CLASS TEST

Sl.: 01SK_EE_ABCDEFG_100824

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ELECTROMAGNETICS THEORY

ELECTRICAL ENGINEERING

Date of Test : 10/08/2024

ANSWER KEY

1.	(d)	7.	(a)	13.	(a)	19.	(c)	25.	(c)
2.	(d)	8.	(d)	14.	(a)	20.	(d)	26.	(b)
3.	(d)	9.	(a)	15.	(a)	21.	(b)	27.	(c)
4.	(c)	10.	(b)	16.	(d)	22.	(a)	28.	(d)
5.	(b)	11.	(b)	17.	(a)	23.	(d)	29.	(d)
6.	(c)	12.	(a)	18.	(a)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. **(d)**

Field lines are always from higher potential point to lower potential point. So, at equipotential surface, no component of field can be tangential to surface as otherwise potential difference will produced.

Hence, electric field lines are orthogonal to equipotential surface.

2. **(d)**

3. **(d)**

$$\nabla \cdot \vec{D} = \rho_v$$

Given $$\vec{D} = \hat{a}_x \sin y + \hat{a}_y\, y$$

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0 + 1 + 0 = 1\,(\text{C/m}^3)$$

4. **(c)**

This is Maxwell's third law derived from Gauss's law which says that total electric displacement i.e., $\bar{D}$ through a surface enclosing a volume is equal to the charge in the volume.

5. **(b)**

$$\phi_m = \iint_s \vec{B} \cdot \vec{ds}$$

$$\vec{ds} = d\rho\, dz = \hat{a}_\phi$$

$$= \int_0^2 \int_{0.5}^{2.5} \frac{20}{\rho}\, d\rho\, dz = 2 \times 20 [\ln \rho]_{0.5}^{2.5}$$

$$= 20 \times 2 \ln 5$$

$$= 40 \ln 5 = 64.40 \text{ Wb}$$

6. **(c)**

From the gauss law magnetic fields

$$\oint \vec{B} \cdot \vec{dS} = 0 \quad \text{or} \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \mu \vec{H} = 0$$

So, $$\nabla \cdot \vec{H} = 0$$

7. **(a)**

∇ depends on space (x, y, z) and has direction

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

$\therefore$ del (∇) is a vector space function.

8. **(d)**

From Ampere's circuital law

$$\oint \vec{H}\cdot\vec{dl} = I_{enc} = I - I + I = I$$

$$\oint \vec{B}\cdot\vec{dl} = \oint \mu_o \vec{H}\cdot\vec{dl} = \mu_o I$$

9. **(a)**

$$\text{Energy} = \frac{1}{2}CV^2 \text{ or } \frac{1}{2}\times\frac{Q^2}{C}$$

$$4.8 = \frac{1}{2}\times\frac{20\times 20\times 10^{-6}}{C}$$

$$C = \frac{20\times 20\times 10^{-6}}{2\times 4.8} = 41.66\ \mu\text{F}$$

10. **(b)**

For a uniformly charged sphere of radius 'R' electric field intensity inside the sphere is given by,

$$E = \frac{1}{4\pi\in_0}\frac{Qr}{R^3}$$

For $r = \frac{R}{2}$, point will be inside the sphere,

$$E = \frac{1}{4\pi\in_0}\frac{Q\left(\frac{R}{2}\right)}{R^3} = \frac{QR}{8\pi\in_0 R^3} \qquad \text{...(i)}$$

For a uniformly charged sphere of radius 'R', electric field intensity outside the sphere is given by,

$$E = \frac{1}{4\pi\in_0}\frac{Q}{r^2}$$

For $r = 2R$, point will be outside the sphere,

$$E = \frac{1}{4\pi\in_0}\frac{Q}{(2R)^2} = \frac{Q}{4\pi\in_0 (4R^2)} \qquad \text{...(ii)}$$

From equation (i) and (ii),

$$\frac{E(\text{at } r = 2R)}{E\left(\text{at } r = \frac{R}{2}\right)} = \frac{\frac{Q}{4\pi\in_0 (4R^2)}}{\frac{Q}{8\pi\in_0 R^2}} = \frac{1}{2} = 0.5$$

