CLASS TEST

Sl.: 01SK_EC_ABCDEFG_270724

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ELECTROMAGNETICS THEORY

ELECTRONICS ENGINEERING

Date of Test : 27/07/2024

ANSWER KEY

1.	(c)	7.	(a)	13.	(b)	19.	(a)	25.	(c)
2.	(d)	8.	(a)	14.	(a)	20.	(c)	26.	(d)
3.	(a)	9.	(c)	15.	(b)	21.	(b)	27.	(b)
4.	(a)	10.	(d)	16.	(a)	22.	(c)	28.	(a)
5.	(b)	11.	(a)	17.	(b)	23.	(c)	29.	(a)
6.	(b)	12.	(d)	18.	(a)	24.	(a)	30.	(a)

DETAILED EXPLANATIONS

1. (c)

$$P_r = \frac{1}{2}(E_r)(H_r)$$

$$E_r = E_i\left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) = 5 \text{ V/m}$$

$$H_r = -\frac{E_r}{\eta_1} = \frac{-1}{20} \text{ A/m}$$

$$|P_r| = \frac{1}{2}\times 5\times\frac{1}{20} = \frac{1}{8} \text{ W/m}^2$$

2. (d)

$$v_p \times v_g = c^2$$

$$3.5 \times 10^8 \times v_g = (3 \times 10^8)^2$$

$$v_g = \frac{9\times10^{16}}{3.5\times10^8} = 2.57\times10^8 \text{ m/s}$$

3. (a)

$$\eta_{TE} = \frac{120\pi}{\cos\theta} > 120\pi$$

$$\eta_{TM} = 120\pi\times\cos\theta < 120\pi$$

$$\eta_{TEM} = 120\pi$$

4. (a)

5. (b)

Given $\vec{H} = (\hat{x} + j\hat{y})e^{j(\omega t - \beta z)}$ mA/m

$$\vec{E} = 120\pi(-\hat{y} + j\hat{x})e^{j(\omega t - \beta z)} \text{ mV}$$

[$\vec{E}\times\vec{H} =$ direction of propagation]

$\vec{E}$ contains two orthogonal components with equal in magnitude and y component leads x component by 90°

∴ Wave is left circularly polarized.

6. (b)

Both are nonmagnetic medium. Brewester angle is given by,

$$\tan\theta_B = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}} = \sqrt{\frac{2.6\varepsilon_0}{\varepsilon_0}} = 1.612 = 58.18°$$

7. (a)

$$\nabla\times\vec{A} = \vec{B}$$

$$\nabla\times\vec{A} = \mu\vec{H}$$

$$\Rightarrow \quad \vec{H} = \frac{1}{\mu}\left(\nabla\times\vec{A}\right)$$

8. (a)

$$Z_{in} = Z_0\left.\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right|_{l = \frac{\lambda}{8},\ Z_0 = 50,\ Z_L = 0} = 50\frac{j50\tan 45°}{50} = j50\ \Omega$$

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9. (c)
Given,

$$\left|\frac{J_C}{J_D}\right| = \left|\frac{\sigma E}{\omega\varepsilon E}\right| = \frac{\sigma}{\omega\varepsilon} = 10$$

or, $$\omega = \frac{\sigma}{10\varepsilon}$$

∴ $$2\pi f = \frac{\sigma}{10\varepsilon} \Rightarrow f = \frac{\sigma}{20\pi\varepsilon} = \frac{20}{20\pi\times81\times8.854\times10^{-12}}$$

$$f = 443.84 \text{ MHz}$$

10. (d)
From Maxwell's equation

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\frac{\partial}{\partial x}8x - \frac{\partial}{\partial y}2ky + \frac{\partial}{\partial z}4z = 0$$

$$8 - 2k + 4 = 0$$

or, $$k = \frac{12}{2} = 6$$

11. (a)
At junction input impedance of $\frac{\lambda}{2}$ line

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right) = 0$$

$$Z_{\text{in }L} = Z_o\left(\frac{Z_L + jZ_o\tan\beta l}{Z_o + jZ_L\tan\beta l}\right) = Z_L = 100\ \Omega$$

For input impedance of S.C. stub
As we know for SC stub, $Z_L = 0$

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) = 1$$

$$Z_{\text{in }S} = Z_o\left(\frac{Z_L + jZ_o\tan\beta l}{Z_o + jZ_L\tan\beta l}\right) = jZ_o = j50\ \Omega$$

