CLASS TEST

S.No.: 02SK_ECEE_HIJKLM_19824

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# CONTROL SYSTEM

## EC-EE

**Date of Test : 19/08/2024**

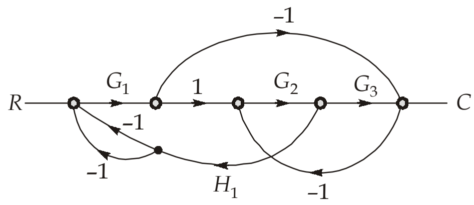
## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (b) | 7. | (b) | 13. | (b) | 19. | (b) | 25. | (d) |
| 2. | (b) | 8. | (b) | 14. | (a) | 20. | (a) | 26. | (b) |
| 3. | (a) | 9. | (a) | 15. | (d) | 21. | (a) | 27. | (b) |
| 4. | (d) | 10. | (d) | 16. | (d) | 22. | (a) | 28. | (d) |
| 5. | (c) | 11. | (d) | 17. | (b) | 23. | (a) | 29. | (a) |
| 6. | (b) | 12. | (d) | 18. | (c) | 24. | (c) | 30. | (b) |

## DETAILED EXPLANATIONS

**1. (b)**

The signal flow graph for the given block diagram can be drawn as




As per signal flow graph, the loops are

$$L_1 = G_2G_3(-1) = -G_2G_3$$
$$L_2 = G_1G_2H_1(-1) = -G_1G_2H_1$$
$$L_3 = G_1G_2H_1(-1) = -G_1G_2H_1$$
$$L_4 = G_1(-1)(-1)G_2H_1(-1) = -G_1G_2H_1$$
$$L_5 = G_1(-1)(-1)G_2H_1(-1) = -G_1G_2H_1$$

$\therefore$ $$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$$
$$= 1 - (-G_2G_3 - G_1G_2H_1 - G_1G_2H_1 - G_1G_2H_1 - G_1G_2H_1)$$
$$= 1 + G_2G_3 + 2G_1G_2H_1 + 2G_1G_2H_1$$
$$= 1 + G_2G_3 + 4G_1G_2H_1$$

and $$P_1 = G_1G_2G_3$$
$$\Delta_1 = 1$$
$$P_2 = G_1(-1) = -G_1$$
$$\Delta_2 = 1$$

$\therefore$ $$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1(G_2G_3 - 1)}{1 + G_2G_3 + 4G_1G_2H_1}$$

**2. (b)**

The characteristic equation of the given system is,

$$1 + G(s)H(s) = 1 + \frac{K}{4s^3 + 2s^2 + 3s} = 0$$

or $$4s^3 + 2s^2 + 3s + K = 0$$

Using Routh's tabular form,

| | | |
|---|---|---|
| $s^3$ | 4 | 3 |
| $s^2$ | 2 | $K$ |
| $s^1$ | $\frac{6-4K}{2}$ | |
| $s^0$ | $K$ | |

For stability,

$$K > 0$$

and $$\frac{6-4K}{2} > 0$$

or $$4K < 6$$
$$K < \frac{3}{2}$$

So, the required condition is $0 < K < \frac{3}{2}$.

**3. (a)**

The closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{12}{s(s+5)+12} = \frac{12}{s^2+5s+12}$$

Here, on comparing with standard second order transfer function, we get,

$$\omega_n = \sqrt{12}$$

and $$\xi\omega_n = \frac{5}{2}$$

For 2% tolerance,

$$\tau_s = \frac{4}{\xi\omega_n} = \frac{4}{5/2} = \frac{8}{5} = 1.6 \text{ sec}$$

**4. (d)**

$$G(s)H(s) = \frac{K(s^2+2s+10)}{(s^2+6s+10)}$$

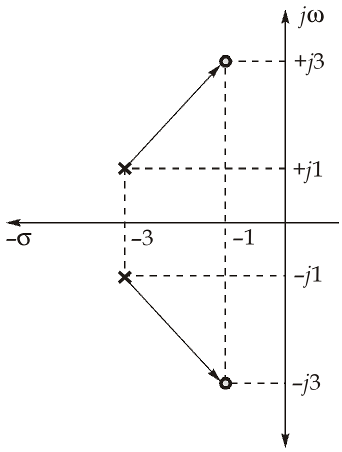
For, $s^2 + 6s + 10 = 0$

$$s = \frac{-6 \pm \sqrt{36-40}}{2} = -3 \pm j$$

For, $s^2 + 2s + 10 = 0$

$$s = \frac{-2 \pm \sqrt{4-40}}{2} = -1 \pm j3$$

$\therefore$ $$G(s)H(s) = \frac{K(s+1+j3)(s+1-j3)}{(s+3+j)(s+3-j)}$$




$\therefore$ There will be no break points.

