Electrical Engineering

Electromagnetic Theory

Comprehensive Theory

with **Solved Examples** and **Practice Questions**

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Electromagnetic Theory

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CHAPTER 1

Vector Analysis

1.1 Introduction

This introductory chapter provides an elegant mathematical language in which electromagnetic (EM) theory is conveniently expressed and best understood. The quantities of interest appearing in the study of EM theory can almost be classified as either a scalar or a vector.

Quantities that can be described by a magnitude alone are called **scalars**. Distance, temperature, mass etc. are examples of scalar quantities. Other quantities, called **vectors**, require both a magnitude and a direction to fully characterize them. Examples of vector quantities include velocity, force, acceleration etc.

In electromagnetics, we frequently use the concept of a **field**. A field is a function that assigns a particular physical quantity to every point in a region. In general, a field varies with both position and time. There are scalar fields and vector fields. Temperature distribution in a room and electric potential are examples of scalar fields. Electric field and magnetic flux density are examples of vector fields.

NOTE: Vectors are denoted by an arrow over a letter ($\vec{A}$) and scalars are denoted by simple letter (A).

1.1.1 Unit Vector

A unit vector $\hat{a}_A$ along $\vec{A}$ is defined as a vector whose magnitude is unity (*i.e.*, 1) and its direction is along $\vec{A}$, that is

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A} \quad ...(1.1)$$

Thus we can write $\vec{A}$ as

$$\vec{A} = A\,\hat{a}_A = |\vec{A}|\hat{a}_A \quad ...(1.2)$$

Remember: Any vector can be written as product of its magnitude and its unit vector.

1.1.2 Vector Addition and Subtraction

Two vectors $\vec{A}$ and $\vec{B}$ can be added together to give another vector $\vec{C}$; that is,

$$\vec{C} = \vec{A}+\vec{B} \quad ...(1.3)$$

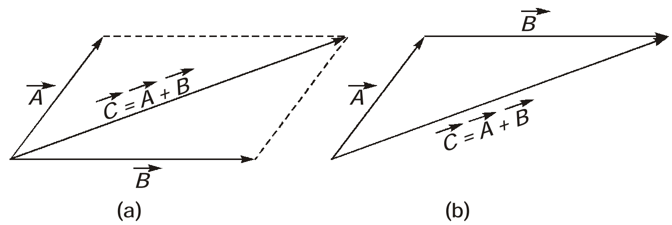


***Figure.* 1.1:** *Vector addition (a) parallelogram rule, (b) head-to-tail rule.*

NOTE

- $\vec{A}+\vec{B}=\vec{B}+\vec{A}$ (Commutative law)
- $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$ (Associative law)

Vector subtraction is similarly carried out as

$$\vec{D} = \vec{A}-\vec{B}=\vec{A}+(-\vec{B}) \quad \text{...(1.4)}$$

Remember: Graphically, vector addition and subtraction are obtained by either the parallelogram rule or the head-to-tail rule as potrayed in figure 1.1.

NOTE

- $k(\vec{A}+\vec{B})=k\vec{A}+k\vec{B}$ (Distributive law)
- $\dfrac{\vec{A}+\vec{B}}{k}=\dfrac{1}{k}\vec{A}+\dfrac{1}{k}\vec{B}$

1.1.3 Position and Distance Vectors:

A point P in cartesian coordinates may be represented by (x, y, z).

The position vector $\vec{r}_p$ (or radius vector) of point P is defined as the directed distance from origin O to P.

$$\vec{r}_p = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z \quad \text{...(1.5)}$$

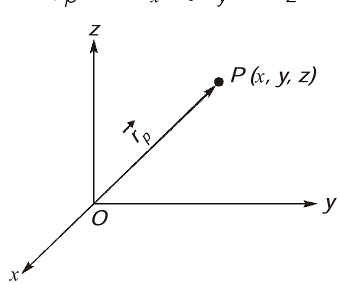


Figure 1.2: *Illustration of position vector* $\vec{r}_p = x\hat{a}_x + \hat{y}a_y + z\hat{a}_z$

The distance vector is the displacement from one point to another.

