CLASS TEST

S.No.:01JPCS_ALGO_22072024

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COMPUTER SCIENCE & IT

Algorithms

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I
A. Binary search
B. Quick sort
C. Insertion sort
D. 0/1 Knapsack problem

List-II
1. Divide and conquer
2. Comparison based
3. Greedy algorithm
4. Dynamic programming

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	1	1	2	4
(c)	2	1	2	4
(d)	1	1	2	3

Q.2 $T(n) = T(\sqrt{n}) + T(\log n) + 1;\ \ n \geq 2$

When you solve the above recurrence relation using recursive tree method, what is the maximum height of the tree obtained?
(a) $\theta(\log\log n)$ (b) $\theta(\log n)$
(c) $\theta(\sqrt{n})$ (d) $\theta(n)$

Q.3 Select all the correct statements:
S_1: Quick sort has combine step of divide and conquer technique.
S_2: Merge sort always has more number of swaps that comparisons.
S_3: Number of comparisons in partition algorithm are same for best case and worst case.
S_4: Quick sort is an inplace algorithm while merge sort is an out place algorithm.
(a) Only S_1 and S_2
(b) Only S_1 and S_3
(c) Only S_2, S_3 and S_4
(d) Only S_1, S_2, S_3 and S_4

Q.4 Which of the following statements are correct?
S_1: Applying binary search on a sorted linked list takes O(log$n$) time in worst case.
S_2: Applying binary search on a sorted linked list takes O(n) time in worst case.
S_3: Applying binary search in an unsorted array will take O(n log$n$) time in the worst case.
S_4: Searching an element in an unsorted array will take O(n) time in the worst case.
(a) Only S_1 and S_4
(b) Only S_2 and S_3
(c) Only S_2, S_3 and S_4
(d) Only S_1, S_2, S_3 and S_4

Q.5 Consider a weighted complete graph on vertex set $\{V_1, V_2 \ldots.. V_n\}$ such that weight of the edge (V_i, V_j) is $5|i-j|$. The weight of a minimum spanning tree using Prim's algorithm is
(a) $n - 1$ (b) $5(n - 1)$
(c) $5n$ (d) n^2

Q.6 Which of the following statement are correct?
S_1: Linear probing is easy to implement but it suffers from primary clustering.
S_2: Inserting an element into an open address hash table with load factor α require at least $\frac{1}{1-\alpha}$ probes on average, assuming uniform hashing.
(a) Only S_1 (b) Both S_1 and S_2
(c) Only S_2 (d) None of these

Q.7 Consider the following matrices with their dimensions:
A → 15 × 20
B → 20 × 10
C → 10 × 25
D → 25 × 15
E → 15 × 500
Apply matrix chain multiplication algorithm to find the minimum number of operations required.
(a) 121500 (b) 123750
(c) 124875 (d) 130625

Q.8 String 1 → *a a b b c p q r q p*
String 2 → *s a b a b t q c p t s p q*
Find the maximum length of the longest common subsequence.
(a) 4 (b) 6
(c) 7 (d) 5

Q.9 Consider a complete graph G with V vertices and E edges. Now, assume two nodes *u* and *v*, the shortest distance between them is 8. Assuming that all edge weights are positive, how many such path are possible if atmost 4 edges can be used in the shortest path?
(a) 180 (b) 121
(c) 150 (d) 165

Q.10 Match the following:
[Let G be the graph with V vertices and E edges]
[Time complexities of implementing Dijkstra's algorithm using heap data structure]

A.	O(V)	**(i)**	Build min heap of vertices
B.	O(V logV)	**(ii)**	Extracting minimum value weighted vertex
C.	O(E logV)	**(iii)**	Edge update in the min heap

(a) A → (ii), B → (i), C → (iii)
(b) A → (i), B → (ii), C → (iii)
(c) A → (i), B → (iii), C → (ii)
(d) None of these

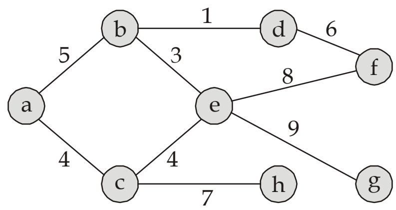
Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 A message is made up entirely of characters from the set P = {W, X, Y, Z}. The table of probability for each characters given below:

