CLASS TEST

S.No.:01JP_CSDM_07082024

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COMPUTER SCIENCE & IT

Discrete Mathematics

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 How many committees of five people can be chosen from 20 men and 12 women such that each committee contains atleast three women?
(a) 75240 (b) 52492
(c) 41800 (d) 9900

Q.2 Consider of the following is/are valid:
1. $((p \Rightarrow q) \wedge (r \Rightarrow s) \wedge (p \vee r)) \Rightarrow (q \vee s)$
2. $((p \Rightarrow q) \wedge (r \Rightarrow s) \wedge (\sim q \vee \sim s) \Rightarrow (\sim p \vee \sim r)$
3. $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

Q.3 Let f and $g : R \to R$ be defined on R (R is a set of real numbers) as: $f(x) = x + 2$, $g(x) = (1 + x^2)^{-1}$
Find the value of $f^{-1}g$ (3) _______?
(a) 1.8 (b) –1.9
(c) 1.9 (d) –1.8

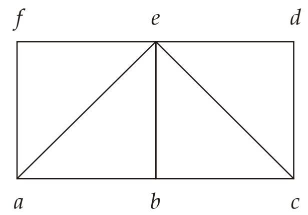
Q.4 Which of the following statement is incorrect?
(a) A graph of 6 vertices can be 1-chromatic.
(b) Every tree with 2 or more vertices is 2-chromatic.
(c) A wheel graph of n-vertices is $\left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right)$ chromatic.
(d) A graph which has no circuit of odd-length and has atleast 1 edge is 2-chromatic.

Q.5 Which of the following statements are equivalent for graph with atleast 1 edge?
1. G is bipartite.
2. A graph G is 2 colorable.
3. Graph G has a Hamiltonian circuit.
4. Every cycle of G is of even length.
(a) 1 and 3 (b) 1, 2 and 3
(c) 1, 2 and 4 (d) 3 and 4

Q.6 Consider $a_n = -5_{a_{n-1}} + 6_{a_{n-2}}$. Then which of the following represents a_n?
(a) $a_n = A(6)^n + B \cdot (3)^n$
(b) $a_n = A(-6)^n + B \cdot (2)^n$
(c) $a_n = A(-6)^n + B$
(d) $a_n = A(6)^n + B \cdot (1)^n$

Q.7 The number of permutations of the string "MADEEASY" in which not all vowels are together are ________.
(a) 10080 (b) 9360
(c) 10040 (d) 9560

Q.8 Which of the following statements is true regarding G?



(a) G has Hamiltonian cycle
(b) G has euler circuit
(c) G is traversable
(d) G has euler path

Q.9 Which of the following is correct statement?
(a) Every distributive lattice is complemented lattice.
(b) Every complemented lattice is distributive lattice.
(c) Every finite lattice is bounded.
(d) Both (b) and (c)

Q.10 Which of the following statement is false?
(a) $L(L, *, \oplus)$ be a lattice and $S \subseteq L$ be a subset of L. The algebra $(S, *, \oplus)$ is a sublattice of $L(L, *, \oplus)$ if and only if S is closed under both operations * and ⊕.
(b) A lattice is called complete if each of its non empty subsets has least upper bound and a great lower bound.
(c) Every chain is distributive lattice.
(d) If a lattice is said to be complementary lattice then every element of L has only one complement.

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 Consider two sets A and B such that:

$A \cup B \subseteq A \cap B$

Then, which of the following is incorrect?

(a) $A = \{\ \}$, $B = \{\ \}$ always
(b) $|A| = |B|$
(c) $A = B$
(d) None of these

Q.12 A binary relation R on $Z \times Z$ is defined as follows:

$(a, b)\ R\ (c, d)$ iff $a = c$ or $b = d$

Consider the following propositions:

1. R is reflexive.
2. R is symmetric.
3. R is antisymmetric.

Which one of the following statements is True?

(a) Both 1 and 2 are true
(b) 1 is true and 2 is false
(c) 1 is false and 3 is true
(d) Both 2 and 3 are true

Q.13 Consider the set $S = \{1, 2, 3, \ldots, 25\}$. The number of subsets $T \subseteq S$ of size five such that T has at least one odd number in it is

(a) 52338 (b) 51843
(c) 53432 (d) 52340

Q.14 The number of positive integer less than or equal to 1000 that are relatively prime to 15 are _____.