Consider point P with position vector $\vec{r}_p$ and point Q with position vector $\vec{r}_q$. The displacement from P to Q is written as

$$\vec{R}_{PQ} = \vec{r}_q-\vec{r}_p \quad \text{...(1.6)}$$

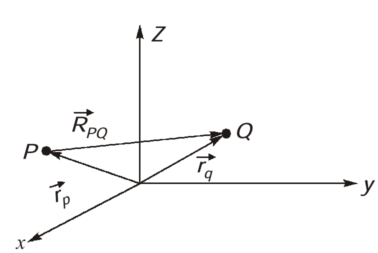


Figure 1.3: *Vector distance* $\vec{R}_{PQ}$

Example 1.1 Point P and Q are located at (0, 2, 4) and (–3, 1, 5). Calculate:

(a) The position vector P

(b) The distance vector from P to Q

(c) The distance between P and Q

(d) A vector parallel to PQ with magnitude of 10.

Solution:

(a) $$\vec{r}_P = 0\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z = 2\hat{a}_x + 4\hat{a}_z$$

(b) $$\vec{R}_{PQ} = \vec{r}_q - \vec{r}_p = (-3, 1, 5) - (0, 2, 4) = (-3, -1, 1)$$
$$= -3\hat{a}_x - \hat{a}_y + \hat{a}_z$$

(c) The distance between P and Q is the magnitude of $\vec{R}_{PQ}$; that is

$$d = |\vec{R}_{PQ}| = \sqrt{9+1+1} = 3.317$$

(d) Let the required vector be $\vec{A}$, then

$$\vec{A} = A\hat{a}_A$$

where $A = 10$ is magnitude of $\vec{A}$

and $$\hat{a}_A = \frac{\vec{R}_{PQ}}{|\vec{R}_{PQ}|} = \pm\frac{(-3,-1,1)}{3.317}$$

then $$\vec{A} = \pm\frac{10(-3,-1,1)}{3.317} = \pm(-9.045\,\hat{a}_x - 3.015\,\hat{a}_y + 3.015\,\hat{a}_z)$$

1.1.4 Vector Multiplication

When two vectors are multiplied, the result is either a scalar or a vector depending on how they are multiplied. Thus there are two types of vector multiplication.

1. Scalar (or dot) product : $\vec{A} \cdot \vec{B}$
2. Vector (or cross) product : $\vec{A} \times \vec{B}$

Multiplication or three vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ can result in either

3. Scaler triple product : $\vec{A} \cdot (\vec{B} \times \vec{C})$
4. Vector triple product : $\vec{A} \times (\vec{B} \times \vec{C})$

Dot Product:

The dot product, or the scalar product of two vectors $\vec{A}$ and $\vec{B}$, written as $\vec{A} \cdot \vec{B}$ is defined geometrically as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and the cosine of the angle between them.

Thus $$\vec{A} \cdot \vec{B} = A\,B\cos\theta_{AB} \quad ...(1.7)$$

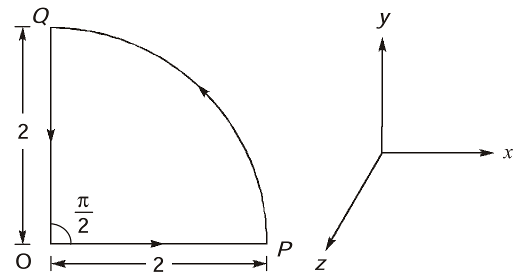
Where θ_{AB} is the smaller angle between $\vec{A}$ and $\vec{B}$. The result of $\vec{A} \cdot \vec{B}$ is called either the scalar product because it is scalar, or the dot product due to the dot sign.

