CLASS TEST

S.No.: 01JP_ECEE_22072024

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EC-EE

CONTROL SYSTEM

Duration : 1:00 hr. **Maximum Marks : 50**

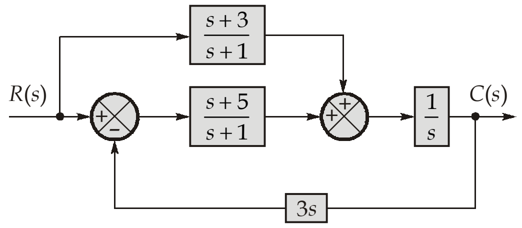
Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

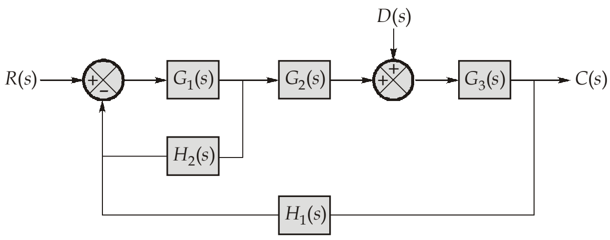
Q.1 Block diagram of a system is shown in the figure,



Then whole system is equivalent to

(a) An integrator with gain = 0.5
(b) A differentiator with gain = 2
(c) First order system
(d) Zero order system

Q.2 Block diagram of a control system is shown in the figure



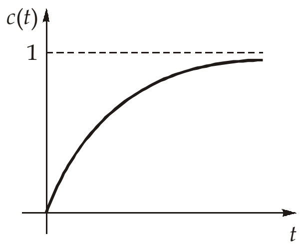
The condition that must be satisfied to remove the effect of disturbance on the response of the system.

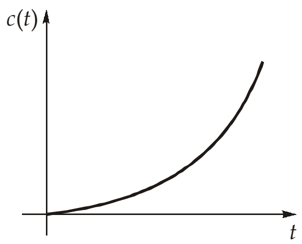
(a) $G_2H_2 = 1$ (b) $G_1H_1 = -1$
(c) $G_1G_2G_3H_1 = -1$ (d) $G_1H_2 = -1$

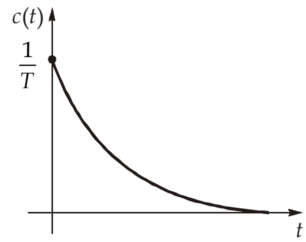
Q.3 Transfer function of a first order system is given by

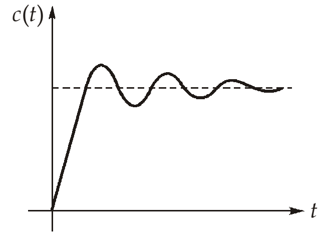
$$G(s) = \frac{1}{Ts+1}$$

Impulse response of the system is,

(a)


(b)


(c)


(d)


Q.4 Valid state transmission matrix $\phi(t)$ is,

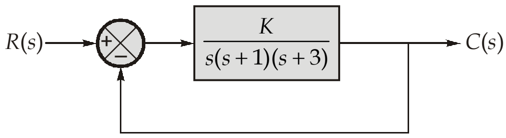
(a) $\phi(t) = \begin{bmatrix} 3te^{-t} & te^{-t} \\ -e^{-t} & e^{-t} \end{bmatrix}$

(b) $\phi(t) = \begin{bmatrix} e^{-t} + te^{-t} & te^{-t} \\ -te^{-t} & e^{-t} - te^{-t} \end{bmatrix}$

(c) $\phi(t) = \begin{bmatrix} te^{-t} & e^{-t} \\ -e^{-t} + te^{-t} & 2te^{-t} \end{bmatrix}$

(d) $\phi(t) = \begin{bmatrix} e^{-t} + te^{-t} & e^{-t} \\ -te^{-t} & e^{-t} - 3e^{-t} \end{bmatrix}$

Q.5 A closed loop system is shown in figure



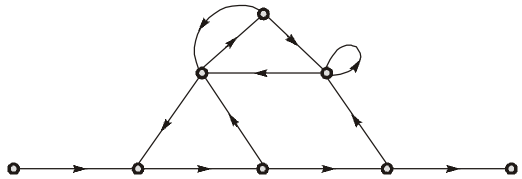
Range of K, in which system is stable

(a) $K > 0$ (b) $K < 12$
(c) $K > 12$ (d) $0 < K < 12$

Q.6 The transfer function of a system is $H(s) = \dfrac{(s+5)}{(s^2+2s-3)}$ and the output of the system for a unit step input is $y(t)$. The value of $\lim_{t \to \infty} y(t)$ is _____.

(a) $-\dfrac{5}{3}$
(b) –2.5
(c) 1
(d) None of the above

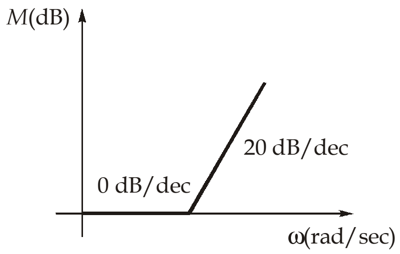
Q.7 The signal flow graph of a control system is given below :

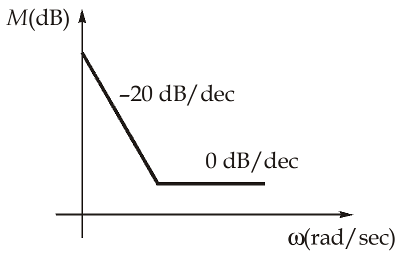


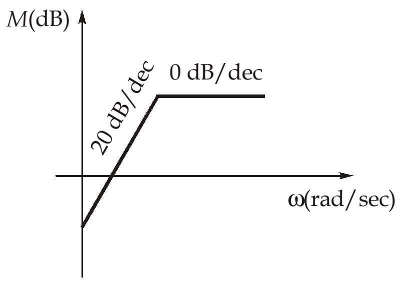
The number of feedback loops in the system are ______.