**5. (c)**

$$c(t) = 1 + 0.5e^{-4t} - 1.5e^{-8t}$$

$$C(s) = \frac{1}{s} + \frac{0.5}{(s+4)} - \frac{1.5}{(s+8)}$$

$$= \frac{(s+4)(s+8) + 0.5(s)(s+8) - 1.5(s)(s+4)}{s(s+4)(s+8)}$$

$$= \frac{s^2+12s+32+0.5s^2+4s-1.5s^2-6s}{s(s+4)(s+8)}$$

$$= \frac{10s+32}{s(s+4)(s+8)}$$

$$= \frac{\frac{(10s+32)}{s}}{(s+4)(s+8)}$$

For unit step input, $$\frac{C(s)}{R(s)} = \frac{10s+32}{s^2+12s+32}$$

Comparing with standard transfer function, we get,

$$\omega_n = \sqrt{32}$$

and $$\xi\omega_n = \frac{12}{2} = 6$$

$$\therefore \quad \xi = \frac{6}{\sqrt{32}} = 1.06$$

**6. (b)**

The standard form of transfer function of the compensator is

$$G_c(s) = \alpha \frac{1+sT}{1+\alpha sT}$$

∴ In time constant form,

$$G_c(s) = \frac{1(1+100s)}{10(1+10s)}$$

Here, T = 100 and $\alpha T = 10$

or $$\alpha = \frac{10}{100} = 0.1$$

∵ $\alpha < 1$ ∴ Lead compensator.

**7. (b)**

The characteristic equation is given by

$$|sI - A| = 0$$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}$$

$$\therefore \quad |sI - A| = \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix} = s^2 + 1$$

$$s = \pm j$$

As the roots are purely imaginary, thus the system has undamped response.

**8. (b)**

For ω = 0, the plot starts at 0°, that means there will be no pole at origin, hence the type of the system is 0.

For ω = ∞, the plot terminates at 270° i.e., the order of the system should be 3.

**9. (a)**

As the initial slope is 0 dB/dec.

$$20\log K = -20$$

$$\log K = -1$$

or $$K = 0.1$$

at ω = 1 rad/sec, the slope changes to 20 dB/dec, thereby adds a zero at ω = 1 rad/sec

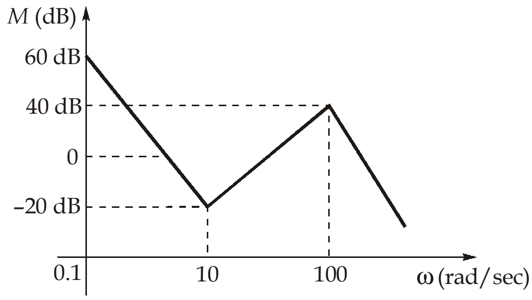
at ω = 10 rad/sec, the slope changes to 0 dB/dec thereby adds a pole at ω = 10 rad/sec.

at ω = 100 rad/sec, the slope changes to –40 dB/dec thereby adds two poles at ω = 100 rad/sec

∴ Resultant transfer function is,

$$T(s) = \frac{0.1\left(\frac{s}{1}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)^2} = \frac{10^4(s+1)}{(s+10)(s+100)^2}$$

**10. (d)**




$$\text{Slope} = \frac{\text{Amount of rise}}{\text{Duration}} = \frac{40\text{ dB} - (-20\text{ dB})}{\log 100 - \log 10}$$

$$= \frac{60\text{ dB}}{\log\left(\frac{100}{10}\right)} = \frac{60\text{ dB}}{1} = 60\text{ dB/dec}$$

**11. (d)**

The location of the poles are given by, $-\xi\omega_n \pm j\omega_d$ ...(i)

where, $\xi$ = damping ratio

$\omega_n$ = natural frequency of oscillation

$\omega_d$ = damped frequency of oscillation

Using maximum peak overshoot, the value of $\xi$ can be obtained as

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.15$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = 0.604$$