If $$\vec{A} = (A_x, A_y, A_z)$$

and $$\vec{B} = (B_x, B_y, B_z)$$

then $$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad ...(1.8)$$

Example 1.9 If $\vec{A} = \hat{a}_\rho + \hat{a}_\phi + \hat{a}_z$, the value of $\oint \vec{A} \cdot d\vec{l}$ around the closed circular quadrant shown in the given figure is



Solution:

$$\oint \vec{A} \cdot d\vec{l} = \oint (\hat{a}_\rho + \hat{a}_\phi + \hat{a}_z).(d\rho\hat{a}_\rho + \rho d\phi\hat{a}_\phi + dz\hat{a}_z)$$

For path OP $$\oint \vec{A} \cdot d\vec{l} = \int_0^2 d\rho = 2$$

For path PQ $$\oint \vec{A} \cdot d\vec{l} = \int_0^{\pi/2} \rho d\,\phi = 2 \times \frac{\pi}{2} = \pi$$

For path QO $$\oint \vec{A} \cdot d\vec{l} = \int_2^0 d\rho = -2$$

$\therefore$ $$\oint \vec{A} \cdot d\vec{l} = 2 + \pi - 2 = \pi$$

Surface Integral

Another integral that will be encountered in the study of electromagnetic fields is the surface intetgral. Given a vector field $\vec{A}$, continuous in a region containing the smooth surface S, we define the surface integral or the flux of $\vec{A}$ through S (see figure 1.15)

$$\psi = \int_S |\vec{A}|\cos\theta \, dS = \int_S \vec{A} \cdot \hat{a}_n dS \quad ...(1.66)$$

Or simply $$\psi = \int_S \vec{A} \cdot d\vec{S} \quad ...(1.67)$$

Where, at any point on S, $\hat{a}_n$ is the unit normal to S. For a closed surface (defining a volume), precedent equation becomes:

$$\psi = \oint_S \vec{A} \cdot d\vec{S} \quad ...(1.68)$$

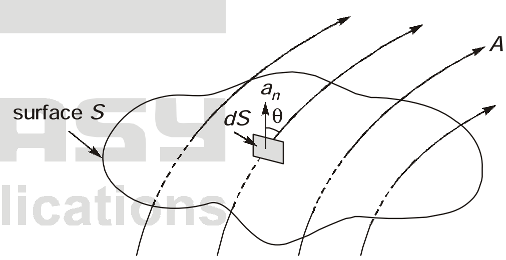


Figure 1.15: *The flux of a vector field $\vec{A}$ through surface S.*

Which is referred to as the net outward flux of $\vec{A}$ from S.

NOTE: Notice that a closed path defines an open surface whereas a closed surface defines a volume.

Volume Integral

Finally, we will encounter various volume integrals of scalar quantities, such as a volume charge density ρ_v. A typical integration would involve the computation of the total charge if the volume charge density was known. It is written as:

$$Q = \int_v \rho_v \, dv \quad ...(1.69)$$

CHAPTER 3

Magnetostatics

3.1 Introduction

According to Coulomb's law, a distribution of stationary charge produces a static electric field (electrostatic field). The analogous equation to Coulomb's law in electrostatics is the Biot-Savart law in magnetostatics. The Biot-Savart law shows that when charge moves at a constant rate (direct current - DC), a static magnetic field (magnetostatic field) is produced.

NOTE : A magnetostatic field is produced by a constant current flow (or direct current).

Static magnetic field are also produced by stationary permanent magnets. When permanent magnets are set in motion such that a time varying magnetic field is produced, a time varying electric field is simultaneously produced. A time varying electric field cannot exist without a corresponding time varying magnetic field and vice versa.

Table 3.1 shows the analogy between electric and magnetic field quantities. Some of the magnetic field quantities will be introduced later in this chapter. The analogy is presented here to show that most of the equations we have derived for the electric fields may be readily used to obtain corresponding equations for magnetic field if the equivalent analogous quantities are used.

Term	Electric	Magnetic
Basic laws	$\vec{F} = \dfrac{Q_1Q_2}{4\pi\varepsilon_0^2}\hat{a}_r$	$d\vec{B} = \dfrac{\mu_0 I d\vec{l} \times \hat{a}_r}{4\pi R^2}$
	$\int \vec{D}.d\vec{S} = Q_{enc}$	$\int \vec{H}.d\vec{l} = I_{enc}$
Force law	$\vec{F} = Q\vec{E}$	$F = Q(\vec{v} \times \vec{B})$
Source element	dQ	Idl
Field intensity	$E = \dfrac{V}{l}(V/m)$	$H = \dfrac{I}{l}$ (A/m)
Flux density	$D = \dfrac{\Psi}{S}$ (C/m^2)	$B = \dfrac{\Psi}{S}$ (Wb/m^2)
Relationship between field	$\vec{D} = \varepsilon\vec{E}$	$\vec{B} = \mu\vec{H}$