(a) 560 (b) 540
(c) 533 (d) 529

Q.15 Let satisfiable (x) be a predicate which denotes that x is satisfiable logic. Let Valid (x) be a predicate which denotes that x is valid logic. Which of the following first order logic sentences does not represents the statements:

"Not every satisfiable logic is Valid"

(a) $\neg\forall x\,(\text{satisfiable}\,(x) \Rightarrow \text{Valid}\,(x))$
(b) $\exists x\,(\text{satisfiable}\,(x) \wedge \neg\,\text{Valid}\,(x))$
(c) $\neg\forall x\,(\neg\text{satisfiable}\,(x) \vee \text{Valid}\,(x))$
(d) $\forall x\,(\text{satisfiable}\,(x) \Rightarrow \neg\,\text{Valid}\,(x))$

Q.16 Consider the following POSETs:

I. $(\{1, 2, 3, 6, 14, 21, 42\}, /)$
II. $(\{1, 2, 3, 6, 11, 22, 33, 66\}, /)$
III. $(\{1, 2, 5, 7, 10, 14, 35, 70\}, \leq)$

Which of the above POSETs are isomorphic to $(P(s), \subseteq)$, where $S = \{a, b, c\}$?

(a) I and II only (b) II only
(c) II and III only (d) III only

Q.17 Solve the following recurrence relation:
$T(n) = 9T(n-1) - 20T(n-2)$, $T(0) = -3$, $T(1) = -10$

(a) $2.5^n - 5.4^n$ (b) $3.5^n - 4.3^n$
(c) $3.4^n - 2.5^n$ (d) $4.5^n - 2.3^n$

Q.18 Which of the following statement(s) is/are false?

S_1 : A connected multigraph has an Euler circuit if and only if each of its vertices has even degree.
S_2 : A connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.
S_3 : A complete graph (K_n) has a Hamiltonian circuit whenever $n \geq 3$.
S_4 : A cycle over six vertices (C_6) is not a bipartite graph but a complete graph over 3 vertices is bipartite.

(a) S_1 only (b) S_2 and S_3
(c) S_3 only (d) S_4 only

Q.19 Consider the collection of all undirected graph with 10 nodes and 6 edges. If a graph has no self loops and their is atmost one edge between any pair of node. The maximum number of connected components is

(a) 7 (b) 8
(c) 9 (d) 10

Q.20 Consider the undirected graph G defined as follows. The vertices are bit string of length 5. We have an edge between vertex "a" and vertex "b" iff "a" and "b" differ only in one bit position (i.e., hamming distance 1).
If the ratio of chromatic number of G to the diameter of G is $\frac{X}{Y}$ then (Y – X) is ________.

(a) 2 (b) 3
(c) 4 (d) 5

Q.21 Find the middle term in the expansion of $\left(\frac{\sqrt{x}}{3} - \frac{3}{x\sqrt{y}}\right)^6$

(a) $20 \cdot \left(\frac{x}{y}\right)^{\frac{3}{2}}$ (b) $56 \cdot \left(\frac{x}{y}\right)^{\frac{2}{3}} \cdot \frac{1}{x^3}$

(c) $-20\left(\frac{x}{y}\right)^{\frac{3}{2}} \cdot \frac{1}{x^3}$ (d) $-56 \cdot \left(\frac{x}{y}\right)^{\frac{2}{3}} \cdot \frac{1}{x^3}$

Q.22 Consider the following statements with respect to relations R and S. Note that R^{-1} denotes the inverse relation of R, and R' denotes the complementary relation of R respectively.

