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STRUCTURAL ANALYSIS

CIVIL ENGINEERING

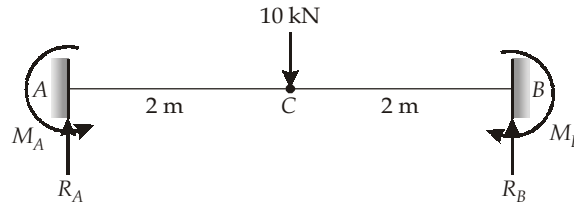
Date of Test : 22/07/2024

ANSWER KEY >

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (b) | 13. (c) | 19. (d) | 25. (b) |
| 2. (c) | 8. (c) | 14. (d) | 20. (b) | 26. (d) |
| 3. (a) | 9. (a) | 15. (b) | 21. (d) | 27. (a) |
| 4. (c) | 10. (b) | 16. (b) | 22. (b) | 28. (b) |
| 5. (c) | 11. (d) | 17. (b) | 23. (b) | 29. (d) |
| 6. (c) | 12. (d) | 18. (d) | 24. (c) | 30. (d) |

DETAILED EXPLANATIONS

1. (b)



$$M_A = M_B = M \text{ (say)} \quad [\because \text{Due to symmetry}]$$

Taking moment about C (from left)

$$R_A \times 2 - M = 0$$

$$\Rightarrow M = 2R_A$$

$$\text{Also; } R_A = R_B = 5 \text{ kN} \quad (\text{due to symmetry})$$

$$M = 2 \times 5 = 10 \text{ kNm}$$

2. (c)

Taking moments about crown i.e. C (from left),

$$H_A \times 2R = V_A \times 2R$$

$$\Rightarrow H_A = V_A$$

$$\text{Similarly, for BC, } H_B = V_B$$

$$\text{Now, as } H_A = H_B \text{ and thus,}$$

$$\therefore V_A = V_B = \frac{W}{2}$$

3. (a)

Let the force in each wire be S .

$$\therefore (2 \sin 45^\circ) \times S = P$$

$$S = \frac{P}{\sqrt{2}}$$

So strain energy stored in wire AO

$$= \left(\frac{P}{\sqrt{2}} \right)^2 \times \frac{l}{2AE} = \frac{P^2 l}{4AE} = \frac{P^2 l}{kAE} \quad \therefore k = 4$$

4. (c)

5. (c)

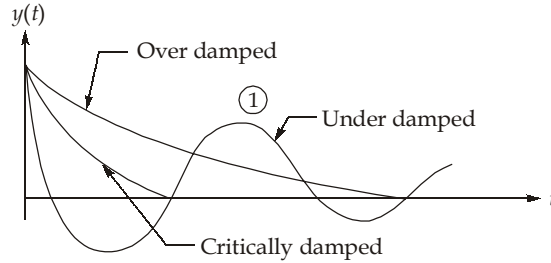
If concentrated load 'W' acts at any point which makes angle α with the horizontal on a semi-circular two hinged arch, the horizontal thrust is given by,

$$\begin{aligned} H &= \frac{W}{\pi} \sin^2 \alpha \\ &= \frac{10}{\pi} \times \sin^2 60 = \frac{7.5}{\pi} \text{ kN} \end{aligned}$$

6. (c)

Horizontal force remains constant while tension along length changes.

7. (b)



8. (c)

9. (a)

Using Maxwell-Betti's theorem,

$$300 \times \Delta_A = 100 \times \Delta_B + 200 \times \Delta_C$$

$$\Delta_A = \frac{100 \times 10}{300} + \frac{200 \times 15}{300} = \frac{10}{3} + 10 = \frac{40}{3} \text{ mm}$$

10. (b)

Internal static indeterminacy = 0

External static indeterminacy = 9 - 3 = 6

$$\begin{aligned} \therefore D_s &= D_{se} + D_{si} \\ &= 6 + 0 \\ &= 6 \end{aligned}$$

11. (d)

$$\text{Each vertical reaction} = V = \frac{wl}{2}$$

$$\text{Horizontal reaction} = H = \frac{wl^2}{8h}$$

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{\left(\frac{wl}{2}\right)^2 + \left(\frac{wl^2}{8h}\right)^2}$$

$$\simeq \frac{wl^2}{8h} \left[1 + \frac{1}{2} \times \frac{16h^2}{l^2} \right] \quad (\because l \gg h)$$