Potentials	$\vec{E} = -\nabla V$	$\vec{H} = \nabla V_m (\vec{H} = 0)$
	$V = \int \dfrac{\rho_L dl}{4\pi \varepsilon_r}$	$\vec{A} = \int \dfrac{\mu I d\vec{l}}{4\pi R}$
Flux	$\psi = \int \vec{D} \cdot d\vec{S}$	$\psi = \int \vec{B} \cdot d\vec{S}$
	$\psi = Q = CV$	$\psi = LI$
	$I = C\dfrac{dV}{dt}$	$V = L\dfrac{dI}{dt}$
Energy density	$W_E = \dfrac{1}{2}\vec{D}\cdot\vec{E}$	$w_m = \dfrac{1}{2}\vec{B}.\vec{H}$
Poisson's equation	$\nabla^2 V = -\dfrac{\rho_v}{\varepsilon}$	$\nabla^2 A = -\mu\vec{J}$

Table 3.1: *Analogy beteen electric and magnetic fields.*

3.2 Biot-Savart's Law

The source of the steady magnetic field may be a permanent magnet, an electric field changing linearly with time, or a direct current. We shall largely ignore the permanent magnet and save the time-varying electric field for a later discussion. Our present relationships will concern the magnetic field produced by a differential dc element in free space.

Biot-Savart's law states that the magnetic field intensity dH produced at a point P, as shown in figure 3.1, by the differential current element Idl is proportional to the product Idl and the sine of the angle α between the element and the line joining P to the element and is inversely proportional to the square to the distance R between P and the element.

That is $\quad dH \propto \dfrac{Idl \sin\alpha}{R^2}$ ...(3.1)

The unit of magnetic field intensity H is evidently amperes per meter (A/m).

The constant of proportionality is equal to $\dfrac{1}{4\pi}$. Thus, from the definition of the cross product we can write

$$d\vec{H} = \frac{I\,d\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{I\,d\vec{l} \times \vec{R}}{4\pi R^3} \quad ...(3.2)$$

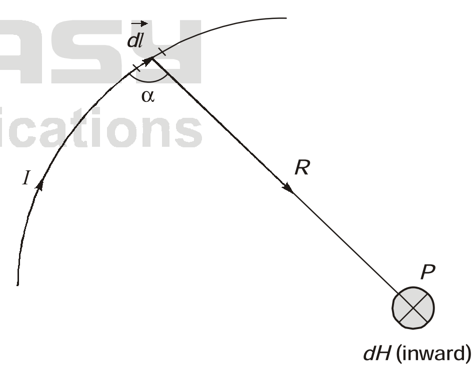


Figure 3.1: *Magnetic field dH at P due to current element Idl.*

where $R = |\vec{R}|$ and $\hat{a}_R = \dfrac{\vec{R}}{R}$. Thus the direction of $d\vec{H}$ can be determined by the right-hand rule with the right-hand thumb pointing in the direction of the current, the right-hand fingers encircling the wire in the direction of $d\vec{H}$ as shown in figure 3.2 (a). Alternatively, we can use the right-handed screw rule to determine the direction of $d\vec{H}$: with the screw placed along the wire and pointed in the direction of current flow, the direction of advance of the screw is the direction of $d\vec{H}$ as in figure 3.2 (b).

CHAPTER 6

Transmission Lines

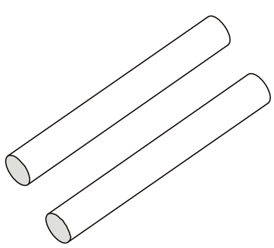
6.1 Introduction

Transmission lines may be defined as devices used to guide energy from one point to another (from a source to a load). Transmission lines can consist of a set of conductors, dielectrics or combination thereof. Transmission lines are normally used in power distribution at low frequencies and in communications at high frequencies.

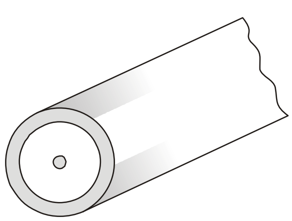
In the last chapter, we have shown that using Maxwell's equations, we can transmit energy in the form of an unguided wave (plane wave) through space. In a similar manner, Maxwell's equations show that we can transmit energy in the form of a guided wave on a transmission line.