I. If $R \subseteq S$, then $R^{-1} \subseteq S^{-1}$
II. If $S \subseteq R$, then $R^{-1} \subseteq S^{-1}$
III. If $R \subseteq S$, then $R' \subseteq S'$
IV. If $S \subseteq R$, then $R' \subseteq S'$

Which of the above statements are correct?
(a) I and III (b) II and IV
(c) I and IV (d) II and III

Q.23 Find the number of ways 10 balls to be chosen from a box containing 10 identical blue balls, 5 identical red balls and 3 identical yellow balls _______?
(a) 20 (b) 21
(c) 24 (d) 28

Q.24 Consider set $R = \{(1, 1), (2, 1), (3, 1), (4, 2)\}$. If set $T = R^1 \cup R^2 \cup R^3$, then find the cardinality of set T ____________? ($R^2 = R^1 \cdot R^1$ i.e composition of R with R)
(a) 3 (b) 4
(c) 5 (d) 6

Q.25 Let f be a function from set A to B. Consider the following statement:

P **:** For each $a \in A$, there exists unique $b \in B$ such that $f(a) = b$.
Q **:** For each $b \in B$, there exists $a \in A$ such that $f(a) = b$.
R **:** There exists $a_1, a_2 \in A$ such that $a_1 \neq a_2$ and $f(a_1) = f(a_2)$.

The negation of the statement "f is one-one and onto" is
(a) P or not R (b) R or not P
(c) R or not Q (d) P and not R

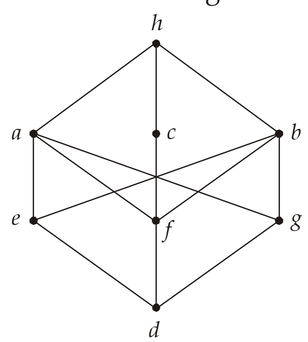
Q.26 Let $(Q, +)$ be a group where Q is the set of rational numbers. Which of the following is a subgroup of Q?
(a) $(Z, +)$ where Z is set of integers.
(b) $(A, +)$ where A is set of positive integers.
(c) $(B, +)$ where B is set of negative integers.
(d) All of these

Q.27 Let G be a planar graph such that every face is bordered by exactly three edges. What are the possible values for chromatic number of G?
(a) Only 3 (b) Only 4
(c) 3 or 4 (d) None of these

Q.28 Let G be a graph with V(G) = {1, 2, ... 10} where V(G) is the set of vertices of G. Two numbers i and j are adjacent in V(G) if and only if $(i \times j)$ is a multiple of 20. The number of edges in G are ________.
(a) 4 (b) 6
(c) 8 (d) 9

Q.29 A connected cubic planar graph with no bridges has exactly 4 edges in boundary of each region. The number of edges of the graph is ________.
(a) 8 (b) 10
(c) 12 (d) 16

Q.30 Consider the following Hasse diagram:



Find the lower bound and upper bound for the set $\{e, f, g\}$?
(a) $\{d\}$ $\{a, c, b\}$ (b) $\{d\}$ $\{a, b, h\}$
(c) $\{d\}$ $\{a, c, b, f\}$ (d) $\{d, e\}$ $\{f\}$

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Date of Test : 07/08/2024

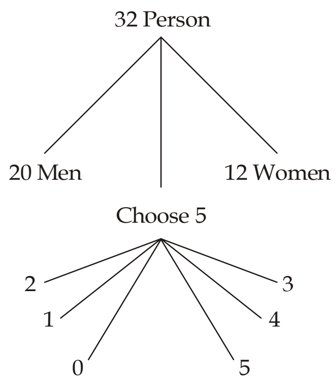
ANSWER KEY

1.	(b)	7.	(b)	13.	(a)	19.	(a)	25.	(c)
2.	(d)	8.	(a)	14.	(c)	20.	(b)	26.	(a)
3.	(b)	9.	(c)	15.	(d)	21.	(c)	27.	(c)
4.	(c)	10.	(d)	16.	(b)	22.	(c)	28.	(b)
5.	(c)	11.	(a)	17.	(a)	23.	(c)	29.	(c)
6.	(c)	12.	(a)	18.	(d)	24.	(c)	30.	(b)

DETAILED EXPLANATIONS

1. **(b)**

Given,



We must choose at least 3 women, so, we calculate 3 women, 4 women and 5 women and by addition rule add the results:

$$= {}^{12}C_3 \times {}^{20}C_2 + {}^{12}C_4 \times {}^{20}C_1 + {}^{12}C_5 \times {}^{20}C_0$$
$$= 220 \times 190 + 495 \times 20 + 792 \times 1 = 52492$$

2. **(d)**

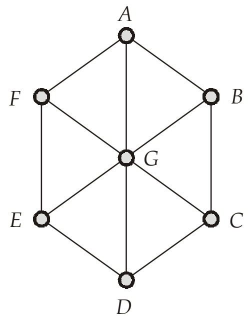
1. is valid by constructive dilemma.
2. is valid by destructive dilemma.
3. is valid by hypothetical syllogism.
All of the above are known rules of inference.

