CLASS TEST

S.No.:01SKCS_ABCDEFGHI_22724

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ENGINEERING MATHEMATICS

COMPUTER SCIENCE & IT

Date of Test : 22/07/2024

ANSWER KEY

1.	(a)	7.	(a)	13.	(a)	19.	(b)	25.	(c)
2.	(b)	8.	(c)	14.	(d)	20.	(c)	26.	(c)
3.	(c)	9.	(c)	15.	(d)	21.	(c)	27.	(c)
4.	(b)	10.	(b)	16.	(a)	22.	(b)	28.	(d)
5.	(a)	11.	(c)	17.	(a)	23.	(b)	29.	(d)
6.	(c)	12.	(c)	18.	(a)	24.	(d)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

$$\text{Probability} = \int_2^\infty f(x)dx$$

$$= \int_2^\infty \left[\frac{1}{2}e^{\frac{-x}{2}}\right]dx = \left[-e^{\frac{-x}{2}}\right]_2^\infty = e^{-1} = 0.368$$

2. (b)

Probability density function:

$$f(x) = \lambda \cdot e^{-\lambda x}, x > 0$$

$$E(X) = \int_0^\infty x \cdot f(x) \cdot dx$$

$$= \int_0^\infty x\lambda \cdot e^{-\lambda x} \cdot dx = \frac{1}{\lambda}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_0^\infty x^2 \cdot f(x) \cdot dx$$

$$= \int_0^\infty x^2 \cdot \lambda \cdot e^{-\lambda x} \cdot dx = \frac{2}{\lambda^2}$$

$$\Rightarrow \quad \text{Var}(X) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

3. (c)

The event can be considered as Binomial distribution

$$E(X) = np$$

$$n = 100$$

$$p = p(3) + p(6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$E(X) = \frac{100}{3} = 33.33$$

33.33 fall either a 6 or 3.

4. (b)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x - 1) = 0$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 - 1) = 0$$

Also $f(1) = 0$

Thus $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow$ f is continuous at $x = 1$

And $Lf'(1) = 2$, $Rf'(1) = 1$

$\Rightarrow f$ is not differentiable at $x = 1$

5. (a)

$$I = \int_{-2}^{2} |1 - x^4|\, dx$$

The given function is an even function i.e., $f(x) = f(-x)$

$$\Rightarrow \quad I = 2\int_{0}^{2} |1 - x^4|\, dx$$

$$= 2\left\{\int_0^1 (1 - x^4)\, dx + \int_1^2 (x^4 - 1) dx\right\}$$

$$= 2\left\{\left[x - \frac{x^5}{5}\right]_0^1 + \left[\frac{x^5}{5} - x\right]_1^2\right\} = 12$$

6. (c)

7. (a)

$$A^2 + B^2 = AA + BB$$

$$\downarrow$$

$$= \underline{ABA} + \underline{BAB} \qquad \text{(Using values of } A \text{ and } B)$$

$$= BA + AB = \underbrace{A + B}$$

8. (c)

Here binomial distribution can be used

$$P(H) = 0.5$$

Probability of getting head exactly 4 time.

$$P(X = 4) = {}^5C_4\, (0.5)^4\, (0.5)^1$$

$$= 5 \times (0.5)^5$$

$$= \frac{5}{32}$$

9. (c)

Given, $a = 0.6$

X	0	1
P(X)	0.6	0.4

Required value $= V(X) = E(X^2) - [E(X)]^2$

$$E(X) = \sum_i X_i P_i = 0 \times 0.6 + 1 \times 0.4 = 0.4$$

$$E(X^2) = \sum_i X_i^2 P_i = 0^2 \times 0.6 + 1^2 \times 0.4 = 0.4$$

$$\therefore \quad V(X) = E(X^2) - [E(X)]^2$$

$$= 0.4 - 0.16 = 0.24$$

10. (b)

Given function is continuous at $x = 0$

$\therefore$ $(\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0)$

Now, $\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \dfrac{1-\cos 4x}{8x^2}$

Put, $x = 0 - h = -h$ when $x \to 0, h \to 0$

So, $\lim_{h\to 0} \dfrac{1-\cos(-4h)}{8h^2}$

$\lim_{h\to 0} \dfrac{2\sin^2 2h}{8h^2}$

$\lim_{h\to 0} \left(\dfrac{\sin 2h}{2h}\right)^2 = 1$

At, $x = 0,\ f(0) = K$

Also $\text{LHL} = f(0)$, therefore $K = 1$

11. (c)