At junction, $$Y = \frac{1}{Z} = \frac{1}{j50} + \frac{1}{100} = 0.01 - j0.02$$

12. (d)
For a distortionless line, $$RC = GL$$

$$G = \frac{RC}{L}$$

- $$Z_0 = \sqrt{\frac{L}{C}}$$
- $$\alpha = \sqrt{RG} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$$

⇒ $$R = \alpha Z_0 = 10 \times 10^{-3} \times 100 = 1\ \Omega/\text{m}$$

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{LC}} \times \sqrt{\frac{L}{L}} = \frac{1}{L}\sqrt{\frac{L}{C}} = \frac{1}{L} \times Z_0$$

$$\Rightarrow \quad L = \frac{Z_0}{v} = \frac{100}{2\times10^8} = 50\times10^{-8} = 0.5\,\mu\text{H/m}$$

13. (b)

- $$Z_{in_3} = \frac{Z_{03}^2}{Z_{L3}} = \frac{300^2}{200} = 450\ \Omega$$
- $$Z_{in_2} = \frac{Z_{02}^2}{Z_{L2}} = \frac{100^2}{0} = \infty \ \text{(open)}$$
- $$Z_{L(\text{eff})} = Z_{in_3} \,||\, Z_{in_2} = 450\ \Omega$$
- $$Z_{in} = Z_{in_1} = \frac{100^2}{450} = 22.22\ \Omega$$

14. (a)

The cut-off frequency for the TE_{mn} mode is,

$$f_c = \frac{C}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

We need the frequency lie between the cut-off frequencies of the TE_{10} and TE_{01} modes.

These will be,
$$f_{c,\,10} = \frac{C}{2\sqrt{\epsilon_r}\,a} = \frac{3\times10^8}{2\sqrt{\epsilon_r}(0.06)} = \frac{2.5\times10^9}{\sqrt{\epsilon_r}}$$

$$f_{c,\,01} = \frac{C}{2\sqrt{\epsilon_r}\,b} = \frac{3\times10^8}{2\sqrt{\epsilon_r}(0.04)} = \frac{3.75\times10^9}{\sqrt{\epsilon_r}}$$

∴ The range of frequencies over which single mode operation will occur is

$$\frac{2.5}{\sqrt{\epsilon_r}}\,\text{GHz} < f < \frac{3.75}{\sqrt{\epsilon_r}}\,\text{GHz}$$

15. (b)

$a = 1$ cm, $b = 3$ cm

Cut-off frequency for TE_{12} mode

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{1}\right)^2 + \left(\frac{2}{3}\right)^2} \times 100$$

$$= 18.03\ \text{GHz}$$

$f = 1.2 \times f_c = 21.6$ GHz

16. (a)

Minimum SWR ⇒ Minimum possible $|\Gamma|^2$

$$\Rightarrow \quad \frac{\partial|\Gamma|^2}{\partial Z_0} = \frac{\partial}{\partial Z_0}\frac{|R_L + jX_L - Z_0|^2}{|R_L + jX_L + Z_0|^2} = 0$$

$$= \frac{\partial}{\partial Z_0}\frac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2} = \frac{4R_L\left(Z_0^2 - \left(R_L^2 + X_L^2\right)\right)}{\left(\left(R_L + Z_0\right)^2 + X_L^2\right)^2} = 0$$

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$$\Rightarrow \quad Z_0^2 = R_L^2 + X_L^2$$

$$\Rightarrow \quad Z_0 = \sqrt{(R_L)^2 + (X_L)^2} = \sqrt{25^2 + (-50)^2} = 55.9\ \Omega$$

17. (b)

$$f_c(\text{TE}_{10}) = \frac{c}{2a} = \frac{3\times 10^{10}}{2\times \frac{1}{3}} = 45\text{ GHz}$$

$$f_c(\text{TE}_{01}) = \frac{c}{2b} = \frac{3\times 10^{10}}{2\times \frac{1}{4}} = 60\text{ GHz}$$

$$f_c(\text{TE}_{11}) = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\Bigg|_{\substack{m=1\\n=1}} = \frac{3\times 10^{10}}{2}\sqrt{\left(\frac{1}{1/3}\right)^2 + \left(\frac{1}{1/4}\right)^2} = 75\text{ GHz}$$

$$f_c(\text{TE}_{21}) = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\Bigg|_{\substack{m=2\\n=1}} = \frac{3\times 10^{10}}{2}\sqrt{\left(\frac{2}{1/3}\right)^2 + \left(\frac{1}{1/4}\right)^2} = 108.17\text{ GHz}$$

$$f_c(\text{TE}_{12}) = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\Bigg|_{\substack{m=1\\n=2}} = \frac{3\times 10^{10}}{2}\sqrt{\left(\frac{1}{1/3}\right)^2 + \left(\frac{2}{1/4}\right)^2} = 128.16\text{ GHz}$$