3. **(b)**

$$f(x) = x + 2 \qquad \text{Let } y = f(x)$$
$$y = x + 2 \qquad \Rightarrow x = f^{-1}(y)$$
$$\Rightarrow \quad x = y - 2$$
$$\Rightarrow \quad f^{-1}(y) = y - 2 \qquad \textbf{or } f^{-1}(x) = x - 2$$
$$g(3) = (1 + (3)^2)^{-1} = (1 + 9)^{-1} = \frac{1}{10}$$
$$f^{-1}\, g(3) = f^{-1}\,(g(3)) = g(3) - 2$$
$$\Rightarrow \quad f^{-1}\, g(3) = \frac{1}{10} - 2 = -1.9$$

4. **(c)**

Consider each options:
(a) Null graph of 6 vertices is 1-chromatic so it is correct.
(b) It is correct because tree with 2 or more vertices is always bichromatic.
(c) It is incorrect. Consider a wheel graph of 7 vertices.



The chromatic number of graph is 3.

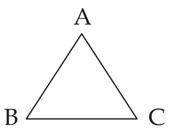
- Color 1 for G
- Color 2 for A, E, C
- Color 3 for F, B, D
- A wheel graph is 3-chromatic when n-vertices are odd and 4-chromatic when n-vertices is even.
- So here $n = 7$, $\left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right) = \left(\left\lfloor \frac{7}{2} \right\rfloor + 1\right) = 4$ which is incorrect because only 3 colors are required to color the above wheel graph.

(d) This statement is correct because graph without odd length cycle having atleast 1 edge is bichromatic.

All other statements are true except option (c).

5. **(c)**

Considering each statements:

- If a graph is bipartite, then its two colourable. Because a bipartite graph can be represented as two groups of vertices such that vertices in same graph are not adjacent.
  Similarly, statement 2 is equivalent to statement 1.
- If a bipartite graph has a cycle, then it has to be of even length. Graph G is bipartite iff no odd length cycle.
- 

  This graph has a Hamiltonian circuit, but the cycle is of odd length and not bipartite.

$\therefore$ $3 \not\equiv 4$

- 

  This graph is bipartite and 2 colorable but does not have Hamiltonian circuit.

So 1, 2 and 4 are equivalent statements.

6. **(c)**

$$a_n = -5_{a_{n-1}} + 6_{a_{n-2}}$$

$$a_n + 5_{a_{n-1}} - 6_{a_{n-2}} = 0$$

$$x^2 + 5x - 6 = 0$$

$$x^2 + 6x - x - 6 = 0$$
$$x(x + 6) - 1(x + 6) = 0$$
$$(x + 6)(x - 1) = 0$$
$$x = -6, \quad x = 1$$
$$\therefore \quad a_n = A(-6)^n + B \cdot (1)^n$$
$$a_n = A(-6)^n + B$$

7. **(b)**

Number of ways all vowels are together = $\dfrac{5! \times 4!}{2!\,2!} = 720$

Number of ways not all vowels are together = Total number of permutation – Number of ways all vowels are together

$$= \frac{8!}{2!\,2!} - 720$$

$\Rightarrow$ Number of not all vowels together = (10080 – 720) = 9360

8. **(a)**
Hamiltonian cycle for the above graph G is *abcdefa*.
Condition: Each node should be visited exactly once.

9. **(c)**
The theorem is every finite lattice is bounded but a bounded lattice may not be finite.

10. **(d)**
Complementary lattice may not have unique complement for every element.

11. **(a)**
$A \cup B \subseteq A \cap B$ holds true when $A = B$. It is true for empty as well as nonempty sets.
$\Rightarrow$ $|A| = |B|$ is true $|A| \geq 0$ ***eg.*** $A = B\ \{a, b\}$
Hence $A = \{\ \}$, $B = \{\ \}$ "always" is false.