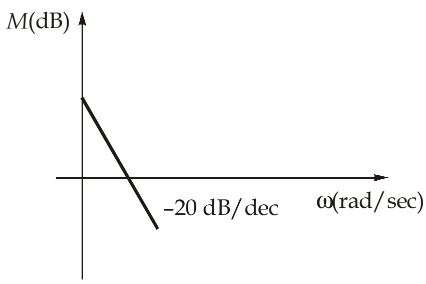
(a) 4 (b) 5
(c) 6 (d) 7

Q.8 Which of the following Bode plot corresponds to the proportional integral controller?

(a)


(b)


(c)


(d)


Q.9 The state variable model of a system is given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

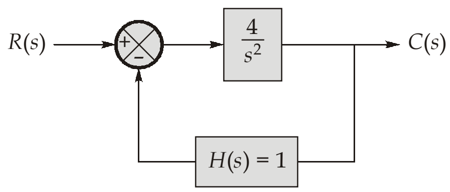
$$[x(0)] = [1 \quad 0]^T$$

The solution of homogeneous equation is given by,

(a) $\begin{bmatrix} e^t \\ te^t \end{bmatrix}$ (b) $\begin{bmatrix} e^t \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ e^t \end{bmatrix}$ (d) $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$

Q.10 Consider the system given below :



The resonant peak of the above system for unit step input is

(a) 0 (b) ∞
(c) 4 (d) undefined

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 State space representation of a system is given by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$y(t) = x_2(t)$$

Where, $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $r(t) = t\,u(t)$;

then zero state response is

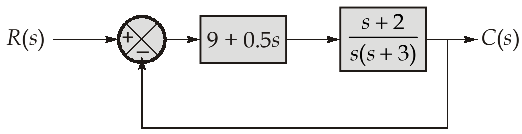
(a) $\dfrac{1}{s(s^2+s+2)}$

(b) $\dfrac{1}{s^2(s^2+s+2)}$

(c) $\dfrac{2}{s(s^2+s+2)}$

(d) None of these

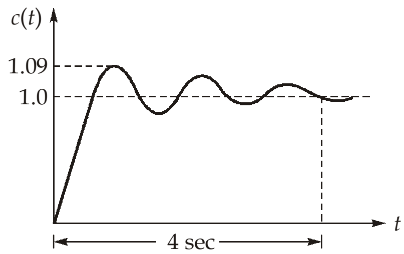
Q.12 A PD controller is tuned to get desired phase margin for second order control system.



Phase crossover frequency for compensated system is

(a) 2.52 rad/sec (b) 3.94 rad/sec
(c) 1.45 rad/sec (d) does not exist

Q.13 Unit step response of second order system is shown in figure

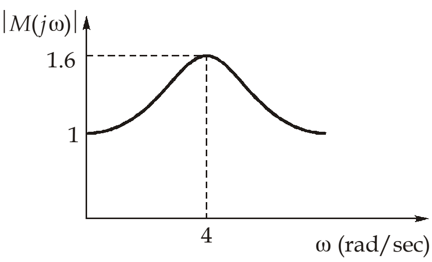


Transfer function of the system is

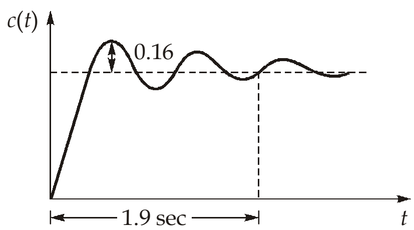
(a) $\dfrac{5}{s^2+2s+5}$ (b) $\dfrac{9.3}{s^2+2s+9.3}$

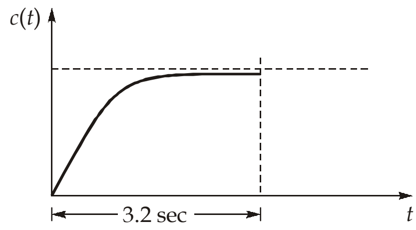
(c) $\dfrac{2.7}{s^2+2s+2.7}$ (d) $\dfrac{10}{s^2+2s+10}$

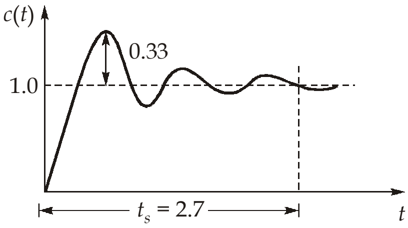
Q.14 The closed loop frequency response $|G(j\omega)|$ versus frequency of a second order prototype system is shown in figure below.

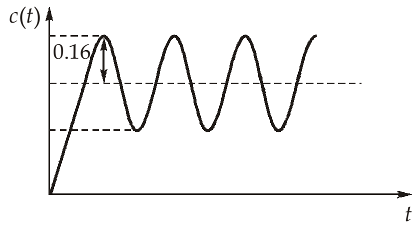


The corresponding unit step response of the system is

(a)


(b)


(c)


(d)


Q.15 The state equations of a control system are given below:

$\dot{x}_1 = x_2$

$\dot{x}_2 = -20\,x_1 - 9x_2 + u$

It is desired that the closed loop poles are to be placed at $s = (-1 \pm j2)$. The value of feedback gain matrix K is

(a) $\begin{bmatrix} 13 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} 15 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} -15 & -7 \end{bmatrix}$