11. **(b)**

Gauss's law for magnetic fields states that the magnetic flux flowing through the closed surface is equal to zero as free magnetic monopoles do not exist and magnetic flux lines are closed,

$$\nabla\cdot\vec{B} = 0$$

12. (a)

$$J = \nabla \times \vec{H} = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3 & 7y & 2x \end{bmatrix}$$

$$J = -2\hat{a}_y$$

13. (a)

$$\vec{A} = 3x^2yz\,\hat{a}_x + x^3z\,\hat{a}_y + (x^3y - 2z)\hat{a}_z$$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(3x^2yz) + \frac{\partial}{\partial y}(x^3z) + \frac{\partial}{\partial z}(x^3y - 2z)$$

$$= 6\,xyz - 2$$

$$\nabla \cdot \vec{A} \neq 0$$

$\vec{A}$ is not a solenoidal

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2yz & x^3z & x^3y - 2z \end{vmatrix}$$

$$= (x^3 - x^3)\hat{a}_x - (3x^2y - 3x^2y)\hat{a}_y + (3x^2z - 3x^2z)\hat{a}_z = 0$$

$\vec{A}$ is irrotational.

As $\nabla \cdot \vec{A}$ is nonzero, this means A is having some divergence. It is not a divergenceless function.

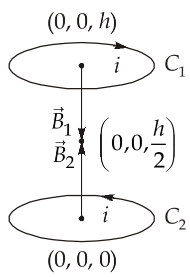
So, $\vec{A}$ is a non solenoidal field. As $\nabla \times \vec{A} = 0$, A is an irrotational field.

14. (a)

$\because$ C_1 and C_2 are identical,

$\therefore$ $\vec{B}_1 = -\vec{B}_2$

$\therefore$ $\vec{B}_{net} = 0$

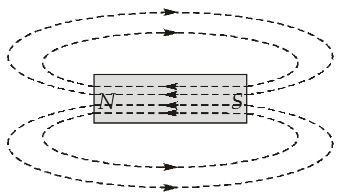


As both the coils are same axis and carrying currents in opposite directions, the field components produced by both the coils are in opposite direction and they cancel out each other. So, the net field at the point on the axis midway between the coils is zero.

15. (a)

$$\vec{\nabla}\cdot\vec{A} = +\text{ve},\ \vec{A} \text{ going away from source}$$
$$\vec{\nabla}\cdot\vec{A} = -\text{ve},\ \vec{A} \text{ is coming towards the sink}$$
$$\vec{\nabla}\cdot\vec{A} = 0, \text{ neither source or sink of } \vec{A} \text{ is present.}$$

16. (d)

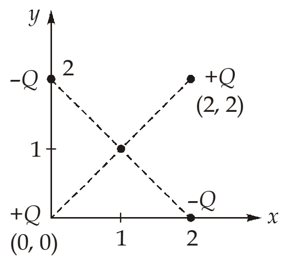


17. (a)

Laplace equation in cylindrical coordinates

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho\partial V}{\partial\rho}\right)+\frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial\phi^2}\right)+\frac{\partial^2 V}{\partial z^2}=0$$

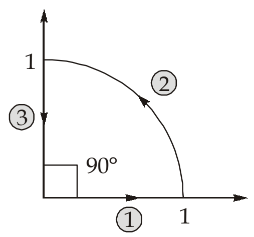
18. (a)



Two positive charges Q are diagonally opposite in position and at the same distance from the point (1, 1, 0) fields produce by them are equal and opposite and so their resultant field is zero. Similarly for negative charges.

19. (c)

$$\nabla\times\vec{E} = \frac{1}{\rho}\begin{vmatrix}\hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ 4\rho\sin\phi & 0 & 0\end{vmatrix}$$



$$= \frac{1}{\rho}\left(-\frac{\partial}{\partial\phi}(4\rho\sin\phi)\right)\hat{a}_z$$

$$= -\frac{1}{\rho}4\rho\cos\phi\,\hat{a}_z = -4\cos\phi\,\hat{a}_z$$

$$\oint\vec{E}\cdot d\vec{l} = \int\nabla\times\vec{E}\cdot d\vec{s}$$

Where, $d\vec{s} = \rho\, d\rho\, d\phi\, \hat{a}_z$

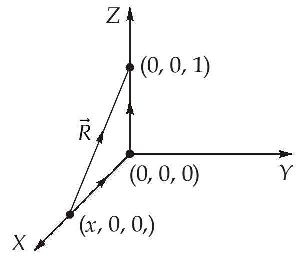
$$= \int -4\cos\phi\,\rho\, d\rho\, d\phi$$

$$= -4\int_0^{\pi/2}\cos\phi\, d\phi\int_0^1\rho\, d\rho = -4\sin\phi\Big|_0^{\pi/2}\times\frac{\rho^2}{2}\Big|_0^1 = -4\times1\times\frac{1}{2} = -2$$