Character	Probability
W	0.01
X	0.30
Y	0.34
Z	0.35

The expected length of the encoded message in bits, if a message of 200 characters over set P encoded using Huffman coding ________ in bits.
(a) 196 (b) 392
(c) 137 (d) 292

Q.12 Consider the following graph:



Which one of the following represents the sequence of edges added in order to make a minimum spanning tree using Prim's algorithm?
(a) (b – d), (b – e), (e – c), (a – c), (d – f), (c – h), (e – g)
(b) (b – d), (c – e), (b – e), (a – c), (c – h), (d – f), (e – g)
(c) (b – e), (a – c), (b – d), (e – c), (d – f), (c – b), (e – g)
(d) (b – e), (b – d), (a – c), (e – c), (c – b), (d – f), (e – g)

Q.13 A certain permutations of integers stored in an array is provided as an input to the procedure of quicksort. After one pass of the algorithm the status of the array is as follows:
9, 6, 11, 13, 18, 15, 17, 24
The sum of all the possible values that could have been used as a pivot is ________.
(a) 45 (b) 47
(c) 46 (d) 48

Q.14 A min heap having 1024 distinct elements with keys ranging from 0 to 1023 is stored in array of 1024 indices. The maximum difference between element 512 present at maximum level and minimum level is ________.
[Assume root is present at level-1]
(a) 8 (b) 9
(c) 7 (d) 6

Q.15 Which of the following represents the number of elements that can be sorted in $\Theta(n)$ times using merge sort?

(a) $\Theta(\log n)$
(b) $\Theta(n)$
(c) $\Theta\left(\frac{n}{\log n}\right)$
(d) $\Theta(\sqrt{n})$

Q.16 Match **List-I (Dynamic algorithm)** with **List-II (Average case running time)** and select the correct answer using the codes given below the lists:

List-I (Dynamic algorithm)
A. Matrix chain multiplication
B. Travelling salesman problem
C. 0/1 knapsack
D. Fibonacci series

List-II (Average case running time)
1. $O(mn)$
2. $O(n^3)$
3. $O(n^n)$
4. $O(n)$

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	1	3	3	2
(c)	2	3	3	2
(d)	2	3	1	4

Q.17 The post order traversal of binary search tree is given by 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12. The height of the tree is ________.

(a) 4
(b) 5
(c) 6
(d) 3

Q.18 The number of different orders are possible for elements 1, 2, 3, 4, 5, 6, 7 to be inserted in to empty AVL tree such that no rotation will be done and element '4' is root are ________.

(a) 72
(b) 80
(c) 40
(d) 82

Q.19 The number of distinct BFS traversal possible on complete graph of 5 vertices are ________ [vertices are labeled as A, B, C, D and E].

(a) 72
(b) 60
(c) 120
(d) 48

Q.20 Consider an initially empty hash table of length 10. Following set of keys are inserted using open addressing with hash function $h(k) = k \bmod 10$ and linear probing.

	0
91	1
	2
33	3
44	4
23	5
64	6
77	7
	8
	9

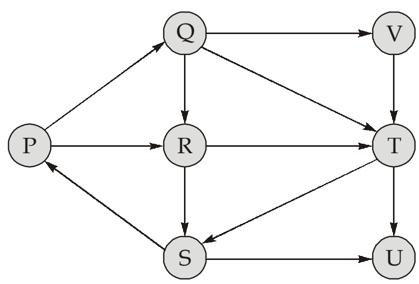
The number of different insertion sequence of the key values using the given hash function and linear probing will result in the hash table shown in above ________.

(a) 70
(b) 60
(c) 30
(d) 45

Q.21 Which of the following has best lower bound time complexity?

(a) Merge sort
(b) Quick sort
(c) Selection sort
(d) Insertion sort

Q.22 Which of the following is the correct decomposition of the directed graph given below into its strongly connected components?