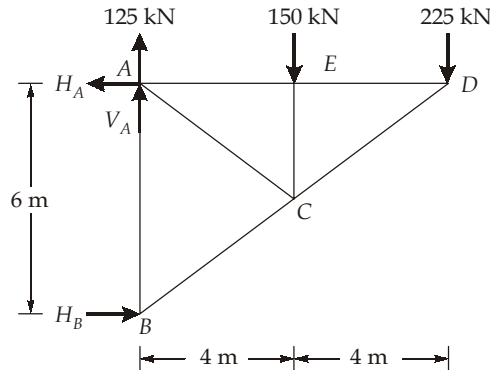
$$= \left(\frac{wl^2}{8h} + wh \right)$$

$$T_{min} = H = \frac{wl^2}{8h}$$

$$\therefore T_{max} - T_{min} = \left(\frac{wl^2}{8h} + wh \right) - \left(\frac{wl^2}{8h} \right)$$

= wh

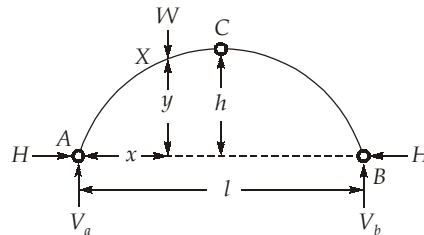
12. (d)



$$\begin{aligned} \Sigma M_B = 0; & \quad H_A \times 6 = 150 \times 4 + 225 \times 8 \\ \Rightarrow & \quad H_A = 400 \text{ kN } (\leftarrow) \\ \Sigma F_x = 0; \text{ So,} & \quad H_B = 400 \text{ kN } (\rightarrow) \\ \Sigma F_y = 0; & \quad V_A + 125 = 150 + 225 \\ \Rightarrow & \quad V_A = 250 \text{ kN} \end{aligned}$$

\therefore Reaction at A = $\sqrt{V_A^2 + H_A^2} = \sqrt{250^2 + 400^2} = 471.7 \text{ kN}$

13. (c)



$$V_a = \frac{W(l-x)}{l}, \quad V_b = \frac{Wx}{l}$$

$$\Sigma M_C = 0 \text{ (From right end)} \Rightarrow H \times h = \frac{Wx}{l} \times \frac{l}{2}$$

$$\Rightarrow H = \frac{Wx}{2h}$$

The maximum B.M. occurs under the load.

$$\therefore BM_x = \frac{W(l-x)}{l} \times x - \frac{Wx}{2h} \times \frac{4h}{l^2} x(l-x)$$

$$\Rightarrow BM_x = \frac{Wx(l-x)}{l} - \frac{2W}{l^2} x^2(l-x)$$

For absolute maximum B.M.,

$$\frac{d(BM_x)}{dx} = 0;$$

$$\Rightarrow \frac{W}{l}(l-2x) = \frac{2W}{l^2}(2lx-3x^2)$$

$$\Rightarrow 6x^2 - 6lx + l^2 = 0$$

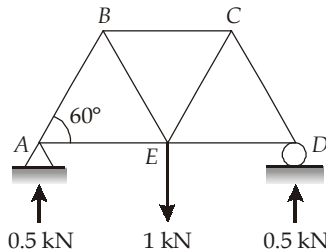
$$\Rightarrow x = \frac{6 - 2\sqrt{3}}{12}l \quad \left(\text{since } x < \frac{l}{2}\right)$$

$$\Rightarrow x = \frac{l}{2} - \frac{l}{2\sqrt{3}} \text{ from } A$$

$$\therefore \text{From crown, distance of maximum B.M. on either side} = \frac{l}{2} - x = \frac{l}{2\sqrt{3}}$$

14. (d)

Apply a unit load at joint E



$$\delta_E = \Sigma K(L\alpha\Delta T) \quad \dots(1)$$

At joint A; $\Sigma F_y = 0$

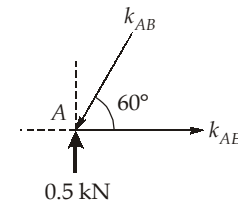
$$\Rightarrow 0.5 - K_{AB} \sin 60^\circ = 0$$

$$\therefore K_{AB} = \frac{1}{\sqrt{3}} \text{ (compressive)}$$

Also; $K_{AB} = K_{CD}$ (Due to symmetry)

From (1)

$$\therefore \delta_E = \left(\frac{1}{\sqrt{3}} \times 2 \times 12 \times 10^{-6} \times 20\right) \times 2 \text{ m} = 0.55 \text{ mm}$$



15. (b)