(d) $\begin{bmatrix} 13 & -7 \end{bmatrix}$

Q.16 The transfer function of a unity feedback control system is given below:

$$G(s) = \frac{K(s+4)}{(s+2)^2}$$

The value of K such that ω_d = 2 rad/sec is

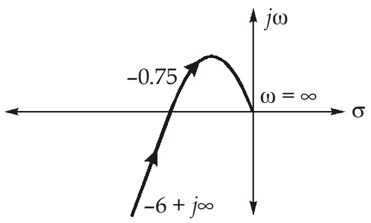
(a) 2
(b) $2\sqrt{2}$
(c) 4
(d) 8

Q.17 The frequency response of

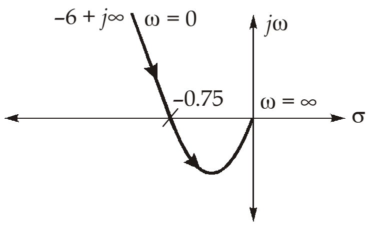
$$G(s) = \frac{1}{s(1+2s)(1+4s)}$$

plotted in the complex $G(j\omega)$ plane (for $0 < \omega < \infty$) is

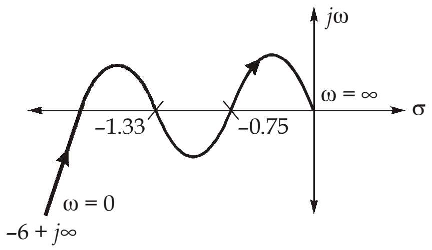
(a)



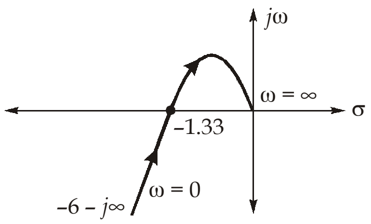
(b)



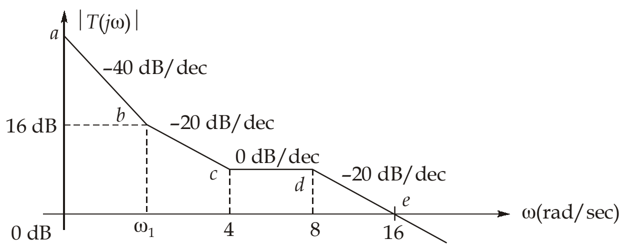
(c)



(d)



Q.18 The asymptotic log-magnitude curve for open loop transfer function is sketched below,



Open loop transfer function is

(a) $T(s) = \dfrac{10(s+8)(s+4)}{s^2(s+1.268)}$

(b) $T(s) = \dfrac{16(s+1.268)(s+4)}{s^2(s+8)}$

(c) $T(s) = \dfrac{10(s+1.268)(s+8)}{s^2(s+4)}$

(d) $T(s) = \dfrac{8(s+1.268)(s+8)}{s^2(s+4)}$

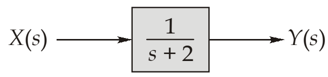
Q.19 A unity feedback control system has an open loop transfer function represented by

$$G(s) = \frac{K}{s(s+4)}$$

If a unit step input is given and value of damping ratio for system is 0.5, then the value of gain K and its peak overshoot will be respectively:

(a) $K = 16$, $M_p = 0.163$
(b) $K = 4$, $M_p = 0.163$
(c) $K = 16$, $M_p = 0.288$
(d) $K = 2$, $M_p = 0.288$

Q.20 In the system shown below, input $x(t) = 2\sin 2t$.



The steady state response $y(t)$ will be

(a) $\dfrac{1}{\sqrt{2}}\sin\left(2t+\dfrac{\pi}{4}\right)$ (b) $\dfrac{1}{2\sqrt{2}}\sin\left(2t-\dfrac{\pi}{4}\right)$

(c) $\dfrac{1}{\sqrt{2}}\sin\left(2t-\dfrac{\pi}{4}\right)$ (d) $\dfrac{1}{2\sqrt{2}}\sin\left(2t+\dfrac{\pi}{4}\right)$

Q.21 Consider the characteristic equation $D(s) = s^6 + s^5 + 6s^4 + 5s^3 + 10s^2 + 5s + 5$. The number of roots in the right side of s-plane is ______.

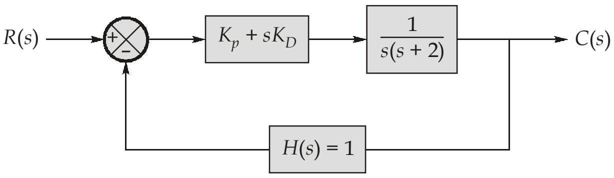
(a) Three (b) Two
(c) One (d) Zero

Q.22 The Bode plot of a low pass system with steady state gain as unity shows an absolute magnitude of 0.5 and phase angle of –90° at the frequency of ω = 1 rad/sec. The transfer function of such system will be

(a) $G(s)H(s) = \dfrac{0.5}{s(s+1)}$ (b) $G(s) = \dfrac{1}{s(s+1)}$

(c) $G(s) = \dfrac{0.5}{(s+1)^2}$ (d) $G(s) = \dfrac{1}{(s+1)^2}$

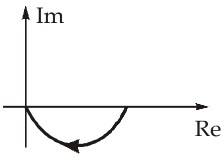
Q.23 The block diagram of a control system is shown below :

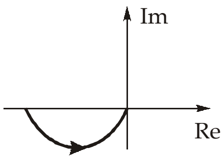


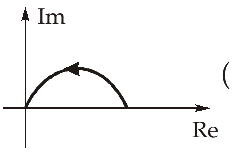
The control specifications are such that the damping ratio of the closed loop system is 0.6 and damped frequency of oscillations is 10 rad/sec. The parameters of PD controller K_P and K_D are respectively

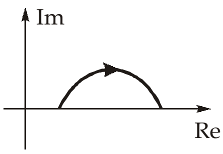
(a) 156.25, 14 (b) 13, 156.25
(c) 12.5, 14 (d) 156.25, 13

Q.24 The polar plot of $G(s) = \frac{1+4s}{1+2s}$ for $0 \le \omega \le \infty$ is

(a)


(b)


(c)


