = 4x$

$$\therefore \quad \text{Volume} = \int_2^3 \pi 4x dx = 4\pi\left[\frac{x^2}{2}\right]_2^3 = 2\pi[x^2]_2^3 = 2\pi\,[9 - 4] = 10\pi$$

20. (c)

Given,

$$\text{Trace } A = 9$$

$$|A| = 24$$

$$\lambda_1 = 3$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 9$$

$$\Rightarrow \quad 3 + \lambda_2 + \lambda_3 = 9$$

$$\Rightarrow \quad \lambda_2 + \lambda_3 = 6$$

21. (a)

$$\int_0^\pi \frac{dx}{c\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) + d\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)} = \int_0^\pi \frac{dx}{(c+d)\cos^2\frac{x}{2} + (c-d)\sin^2\frac{x}{2}}$$

$$= \int_0^\pi \frac{\sec^2\frac{x}{2}dx}{(c+d) + (c-d)\tan^2\frac{x}{2}} = \frac{1}{(c-d)}\int_0^\pi \frac{\sec^2\frac{x}{2}dx}{\frac{(c+d)}{(c-d)} + \tan^2\frac{x}{2}}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\left\{\tan\frac{x}{2}\sqrt{\frac{c-d}{c+d}}\right\}\right]_0^\pi$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{2}{\sqrt{(c-d)(c+d)}} \times \frac{\pi}{2} = \frac{\pi}{\sqrt{c^2-d^2}}$$

22. (b)

Let $$I = \int_0^\infty \frac{e^{-x}\sin bx}{x}dx$$

$$\frac{dI}{db} = \int_0^\infty \frac{\partial}{\partial b}\left(\frac{e^{-x}\sin bx}{x}\right)dx = \int_0^\infty \frac{e^{-x}x\cos bx}{x}dx$$

$$= \int_0^\infty e^{-x}\cos bx\,dx$$

We know that

$$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int_0^\infty e^{-x}\cos bx\,dx = \left[\frac{e^{-x}}{1+b^2}[-\cos bx + b\sin bx]\right]_0^\infty$$

$$\frac{dI}{db} = \frac{1}{1+b^2}$$

Integrating both sides, $I = \tan^{-1}b$

23. (d)

$$y'' + 4y' + 4y = 0$$

$$(D^2 + 4D + 4)y = 0$$

$\Rightarrow$ $$(D+2)(D+2) = 0$$

$\Rightarrow$ $$D = -2, -2$$

$\therefore$ $$y = C_1e^{-2x} + C_2xe^{-2x}$$

$$y(0) = 0 \Rightarrow 0 = C_1$$

$$y(1) = 0 \Rightarrow 0 = C_1 + C_2$$

$\Rightarrow$ $$C_2 = 0$$

$y = 0$ is the solution

$\therefore$ $$y(2) = 0$$

24. (d)

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 3-\lambda & 1 \\ -2 & -\lambda \end{vmatrix} = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$A^2 - 3A + 2 = 0$$
$$A - 3I + 2A^{-1} = 0$$

25. (c)

$$AX = \lambda X$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -6 \end{bmatrix} = (-6)\begin{bmatrix} 3 \\ -6 \end{bmatrix}$$
$$3a - 6b = -18 \quad \text{... (i)}$$
$$3c - 6d = 36 \quad \text{... (ii)}$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -3 \end{bmatrix} = (-3)\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$
$$3a - 3b = -9 \quad \text{... (iii)}$$
$$3c - 3d = 9 \quad \text{... (iv)}$$

From equation (i) and (ii), $a = 0$ and $b = 3$.
From equation (ii) and (iv), $c = -6$ and $d = -9$.

$$\therefore \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -6 & -9 \end{bmatrix}$$

26. (b)

$$y = 7x^2 + 12x$$
$$\frac{dy}{dx} = 14x + 12$$
$$\left.\frac{dy}{dx}\right|_{x=1} = 26$$
$$\left.\frac{dy}{dx}\right|_{x=3} = 54$$

$\because$ x is defined in open interval $x = (1, 3)$

$$\therefore \quad 1 < x < 3$$
$$\therefore \quad 26 < \frac{dy}{dx} < 54$$

27. (a)

Since $$\sum_{x=0}^{4} P(x) = 1$$
$$c + 2c + 2c + c^2 + 5c^2 = 1$$
$$6c^2 + 5c - 1 = 0$$
$$c = \frac{1}{6}, -1$$

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{5}{36}$
$xP(x)$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{3}{36}$	$\frac{20}{36}$

Since $P(x) \geq 0$, the possible value of

$$c = \frac{1}{6}$$

$$\text{Mean} = \sum_{x=0}^{4} xP(x) = 0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{36} + \frac{20}{36}$$

$$= \frac{59}{36} = 1.638$$

$$\text{Variance} = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= \left[0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + 9\left(\frac{1}{36}\right) + 16\left(\frac{5}{36}\right) - \left(\frac{59}{36}\right)^2\right]$$

$$= 1.45$$

28. (d)

Given differential equation is

$$x\frac{dy}{dx} + y = x^4$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{y}{x}\right) = x^3 \quad \text{... (i)}$$

Standard form of liebnitz linear equation is

$$\frac{dy}{dx} + Py = Q \quad \text{... (ii)}$$

where P and Q function of x only and solution is given by

$$y(IF) = \int Q(IF)dx + c$$

where, integrating factor (IF) = $e^{\int Pdx}$

here in equation (ii),

$$P = \frac{1}{x}, \text{ and } Q = x^3$$

$$IF = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

Solution $\quad y(x) = \int x^3 \cdot xdx + c = \frac{x^5}{5} + c$

Given condition, $\quad y(2) = \frac{21}{5}$

$$\therefore \quad \frac{21}{5} \times 2 = \frac{2^5}{5} + c$$

$$\Rightarrow \quad c = \frac{42 - 32}{5} = 2$$

$$\therefore \quad yx = \frac{x^5}{5} + 2$$

$$\Rightarrow \quad y = \frac{x^4}{5} + \frac{2}{x}$$

29. (c)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x = 3,\ f(x_0) = 2\sqrt{3} + 9 - 6 = 3 + 2\sqrt{3}$$

$$f'(x) = 3 + \frac{2}{2\sqrt{x}} = 3 + \frac{1}{\sqrt{x}}$$

$$f'(x_0) = 3 + \frac{1}{\sqrt{3}}$$

Then,
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{3 + 2\sqrt{3}}{3 + \frac{1}{\sqrt{3}}} = 1.192$$

30. (a)

Given,
$$\frac{dy}{dx} + 3y = 0 \text{ and } y(1) = 4$$

$$\frac{dy}{dx} = -3y$$

$$\int \frac{dy}{y} = \int -3dx$$

$\Rightarrow$ $$\ln y = -3x + c$$

$\Rightarrow$ $$y = e^{-3x}.\ e^{c} = c_1 e^{-3x}$$

$$y(1) = c_1 e^{-3} = 4$$

$\Rightarrow$ $$c_1 = \frac{4}{e^{-3}}$$

So,
$$y = \frac{4}{e^{-3}} e^{-3x} = 4e^{3}e^{-3x} = 80.34e^{-3x}$$

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