12. **(a)**
***R* is reflexive:** Since $(a, b)\ R\ (a, b)$ for all elements (a, b) because $a = a$ and $b = b$ are always true.
***R* is symmetric:** Since $(a, b)\ R\ (c, d)$ and $a = c$ or $b = d$ which can be written as $c = a$ or $d = b$.
So, $(a, b)\ R\ (a, b)$ is true.
***R* is not antisymmetric:** Since $(1, 2)\ R\ (1, 3)$ and $1 = 1$ or $2 = 3$ true b/c $1 = 1$.
So $(1, 3)\ R\ (1, 2)$ but here $2 \neq 3$ so $(1, 2) \neq (1, 3)$.
So, only statement 1 and 2 are correct.

13. **(a)**
Total number of subset of 5 element = ${}^{25}C_5$

$$= \frac{25 \times 24 \times 23 \times 22 \times 21}{5 \times 4 \times 3 \times 2 \times 1} = 23 \times 22 \times 21 \times 5 = 53130$$

T be a 5 element subset contain no odd number = ${}^{12}C_5$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$

So number of 5 element subset with atleast 1 odd number

$$T \subseteq S = {}^{25}C_5 - {}^{12}C_5$$
$$= 53130 - 792 = 52338$$

14. (c)

A number is relatively prime to 15 iff it is not divisible by 3 and not divisible by 5.

Set of integer from 1 to 1000 divisible by $3 = \left\lfloor \frac{1000}{3} \right\rfloor = 333.$

Set of integer from 1 to 1000 divisible by $5 = \left\lfloor \frac{1000}{5} \right\rfloor = 200.$

So, number of integer not relatively prime to 15 are

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= \left\lfloor \frac{1000}{3} \right\rfloor + \left\lfloor \frac{1000}{5} \right\rfloor - \left\lfloor \frac{1000}{15} \right\rfloor$$
$$= 333 + 200 - 66 = 467$$

So, number of integer relatively prime to 15 are

$$\overline{|A \cup B|} = 1000 - 467 = 533$$

15. (d)

"Not every satisfiable logic is Valid"

= Not (every satisfiable logic is Valid)

$= \text{Not } (\forall x (\text{satisfiable } (x) \Rightarrow \text{Valid } (x))$ option (a)

$= \text{Not } (\forall x (\neg \text{satisfiable} \vee \text{Valid } (x))$ option (c)

$= \exists x (\text{satisfiable } (x) \wedge \neg \text{Valid } (x))$ option (b)

Statement (d) says every satisfiable logic is invalid.
So option (d) is not represent given statement.

16. (b)

- I is not D_{42} because the divisor 7 is missing. So, there is no way for I to be isomorphic to $(P\{a,b,c\}, \subseteq)$ as it needs to have 8 divisors but right now it has only 7.
- II is D_{66} a well known boolean algebra and has 8 vertices and its masses diagram will be isomorphic $(P(\{a,b,c\}), \subseteq)$.
- III is not isomorphic even though it looks like D_{70}, it is on the relation ≤, resulting in a chain, which won't be boolean algebra.

17. (a)

$T(n) - 9T(n-1) + 20T(n-2) = 0$

Let $a_n = T(n)$

$\Rightarrow a_n - 9a_{n-1} + 20a_{n-2} = 0$

$$t^2 - 9t + 20 = 0$$

$$t^2 + 5t - 4t + 20 = 0$$
$$t(t-5) - 4(t-5) = 0$$
$$(t-4)(t-5) = 0$$
$$t = 4, 5$$

Homogenous equation become

$$a_n = c_1 . 5^n + c_2 . 4^n \quad ...(1)$$

Put $n = 0$ in equation (1)

$$a_0 = c_1 . 5^0 + c_2 . 4^0$$
$$-3 = c_1 + c_2 \quad ...(2)$$

Put n = 1 in equation (1)

$$a_1 = c_1 . 5^1 + c_2 . 4^1$$
$$-10 = 5c_1 + 4c_2 \quad ...(3)$$

Solving equation (2) and (3) and get c_1 and c_2

$$(c_1 + c_2 = -3) \times 5$$
$$5c_1 + 4c_2 = -10$$

$$5c_1 + 5c_2 = -15$$
$$5c_1 + 4c_2 = -10$$
$$c_2 = -5 \quad \text{and} \quad c_1 = 2$$