So, only three modes can propagate at 80 GHz.

18. (a)

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{1\times 10^6} = 300\text{ m}$$

$$\frac{l}{\lambda} = \frac{2}{300} = \frac{1}{150} \qquad \Rightarrow \text{short dipole}$$

$$R_{\text{rad}} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{150}\right)^2 = 35\text{ m}\Omega$$

$$R_{\text{loss}} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu}{\sigma}} = \frac{2}{2\pi\times 10^{-3}}\sqrt{\frac{\pi\times 10^6\times \mu_0}{5.8\times 10^7}} = 83\text{ m}\Omega$$

$$\eta = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{35}{35+83} \simeq 30\%$$

19. (a)

$$\text{Cut-off frequency, } f_c = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \left[u = \frac{c}{\sqrt{u_r\varepsilon_r}} = c\right]$$

$$f_{c11} = f_{c03} = 12\text{ GHz}$$

$$\frac{u}{2}\sqrt{\left(\frac{1}{a^2} + \frac{1}{b^2}\right)} = \frac{u}{2}\left(\frac{3}{b}\right)$$

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{9}{b^2}, \qquad \frac{8}{b^2} = \frac{1}{a^2}$$

$$\therefore \quad b = \sqrt{8}a, \quad b > a$$

$$f_{c03} = \frac{u}{2}\frac{3}{b} = 12 \text{ GHz}$$

$\therefore$
$$b = \frac{3\times10^8\times3}{2\times12\times10^9} = 3.75 \text{ cm}$$

20. (c)

We have,
$$\frac{m\pi x}{a} = \frac{2\pi x}{a}, \quad m = 2$$
$$\frac{n\pi x}{b} = \frac{3\pi y}{b}, \quad n = 3$$

$\therefore$ it is TE_{23} node

Cut off frequency,
$$f_c = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{2}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{2}{286}\right)^2+\left(\frac{3}{1.016}\right)^2}\times100$$
$$= 46.19 \text{ GHz}$$

$\omega = 10\pi\times10^{10}$, $\quad f = 50$ GHz

$$\beta = \frac{2\pi f}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi\times50\times10^9}{3\times10^8}\sqrt{1-\left(\frac{46.19}{50}\right)^2} = 400.68 \text{ rad/m}$$

$\because f > f_c$ $\quad \alpha = 0$

$\gamma = \alpha + j\beta = j400.7$ rad/m

21. (b)

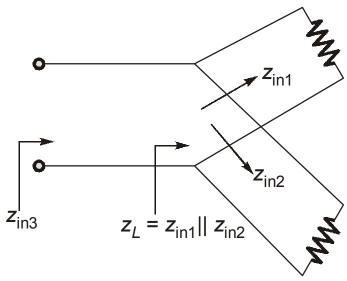
For quarter wave transformer, $z_0^2 = z_{in}\cdot z_L$

$$z_{in1} = \frac{z_0^2}{200} = \frac{50^2}{200} = 12.5\,\Omega$$

$$z_{in2} = \frac{z_0^2}{0} = \infty$$

$$z_L = z_{in1} \| z_{in2} = \infty \| 12.5 = 12.5\,\Omega$$

$$z_{in3} = \frac{z_0^2}{z_L} = \frac{50^2}{12.5} = 200\,\Omega$$



22. (c)

From boundary condition of dielectric-dielectric medium.

$$E_{t_1} = E_{t_2}$$

and
$$D_{n_1} = D_{n_2}$$
$$\varepsilon_{r_1}E_{n_1} = \varepsilon_{r_2}E_{n_2}$$

or
$$E_{n_2} = \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}E_{n_1} = \frac{2}{8}\times100 = 25$$

$\therefore$
$$\vec{E}_2 = 25\hat{a}_x + 200\hat{a}_y - 50\hat{a}_z$$

23. (c)

- Field contains orthogonal components with unequal amplitudes $\Rightarrow$ Elliptical polarization
- y component leads x component by 90° and wave is travelling in positive-z direction $\Rightarrow$ Left elliptically polarized.