(a) {P, Q, R, S} {T} {U} {V}
(b) {P, Q, R, S, T, V} {U}
(c) {P, Q, S, T, V} {R} {U}
(d) {P, Q, R, S, T, U, V}

Q.23 consider an array A of length n, array contain number between (1 – 10), in any arbitrary order, best sorting algorithm takes 650 ns if n = 50, the time required by the algorithm if n = 300.

(a) 4200 (b) 3900
(c) 3800 (d) 4500

Q.24 Consider the following cases for quick sort to sort an array of n element $a[0 \ldots n-1]$

(i) Choosing the pivot element randomly from the given array.
(ii) Choosing median element as pivot.
(iii) Choosing middle element as pivot.

For which of the above cases quick sort always gives O($n$log$n$) time complexity?

(a) (i) and (iii) (b) (ii) and (iii)
(c) (i) and (ii) (d) None of these

Q.25 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

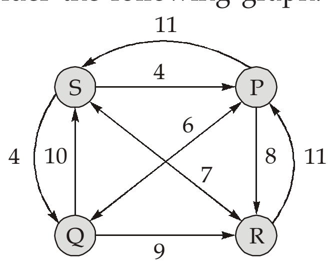
List-I
A. Quick sort
B. Longest common subsequence
C. Kruskal's algorithm
D. Shortest distance from a given node to every node

List-II
1. Greedy algorithm
2. Divide and conquer
3. Dynamic programming

Codes:

	A	B	C	D
(a)	2	3	1	3
(b)	2	1	3	2
(c)	2	3	1	1
(d)	2	1	2	1

Q.26 Consider the following graph:



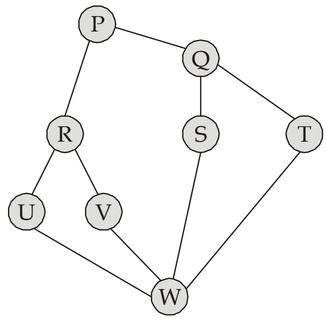
The optimal cost of tour by travelling salesman using dynamic algorithm, if S is the starting vertex ________.

(a) 24 (b) 25
(c) 26 (d) 28

Q.27 Consider two string p = "abcbdab" and q = "bdcabc" then the number of longest common subsequence of p and q is ________.

(a) 1 (b) 2
(c) 0 (d) 3

Q.28 Consider the following graph:



Which of the following is not a depth first search traversal of the given graph?

(a) P Q S W V R U T
(b) P Q S W T V R U
(c) P R U V W S Q T
(d) P R V W S Q T U

Q.29 Which of the following statements is true?

(a) For a directed graph the absence of back edges in a DFS tree means the graph has no cycle.
(b) If all the edges in a graph have distinct weight then the shortest path between two vertices is unique.
(c) A complete graph with 4 vertices can have maximum 20 minimum cost spanning tree.
(d) Both (a) and (b)

Q.30 Consider a hash table with hash function $H(i) = i \bmod 11$ and following keys are hashed into the 24, 49, 20, 16, 23, 36, 34, 60 hash table, to handle the collision chaining is used, after inserting all the keys if new key is inserted then what is the probability that it hashed into empty slot ____.

(a) 0.60 (b) 0.48
(c) 0.56 (d) 0.54

■■■■

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ALGORITHMS

COMPUTER SCIENCE & IT

Date of Test : 22/07/2024

ANSWER KEY

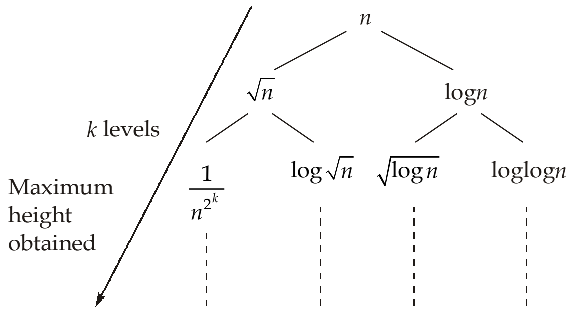
1.	(b)	7.	(a)	13.	(d)	19.	(c)	25.	(c)
2.	(a)	8.	(b)	14.	(b)	20.	(b)	26.	(b)
3.	(c)	9.	(d)	15.	(c)	21.	(d)	27.	(b)
4.	(c)	10.	(b)	16.	(d)	22.	(b)	28.	(c)
5.	(b)	11.	(b)	17.	(d)	23.	(b)	29.	(a)
6.	(a)	12.	(a)	18.	(b)	24.	(d)	30.	(d)