(d)


Q.25 The open loop transfer function of a non-unity feedback transfer function is

$$G(s)H(s) = \frac{1.25(s+1)}{(s+0.5)(s-2)}.$$ If the Nyquist contour is in anti-clockwise direction, then the Nyquist plot of $G(s)H(s)$ encircles $-1 + j0$

(a) once in clockwise direction
(b) twice in clockwise direction
(c) once in anticlockwise direction
(d) twice in anticlockwise direction

Q.26 For a second order system, the peak overshoot is given by 15% at time τ_p = 3 sec, the location of the poles are

(a) $-0.289 \pm j1.217$ (b) $-0.289 \pm j1.047$
(c) $-0.632 \pm j1.217$ (d) $-0.632 \pm j1.047$

Q.27 The open loop transfer function of a unity negative feedback control system of type-1 has pole located at $s = -3$. The system gain K is adjusted such that the steady state error of the system due to the application of $(2 + 5t)\,u(t)$ is 2.75. Then the value of gain K is

(a) 15.00 (b) 10.61
(c) 7.52 (d) 5.45

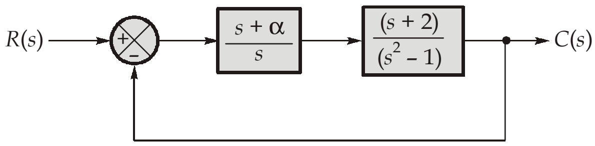
Q.28 The open loop transfer function of a unity feedback control system is

$$G(s)H(s) = \frac{K(s+1)}{(s+9)(s+3)}$$

The value of 'K' for which point $(-1 + j2)$ lies on the root locus is

(a) 5.21 (b) 27
(c) 11.66 (d) None of these

Q.29 Consider the block diagram shown below,



The sensitivity of steady state error for ramp input with respect to 'α' is

(a) 0 (b) –1
(c) $\frac{-1}{2\alpha}$ (d) ∞

Q.30 Given the system represented in state space by equations:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-2t}$$

$$y = [2 \quad 1]x$$

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The state-transition matrix of the system is

(a) $\phi(t) = \begin{bmatrix} (e^{-2t} - 2e^{-4t}) & (e^{-2t} - e^{-4t}) \\ (-4e^{-2t} + 4e^{-4t}) & (-e^{-2t} + 2e^{-4t}) \end{bmatrix}$

(b) $\phi(t) = \begin{bmatrix} (2e^{-2t} - e^{-4t}) & \frac{1}{2}(e^{-2t} - e^{-4t}) \\ (-4e^{-2t} + 4e^{-4t}) & (-e^{-2t} + 2e^{-4t}) \end{bmatrix}$

(c) $\phi(t) = \begin{bmatrix} (2e^{-2t} - e^{-4t}) & \frac{1}{2}(e^{-2t} - e^{-4t}) \\ (-e^{-2t} + e^{-4t}) & \frac{1}{4}(e^{-2t} - e^{-4t}) \end{bmatrix}$

(d) $\phi(t) = \begin{bmatrix} (e^{-2t} - 2e^{-4t}) & (2e^{-2t} - 2e^{-4t}) \\ (-e^{-2t} + e^{-4t}) & (-e^{-2t} + 2e^{-4t}) \end{bmatrix}$

■■■■

CLASS TEST

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CONTROL SYSTEM

EC-EE

Date of Test : 22/07/2024

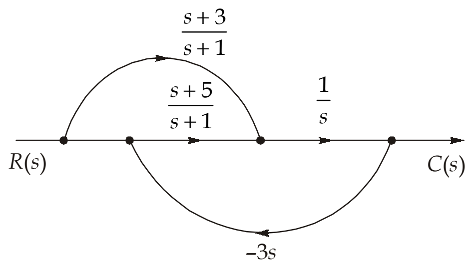
ANSWER KEY

1.	(a)	7.	(b)	13.	(c)	19.	(a)	25.	(a)
2.	(d)	8.	(b)	14.	(c)	20.	(c)	26.	(d)
3.	(c)	9.	(d)	15.	(c)	21.	(d)	27.	(d)
4.	(b)	10.	(b)	16.	(c)	22.	(d)	28.	(d)
5.	(d)	11.	(a)	17.	(d)	23.	(d)	29.	(b)
6.	(d)	12.	(d)	18.	(b)	24.	(d)	30.	(b)

DETAILED EXPLANATIONS

1. (a)

Signal flow graph of the system is



Mason's gain formula,

$$\frac{C(s)}{R(s)} = \frac{P_k \Delta_k}{\Delta}$$

Where, P_k = forward path gain

Δ_k = 1 – (sum of individual loops) + (sum of two non touching loops)....

$$P_1 = \left(\frac{s+5}{s+1}\right) \cdot \frac{1}{s}$$

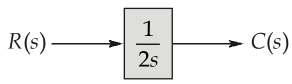
$$P_2 = \left(\frac{s+3}{s+1}\right) \cdot \frac{1}{s}$$

Loops: $$L_1 = \left(\frac{s+5}{s+1}\right)\left(\frac{1}{s}\right)(-3s)$$

$$\frac{C(s)}{R(s)} = \frac{\left(\frac{s+5}{s+1}\right)\times\frac{1}{s}+\left(\frac{s+3}{s+1}\right)\times\frac{1}{s}}{1+\left(\frac{s+5}{s+1}\right)\times\frac{1}{s}\times(3s)} = \frac{s+5+s+3}{s[s+1+3s+15]}$$

$$= \frac{2s+8}{s(4s+16)} = \frac{1}{2s}$$

Thus system can be represented as



It is an integrator with gain = 0.5

2. (d)