Squaring both the sides,

$$\xi^2 = 0.364(1 - \xi^2)$$

or $$\xi^2 = \frac{0.364}{1.364} = 0.267$$

or $$\xi = 0.517 \quad \text{...(ii)}$$

now, peak time, $$\tau_p = \frac{\pi}{\omega_d} = 3$$

or $$\omega_d = \frac{\pi}{3} = 1.047\text{ rad/sec}$$

$\because$ $$\omega_d = \omega_n\sqrt{1-\xi^2} \quad \text{...(iii)}$$

∴ From equation (ii) and (iii), we have

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{1.047}{\sqrt{1-0.517^2}}$$

$$\omega_n = 1.223 \text{ rad/sec} \qquad \text{...(iv)}$$

∴ Location of poles are,

$$P = -\xi\omega_n \pm j\omega_d$$
$$= -(0.517 \times 1.223) \pm j1.047$$
$$= -0.632 \pm j1.047$$

**12. (d)**

Steady state error,

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s \times \frac{R(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{s \times \left(2+\frac{5}{s}\right) \times \frac{1}{s}}{1+\frac{K}{s(s+3)}}$$

$$= \lim_{s\to 0} \frac{\frac{s(2s+5)}{s}}{s[s(s+3)+K]}$$

$$= \lim_{s\to 0} \frac{\frac{(2s+5)}{s} \times s(s+3)}{(s^2+3s+K)}$$

$$= \lim_{s\to 0} \frac{(2s+5)(s+3)}{s^2+3s+K}$$

$$2.75 = \frac{15}{K}$$

or $$K = \frac{15}{2.75} = 5.45$$

**13. (b)**

The steady state error is defined by

$$e_{ss} = \lim_{s\to 0} \frac{s \times \frac{1}{s^2}}{1+\frac{(s+\alpha)}{s} \times \frac{(s+2)}{s^2-1}}$$

$$= \lim_{s\to 0} \frac{(s^2-1)}{s(s^2-1)+(s+\alpha)(s+2)}$$

$$e_{ss} = -\frac{1}{2\alpha}$$

$$\therefore \qquad S_{\alpha}^{e_{ss}} = \frac{\dfrac{\partial e_{ss}}{e_{ss}}}{\dfrac{\partial \alpha}{\alpha}} = \frac{\partial e_{ss}}{\partial \alpha} \times \frac{\alpha}{e_{ss}} = \frac{\partial}{\partial \alpha}\left(\frac{-1}{2\alpha}\right) \times \frac{\alpha}{-\dfrac{1}{2\alpha}}$$

$$= -\frac{\alpha^2}{\alpha^2} = -1$$

**14. (a)**

The characteristic equation is,

$$1 + G(s)H(s) = 0$$

$$s^2(s+a) + K\left(s + \frac{4}{3}\right) = 0$$

For $K = 6$

$$s^3 + as^2 + 6s + 8 = 0$$

Using Routh's method for 3rd order system for stability,

$$6a \geq 8$$

$$a \geq \frac{8}{6}$$

$$a \geq \frac{4}{3}$$

$\therefore$ For system to be unstable $a < \frac{4}{3} = 1.33$

**15. (d)**

The characteristic equation is,

$$1 + G(s) = 0$$

$$s(s+1)(s+2)(s+4) + K = 0$$

$$s^4 + 7s^3 + 14s^2 + 8s + K = 0$$

Using Routh's tabular form, we have

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 14 | $K$ |
| $s^3$ | 7 | 8 | |
| $s^2$ | 12.86 | $K$ | |
| $s^1$ | $\frac{102.88 - 7K}{12.86}$ | | |
| $s^0$ | $K$ | | |

For system to be oscillatory

$$\frac{102.8 - 7K}{12.86} = 0 \qquad \Rightarrow K \approx 14.697$$

$\therefore$ Auxiliary equation,

$$12.86s^2 + K = 0$$

$$s^2 = \frac{-K}{12.86} = -\frac{14.697}{12.86} = -1.142$$

or $\qquad s = \pm j1.07$

$\therefore \qquad \omega = 1.07$ rad/sec

**16. (d)**

$$\frac{V_0(s)}{V_s(s)} = \frac{R_2}{\frac{R_1}{R_1Cs+1}+R_2} = \frac{R_2(R_1Cs+1)}{R_1+R_2+R_1R_2Cs}$$

By rearranging, we get,

$$\frac{V_0(s)}{V_s(s)} = \frac{R_2}{R_1+R_2}\times\left(\frac{1+R_1Cs}{1+\frac{R_2}{R_1+R_2}\cdot R_1Cs}\right)$$