DETAILED EXPLANATIONS

1. **(b)**

Binary search → Divide and conquer
Quick sort → Divide and conquer
Insertion sort → Comparison based
0/1 Knapsack problem → Dynamic programming

2. **(a)**



So,
$$n^{\frac{1}{2^k}} = 2$$
$$\frac{1}{2^k}\log n = 1$$
$$\Rightarrow \quad \log n = 2^k$$
$$k = \log\log n = \theta(\log\log n)$$

3. **(c)**

S_1: Quick sort only has divide step and conquer step.
S_2: True: Since merge sort is an out place algorithm.
S_3: True: Number of comparisons in partition algorithm do not change for best case or worst case.
S_4: True

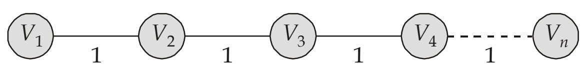
4. **(c)**

S_1: False
S_2: True; as the memory is not contiguous searching an element will take O(n) time even if we apply binary search [because of traversing].
S_3: True: First sort the array using merge sort then apply binary search.
S_4: True: Apply linear search.

5. **(b)**

Prim's algorithm will pick up the edge with least weight for a particular node such that it does not form a cycle.

∴ MST will be



Total n – 1 edges will be required.

So, weight = $5(n - 1)$

6. **(a)**

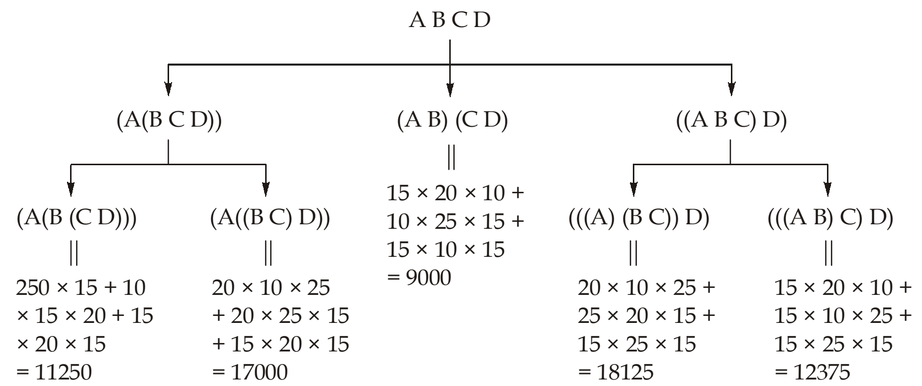
- Linear probing suffers from primary clustering.
- Inserting a key require unsuccessful search by placing the key into the first empty slot found.

Thus the expected number of probes is at most $\frac{1}{1-\alpha}$.

7. **(a)**

$$9000 + 112500 = 121500$$

E $\rightarrow$ has 500 columns so it should be multiplied only once, in the last.



8. **(b)**

String 1 $\rightarrow$ *a a b b c p q r q p*

String 2 $\rightarrow$ *s a b a b t q c p t s p q*

		a	*a*	*b*	*b*	*c*	*p*	*q*	*q*	*p*
	0	0	0	0	0	0	0	0	0	0
a	0	1	1	1	1	1	1	1	1	1
b	0	1	1	2	2	2	2	2	2	2
a	0	1	2	2	2	2	2	2	2	2
b	0	1	2	3	3	3	3	3	3	3
q	0	1	2	3	3	3	3	4	4	4
c	0	1	2	3	3	4	4	4	4	4
p	0	1	2	3	3	4	5	5	5	5
p	0	1	2	3	3	4	5	5	5	6
q	*o*	1	2	3	3	4	5	6	6	6