It is desirable to remove the effect of disturbance on response. So $\frac{C(s)}{D(s)}$ ratio can be calculated as follows:

$$\frac{C(s)}{D(s)} = \frac{G_3(1+G_1H_2)}{1+G_1G_2G_3H_1+G_1H_2}$$

Response due to disturbance,

$$C(s) = D(s) \cdot \frac{G_3(1+G_1H_2)}{1+G_1G_2G_3H_1+G_1H_2}$$

$C(s)$ should be zero for $D(s) \neq 0$

$\therefore \quad G_1H_2 = -1$

3. (c)

$$G(s) = \frac{1}{(Ts+1)}$$

Given that, $R(s) = 1$

$$C(s) = R(s).G(s)$$

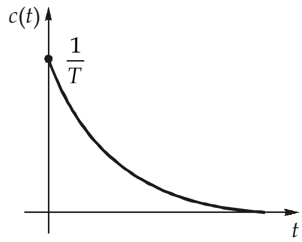
$$C(s) = \frac{1}{(Ts+1)} = \frac{1}{T\left(s+\frac{1}{T}\right)}$$

Taking inverse Laplace transform of above equation, we get

$$C(t) = \frac{1}{T}e^{-t/T}$$

At, $t = 0, \quad C(t) = \frac{1}{T}$

$t = \infty, \quad C(t) = 0$



4. (b)

State transition matrix, $\phi(t) = e^{At}$

$\phi(0) = I$

From property of state transmission matrix at $t = 0$,

$$\phi(0) = I$$

$$= \begin{bmatrix} 1+0 & 0 \\ -0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Option (b) is correct

5. (d)

Characteristic equation of system is,

$$1 + G(s)\,H(s) = 0$$

$$1+\frac{K}{s(s+1)(s+3)} = 0$$

$$s^3 + 4s^2 + 3s + K = 0$$

Routh-Hurwitz method,

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 4 & K \\ s^1 & \dfrac{12-K}{4} & 0 \\ s^0 & K & \end{array}$$

According to Routh-Hurwitz criteria, for a stable system, first column of Routh array should have positive sign

$$\frac{12-K}{4} > 0$$

and $K < 12$

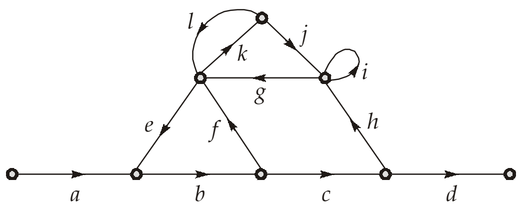
$K > 0$

Common range is $0 < K < 12$

6. (d)

Since the transfer function has one pole on the R.H.S. thus the system is unstable. So, the final value theorem is not applicable in this case. The output will be unbounded.

7. (b)



$L_1 = bfe$, $L_2 = bchge$, $L_3 = i$, $L_4 = kjg$ and $L_5 = kl$.

8. (b)

Transfer function of proportional integral controller is

$$T(s) = K_p + \frac{K_I}{s} = \frac{K_I + sK_p}{s}$$

Initial slope = –20 dB/dec, due to pole at origin

Final slope = 0 dB/dec, due to finite zero.

9. (d)

Solution of homogeneous equation is given by,

$$x(t) = \phi(t)\, x(0)$$

$\phi(t)$ = State transition matrix

$x(0)$ = Initial conditions of system

Given : $$[A] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & -1 \\ 0 & s-1 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \dfrac{1}{s-1} & \dfrac{1}{(s-1)^2} \\ 0 & \dfrac{1}{s-1} \end{bmatrix}$$

$$\phi(t) = L^{-1}[sI - A]^{-1} = \begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}$$

$$x(t) = \phi(t)\, x(0) = \begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ 0 \end{bmatrix}$$

10. (b)

$$\text{C.L.T.F. } T(s) = \frac{4}{s^2+4}$$

$$\text{Characteristic equation} = s^2 + 4 = 0$$

On comparing, $s^2 + 2\xi\omega_n s + \omega_n^2 = 0, \quad \xi = 0$

So, $\quad \text{Resonant peak } (M_r) = \dfrac{1}{2\xi\sqrt{1-\xi^2}} = \infty$

11. (a)

Response of the system in Laplace form is,

$$Y(s) = X_2(s) \qquad \text{...(i)}$$

For zero state response,

$$X(s) = \phi(s)\, BR(s) \qquad \text{...(ii)}$$

Where, $\quad \phi(s) = [sI - A]^{-1}$

$$R(s) = \frac{1}{s^2}$$

State space representation is

$$\dot{x}(t) = A\, x(t) + Br(t)$$

$$y(t) = C\, x(t)$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+1 & 1 \\ -2 & s \end{bmatrix}}{s^2+s+2}$$

From equation (ii), we get

$$X(s) = \frac{1}{s^2+s+2}\begin{bmatrix} s+1 & 1 \\ -2 & s \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{s^2+s+2} \\ \frac{s}{s^2+s+2} \end{bmatrix}\cdot\begin{bmatrix} \frac{1}{s^2} \end{bmatrix}$$

$$X(s) = \begin{bmatrix} \frac{1}{s^2(s^2+s+2)} \\ \frac{1}{s(s^2+s+2)} \end{bmatrix}$$

From equation (i), we get

$$Y(s) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

$$Y(s) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s^2(s^2+s+2)} \\ \frac{1}{s(s^2+s+2)} \end{bmatrix}$$

$$Y(s) = \frac{1}{s(s^2+s+2)}$$

12. (d)

At phase crossover frequency,

$$\angle G(j\omega)H(j\omega) = -180°$$

Given, $$G(s)\,H(s) = \frac{(9+0.5s)(s+2)}{s(s+3)}$$

$$G(j\omega)\,H(j\omega) = \frac{(9+j0.5\omega)(j\omega+2)}{j\omega(3+j\omega)}$$

$$\angle G(j\omega)\,H(j\omega) = \tan^{-1}\left(\frac{0.5\omega}{9}\right) + \tan^{-1}\left(\frac{\omega}{2}\right) - 90° - \tan^{-1}\left(\frac{\omega}{3}\right)$$