$\therefore$
$$\alpha = \frac{R_2}{R_1+R_2}$$

**17. (b)**

The phase margin is given by,

$$\text{PM} = 180° + \phi$$

$$60° = 180° + \left[-90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}\right]$$

$$-30° = -\tan^{-1}\left(\frac{\frac{\omega}{1}+\frac{\omega}{3}}{1-\frac{\omega^2}{3}}\right)$$

$$\tan 30° = \frac{\omega+\frac{\omega}{3}}{1-\frac{\omega^2}{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{\frac{4\omega}{3}}{1-\frac{\omega^2}{3}}$$

or
$$1-\frac{\omega^2}{3} = \frac{4\omega}{\sqrt{3}}$$

$\Rightarrow$
$$\omega^2+4\sqrt{3}\,\omega-3 = 0$$

On solving the above equation, we get,

$$\omega = 0.408 \text{ rad/sec and } -7.33 \text{ rad/sec}$$

Considering positive value of frequency for $\omega = \omega_{gc}$, we have,

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1$$

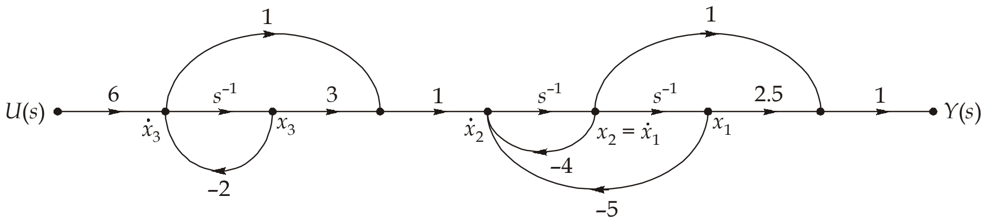
$$\frac{K}{\omega\sqrt{\omega^2+1}\sqrt{\omega^2+9}} = 1$$

$$K = 0.408\sqrt{1.166}\times\sqrt{9.166}$$

$$K = 1.33$$

**18. (c)**

The state diagram of the system shown in below figure is the cascade of the state diagram of two sub-systems.




From this state diagram, the input output equations can be written as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -5x_1 - 4x_2 + 3x_3 + 6u$$

$$\dot{x}_3 = -2x_3 + 6u$$

$$y = 2.5x_1 + x_2$$

$$\therefore \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -4 & 3 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix} u$$

$$y = [2.5 \quad 1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

**19. (b)**

For crossing the ±180° line,

$$\pm 180° = \tan^{-1}(4\omega) - 90° - \tan^{-1}(2\omega) - \tan^{-1}(P\omega)$$

or

$$90° = \tan^{-1}(4\omega) - \tan^{-1}\left(\frac{2\omega + P\omega}{1 - 2P\omega^2}\right)$$

$$= \tan^{-1}\left[\frac{4\omega - \dfrac{2\omega + P\omega}{1 - 2P\omega^2}}{1 + \dfrac{4\omega(2\omega + P\omega)}{1 - 2P\omega^2}}\right]$$

$$1 - 2P\omega^2 + 8\omega^2 + 4P\omega^2 = 0$$

$$2P\omega^2 + 8\omega^2 + 1 = 0$$

$$(2P + 8)\omega^2 = -1$$

$$\omega^2 = -\frac{1}{(2P + 8)}$$

$$\frac{1}{4 \times 3} = \frac{-1}{2P + 8}$$

$$2P + 8 = -12$$

$$2P = -20$$

$$P = -10$$

**20. (a)**

Given $M_r = 4.2$

$\because$ $$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$\therefore$ $$4.2 = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

or $$2\xi\sqrt{1-\xi^2} = \frac{1}{4.2} = 0.238$$

or $$\xi^2(1-\xi^2) = \left(\frac{0.238}{2}\right)^2 = 0.0141$$

$$\xi^4 - \xi^2 + 0.0141 = 0$$

On solving, we get,

$$\xi^2 = 0.985,\ 0.0143$$

or $$\xi = 0.992,\ 0.119$$

$\therefore$ The valid answer is 0.119.