Let the reaction at the roller (R) be redundant

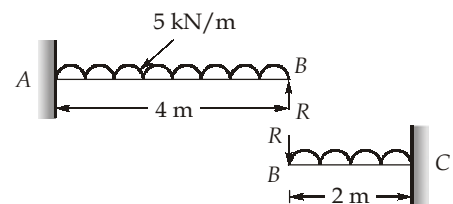
$$\therefore (\Delta_B)_{AB} = (\Delta_B)_{BC}$$

$$\Rightarrow \frac{5 \times (4)^4}{8EI} - \frac{R(4)^3}{3EI} = \frac{5(2)^4}{8EI} + \frac{R(2)^3}{3EI}$$

$$\Rightarrow \frac{160}{EI} - \frac{64R}{3EI} = \frac{10}{EI} + \frac{8R}{3EI} \quad \Rightarrow R = 6.25 \text{ kN}$$

$$\begin{aligned} \therefore \text{Moment at } A, M_A &= R \times 4 - 5 \times 4 \times 2 = 6.25 \times 4 - 5 \times 4 \times 2 \\ &= 25 - 40 \\ &= -15 \text{ kN-m} \end{aligned}$$

Therefore, the magnitude of moment at A = 15 kN-m.



16. (b)

Since stiffness matrix is inverse of flexibility matrix.

$$\therefore \text{If } [A] = k \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then $[A]^{-1} = \frac{1}{k|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

\therefore Stiffness matrix, $[k] = \frac{6EI}{7L^3} \begin{bmatrix} 16 & -5 \\ -5 & 2 \end{bmatrix}$

17. (b)

\therefore k_{12} = Force developed at (1) due to unit displacement at (2) while restraining other displacements

$$k_{12} = \frac{2EI}{4} = \frac{EI}{2}$$

18. (d)

Stiffness of member OA = $\frac{3EI}{L}$

Stiffness of member OB = $\frac{4EI}{L}$

Stiffness of member OC = $\frac{4EI}{L}$

\therefore $\Sigma k = \frac{4EI}{L} + \frac{3EI}{L} + \frac{4EI}{L}$
 $= \frac{11EI}{L}$

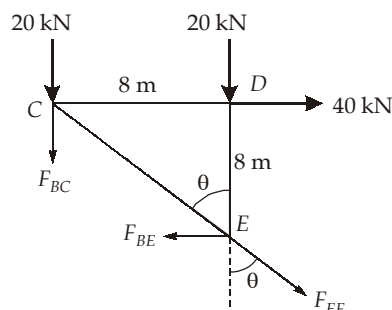
\therefore $\frac{M}{\theta} = \frac{11EI}{L}$

$\Rightarrow M = \frac{11EI}{L} \theta$

\therefore For $\theta = 1$ unit

$$M = \frac{11EI}{L}$$

19. (d)



Cut a section through BC, BE and EF and considering its right portion

$$\Sigma F_y = 0; \Rightarrow 20 + 20 + F_{EF} \cos \theta + F_{BC} = 0 \quad \dots(1)$$

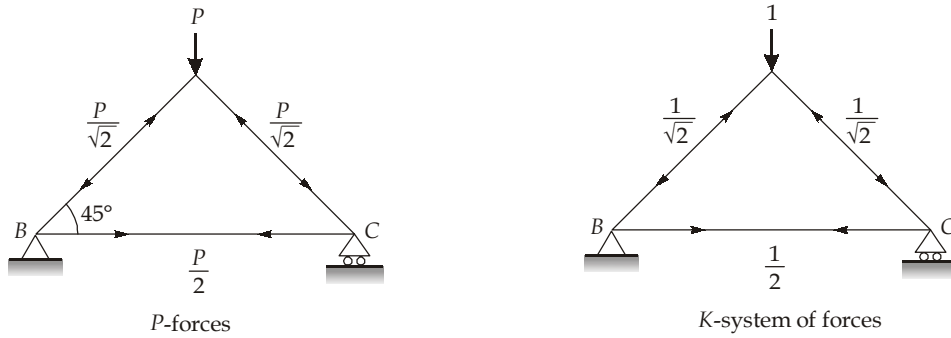
$$\Sigma M_E = 0; \Rightarrow F_{BC} \times 8 + 20 \times 8 = 40 \times 8$$

$\Rightarrow F_{BC} = 20 \text{ kN}; (\sin \theta = \cos \theta = \frac{1}{\sqrt{2}}) \quad (\because \theta = 45^\circ)$

From (1); $F_{EF} = -60\sqrt{2}$ kN (-ve i.e. compression)

∴ Magnitude of $F_{EF} = 60\sqrt{2}$ kN

20. (b)



(All other remaining members will have zero force)

