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# HEAT TRANSFER

## MECHANICAL ENGINEERING

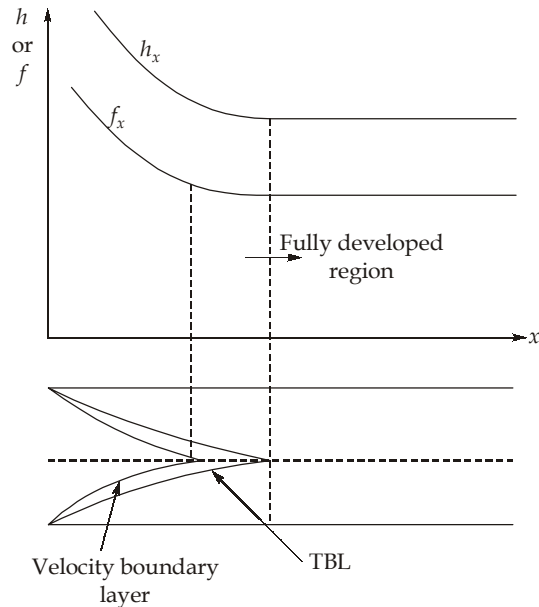
Date of Test : 27/07/2024

### ANSWER KEY >

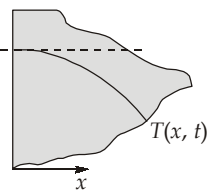
- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (a) | 7. (b)  | 13. (b) | 19. (b) | 25. (b) |
| 2. (c) | 8. (c)  | 14. (b) | 20. (a) | 26. (a) |
| 3. (d) | 9. (b)  | 15. (c) | 21. (b) | 27. (b) |
| 4. (d) | 10. (d) | 16. (b) | 22. (c) | 28. (a) |
| 5. (c) | 11. (c) | 17. (c) | 23. (a) | 29. (c) |
| 6. (b) | 12. (b) | 18. (a) | 24. (b) | 30. (d) |

## DETAILED EXPLANATIONS

1. (a)



2. (c)



As  $\left. \frac{\partial T}{\partial x} \right|_{x=0} = 0$ , the boundary is an adiabatic or insulated surface.

3. (d)

$$\begin{aligned}
 (Q)_{x=1} &= -kA \left( \frac{dT}{dx} \right)_{x=1} \\
 &= -25 \times 0.005 \times (-150 + 20x)_{x=1} \\
 &= 16.25 \text{ W}
 \end{aligned}$$

4. (d)

$$\begin{aligned}
 d &= 5 \text{ mm} \\
 p &= \pi d = \pi \times 0.005 = 0.0157 \text{ m} \\
 T_0 &= 100^\circ\text{C} \\
 h &= 100 \text{ W/m}^2\text{K} \\
 k &= 398 \text{ W/mK} \\
 T_\infty &= 25^\circ\text{C}
 \end{aligned}$$

Assume infinite length of rod

$$Q = \sqrt{hpkA_c} (T_0 - T_\infty)$$

$$= \sqrt{100 \times 0.0157 \times 398 \times \frac{\pi \times (0.005)^2}{4}} (100 - 25)$$

$$= 8.3 \text{ W}$$

5. (c)

In case of natural convection,

$$\text{Nu} = C(\text{GrPr})^m$$

6. (b)

For parallel flow heat exchanger,

$$\varepsilon = \frac{1 - \exp(-NTU(1+C))}{1+C}$$

Put  $C = 0$ ,

$$\varepsilon = \frac{1 - \exp(-NTU)}{1}$$

$$\varepsilon = 1 - \exp(-NTU)$$

For counter flow heat exchanger,

$$\varepsilon = \frac{1 - \exp\{-NTU(1-C)\}}{1 - C \exp\{-NTU(1-C)\}}$$

Put  $C = 0$ ,

$$\varepsilon = \frac{1 - \exp\{-NTU(1-0)\}}{1 - 0 \times \exp\{-NTU(1-0)\}}$$

$$\varepsilon = 1 - \exp(-NTU)$$

So, expression:

$$\varepsilon = 1 - \exp(-NTU)$$

is valid for all the heat exchangers having zero capacity ratio.