$A = LU$

Given, $L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix}$

$\therefore$ $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$a_{ij} = j$, if $i = 1$

$= 3 + a_{(i-1)j}$; otherwise

$\therefore$ $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$A = LU$

$\Rightarrow$ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

(*i*) $a = 1$, (*ii*) $b = 2$ (*iii*) $c = 3$

(*iv*) $4b + d = 5 \Rightarrow 4 \times 2 + d = 5 \Rightarrow d = -3$

(*v*) $4c + e = 6 \Rightarrow 4 \times 3 + e = 6 \Rightarrow e = -6$

(*vi*) $7c + 2e + f = 9 \Rightarrow 7 \times 3 + 2 \times (-6) + f = 9 \Rightarrow f = 0$

$\therefore$ $U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

12. (c)

$$P(x = r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}, \text{ where } \lambda = np$$

$$\lambda = 500 \times 0.006 = 3$$

$$P(x \le 1) = P(x = 0) + P(x = 1)$$

$$= \frac{e^{-\lambda} \cdot \lambda^0}{0!} + \frac{e^{-\lambda} \cdot \lambda^1}{1!}$$

$$= e^{-3} + e^{-3} \cdot 3 = 4e^{-3}$$

13. (a)

$$|A - \lambda I| = (1 - \lambda)(\lambda^2 - 2) + (2 - \lambda) - \lambda = -\lambda^3 + \lambda^2$$

$$\Rightarrow \quad -\lambda^3 + \lambda^2 = 0$$

$$\Rightarrow \quad -\lambda^2(\lambda - 1) = 0$$

$$\lambda = 0, \lambda = 1$$

To find eigen vector

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_2 - x_3 = 0 \quad \ldots(1)$$

$$x_1 - 2x_2 + x_3 = 0 \quad \ldots(2)$$

$$-x_1 + x_2 = 0 \quad \ldots(3)$$

To solving (1), (2), (3) we get

$x_1 = 1, x_2 = 1, x_3 = 1$

Then $$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

14. (d)

Given, $$np = 4$$

$$npq = 2$$

$$q = \frac{1}{2}, p = \frac{1}{2}, n = 8$$

$$P(x = 1) = {}^nC_1 \, p^1 q^{n-1}$$

$$= {}^8C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^7 = \frac{8}{2^8} = \frac{1}{2^5} = \frac{1}{32}$$

$$= 0.03125$$

15. (d)

$$E[6X] = 6.E[X] = 6$$

$$Var[6X] = 6^2 \, Var[X] = 36 \times 2 = 72$$

$$E[1 - X] = 1 + (-1)E[X] = 1 - 1 = 0$$

$$Var[1 - X] = (-1)^2 \, Var[X] = Var[X] = 2$$

$$\therefore \quad Var[1 - X] \ne 3$$

16. (a)

$$|x-2| = \begin{cases} -(x-2)\,; & x<2 \\ (x-2)\,; & x>2 \end{cases}$$

$$\int_1^3 \frac{|x-2|}{x}dx = \int_1^2 \frac{-(x-2)}{x}dx + \int_2^3 \frac{x-2}{x}dx$$

$$= \int_1^2 \left(-1+\frac{2}{x}\right)dx + \int_2^3 \left(1-\frac{2}{x}\right)dx$$

$$= [-x]_1^2 + [2\ln x]_1^2 + [x]_2^3 - 2[\ln x]_2^3$$

$$= -(2-1) + 2\ln 2 - 2\ln\frac{3}{2} + (3-2)$$

$$= 2\ln 2 - 2\ln\frac{3}{2}$$

$$= 2\ln\frac{2}{\frac{3}{2}} = 2\ln\frac{4}{3}$$

17. (a)

Augmented matrix:

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

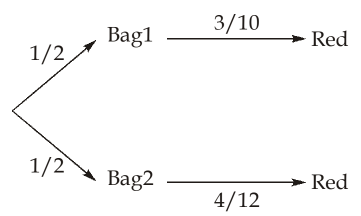
$R_3 \leftarrow R_3 - R_2$
$R_2 \leftarrow 4R_2 - R_1$

$$\left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 0 & 9 & 22 & 28 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

If $\lambda = 5$ and $\mu \neq 9$, then system has no solution because Rank$[A \mid B] \neq$ Rank [A].