Put value of c_1 and c_2 in eq. (1)

$$a_n = 2.5^n - 5.4^n$$

18. **(d)**

- S_1 is correct because connected graph has a Euler circuit if and only if it has number of odd degree vertices is 0.
- A connected graph has a Euler path if and only if it has number of odd degree vertices is either 0 or 2. Therefore a connected graph has Euler path but not euler circuit if and only it has exactly 2 vertices of odd degree therefore S_2 is correct.
- A complete graph of n-vertices contains $n - 1$ degree at each vertex which is greater than $\frac{n}{2}$ for all $n \geq 3$ therefore complete graph has a Hamiltonian circuit. So S_3 is correct
- C_6 is bipartite because any cycle graph with even number of vertices is bipartite. A complete graph with 4 vertices contains complete graph of 3 vertices which contains odd length cycle hence it is not bipartite. So statement S_4 is incorrect.

19. **(a)**

Maximum and minimum number of component given by:

$$n - K \leq e \leq \frac{(n-K+1)(n-K)}{2}$$

1.
$$n - K \leq e$$
$$n - e \leq K$$
$$10 - 6 \leq K \qquad (\because \text{ Minimum number of component})$$

2.
$$e \leq \frac{(n-K+1)(n-K)}{2}$$
$$6 \leq \frac{(10-K+1)(10-K)}{2}$$

$$2 \times 6 \le (11 - K)(10 - K)$$
$$12 \le (10 - K)(11 - K)$$
$$12 \le K^2 + 110 - 21K$$
$$0 \le K^2 + 98 - 21K$$
$$K^2 + 98 - 21K = 0$$
$$K = 14, 7$$

Maximum value of K is 7 because number of components never be larger than nodes.

20. **(b)**

Since bit are '0' and '1' form. The hamming distance relation on bit has a digraph which will be always an 5-cube where 5 is the number of bits.

- Chromatic number of n-cube = 2 (Since n-cube is always bipartite)
  So chromatic number of 5-cube = 2

i.e., '0' = One color
'1' = Second color

- Diameter of n-cube = n
  Diameter of 5 cube = 5

i.e., maximum length between any two vertex.

So ratio $$\frac{2}{5} = \frac{X}{Y}$$
$$Y - X = 5 - 2 = 3$$

21. **(c)**

Total number of terms = 6 + 1 = 7

So middle term is 4th term.

$(x + y)^n$ has $(r + 1)^{th}$ term as ${}^nC_r\, x^{n-r}\, y^r$.

[(3 + 1)th term] 4th term is

$$= {}^6C_3\left(\frac{\sqrt{x}}{3}\right)^{6-3}\left(\frac{-3}{x\sqrt{y}}\right)^3$$

$$= {}^6C_3\cdot\left(\frac{\left(\sqrt{x}\right)^3}{27}\right)\cdot\left(\frac{-27}{x^3\cdot\left(\sqrt{y}\right)^3}\right)$$

$$= 20\cdot\left(\frac{x^{3/2}}{27}\right)\cdot\left(\frac{-27}{x^3\cdot(y)^{3/2}}\right)$$

$$= -20\left(\frac{x}{y}\right)^{\frac{3}{2}}\cdot\frac{1}{x^3}$$

22. **(c)**

I and IV are true. Let's see why IV is true first. We will treat set theory as boolean algebra here, and will demonstrate how to apply this approach.

Given, $S \subseteq R$, which in same as, $\Rightarrow S - R = \varphi$

The same can be written in boolean algebra as, $\Rightarrow S \wedge R' = 0$

Since we know that $\wedge$ is commutative, $\Rightarrow R' \wedge S = 0$

Now $R' \wedge S = 0$ is same as $R' \wedge (S')' = 0 \Rightarrow R' - S' = \varphi$

$\Rightarrow \quad R' \subseteq S'$

Therefore IV is correct.

In order to show that R^{-1} is a subset of S^{-1}, we just need to show that every element in R^{-1} belongs to S^{-1}. So let's assume that R is a subset of S. So if (a, b) is an element of R, then (a, b) belongs to S as well. As (a, b) belongs to R, (b, a) belongs to R^{-1}. Also, (a, b) belongs to S, (b, a) will belong to S^{-1}. Since we can show this presence of every element in R^{-1} in S^{-1}, we see that R^{-1} is a subset of S^{-1}. However II is clearly not true.