7. (b)

Temperature of body,  $T = 2000 \text{ K}$

From Wien's Displacement law:

$$\lambda_m T = 2898 \text{ } \mu\text{m-K}$$

$$\lambda_m \times 2000 = 2898$$

$$\lambda_m = 1.449 \text{ } \mu\text{m}$$

$$\lambda_m \simeq 1.45 \text{ } \mu\text{m}$$

8. (c)

Minimum heat capacity rate,

$$C_{\min} = \dot{m}_c c_{pc} = 3 \times 4.2 = 12.6 \text{ kW/K}$$

Overall heat transfer coefficient,  $U = 1600 \text{ W/m}^2\text{°C}$

Heat transfer area,  $A = 12 \text{ m}^2$

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{1600 \times 12}{12.6 \times 1000} = 1.5238$$

$$\text{Effectiveness, } \varepsilon = 1 - e^{-\text{NTU}} = 1 - e^{-(1.5238)} = 0.782$$

9. (b)

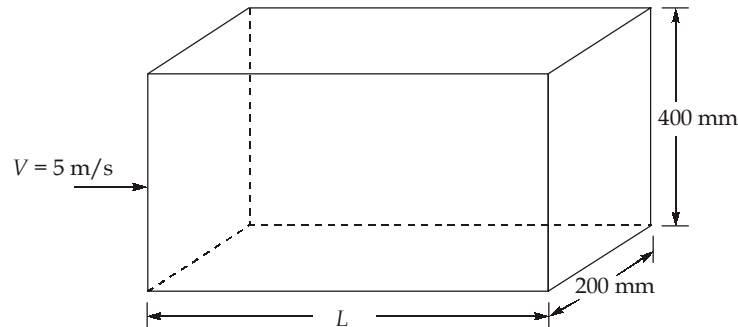
$$F_{12} = 1.0$$

$$A_2 F_{21} = A_1 F_{12}$$

$$F_{21} = \frac{A_1}{A_2} \times 1 = \frac{2R \times L}{\frac{3}{4} \times (2\pi RL)} \times 1.0 = \frac{4}{3\pi} = 0.424$$

10. (d)

11. (c)



$$\text{Hydraulic diameter, } D_h = \frac{4A}{P} = \frac{4 \times 200 \times 400}{2(200 + 400)}$$

$$D_h = 266.667 \text{ mm}$$

$$\text{Reynolds number, } Re = \frac{VD_h}{\nu} = \frac{5 \times 0.266667}{15.06 \times 10^{-6}} = 88.535 \times 10^3 > 2000$$

So, flow is turbulent,

$$\text{Prandtl number, } Pr = \frac{\nu}{\alpha} = \frac{15.06 \times 10^{-6}}{7.71 \times 10^{-2} / 3600} = 0.7032$$

For heating of fluid case,

$$\begin{aligned} Nu &= 0.023(Re)^{0.8}(Pr)^{0.4} \\ &= 0.023(88.535 \times 10^3)^{0.8}(0.7032)^{0.4} \\ Nu &= 181.239 \end{aligned}$$

$$\Rightarrow \frac{h \times D_h}{k} = 181.239$$

$$\frac{h \times 0.266667}{0.026} = 181.239$$

$$\Rightarrow h = 17.671 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

Heat transfer rate per unit length per unit temperature difference,

$$Q = h(PL)(\Delta T)$$

$$\frac{Q}{L\Delta T} = 17.671 \times 2(0.2 + 0.4)$$

$$\frac{Q}{L\Delta T} = 21.205 \text{ W/m}^\circ\text{C}$$

12. (b)

$$q_{\text{net}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1 - \epsilon_1}{\epsilon_1 A_1} + \frac{1 - \epsilon_2}{\epsilon_2 A_2} + \frac{1}{A_1 F_{12}}}$$