NOTE : Plane wave propagation in air: Unguided wave propagation
Transmission Lines: Guided wave propagation

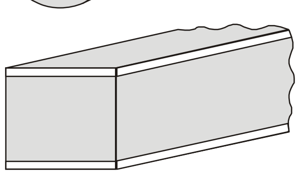
6.1.1 Transmission Lines Examples



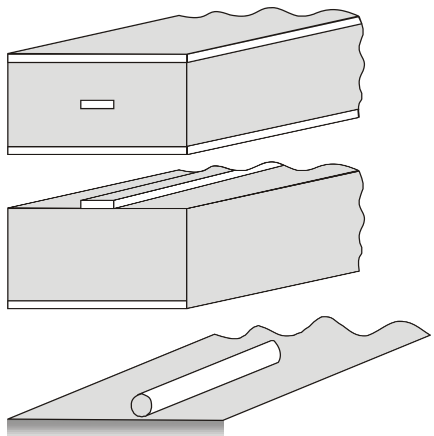
(a) Two-wire line (twisted pair is a variation of the two-wire line)



(b) Coaxial line



(c) Parallel plate or planar line



(d) Strip line

(e) Microstrip line

(f) Wire above conducting plane

***Figure. 6.1 :** Transmission line types*

NOTE

- Parallel - wire line is used where balanced properties are required (example: in connecting a folded dipole antenna to a TV receiver).
- Coaxial line is used when unbalanced properties are required (Example: in connecting a broadcast transmitter to its grounded antenna).

6.1.2 Transmission Lines Definitions

Uniform Transmission Line: Conductors and dielectrics maintain the same cross-sectional geometry along the transmission line in the direction of wave propagation.

NOTE : Most of the transmission lines have this type of geometry (e.g. Two wire coaxial etc.)

Transmission Line Mode: A distinct pattern of electric and magnetic field induced on a transmission line under source excitation.

NOTE : Throughout the chapter we assume TEM mode of wave propagation.

6.2 Transmission Line Equations

Transmission line are typically electrically long (several wavelengths) such that we cannot accurately describe the voltages and currents along the transmission line using a simple lumped-element equivalent circuit. We must use a distributed-element equivalent circuit which describes each short segment of the transmission line by a lumped-element equivalent circuit.

Consider a simple uniform two-wire transmission line with its conductors parallel to the *z*-axis as shown below.

Uniform Transmission Line: Conductors and insulating medium maintain the same cross-sectional geometry along the entire transmission line.

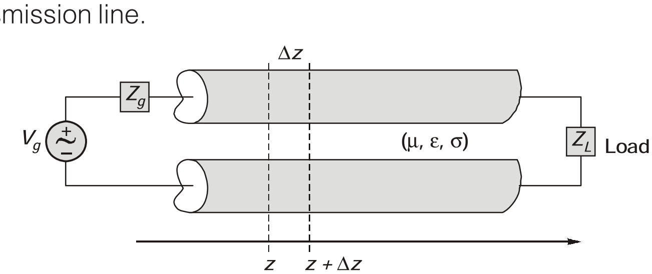


***Figure 6.2:** Transmission line connecting source to a load*

6.3 Transmission Line Circuit (Input impedance, Reflection coefficient, SWR)

The most commonly encountered transmission line configuration is the simple connection of a source (or generator) to a load (Z_L) through the transmission line. The generator is characterized by a propagation constant γ and characteristic impedance Z_o. In this section we will determine the input impedance, the standing wave ratio (SWR), and the power flow on the line.

6.3.1 Input Impedance (Z_{in})

Consider a transmission line of length l, characterized by γ and Z_0, connected to a load Z_L as shown in figure (6.6). Looking into the line, the generator sees the line with the load as an input impedance Z_{in}.

Let the transmission line extend from $z = 0$ at the generator to $z = l$ at the load. The voltage and current waves are:

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad ...(6.52)$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \qquad ...(6.53)$$

To find V_0^+ and V_0^-, the terminal conditions must be given.