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24. (a)

The time average power is given by,

$$P = \frac{E^2}{2\eta}$$

Where, $$\eta = 120\pi\sqrt{\frac{\pi^2}{80}} = \frac{120\pi^2}{\sqrt{80}} = 132.414$$

$\therefore$ $$P = \frac{(15)^2}{2\times 132.414} = 0.849 \text{ W/m}^2 \approx 0.85 \text{ W/m}^2$$

25. (c)

$$\text{Electrical length} = \beta l = 2\pi f\sqrt{LC}\times l$$

$$92^\circ\times\frac{\pi}{180^\circ} = 2\times 40\times 10^6\times 20\times 10^{-2}\sqrt{L\times 20\times 10^{-12}}$$

$$\sqrt{L\times 20\times 10^{-12}} = \frac{92^\circ}{16\times 10^6\times 180^\circ} = 3.194\times 10^{-8}$$

On solving the above equation, we get,

or, $$L = \frac{(3.194\times 10^{-8})^2}{20\times 10^{-12}}$$

$$L = 51.008\ \mu\text{H/m}$$

26. (d)

As we know,

$$\Psi = \beta d\cos\theta + \alpha$$

$$\theta = 60^\circ$$

$\therefore$ $$0 = \frac{2\pi}{\lambda}\times\frac{\lambda}{8}\cos 60^\circ + \alpha$$

$$0 = \frac{\pi}{4}\times\frac{1}{2} + \alpha$$

or $$\alpha = -\frac{\pi}{8}$$

27. (b)

At dominant mode, TE_{10}

$$f_c = \frac{v_p}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{v_p}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2}$$

here, $$v_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{\sqrt{1.44}} = 2.5\times 10^8 \text{ m/s}$$

$$8.5\times 10^9 = \frac{2.5\times 10^8}{2\times a}$$

or $$a = 0.0147 \text{ m} = 1.47 \text{ cm}$$

28. (a)

Given, $$G = 2x^2yz\hat{a}_x - 20y\hat{a}_y + (x^2 - z^2)\hat{a}_z$$

$$\nabla\cdot G = 4xyz - 20 - 2z$$

$$\nabla(\nabla \cdot G) = 4yz\hat{a}_x + 4xz\hat{a}_y + (4xy - 2)\hat{a}_z$$

$$\nabla \times [\nabla(\nabla \cdot G)] = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 4yz & 4xz & 4xy - 2 \end{vmatrix}$$

$$= \hat{a}_x\left[\frac{\partial}{\partial y}(4xy-2) - \frac{\partial}{\partial z}(4xz)\right] - \hat{a}_y\left[\frac{\partial}{\partial x}(4xy-2) - \frac{\partial}{\partial z}4yz\right] + \hat{a}_z\left[\frac{\partial}{\partial x}(4xz) - \frac{\partial}{\partial y}(4yz)\right]$$

$$= (4x - 4x)\hat{a}_x - (4y - 4y)\hat{a}_y + (4z - 4z)\hat{a}_z$$

$$= 0$$

29. (a)

We known that $V_p = f\lambda$

$$\frac{C}{\sqrt{\epsilon}} = f\lambda$$

$$\lambda \propto \frac{1}{\sqrt{\epsilon}}$$

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

$$\Gamma = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}} - 1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}} + 1} = \frac{\left(\frac{3}{5} - 1\right)}{\left(\frac{3}{5} + 1\right)}$$

$$\Gamma = -\frac{1}{4}$$

$$\frac{P_r}{P_i} \times 100 = -|\Gamma|^2 = \frac{1}{16} \times 100 = 6.25\%$$

30. (a)

$$\eta_1 = \eta_0$$

$$\eta_2 = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}} = \frac{\eta_o}{\sqrt{5}} = 0.447\eta_0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.382$$

$$\tau = 1 + \Gamma = 0.618$$

$$E_t = \tau E_i$$

$$E_t = 92.7\cos(\omega t - 8\sqrt{5}\,y)\ \hat{a}_z \text{ V/m}$$

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