18. (a)

The tree diagram for above problem, is shown below:



$$P\,(\text{bag1}\,|\,\text{Red}) = \frac{P(\text{bag1} \cap \text{Red})}{P(\text{Red})}$$

$$= \frac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{3}{10}+\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{3}{20}}{\frac{3}{20}+\frac{1}{6}} = 0.473$$

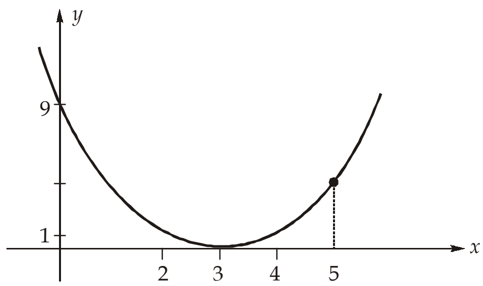
19. (b)

The total number of ways of drawing the 4 components sequentially from the box = 4!.
There are only two possible ways the product can be assembled.

$\therefore$ The required probability = $\frac{2}{4!}$ = 0.083

20. (c)

$$y = x^2 - 6x + 9 = (x-3)^2$$
$$y(2) = 1$$
$$y(5) = 4$$



$\therefore$ maximum value of y over the interval 2 to 5 will be at $x = 5$.

21. (c)

$$x_1 + 2x_2 = b_1 \quad \text{...(i)}$$
$$2x_1 + 4x_2 = b_2 \quad \text{...(ii)}$$
$$3x_1 + 7x_2 = b_3 \quad \text{...(iii)}$$
$$3x_1 + 9x_2 = b_4 \quad \text{...(iv)}$$

From equations (ii) and (i) we can write

$$b_2 = 2(x_1 + 2x_2) = 2b_1$$

From option (b):

$3b_1 - 6b_3 + b_4$

$$= 3[x_1 + 2x_2] - 6[3x_1 + 7x_2] + 3x_1 + 9x_2 \neq 0$$

From option (c):

$$b_2 = 2b_1 \text{ and } 6b_1 - 3b_3 + b_4$$
$$= 6[x_1 + 2x_2] - 3[3x_1 + 7x_2] + [3x_1 + 9x_2] = 0$$
$$6b_1 - 3b_3 + b_4 = 0$$

So, option (c) is correct answer.

22. (b)

Lets take C be any skew symmetric matrix of order 2 × 2 i.e. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Assume, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ then $X^T = [x_1\ x_2]$

So, $X^T CX = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= [x_2 - x_1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= [x_1\, x_2 + (-x_1\, x_2)]$

$= [0]$ = Null matrix

23. (b)

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1,\ R_3 \leftarrow R_3 - 4R_1$

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & -3 & 3 & 9 \end{bmatrix}$$

$R_3 \leftarrow R_3 - \dfrac{3}{2} R_2$

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of non-zero rows = 2

So, Rank of A = 2

24. (d)

Let, $f(x) = ax^2 + bx + c$

$f(-4) = 16a - 4b + c = 8$ ...(i)

$f(1) = a + b + c = 8$ ...(ii)

$f(3) = 9a + 3b + c = 15$ ...(iii)

On solving equation (i), (ii) and (iii) we get,

$$a = \frac{1}{2},\quad b = \frac{3}{2},\quad c = 6$$

So, $f(x) = \dfrac{x^2}{2} + \dfrac{3x}{2} + 6$

Now value of $f(x)$ at $x = 2$

$f(2) = \dfrac{4}{2} + 3 + 6 = 11$ which leads to point (2, 11)

25. (c)

Let, A = First drawn orange is good

B = Second drawn orange is good

$$C = \text{Third drawn orange is good}$$

The oranges are not replaced.

Thus, $$P(A) = \frac{12}{15},\ P(B) = \frac{11}{14},\ P(C) = \frac{10}{13}$$

The box is approved for sale, if all three oranges are good.

Thus, the probability of getting all the oranges good

$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$$

26. (c)

$$f(x) = \sin^4 x$$

Also, $$f(-x) = \sin^4(-x) = \sin^4 x = f(x)$$

So, $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\, dx = 2\int_0^{\frac{\pi}{2}} \sin^4 x\, dx = 2\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1+\cos^2 2x - 2\cos 2x)dx$$

$$\Rightarrow \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)dx$$

$$\Rightarrow \frac{1}{4}\int_0^{\frac{\pi}{2}} (3 - 4\cos 2x + \cos 4x)dx$$

$$\Rightarrow \frac{1}{4}\left[3x - \frac{4\sin 2x}{2} + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

On solving we get $\frac{3\pi}{8}$.