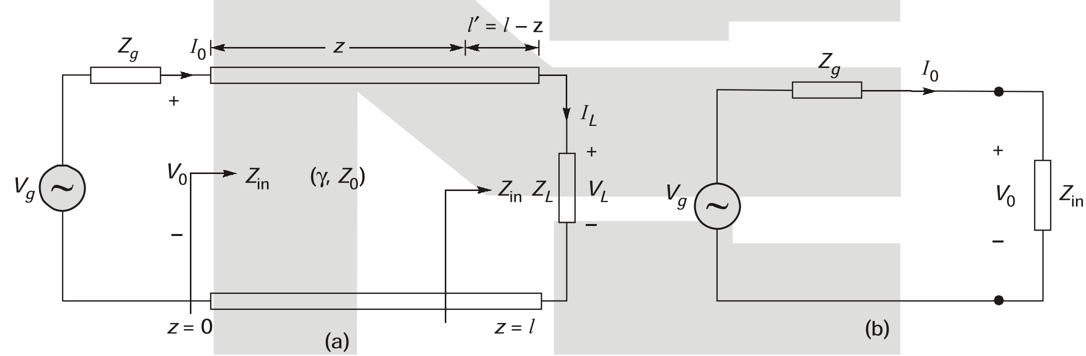


Figure 6.6: *(a) Input impedance due to a line terminated by a load.*
(b) Equivalent circuit for finding V_0 and I_0 in terms of Z_{in} at the input.

If we are given the conditions at the input,

$$V_0 = V(z = 0),\ I_0 = I(z = 0) \qquad ...(6.54)$$

substituting these into equations (6.52) and (6.53) results in

$$V_0^+ = \frac{1}{2}(V_0 + Z_0 I_0) \qquad ...(6.55)$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_0) \qquad ...(6.56)$$

If the input impedance at the input terminals is Z_{in}, the input voltage V_0 and the input current I_0 are easily obtained from figure (6.6 b) as

$$V_0 = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \qquad I_0 = \frac{V_g}{Z_{in} + Z_g} \qquad ...(6.57)$$

On the other hand, if we are given the conditions at the load, say

$$V_L = V(z = l),\ I_L = I(z = l) \qquad ...(6.58)$$

Substituting these into equations (6.23) and (6.24) gives

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{\gamma l} \qquad ...(6.59)$$

$$V_0^- = \frac{1}{2}(V_L - Z_0 I_L)e^{-\gamma l} \quad ...(6.60)$$

Next, we determine the input impedance $Z_{in} = \frac{V_s(z)}{I_s(z)}$ at any point on the line. At the generator, for example, equations (6.23) and (6.24) yield

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_0(V_0^+ + V_0^-)}{(V_0^+ - V_0^-)} \quad ...(6.61)$$

substituting equations (6.59, 6.60) into (6.61) and utilizing the fact that

$$\frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cos h\gamma l, \ \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sin h\gamma l$$

or
$$\tan h\gamma l = \frac{\sin h\gamma l}{\cos h\gamma l} = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} \quad ...(6.62)$$

we get,
$$Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tan h\gamma l}{Z_0 + Z_L \tan h\gamma l}\right] \text{(lossy)} \quad ...(6.63)$$

Note:

⇒ For a lossless line, $\gamma = j\beta$, $\tan hj\beta l = j\tan\beta l$, so equation (6.63) becomes

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] \text{(lossless)} \quad ...(6.64)$$

⇒ The quantity βl in equation (6.64) is usually referred to as the **electrical length** of the line and can be expressed in degrees or radians.

6.3.2 Reflection Coefficient (Γ)

We now define Γ_l as the **voltage reflection coefficient** (at the load). The reflection coefficient Γ_l is the ratio of the voltage reflection wave to the incident wave at the load, that is,

$$\Gamma_l = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} \quad ...(6.65)$$

Substituting V_0^- and V_0^+ from equations (6.59, 6.60) into equation (6.65) and incorporating $V_L = Z_L I_L$ gives,

$$\Gamma_l = \frac{Z_L - Z_0}{Z_L + Z_0} \quad ...(6.66)$$

The voltage reflection coefficient at any point on the line is the ratio of the reflected voltage wave to that of the incident wave.