At $\omega = \omega_{pc}$, $$\angle G(j\omega)H(j\omega) = -180°$$

$$-180° + 90° = \tan^{-1}\left(\frac{0.5\omega}{9}\right) + \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right)$$

$$-90° = \tan^{-1}\left(\frac{0.5\omega}{9}\right) + \tan^{-1}\left(\frac{\frac{\omega}{2}-\frac{\omega}{3}}{1+\frac{\omega^2}{6}}\right)$$

$$-90° = \tan^{-1}\left(\frac{0.5\omega}{9}\right) + \tan^{-1}\left(\frac{\omega}{(6+\omega^2)}\right)$$

$$-90^\circ = \tan^{-1}\left(\frac{\frac{0.5\omega}{9}+\frac{\omega}{6+\omega^2}}{1-\frac{0.5\omega^2}{9(6+\omega^2)}}\right)$$

$$\Rightarrow \quad \left(\frac{\frac{0.5\omega}{9}+\frac{\omega}{6+\omega^2}}{1-\frac{0.5\omega^2}{9(6+\omega^2)}}\right) = -\infty$$

$$1 = \frac{0.5\omega^2}{54+9\omega^2}$$

$$9\,\omega^2 - 0.5\,\omega^2 + 54 = 0$$

$$\omega = \sqrt{-6.35}$$

Frequency can not be imaginary. Thus, phase crossover frequency does not exist.

Alteratively, for second order system, Bode phase plot never crosses –180° axis. Thus phase crossover frequency is infinite.

13. **(c)**

From time response curve,

$$t_s = \frac{4}{\xi\omega_n} \qquad \text{(For 2\% tolerance band)}$$

$$4 = \frac{4}{\xi\omega_n}$$

$$\xi\omega_n = 1$$

and $$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$0.09 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\ln(0.09) = -\frac{\pi\xi}{\sqrt{1-\xi^2}}$$

$$(2.41)^2(1-\xi^2) = \pi^2\xi^2$$

$$(2.41)^2 = \xi^2(\pi^2 + (2.41)^2)$$

$$\sqrt{\frac{(2.41)^2}{\pi^2+(2.41)^2}} = \xi$$

$$\text{Damping ratio, } \xi = 0.61$$

$$\omega_n = \frac{1}{0.61} = 1.64$$

Standard second order transfer function:

$$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$T(s) = \frac{2.7}{s^2+2s+2.7}$$

14. (c)

At resonant frequency, resonant peak is achieved.

$$M_r = 1.6,$$
$$\omega_r = 4 \text{ rad/sec}$$

M_r and ω_r are given by

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

Now,
$$1.6 = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$4\xi^4 - 4\xi^2 + \left(\frac{10}{16}\right)^2 = 0$$

$$\xi = 0.94,\ 0.33 \qquad (\omega_r \text{to exist } 1 - 2\xi^2 > 0;\ \xi \text{ should be less than } \frac{1}{\sqrt{2}})$$

$$\xi = 0.33 \text{ is acceptable}$$

and
$$\omega_n = \frac{4}{\sqrt{1-2(0.33)^2}}$$

$$\omega_n = 4.53 \text{ rad/sec}$$

Settling time for 2% tolerance band is,

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.33\times 4.53} = 2.7 \text{ sec}$$

15. (c)

$$\text{The system matrix } A = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$$

The state equation with stable variable feedback is

$$\dot{x} = (A - BK)x + Br$$

$$(A - BK) = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -20-K_1 & -9-K_2 \end{bmatrix}$$

The desired characteristic equation is

$$s^2 + 2s + 5 = 0 \qquad \text{...(i)}$$

The characteristic equation using state variable feedback is

$$|sI - (A - BK)| = 0$$

$$\begin{vmatrix} s & -1 \\ 20+K_1 & s+9+K_2 \end{vmatrix} = 0$$

$$s^2 + 9s + K_2 s + 20 + K_1 = 0$$

$$s^2 + (9 + K_2)s + (20 + K_1) = 0 \qquad \text{...(ii)}$$

Comparing (i) and (ii),

$$K_2 = -7$$
$$K_1 = -15$$
$$K = [-15 \ \ -7]$$

16. (c)

$$G(s) = \frac{K(s+4)}{(s+2)^2}$$

The open loop poles are,

$$s = -2, -2$$

The open loop zero is $s = -4$

$$P - Z = 2 - 1 = 1$$

So angle of asymptote is 180°

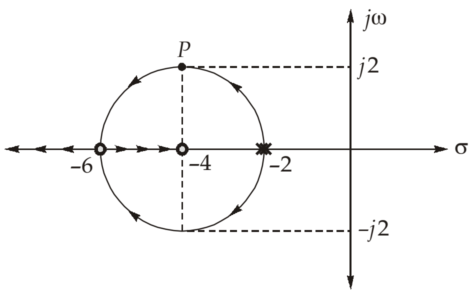
The characterisitic equation is,

$$(s+2)^2 + K(s+4) = 0$$

Put $\frac{dK}{dS} = 0$

$$\frac{dK}{dS} = \frac{(s+2)(s+6)}{(s+4)^2} = 0$$

$s = -2$ and $s = -6$ are the breakaway points.