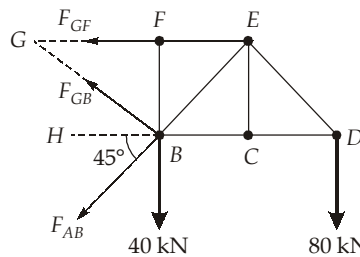
Deflection at A,
$$\Delta_A = \sum \frac{PKL}{AE} = \frac{\frac{P}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{L}{\sqrt{2}}}{AE} \times 2 + \frac{\frac{P}{2} \times \frac{1}{2} \times L}{AE}$$

$$= \frac{PL}{AE} \left(\frac{1}{2\sqrt{2}} \times 2 + \frac{1}{4} \right) = 0.957 \frac{PL}{AE} = k \frac{PL}{AE}$$

∴ $k = 0.957 \approx 0.96$

21. (d)

By method of sections,



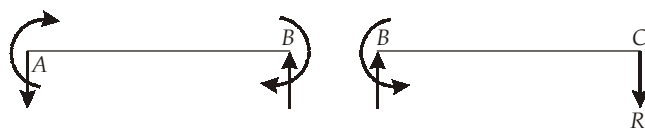
$$\sum M_G = 0; \Rightarrow F_{AB} \cos 45^\circ \times 2 + F_{AB} \sin 45^\circ \times 2 + 40 \times 2 + 80 \times 6 = 0$$

$$\Rightarrow \sqrt{2} F_{AB} + F_{AB} \sqrt{2} + 80 + 480 = 0$$

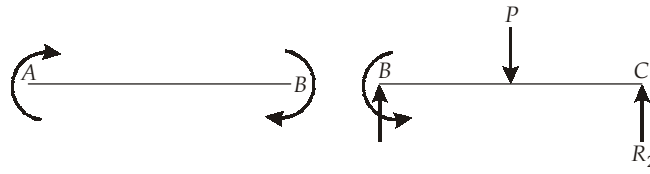
$$\Rightarrow F_{AB} = -197.989 \text{ kN} \approx 197.99 \text{ kN} \approx 198 \text{ kN (compressive)}$$

22. (b)

Due to sinking of support A,

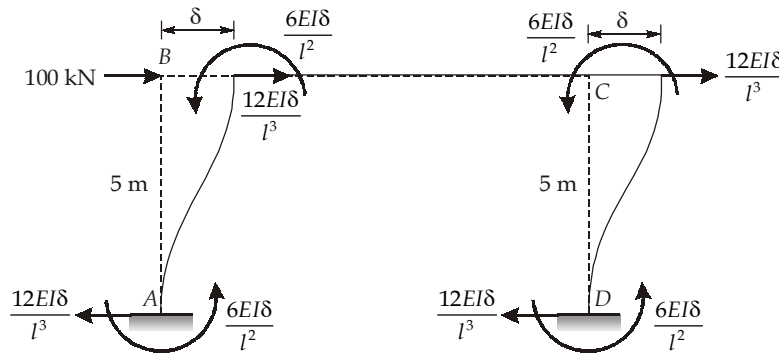


Due to load P



∴ $R_C = R_1(\downarrow) + R_2(\uparrow) = R_2 - R_1 < R_2$
Hence, reaction at C decreases.

23. (b)



$$\frac{12EI\delta}{l^3} + \frac{12EI\delta}{l^3} = 100$$

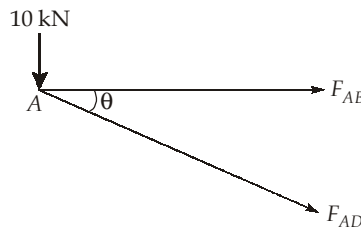
$$\Rightarrow \frac{24EI\delta}{l^3} = 100$$

$$\Rightarrow M_A = \frac{6EI\delta}{l^2}$$

$$= \frac{100 \times l}{4} = \frac{100 \times 5}{4} = 125 \text{ kNm}$$

24. (c)

Applying the method of joints
Consider joint A,



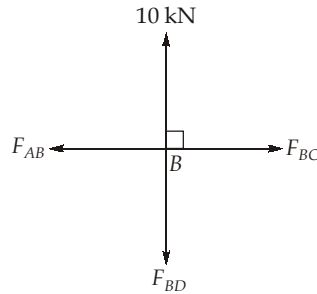
$$\Rightarrow \sum F_y = 0$$

$$F_{AD} \sin \theta = -10$$

$$\Rightarrow F_{AD} = \frac{-10}{\sin \theta}$$

$$\Rightarrow F_{AD} = -\frac{50}{3} \text{ kN} = \frac{50}{3} \text{ kN (C)}$$

$$\begin{aligned} \Rightarrow \quad \sum F_x &= 0 \\ F_{AD} \cos \theta &= F_{AB} \\ \Rightarrow \quad F_{AB} &= \frac{50}{3} \times \frac{4}{5} = \frac{40}{3} (T) \\ \therefore \text{ Now consider joint } B \end{aligned}$$