Maximum length = 6

$$\begin{bmatrix} a\ a\ b\ c\ p\ q \\ a\ a\ b\ c\ p\ p \\ a\ b\ b\ c\ p\ q \\ a\ b\ b\ c\ q\ p \end{bmatrix} \rightarrow \text{possible 6 – length strings.}$$

9. **(d)**

$$e_1 + e_2 + e_3 + e_4 = 8$$

$^{8+4-1}C_{4-1}$

$$e_1, e_2, e_3, e_4 \geq 0$$

$$= {}^{11}C_3 = \frac{11 \times 10 \times 9}{3 \times 2} = 165$$

10. **(b)**

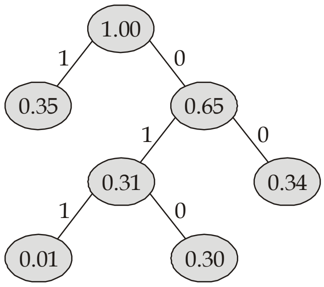
O(V) → Time taken to build the vertices min heap.

O(V logV) → Extract minimum vertex and heapify.

O(E logV) → For each updates have to be mode in the min heap before next extraction.

11. **(b)**

Using min heap data structure:



Expected length: [0.35 × 1 + 0.34 × 2 + 0.30 × 3 + 0.01 × 3] × 200

= [0.35 + 0.68 + 0.90 + 0.03] × 200

= [1.96] × 200

= 392

12. **(a)**

Since by looking through options, we get to know 'b' will be the start vertex.

	a	b	c	d	e	f	g	h
	∞	0	∞	∞	∞	∞	∞	∞
b	5	–	∞	1	3	∞	∞	∞
(b – d) d	5	–	∞	–	3	6	∞	∞
(d – e) e	5	–	4	–	–	6	9	∞
(e – c) c	4	–	–	–	–	6	9	7
(c – a) a	–	–	–	–	–	6	9	7
(d – f) f	–	–	–	–	–	–	9	7
(c – h) h	–	–	–	–	–	–	9	–
(e – g) g	–	–	–	–	–	–	–	–

So, correct sequence will be (b – d), (b – e), (e – c), (c – a), (d – f), (c – h), (e – g).

13. **(d)**

Property of the output of each pass of quicksort:

I. The pivot elements is on its correct position.

II. All the elements on the left of the pivot are smaller than it and elements on the right are greater than it.

If we consider the sorted permutation of the above sequence it will be, 6, 9, 11, 13, 15, 17, 18, 24. Comparing both it can be observed that elements 11, 13 and 24 are on their correct positions. Besides that they are satisfying II property also.

Hence, possible pivots, 11, 13, 24

Sum, 11 + 13 + 24 = 48

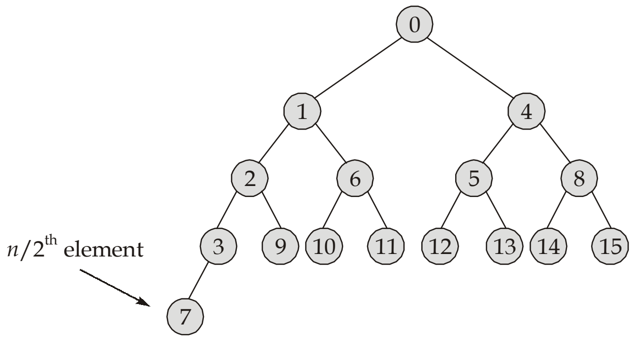
14. (b)

Since there are total 1024 elements hence there will be total 11 levels of the heap. $(n/2)^{th}$ element can be present at last level in worst case i.e. 11^{th} level.

$(n/2)^{th}$ element can also be present in best case level i.e. 2^{nd} level

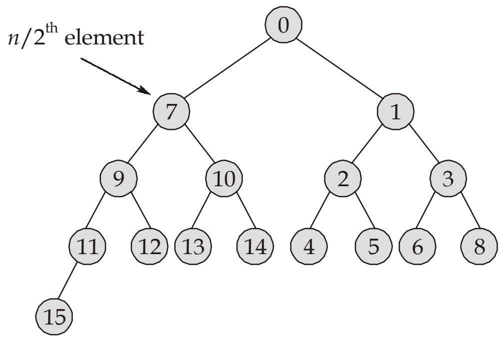
Assume for n = 16

In worst case:



$(n/2)^{th}$ element is at last level.