That is,
$$\Gamma(z) = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^-}{V_0^+} e^{2\gamma z}$$

But $z = l - l'$. Substituting and combining with equation (6.65) we get,

$$\Gamma(z) = \frac{V_0^-}{V_0^+} e^{2\gamma l} e^{-2\gamma l'} = \Gamma_l e^{-2\gamma l'} \quad ...(6.67)$$

The current reflection coefficient at any point on the line is the negative of the voltage reflection coefficient at that point.

Then, the current reflection coefficient at the load is

$$\frac{I_0^- e^{\gamma l}}{I_0^+ e^{-\gamma l}} = -\Gamma_l \quad ...(6.68)$$

6.3.3 Standing Wave Ratio (*s*)

$$V(x) = V_o e^{+j\beta x} + V_o|\Gamma| \cdot e^{j\theta} \cdot e^{-j\beta x} = V_o e^{j\beta x} + V_o|\Gamma| \cdot e^{j(-\beta x+\theta)} \quad ...(6.69)$$

$$= A\sin\psi_1 + B\sin\psi_2$$

- The amplitude plot in a single harmonic mathced line has straight line plot as the amplitude is same everywhere.
- But for a miss matched line the amplitude plot is not the same everywhere. It has periodic maximas and minimas due to additions and cancellations and hence the distribution is non uniform.
- Positions of voltage maximas and impedance maxima also position of current of current minimas.
$$2\beta x_{max} = 2n\pi + \theta$$
- Positions of voltage minima and impedance minima also position of current of current maximas.
$$2\beta x_{min} = (2n + 1)\pi + \theta$$
- The amplitude plot of two harmonics travelling in opposite directions having interference having maxima and minima formation is called as standing wave formation.
- The gap between two consecutive maximas or minimas is $\lambda/2$ as $2\beta x$ is periodic with 2π, x is periodic with $\lambda/2$.
- In current standing as identical pattern to that of voltage maxima coninsides with current minima and vice versa.
- As the load is Z_L the loading effect anywhere on the line leads to periodic impedance maximas and impedance minimas such that where ever impedance is minima current is maxima hence,

$$Z_{max} = \frac{V_{max}}{I_{min}} \quad ; \quad Z_{min} = \frac{V_{min}}{I_{max}}$$

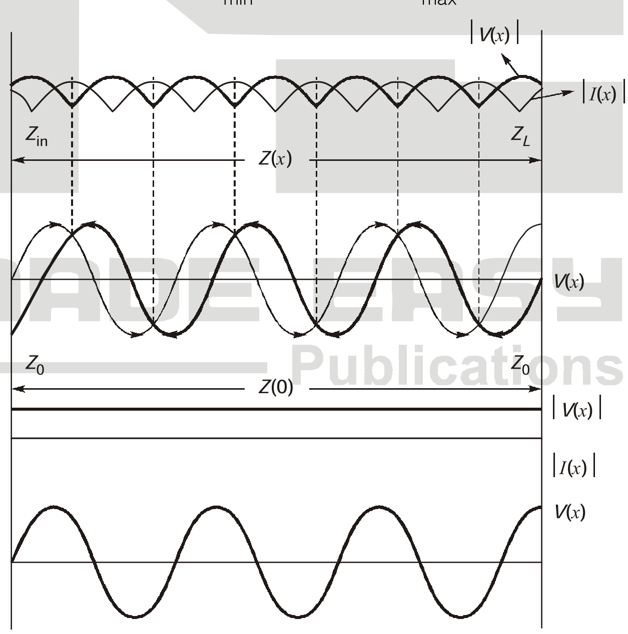


Figure 6.7

- Standing wave ratio SWR = Current SWR or voltage SWR

$$\text{SWR} = \left|\frac{V_{max}}{V_{min}}\right| \text{ (or) } \left|\frac{I_{max}}{I_{min}}\right| = \frac{V_o + V_o|\Gamma|}{V_o - V_o|\Gamma|} \quad ...(6.70)$$

$$\therefore \quad \text{SWR} = \frac{1+|\Gamma|}{1-|\Gamma|} \text{ (or) } |\Gamma| = \frac{SWR-1}{SWR+1}$$