23. **(c)**

This problem corresponds to the number of non-negative integral solution to

$$x_1 + x_2 + x_3 = 10 \text{ with the conditions}$$
$$0 \le x_1 \le 10$$
$$0 \le x_2 \le 5$$
$$0 \le x_3 \le 3$$

Generating functions are required, since the variables have an upper constraint.

Generating function is

$(1 + x + x^2 \, ... + x^{10})\,(1 + x + x^2 \, ... + x^5)\,(1 + x \, ... + x^3)$

$$= \left(\frac{1-x^{11}}{1-x}\right)\left(\frac{1-x^6}{1-x}\right)\left(\frac{1-x^4}{1-x}\right)$$

$$= \frac{(1-x^{11})(1-x^6)(1-x^4)}{(1-x)^3}$$

$$= (1-x^4-x^6+x^{10})\sum_{r=0}^{\infty} {}^{3-1+r}C_r \cdot x^r$$

$$= (1-x^4-x^6+x^{10})\sum_{r=0}^{\infty} {}^{r+2}C_r \cdot x^r$$

The coefficient of x^{10} in above generating function is

$$^{12}C_{10} - {}^{8}C_6 - {}^{6}C_4 - {}^{2}C_0 = 24$$

24. **(c)**

R^1 is nothing but R itself.

Now, $\quad R^2 = R \text{ o } R$ i.e. composite of R with R

If $(a, b) \in R$, then $(a, c) \in R^2$ iff $(b, c) \in R$.

So, $\quad R^2 = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

$\quad R^3 = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

So, $\quad P = R^1 \cup R^2 \cup R^3$

$\quad = \{(1, 1), (2, 1), (3, 1), (4, 1), (4, 2)\}$

$\therefore \quad$ Cardinality $= 5$

25. (c)

"f is one-one and onto".

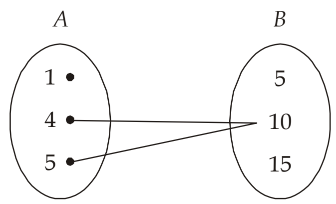
Negation of this statement will be

"f is not one-one or not onto".

Now, according to statement R.

Let,
$$a_1 = 4 \in A$$
$$a_2 = 5 \in A$$
$$f(a_1) = f(a_2) = 10$$

So,



Clearly this is condition of not one-one.

So, R is correct.

Now, Q is definition of onto so we have to take negation of this.

Therefore option (c) is correct answer.

26. (a)

$(Z, +)$ is a group and $Z \subseteq Q$.

$(A, +)$ is not a group. Hence it is not a subgroup of $(Q, +)$.

$(B, +)$ is not a group. Hence it is not a subgroup of $(Q, +)$.

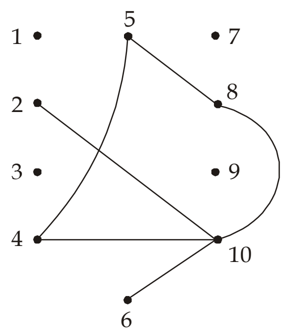
27. (c)

G is a planar graph. Every planar graph is 4 colorable. Every face is bordered by 3 edges.

So graph has possibilities of 3 or 4 colors.

k_3 colored with 3 and k_4 colored with 4 colors.

28. (b)



$$E = \{\{2, 10\}, \{4, 10\}, \{6, 10\}, \{8, 10\}, \{4, 5\}, \{5, 8\}\}$$

$\Rightarrow$ 6 edges are present in G.

29. (c)

$$4.r = 2.e \quad \text{[Planar graph]} \quad \ldots(i)$$
$$3.n = 2.e \quad \text{[Cubic graph]} \quad \ldots(ii)$$
$$n - e + r = 2 \quad \ldots(iii)$$

Substitute (i) and (ii) in (iii)

$$\Rightarrow \quad \frac{2e}{3} - e + \frac{2e}{4} = 2$$
$$\Rightarrow \quad 8e - 12e + 6e = 24$$
$$\Rightarrow \quad 14e - 12e = 24$$
$$\Rightarrow \quad 2e = 24$$
$$\Rightarrow \quad e = 12$$

30. (b)

'd' preceeds the set and $\{a, b, h\}$ suceeds the set.

$\therefore$ Lower bound = d

Upper bound = $\{a, b, h\}$

■■■■