At point P, $S = -4 + j2$,

at this point $\omega_d = 2$ rad/sec

$$\left|\frac{K(s+4)}{(s+2)^2}\right|_{s=-4+j2} = 1$$

$$\left|\frac{K(-4+j2+4)}{(-4+j2+2)^2}\right| = 1$$

$$\frac{2K}{\left(\sqrt{2^2+2^2}\right)^2} = 1$$

$$K = 4$$

17. (d)

Given, $$G(s) = \frac{1}{s(1+2s)(1+4s)}$$

$$G(j\omega) = \frac{1}{j\omega(1+2j\omega)(1+j4\omega)}$$

$$G(j\omega) = \frac{(1-j2\omega)(1-j4\omega)}{j\omega(1+4\omega^2)(1+16\omega^2)} = \frac{1-j6\omega-8\omega^2}{j\omega(1+4\omega^2)(1+16\omega^2)}$$

$$= \frac{-6}{(1+4\omega^2)(1+16\omega^2)} - \frac{j(1-8\omega^2)}{\omega(1+4\omega^2)(1+16\omega^2)}$$

At $\omega = 0$; $\quad G(j\omega) = -\frac{6}{1} - \frac{j(1)}{0} = -6 - j\infty$

At $\omega = \infty$; $\quad G(j\omega) = -\frac{6}{\infty} - \frac{j}{\infty} = -0 - j0$

plot cuts the negative real axis, when

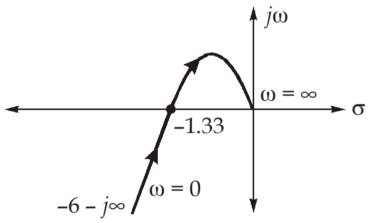
$$\text{Img}\,[G(j\omega)] = 0$$

$$1 - 8\omega^2 = 0$$

$$\omega = \frac{1}{2\sqrt{2}}\ \text{rad/sec}$$

$$G(j\omega) = \frac{-6}{\left(1+4\times\frac{1}{8}\right)\left(1+16\times\frac{1}{8}\right)} = -1.33$$

From above points, polar plot can be drawn as



18. (b)

From the above Bode plot,

For section *de*, slope is –20 dB/dec

$$\therefore \quad -20 = \frac{y-0}{\log 8 - \log 16}$$

$$y = 6.02 \text{ dB}$$

Now, for section *bc*, slope is –20 dB/dec

$$\therefore \quad -20 = \frac{16-6.02}{\log \omega_1 - \log 4}$$

$$\omega_1 = 1.268 \text{ rad/sec}$$

To find value of gain *K*

$$y = mx + c$$

$$16 = -40 \log 1.268 + 20 \log K$$

$$K = 10.14$$

From all the result, transfer function is,

$$T(s) = \frac{10.14\left(\frac{s}{1.268}+1\right)\left(\frac{s}{4}+1\right)}{s^2\left(\frac{s}{8}+1\right)}$$

$$T(s) = \frac{16(s+1.268)(s+4)}{s^2(s+8)}$$

19. (a)

Closed loop transfer function with unity feedback

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$= \frac{\frac{K}{s(s+4)}}{1+\frac{K}{s(s+4)}} = \frac{K}{s^2+4s+K}$$

Comparing $T(s)$ with standard form

$$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

We get, $\omega_n^2 = K$

$\Rightarrow$ $\omega_n = \sqrt{K}$

$2\xi\omega_n = 4$

Given damping ratio, $\xi = 0.5$

$\therefore$ $$\omega_n = \frac{4}{2\xi} = \frac{4}{2\times(0.5)} = 4$$

$$K = \omega_n^2 = 16$$

Peak overshoot is given by,

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$= e^{-\pi(0.5)/\sqrt{1-0.25}}$$

$$= e^{-1.814} = 0.163$$

20. (c)

$$y(t) = AM\sin(2t+\phi)$$

where, $A = 2$, and $M = \left|\frac{1}{j\omega+2}\right|$

At $\omega = 2$, $$M = \frac{1}{2\sqrt{2}}$$

and $$\phi = -\tan^{-1}\left(\frac{\omega}{2}\right) = -\tan^{-1}\left(\frac{1}{1}\right) = -\frac{\pi}{4}$$

$$y(t) = \frac{1}{\sqrt{2}}\sin\left(2t-\frac{\pi}{4}\right)$$

21. (d)

Using Routh table :

s^6	1	6	10	5
s^5	1	5	5	
s^4	1	5	5	
s^3	0	0		

The Routh table construction procedure breaks down here. Since the s^3 row has all zeros. The auxiliary polynomial coefficients are given by the s^4 row. Therefore the auxiliary polynomial is

$$A(s) = s^4 + 5s^2 + 5$$

$$\frac{dA(s)}{ds} = 4s^3 + 10s$$

Replacing the s^3 row in the Routh table with the coefficients of $\frac{dA(s)}{ds}$, we have,

$$\begin{array}{c|llll} s^6 & 1 & 6 & 10 & 5 \\ s^5 & 1 & 5 & 5 & \\ s^4 & 1 & 5 & 5 & \\ s^3 & 4 & 10 & & \\ s^2 & \frac{20-10}{4} = 2.5 & 5 & & \\ s^1 & \frac{25-20}{2.5} = 2 & & & \\ s^0 & 5 & & & \end{array}$$

Examining the first column of this table we see that there are no sign changes. Hence, there is no root lying in the RHS of s-plane.

22. **(d)**

Steady state gain = 1

Given, $\left.\begin{aligned} |G(j\omega)| &= \frac{1}{2} \\ \angle G(j\omega) &= -90^\circ \end{aligned}\right\}$ at $\omega = 1$ rad/sec

In option (d), $G(j\omega)|_{\omega=1}$ $= \frac{1}{\left[\sqrt{1+1}\right]^2}\angle -45^\circ - 45^\circ = \frac{1}{2}\angle -90^\circ$

23. **(d)**

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_n = \frac{10}{\sqrt{1-0.6^2}} = 12.5 \text{ rad/sec}$$

Desired characteristic equation of second order is

$$= s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 15s + 156.25 = 0$$

C.E. of given system is

$$1 + (K_p + sK_D)\left[\frac{1}{s(s+2)}\right] = 0$$

$$s^2 + 2s + K_p + sK_D = 0$$

$$s^2 + (2 + K_D)s + K_p = 0$$

On comparing $K_p = 156.25$

$$K_D = 15 - 2 = 13$$

24. **(d)**

$$G(j\omega) = \frac{1+j4\omega}{1+j2\omega}$$

$$\left.\begin{array}{lrl} \text{At } \omega = 0, & G(j\omega) = & 1\angle 0° \\ \text{At } \omega = 2, & G(j\omega) = & 1.9553\angle 6.91° \\ \text{At } \omega = 10, & G(j\omega) = & 1.99\angle 1.43° \\ \text{At } \omega = \infty, & G(j\omega) = & 2\angle 0° \end{array}\right] \text{First quadrant with clockwise direction}$$

So, correct option is (d).