In best case:



$(n/2)^{th}$ element is at second level.

So, difference = [11 – 2] = 9

15. (c)

Time complexity to sort n elements using merge sort = $\Theta(n\log n)$

$$\Theta(n) = \Theta\left(\frac{n}{\log n}\log\frac{n}{\log n}\right)$$

$$\Theta(n) = \Theta\left(\frac{n}{\log n}[\log n - \log\log n]\right)$$

$$\Theta(n) = \Theta\left(\frac{n}{\log n}\log n\right) \quad [\log n - \log\log n = O(\log n)]$$

$$\Theta(n) = \Theta(n)$$

16. (d)

A. Matrix chain multiplication : (n^3)

B. Travelling salesman problem : (n^n)

C. 0/1 knapsack : (mn)

D. Fibonacci series : $O(n)$

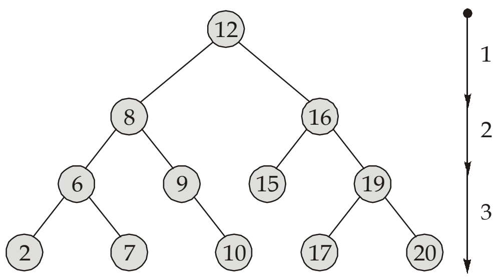
17. (d)

Post order: 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12

Inorder of BST must be sorted order:

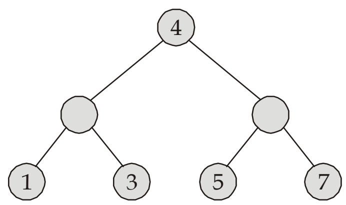
2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20

So, tree will be:



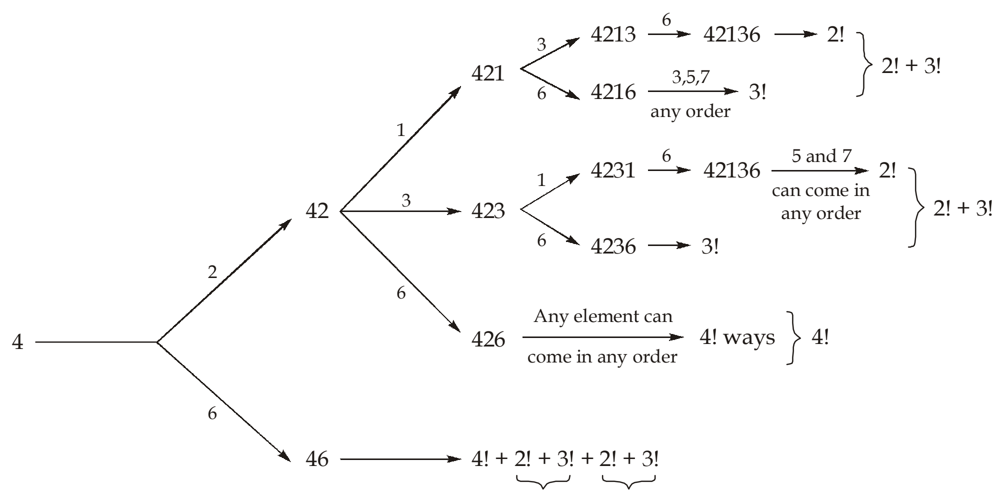
18. (b)

Since 4 is root element, so



(1, 3, 5, 7) can be inserted in any order since these are leaf nodes. However, 6 needs to be inserted before 5 and 7 and 2 needs to be inserted before 1 and 3.