20. (d)

According to the Biot-Savart's law,

$$\vec{H} = \int \frac{Id\vec{L} \times \vec{R}}{4\pi R^3}$$



$$\vec{R} = -x\hat{a}_x + \hat{a}_z$$

$$R = \sqrt{1+x^2}$$

$$\vec{H} = \int_0^{\infty} \frac{10dx(-\hat{a}_x) \times (-x\hat{a}_x + \hat{a}_z)}{4\pi(x^2+1)^{3/2}}$$

$$= \frac{10}{4\pi}\int_0^{\infty} \frac{dx}{(x^2+1)^{3/2}}\hat{a}_y$$

$$\vec{H} = \frac{10}{4\pi}\left.\frac{x}{\sqrt{1+x^2}}\right|_0^{\infty}\hat{a}_y = \frac{10}{4\pi}\hat{a}_y$$

21. (b)

Force on charge is given by,

$$\vec{F} = Q(\vec{E} + \vec{V} \times \vec{B})$$

$$= -e(\vec{E} + \vec{V} \times \vec{B})$$

If $\vec{F} = 0$,

Then, $$\vec{E} = -(\vec{v} \times \vec{B})$$

$$= (\vec{B} \times \vec{V})$$

$$\vec{E} = \vec{B} \times \vec{V} = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 2 & 2 \\ 3 & 12 & -4 \end{bmatrix} \times 10^5 \times 10^{-3}\,\text{V/m}$$

$$= \left[(-8-24)\hat{a}_x - (-4-6)\hat{a}_y + (12-6)\hat{a}_z\right] \times 10^2$$

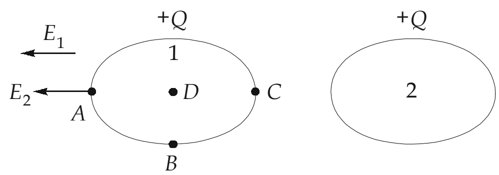
$$= \left[-32\hat{a}_x + 10\hat{a}_y + 6\hat{a}_z\right] \times 10^2\,\text{V/m}$$

$$\vec{E} = -3.2\hat{a}_x + \hat{a}_y + 0.6\hat{a}_z\ \text{kV/m}$$

22. (a)

Since, both the electrodes are at the same potential we can say that both have same nature of charge either positive or negative.

In such a case, of we draw electric field, the field due to both will be in same direction at A and hence get added up. So electric field at A will be maximum.



23. (d)

The ρ_v is dependent on the variable r. Hence though the charge distribution is sphere of radius 'a' we can not obtain Q just by multiplying ρ_v by $\left(\frac{4}{3}\pi a^3\right)$ as ρ_v is not constants. As it depends on r, it is necessary to consider differential volume dV and integrating from $r = 0$ to a, total Q must be obtained.

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\therefore \quad Q = \int_v \rho_v dV = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{a} \rho_0\left(1-\frac{r^2}{a^2}\right) r^2 \sin\theta dr d\theta d\phi$$

$$= \rho_0 [-\cos\theta]_0^{\pi} [\phi]_0^{2\pi} \int_{r=0}^{a}\left(r^2 - \frac{r^4}{a^2}\right) dr$$

$$= \rho_0 [-(-1)-(-1)][2\pi]\left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]_0^a$$

$$= \rho_0 \times 2 \times 2\pi \times \left[\frac{a^3}{3} - \frac{a^3}{5}\right]$$

$$= \rho_0 \times 4\pi \times \frac{2a^3}{15} = \frac{8\pi}{15}\rho_0 a^3 C$$

24. (c)

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r d^2}$$

$$F \propto \frac{1}{\varepsilon_r}$$

$$\therefore \quad \frac{F_2}{F_1} = \frac{\varepsilon_r}{1}$$

$$F_2 = \varepsilon_r F_1 = 2.25 F_1$$

25. (c)