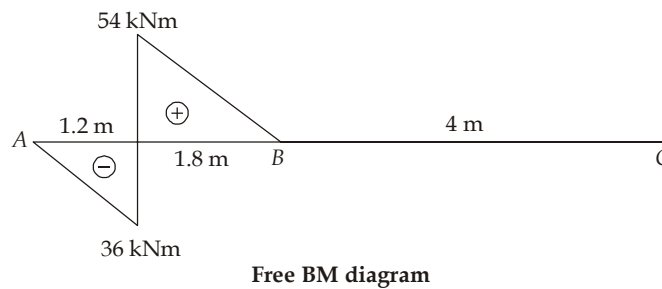


$$\begin{aligned} \therefore \text{ Take } \quad \sum F_x &= 0 \\ \Rightarrow \quad F_{AB} &= F_{BC} \\ \Rightarrow \quad F_{BC} &= \frac{40}{3} \text{ kN (T)} \end{aligned}$$

So, correct option is (c).

25. (b)

Free bending moment diagram for the beam is shown below.



$$M_A = 0$$

$$\begin{aligned} a_1 \bar{x}_1 &= -\frac{1}{2} \times 1.2 \times 36 \times \frac{2}{3} (1.2) + \frac{1}{2} \times 1.8 \times 54 \left(1.2 + \frac{1.8}{3} \right) \\ &= 70.20 \text{ units} \end{aligned}$$

Applying the theorem of three moments for span AB and BC,

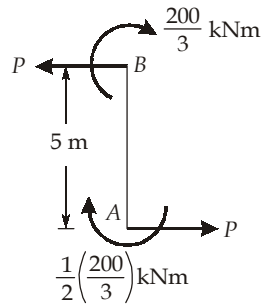
$$0 \times 3 + 2 M_B (3 + 4) + M_C \times 4 = \frac{6}{3} (70.20)$$

$$\Rightarrow \quad 7 M_B + 2 M_C = 70.20$$

26. (d)

Joint	Member	k	Σk	D.F.
B	BA	$\frac{8EI}{5}$	$\frac{12EI}{5}$	$\frac{2}{3}$
	BC	$\frac{4EI}{5}$		$\frac{1}{3}$

$$\therefore M_{BA} = \frac{2}{3} \times 100 = \frac{200}{3} \text{ kNm}$$

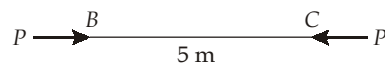


$$P \times 5 = \frac{200}{3} + \frac{100}{3}$$

$$\Rightarrow 5 \times P = \frac{300}{3}$$

$$\Rightarrow P = 20 \text{ kN}$$

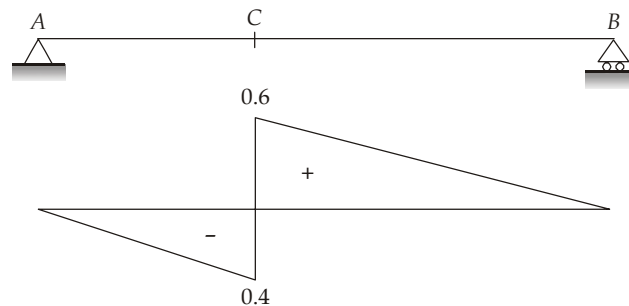
FBD of member BC is as shown below.



Hence force in member BC = 20 kN

27. (a)

ILD for SF at C is shown below



ILD for S.F. at C

For maximum positive shear force at C, BC should be covered with UDL

$$\begin{aligned} \therefore \text{Maximum S.F.} &= \frac{1}{2} \times 0.6 \times 6 \times 15 \\ &= 27 \text{ kN} \end{aligned}$$

28. (b)
Applying Betti's theorem

$$25 \times 0.002 + 15 \times \frac{9}{1000} = 15 \times \theta_A + 22 \times 0.004$$

$$\Rightarrow \theta_A = 0.00647 \text{ radian}$$

29. (d)
Elements in flexibility matrix can be positive or negative but the elements of leading diagonal must be positive since the displacement at any co-ordinate due to a unit force at that co-ordinate is always in the direction of unit force.
30. (d)
Selected joint can have more than two members but it should not have more than two unknown member forces.