Since,

$$F_{12} = 1$$

$$q_{\text{net}} = \frac{5.67 \times 10^{-8} \times [(40 + 273)^4 - (250 + 273)^4]}{\frac{1}{A_2} \left[ \frac{1 - 0.25}{0.25 \times A_1} \times A_2 + \frac{1 - 0.7}{0.7 \times 1} + \frac{A_2}{A_1} \right]}$$

$$\frac{q_{\text{net}}}{A_2} = \frac{-3697.9847}{\frac{0.75}{0.25} \times \frac{\pi D_2^2}{\pi D_1^2} + \frac{0.3}{0.7} + \frac{\pi D_2^2}{\pi D_1^2}} = \frac{-3697.9847}{0.25 \times \left(\frac{1}{0.3}\right)^2 + \frac{0.3}{0.7} + \left(\frac{1}{0.3}\right)^2}$$

$$\frac{q_{\text{net}}}{A_2} = -82.409 \text{ W/m}^2$$

Negative sign shows that there is net heat transfer from sphere 2 to sphere 1.

13. (b)

Effectiveness is lowest when capacity ratio is 1.

So,

$$\epsilon = \frac{1 - \exp\{-NTU(1+C)\}}{1 - \exp\{-NTU(1+1)\}} = \frac{1 - \exp\{-3.5(2)\}}{2} = 0.4995 \approx 0.5$$

14. (b)

Without shield

Radiation heat transfer rate,

$$q = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Let number of shields be  $N$ .

With shield

Radiation heat transfer rate,

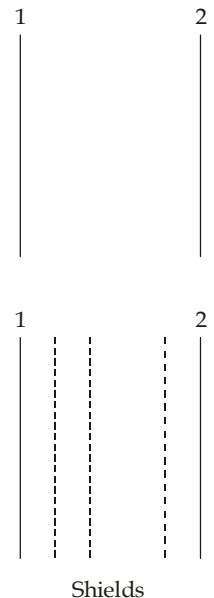
$$q' = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + N\left(\frac{2}{\epsilon} - 1\right)}$$

As per the conditions,

$$q' = (1 - 0.9)q$$

$$\frac{q}{q'} = \frac{1}{0.1} = 10$$

$$\frac{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + N\left(\frac{2}{\epsilon} - 1\right)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)} = 10$$

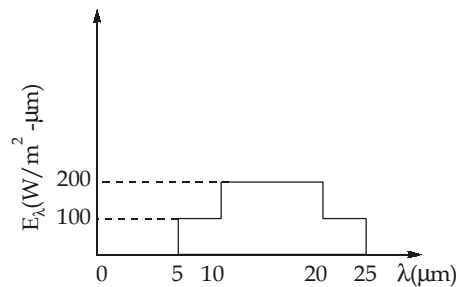


$$\frac{\left(\frac{1}{0.8} + \frac{1}{0.6} - 1\right) + N\left(\frac{2}{0.29} - 1\right)}{\left(\frac{1}{0.8} + \frac{1}{0.6} - 1\right)} = 10$$

Number of shields,  $N = 2.925$

$$N \simeq 3$$

15. (c)  
Refer figure,



$$E = \int_0^{\infty} E_{\lambda} d\lambda = \text{Area under the spectral intensity curve}$$

$$= 100 \times (25 - 5) + 100 \times (20 - 10)$$

$$= 3000 \text{ W/m}^2 \text{ or } 3 \text{ kW/m}^2$$

16. (b)

$$q_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

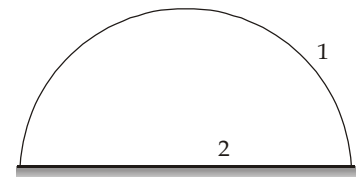
$$A_1 = 2\pi r^2$$

$$F_{12} = F_{21} \frac{A_2}{A_1}$$

$$= 1 \times \frac{\pi r^2}{2\pi r^2} = 0.5$$

$$q_{12} = 2 \times \pi \times (0.5)^2 \times 0.5 \times 5.67 \times 10^{-8} (1100^4 - 330^4)$$