Given, $\quad \phi = 4x^2 + y^2 + cz^2$

In source free region,

$$\nabla \cdot \vec{D} = 0$$

Also $\quad D = \in E$

So, $\quad \in (\nabla \cdot \vec{E}) = 0$

or, $\quad \nabla \cdot \vec{E} = 0$

Also $\quad \vec{E} = -\nabla V = -\nabla\phi$

$$\nabla V = \frac{\partial\phi}{\partial x}\hat{a}_x + \frac{\partial\phi}{\partial_y}\hat{a}_y + \frac{\partial\phi}{\partial z}\hat{a}_z$$

$$-\vec{E} = 8x\,\hat{a}_x + 2y\,\hat{a}_y + 2cz\,\hat{a}_z$$

Again, $\nabla \cdot \vec{E} = 0$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 8 + 2 + 2c = 0$$

$$c = -5$$

26. (b)

When spheres are brought in contact total charge gets redistributed

$$Q = \frac{Q_1 + Q_2}{2} = \frac{4.5 - 1.5}{2} = 1.5 \text{ nC}$$

After separation of 50 cm between spheres

$$\text{Force, } |F| = \frac{Q_1 \times Q_2}{4\pi \in_0 \times r^2}$$

$$= \frac{(1.5 \times 10^{-9}) \times (1.5 \times 10^{-9})}{4\pi \in_0 \times (50 \times 10^{-2})^2}$$

$$= \frac{2.25 \times 10^{-18}}{4\pi \times 8.854 \times 10^{-12} \times (50 \times 10^{-2})^2} = 80.89 \text{ nN}$$

27. (c)

Given, $\vec{H} = 4y\,\hat{a}_x + (z^2 - x^2)\hat{a}_y + 3y\,\hat{a}_z$ A/m

So,
$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4y & z^2 - x^2 & 3y \end{vmatrix} = \hat{a}_x(3 - 2z) + \hat{a}_z(-2x - 4)$$

At origin (0, 0, 0), $\nabla \times H = 3\hat{a}_x - 4\hat{a}_z$

$$|\nabla \times H| = \sqrt{3^2 + 4^2} = 5$$

28. (d)

We know, $\vec{F} = I(d\vec{l} \times \vec{B})$

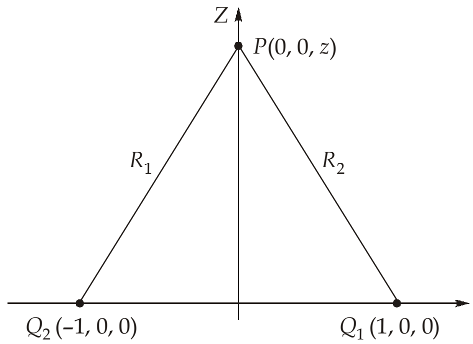
$$= 10(2\hat{a}_z \times 2.5\hat{a}_x) = 50\hat{a}_y \text{ N}$$

$$\vec{T} = \vec{r} \times \vec{F} = (0.4\hat{a}_x) \times (50\hat{a}_y)$$

$$= 20\hat{a}_z \text{ N-m}$$

29. (d)

Consider the layout shown below,



Distance of Q_1 and Q_2 from point P,

$$R = \sqrt{z^2+1}$$

Also, $Q_1 = Q_2$

and $R_1 = R_2 = R$

Potential at P is twice that of due to single charge

$$V = 2\times\frac{Q}{4\pi\epsilon_0 R} = \frac{2\times 3\times 10^{-9}}{4\pi\times\left(\dfrac{10^{-9}}{36\pi}\right)\sqrt{z^2+1}} = \frac{54}{\sqrt{z^2+1}}\text{Volt}$$

$$\frac{dV}{dz} = \frac{d}{dz}\left(\frac{54}{(z^2+1)^{1/2}}\right) = 54\times\left[-\frac{1}{2}(z^2+1)^{-3/2}\right]\times 2z$$

$$\frac{dV}{dz} = \frac{-54z}{(z^2+1)^{3/2}}$$

So, $$\left|\frac{dV}{dz}\right| = \frac{54z}{(z^2+1)^{3/2}}\text{V/m}$$

30. (a)

We know,

Biot Savart's law, $$H = \int\frac{I\,\vec{dl}\times\hat{a}_r}{4\pi R^2} = \int_0^{2\pi}\frac{IR\,d\phi\,\hat{a}_\phi}{4\pi R^2}(-\hat{a}_\rho)$$

$$= \frac{I}{4\pi}\int_0^{2\pi}\frac{R\,d\phi}{R^2}(\hat{a}_z) = \frac{I}{2R}\hat{a}_z$$

■■■■