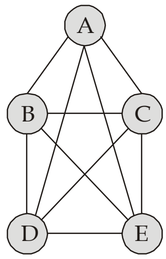
4 being the root node, needs to be inserted first of all.



$$
\begin{aligned}
\text{Total possibilities} &= 2(4! + 2(2! + 3!)) \\
&= 2(24 + 2(2 + 6)) \\
&= 2(24 + 16) = 2(40) \\
&= 80 \text{ ways}
\end{aligned}
$$

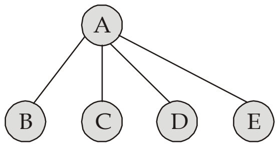
19. **(c)**

Complete graph on 5 vertices:



BFS traversal:

1. We have 5 choices to select initial vertex i.e. A, B, C, D and E.
2. With initial vertex as A

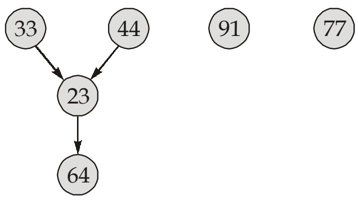


So total number of traversal are $5 \times 4! = 5! = 120$

Note: Number of BFS traversals on complete graph with n vertex are $n!$

20. **(b)**

Here some of the dependencies are presents:



So, number of possibilities are:

1. 2 choices for 33 and 44 either 33 then 44 or 44 then 33.
2. After that 23 will be come.
3. After that 64 will come.

Now, here 91 and 77 can come in any order i.e. 5×6

So, total choices will be $= 2 \times 5 \times 6$

$= 60$

21. **(d)**

In best case merge sort time complexity = $O(n \log n)$

For quick sort in best case = $O(n \log n)$

For selection sort = $O(n^2)$

For insertion sort = $O(n)$

22. **(b)**

If $\forall i\ \forall j\ (P(i \to j) \wedge P(j \to i))$ then i and j belongs to same strongly connected components where i and j are vertices and $P(i \to j)$ means there is a path from i to j.

Solving by elimination

Option (d): $P(U \to T)$ does not exists thus false.

Option (c): From R we can reach P Q V S T and from P Q V S T we can reach R thus R should not be separated thus false.

23. **(b)**

We know the range of the element present in the array A, so can use counting sort also, takes O(n) time.

$$C \cdot n = 650$$
$$C \cdot 50 = 650$$
$$C = 13$$

For $n = 300$

$$300 \times 13 = 3900$$

24. **(d)**

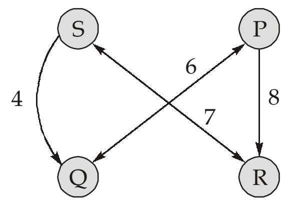
(ii) choosing median element as pivot divide the array into two equal half and time complexity O($n$log$n$).

For (i) and (iii) choosing pivot randomly or middle element does not guaranty O($n$log$n$) time complexity.

25. **(c)**

A : Quick sort uses divide and conquer.
B : Longest common subsequence uses dynamic programming.
C : Kruskal's algorithm is greedy algorithm.
D : Shortest distance from a given node to every node is greedy algorithm.

26. **(b)**



Optimal cost = 4 + 6 + 7 + 8 = 25

27. **(b)**

p = "abcbdab"
q = "bdcabc"
LCS (p, q) = abcbdab, bdcabc
First LCS = "bcab"
Second LCS = "bdab"

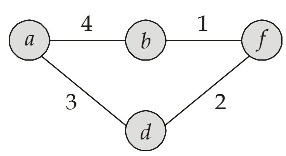
Then total 2 LCS.

28. **(c)**

In P R U V W S Q T is not DFS traversal because after U, V can not come.

29. **(a)**

(a) For a directed graph if DFS tree does not have back edges then there is no cycle.
(b) Shortest path between two vertices may not be unique.

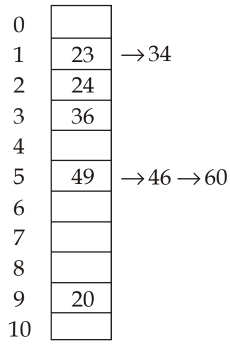


Shortest path between $a - f$ is not unique.

(c) A complete graph can have maximum $n^{n-2} = 4^{4-2} = 16$ MST.

So only option (a) is true.

30. (d)



If the new key is inserted in empty slot 6, total 11 slot.

$$\text{Probability} = \frac{6}{11} = 0.54$$

■■■■