$$= 64.7 \text{ kW} \simeq 65 \text{ kW}$$

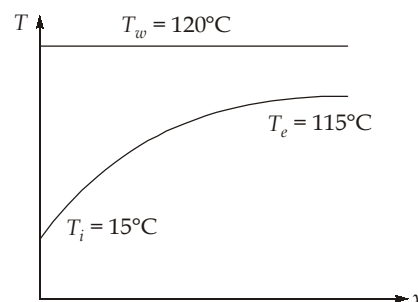


17. (c)

The rate of heat transfer ( $\theta$ ) is calculated as,

$$Q = \dot{m} c_p (T_e - T_i)$$

$$= 0.3 \times 4.187 \times (115 - 15) = 125.61 \text{ kW}$$



$$\Delta T_i = T_w - T_i = 120 - 15 = 105^\circ\text{C}$$

$$\Delta T_e = T_w - T_e = 120 - 115 = 5^\circ\text{C}$$

$$\Delta T_m = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)} = \frac{5 - 105}{\ln\left(\frac{5}{105}\right)} = 32.845^\circ\text{C}$$

$$\begin{aligned} Q &= hA_w \Delta T_m \\ \Rightarrow 125.61 \times 10^3 &= 800 \times \pi \times 2.5 \times 10^{-2} \times L \times 32.845 \\ L &= 60.866 \text{ m} \simeq 61 \text{ m} \end{aligned}$$

18. (a)

$$\begin{aligned} \frac{1}{U} &= \frac{1}{h_i} + R_i + \frac{1}{h_o} + R_o \\ &= \frac{1}{500} + 0.0002 + \frac{1}{2500} + 0.0004 = 0.003 \text{ m}^2\text{C/W} \end{aligned}$$

$$\therefore U = 333.33 \text{ W/m}^2\text{C}$$

19. (b)

$$q'' = h(T_w - T_\infty) = -K \left( \frac{dT}{dy} \right)_{y=0}$$

$$\frac{-dT}{dy} = (T_w - T_\infty) \left[ \frac{a_1}{L} + 2a_2 \frac{y}{L^2} \right]$$

$$\left( \frac{dT}{dy} \right)_{y=0} = -(T_w - T_\infty) \frac{a_1}{L}$$

$$\Rightarrow h(T_w - T_\infty) = K \frac{a_1}{L} (T_w - T_\infty)$$

$$\frac{hL}{K} = a_1$$

$$Nu = \frac{hL}{K} = a_1$$

20. (a)

$$\eta_{\text{fin}} = \left( \frac{\tanh mL}{mL} \right) = 0.7$$

$$\varepsilon_{\text{fin}} = \tanh mL \sqrt{\frac{kP}{hA}} = \frac{\tanh mL}{m} \times \frac{P}{A}$$

$$\frac{\varepsilon_{\text{fin}}}{\eta_{\text{fin}}} = \frac{L \times P}{A}$$

$$\text{Effectiveness} = \frac{3 \times \pi \times 0.6 \times 0.7}{\frac{\pi}{4} (0.6)^2} = 14$$

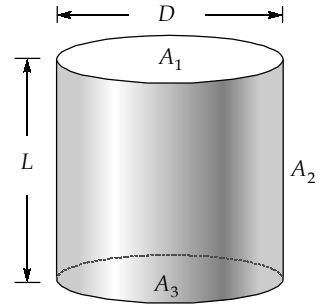
21. (b)

From summation rule,

$$\begin{aligned} F_{11} + F_{12} + F_{13} &= 1 \\ F_{12} &= 1 - F_{13} \\ &= 1 - 3 + 2\sqrt{2} \\ &= 2(-1 + \sqrt{2}) \end{aligned}$$