27. (c)

$$f(x) = \begin{cases} a+bx, & \text{for } 0<x<1 \\ 0, & \text{otherwise} \end{cases}$$

$$E(X) = \frac{2}{3}$$

$$\Rightarrow \int_0^1 x\, f(x)dx = \frac{2}{3}$$

$$\Rightarrow \int_0^1 x(a+bx)\,dx = \frac{2}{3}$$

$$\Rightarrow \qquad a\left(\frac{x^2}{2}\right)_0^1 + b\left(\frac{x^3}{3}\right)_0^1 = \frac{2}{3}$$

$$a\left(\frac{1}{2}\right) + b\left(\frac{1}{3}\right) = \frac{2}{3}$$

$$\Rightarrow \qquad 3a + 2b = 4 \qquad \text{...(i)}$$

Now, $\int_0^1 f(x)dx = 1$

Total probability is always equal to 1.

$$\int_0^1 (a + bx)dx$$

$$\left(ax + \frac{bx^2}{2}\right)_0^1 = 1$$

$$a + \frac{b}{2} = 1$$

$$2a + b = 2 \qquad \text{...(ii)}$$

On solving equation (i) and (ii)

$$a = 0, \quad b = 2$$

So, $$f(x) = \begin{cases} 2x, & \text{for } 0 < x < 1 \\ 0, & \end{cases}$$

Now, we need $\int_0^{\frac{1}{2}} 2x dx = \frac{1}{4} = 0.25$

28. (d)

Total number of 4 digit numbers = 9000 (1000 to 9999)

- At first place 0 and 6 are not allowed, so 8 choices for filling first place.
- For second, third and fourth place all digits are allowed except 6. So 9 choices.

So, n(not containing dight 6) = 8 × 9 × 9 × 9

$$P(\text{not containing the digit } 6) = \frac{8 \times 9 \times 9 \times 9}{9000} = 0.648$$

29. (d)

Given: $P = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ and $Q = \begin{bmatrix} x^2 + y^2 & xz + yw \\ xz + yw & z^2 + w^2 \end{bmatrix}$

$$PP^T = \begin{bmatrix} x & y \\ z & w \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix}^T = \begin{bmatrix} x & y \\ z & w \end{bmatrix} \begin{bmatrix} x & z \\ y & w \end{bmatrix}$$

$$= \begin{bmatrix} x^2 + y^2 & xz + yw \\ xz + yw & z^2 + w^2 \end{bmatrix}$$

$\because \qquad Q = PP^T$

and $r(P) = n$

Then, $r(Q) = \min\{(r(P), r(P^T)\}$ $\quad \because \quad [(r(P) = r(P^T)]$

$r(Q) = \min(n, n)$

$r(Q) = n$

30. (a)

$$P(2) = 9P(4) + 90P(6)$$

For Poisson's distribution,

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \text{ where } \lambda \text{ is the mean of Poisson's distribution}$$

Hence, $$\frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$$

$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^2}{2} = 9\frac{e^{-\lambda}\lambda^4}{24} + 90\frac{e^{-\lambda}\lambda^6}{720}$$

$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^6}{8} + \frac{3e^{-\lambda}\lambda^4}{8} - \frac{e^{-\lambda}\lambda^2}{2} = 0$$

$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^2}{2}\left[\frac{\lambda^4}{4} + \frac{3\lambda^2}{4} - 1\right] = 0$$

Given: $\lambda \neq 0$

$$\therefore \quad \left[\frac{\lambda^4}{4} + \frac{3\lambda^2}{4} - 1\right] = 0$$

$$\Rightarrow \quad \lambda^4 + 3\lambda^2 - 4 = 0$$

$$\Rightarrow \quad \lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0$$

$$\Rightarrow \quad \lambda^2(\lambda^2 + 4) - 1(\lambda^2 + 4) = 0$$

$$\Rightarrow \quad (\lambda^2 - 1)(\lambda^2 + 4) = 0$$

$$\Rightarrow \quad \lambda^2 = 1, \lambda^2 + 4 \neq 0$$

$$\Rightarrow \quad \lambda = \pm 1$$

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