25. **(a)**

Number of open loop poles in R.H.S. of s-plane(P) = 1.

$$\text{C.E.,}(s + 0.5)(s - 2) + 1.25(s + 1) = 0$$
$$s^2 - 1.5s - 1 + 1.25s + 1.25 = 0$$
$$s^2 - 0.25s + 0.25 = 0$$
$$s = 0.125 \pm j0.4841 \text{[Location of closed loop poles]}$$

So, both closed loop poles lies in R.H.S. of s-plane, Z = 2.

Number of encirclement (N)

$$N = Z - P[\because \text{ Nyquist contour in anticlockwise direction}]$$
$$N = 2 - 1 = 1$$

N is positive for clockwise encirclement.

N is negative for anti-clockwise encirclement.

So, Nyquist plot will encircles $-1 + j0$, once in clockwise direction.

26. **(d)**

The location of the poles are given by, $-\xi\omega_n \pm j\omega_d$ ...(i)

where, ξ = damping ratio

ω_n = natural frequency of oscillation

ω_d = damped frequency of oscillation

Using maximum peak overshoot, the value of ξ can be obtained as

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.15$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = 0.604$$

Squaring both the sides,

$$\xi^2 = 0.364(1 - \xi^2)$$

or $$\xi^2 = \frac{0.364}{1.364} = 0.267$$

or $$\xi = 0.517$$ ...(ii)

now, peak time, $$\tau_p = \frac{\pi}{\omega_d} = 3$$

or $$\omega_d = \frac{\pi}{3} = 1.047 \text{ rad/sec}$$

$\because \quad \omega_d = \omega_n\sqrt{1-\xi^2}$ ...(iii)

$\therefore$ From equation (ii) and (iii), we have

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{1.047}{\sqrt{1-0.517^2}}$$

$$\omega_n = 1.223 \text{ rad/sec} \quad \text{...(iv)}$$

$\therefore$ Location of poles are,

$$P = -\xi\omega_n \pm j\omega_d$$
$$= -(0.517 \times 1.223) \pm j1.047$$
$$= -0.632 \pm j1.047$$

27. (d)

Steady state error,

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s \times \frac{R(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{s\times\left(2+\frac{5}{s}\right)\times\frac{1}{s}}{1+\frac{K}{s(s+3)}}$$

$$= \lim_{s\to 0} \frac{\frac{(2s+5)}{s}\times s(s+3)}{(s^2+3s+K)}$$

$$= \lim_{s\to 0} \frac{(2s+5)(s+3)}{s^2+3s+K}$$

$$2.75 = \frac{15}{K}$$

or $\quad K = \frac{15}{2.75} = 5.45$

28. (d)

For any point to lie on the root locus the angle condition must be satisfied.

$$\angle G(s)H(s)\big|_{s=(-1+j2)} = \pm 180°$$

$$\therefore \quad G(s)H(s)\big|_{s=(-1+j2)} = \frac{K(-1+j2+1)}{(-1+j2+9)(-1+j2+3)} = \frac{K(j2)}{(8+j2)(2+j2)}$$

$$\therefore \quad \angle G(s)H(s)\big|_{s=-1+j2} = 90° - \tan^{-1}\left(\frac{2}{8}\right) - \tan^{-1}(1)$$

$$= 90° - 14.036° - 45°$$
$$= 30.96°$$

$$\because \quad \angle G(s)H(s)\big|_{s=s_0} \neq \pm 180°$$

Angle condition does not satisfy.

29. (b)

The steady state error is defined by

$$e_{ss} = \lim_{s \to 0} \frac{s \times \frac{1}{s^2}}{1 + \frac{(s+\alpha)}{s} \times \frac{(s+2)}{s^2 - 1}}$$

$$= \lim_{s \to 0} \frac{(s^2 - 1)}{s(s^2 - 1) + (s+\alpha)(s+2)}$$

$$e_{ss} = -\frac{1}{2\alpha}$$

$$\therefore \quad S_{\alpha}^{e_{ss}} = \frac{\frac{\partial e_{ss}}{e_{ss}}}{\frac{\partial \alpha}{\alpha}} = \frac{\partial e_{ss}}{\partial \alpha} \times \frac{\alpha}{e_{ss}} = \frac{\partial}{\partial \alpha}\left(\frac{-1}{2\alpha}\right) \times \frac{\alpha}{-\frac{1}{2\alpha}}$$

$$= -\frac{\alpha^2}{\alpha^2} = -1$$

30. (b)

$$\phi(t) = L^{-1}[(sI - A)^{-1}]$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & -1 \\ 8 & s+6 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 + 6s + 8} \begin{bmatrix} s+6 & 1 \\ -8 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+6}{(s+2)(s+4)} & \frac{1}{(s+2)(s+4)} \\ \frac{-8}{(s+2)(s+4)} & \frac{s}{(s+2)(s+4)} \end{bmatrix}$$

$$\phi(t) = L^{-1}[(sI - A)^{-1}] = \begin{bmatrix} (2e^{-2t} - e^{-4t}) & \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t}\right) \\ (-4e^{-2t} + 4e^{-4t}) & (-e^{-2t} + 2e^{-4t}) \end{bmatrix}$$

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