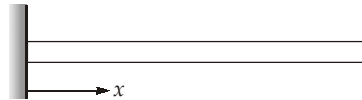
From reciprocal theorem,

$$\begin{aligned} A_2 F_{21} &= A_1 F_{12} \\ \pi D \times L \times F_{21} &= \frac{\pi D^2}{4} \times 2(-1 + \sqrt{2}) \\ F_{21} &= \frac{\sqrt{2} - 1}{2} \end{aligned}$$



22. (c)

$$T(x) - T_\infty = e^{-mx} (T_b - T_\infty)$$



Heat transfer through fin,  $q_{x=0}$

$$\begin{aligned} &= -kA \left. \frac{dT}{dx} \right|_{x=0} = -kA(-m)(T_b - T_\infty)e^{-mx} \Big|_{x=0} \\ &= kA \sqrt{\frac{hP}{kA}} (T_b - T_\infty) = \sqrt{hPkA} (T_b - T_\infty) \end{aligned}$$

23. (a)

Given:

$$\begin{aligned} \alpha &= \epsilon = 0.8 \text{ (for gray surface)} \\ T &= 150^\circ\text{C} = 150 + 273 = 423 \text{ K} \\ G &= 1200 \text{ W/m}^2 \end{aligned}$$

For opaque body,

$$\begin{aligned} \tau &= 0, & \alpha + \rho &= 1 \\ \rho &= 1 - \alpha = 1 - 0.8 = 0.2 \end{aligned}$$

$$\begin{aligned} \text{Radiosity, } J &= E + \rho G = \epsilon E_b + \rho G = \epsilon \sigma T^4 + \rho G \\ &= 0.8 \times 5.67 \times \left( \frac{423}{100} \right)^4 + 0.2 \times 1200 \\ &= 1692.227 \text{ W/m}^2 \end{aligned}$$

24. (b)

Given:

The heat release is uniform along the rod. Maximum temperature occurs at the centre is given by,

$$\begin{aligned} T_{\max} &= T_s + \frac{\dot{q}R^2}{4k} & (T_s = \text{Surface temperature}) \\ T_{\max} - T_s &= \frac{\dot{q}R^2}{4k} & \dots \text{ (i)} \end{aligned}$$

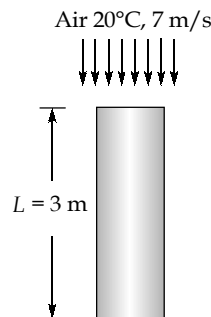


$$\begin{aligned} \therefore \dot{q} \text{ (Heat generated/m}^3\text{)} &= \frac{0.25 \times 10^6}{\pi R^2 L} = \frac{0.25 \times 10^6}{\pi R^2 \times 6} \\ &= \frac{13262.91}{R^2} \text{ W/m}^3 \end{aligned}$$

Now from equation (i)

$$\begin{aligned} T_{\max} - T_s &= \frac{13262.91}{R^2} \times \frac{R^2}{4 \times 30} \\ &= 110.52^\circ\text{C} \end{aligned}$$

25. (b)



The flow is along 3 m side of the plate, and thus the characteristic length is  $L = 3$  m. Both sides are exposed to air flow,

$$\begin{aligned} A &= 2 \times w \times L \\ &= 2 \times 2 \times 3 = 12 \text{ m}^2 \end{aligned}$$

For flat plates, drag force is equivalent to friction force.

$$\begin{aligned} F_f &= C_f A_s \frac{\rho V^2}{2} \\ C_f &= \frac{F_f}{\frac{1}{2} \rho A_s V^2} = \frac{0.86}{1.204 \times 12 \times \frac{1}{2} \times (7)^2} = 0.00243 \end{aligned}$$

From Reynolds Analogy,

$$\begin{aligned} St \times (\text{Pr})^{2/3} &= \frac{C_f}{2} = \frac{0.00243}{2} \\ St &= \frac{h}{\rho V c_p} \\ h &= 0.00149 \times 1.204 \times 7 \times 1007 \\ &= 12.64 \text{ W/m}^2\text{K} \end{aligned}$$

26. (a)

$$\text{Reynolds, number, Re} = \frac{\rho V L}{\mu} = \frac{996.6 \times 0.2 \times 1}{0.854 \times 10^{-3}} = 2.33 \times 10^5 \quad (< 5 \times 10^5, \text{ So laminar flow})$$

$$\text{Nusselt number, Nu} = 0.664 \text{ Re}^{1/2} \text{ Pr}^{1/3}$$

$$\frac{hL}{k} = 0.664 (2.33 \times 10^5)^{1/2} \times (5.85)^{1/3}$$

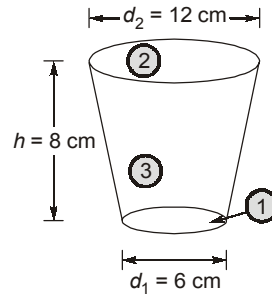
$$= 577.516$$

$$h = 577.516 \times 0.61 = 352.28 \text{ W/m}^2\text{K}$$

Heat transfer,

$$Q = hA\Delta T = 352.28 \times (1 \times 1) \times (40 - 10) \\ = 10568.4 \text{ W} = 10.56 \text{ kW}$$

27. (b)



Given:

$$F_{21} = 0.2$$

$$\therefore F_{21} + F_{22} + F_{23} = 1 \quad (F_{22} = 0)$$

$$F_{21} + F_{23} = 1$$

$$F_{23} = 1 - F_{21} = 1 - 0.2 = 0.8$$

28. (a)

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh mL}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{250 \times 0.11}{20 \times 6 \times 10^{-4}}}$$

$$m = 47.87 \text{ per meter}$$

$$mL = 47.87 \times 0.05 = 2.39$$

$$\dot{Q} = \sqrt{hPkA}\theta_0 \tanh mL$$

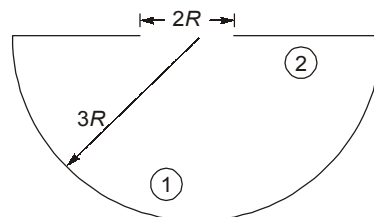
$$= \sqrt{250 \times 0.11 \times 20 \times 6 \times 10^{-4}} \times [1200 - 300] \times \tanh(2.39)$$

$$Q = 517 \times 0.983$$

$$Q = 508 \text{ W}$$

29. (c)

$F_{12}$  needs to be found;

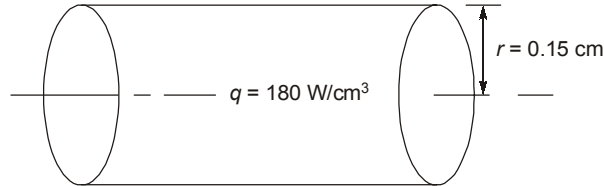


$$A_1 = 2\pi \times (3R)^2 = 18\pi R^2$$

$$A_2 = \pi \times (3R^2) - \pi R^2 = 8\pi R^2$$

$$\begin{aligned} F_{21} + F_{22} &= 1 \\ \Rightarrow F_{21} &= 1 \\ A_1 F_{12} &= A_2 F_{21} \\ \Rightarrow 18\pi R^2 F_{12} &= 8\pi R^2 \times 1 \\ \Rightarrow F_{12} &= 0.44 \end{aligned}$$

30. (d)



Applying energy conservation

Heat generated in the cylinder = Heat conduction at  $x = r$

$$q \times \text{volume} = Q = -kA \frac{dT}{dx}$$

$$Q = \left( 180 \times \frac{\pi}{4} \times 0.3^2 \times L \right) \text{ W}$$

$$\text{Heat flux} = \frac{Q}{A} = \frac{180 \times \frac{\pi}{4} \times 0.3^2 \times L}{\pi \times 0.3 \times L} = 13.5 \text{ W/cm}^2$$