**21. (a)**

From the given state model, the transfer function can be calculated as

$$T(s) = C(sI - A)^{-1} B$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T \begin{bmatrix} s & -1 \\ 2 & s+5 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= [1 \quad 2] \times \frac{1}{s(s+5)+2} \begin{bmatrix} s+5 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= [1 \quad 2] \times \frac{1}{s(s+5)+2} \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$T(s) = \frac{1+2s}{s^2+5s+2}$$

$\therefore$ Open loop transfer function for unity negative feedback system becomes

$$G(s) = \frac{1+2s}{s^2+5s+2-2s-1} = \frac{1+2s}{s^2+3s+1}$$

For unit step input,

$$K_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{1+2s}{s^2+3s+1} = 1$$

$\therefore$ $$e_{ss} = \frac{1}{1+K_p} = \frac{1}{2} = 0.5$$

**22. (a)**

$$T(s) = \frac{C(s)}{R(s)} = \frac{2\alpha}{s(s+1)+2\alpha}$$

$$\therefore \quad S_{\alpha}^{T} = \frac{\partial T/T}{\partial \alpha/\alpha} = \frac{\partial T}{\partial \alpha}\times\frac{\alpha}{T} = \frac{\partial}{\partial \alpha}\left(\frac{2\alpha}{s(s+1)+2\alpha}\right)\times\frac{\alpha}{T}$$

$$= \frac{(s^2+s+2\alpha)2-2\alpha(2)}{(s^2+s+2\alpha)^2}\times\frac{\alpha}{T} = \frac{2s^2+2s+4\alpha-4\alpha}{(s^2+s+2\alpha)^2}\times\frac{\alpha(s^2+s+2\alpha)}{2\alpha}$$

$$= \frac{2s(s+1)}{(s^2+s+2\alpha)}\times\frac{1}{2}$$

$$= \frac{s(s+1)}{s(s+1)+2\alpha} = \frac{1}{1+\dfrac{2\alpha}{s(s+1)}}$$

**23. (a)**

For gain margin of 10 dB,

$$GM = 20\log\frac{1}{|G|_{\omega=\omega_{pc1}}}$$

$$\therefore \quad \frac{1}{|G|_{\omega pc1}} = 10^{\left(\frac{10}{20}\right)} = 3.167$$

$$|G|_{\omega pc_1} = 0.315$$

For new gain margin of 18 dB, the gain at $\omega_{pc2}$ is

$$18 = 20\log\frac{1}{|G|_{\omega pc2}}$$

$$\log\frac{1}{|G|_{\omega pc2}} = \frac{18}{20} = 0.9$$

$$\frac{1}{|G|_{\omega pc2}} = 7.943$$

$$\therefore \quad |G|_{\omega pc_2} = 0.126$$

$\therefore$ The gain must be multiplied by a factor of $\dfrac{0.126}{0.315} = 0.4$

**24. (c)**

State transition matrix $= e^{At} = L^{-1}(sI - A)^{-1}$

$$\therefore \quad L^{-1}\begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}^{-1} = L^{-1}\left[\frac{Adj(sI-A)}{|sI-A|}\right]$$

$$= L^{-1}\left[\frac{\begin{matrix} s+2 & 1 \\ -1 & s \end{matrix}}{s(s+2)+1}\right]$$

$$= L^{-1}\begin{bmatrix} \frac{s+2}{s^2+2s+1} & \frac{1}{s^2+2s+1} \\ \frac{-1}{s^2+2s+1} & \frac{s}{s^2+2s+1} \end{bmatrix}$$

$$= L^{-1}\begin{bmatrix} \frac{s+2}{(s+1)^2} & \frac{1}{(s+1)^2} \\ \frac{-1}{(s+1)^2} & \frac{s}{(s+1)^2} \end{bmatrix}$$

$$= L^{-1}\begin{bmatrix} \frac{s}{(s+1)^2}+\frac{2}{(s+1)^2} & \frac{1}{(s+1)^2} \\ \frac{-1}{(s+1)^2} & \frac{s}{(s+1)^2} \end{bmatrix}$$

$$= \begin{bmatrix} (1+t)e^{-t} & te^{-t} \\ -te^{-t} & (1-t)e^{-t} \end{bmatrix}$$

**25. (d)**

The state model form is

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}u$$

$$Y = \begin{bmatrix} 3 & -2 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$\therefore$ Characteristic equation becomes,

$$[sI - A] = 0$$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 0 & -4 \end{bmatrix} = 0$$

$$\begin{bmatrix} s+2 & 1 \\ 0 & s+4 \end{bmatrix} = 0$$

$$(s + 2)(s + 4) = 0$$

$$s^2 + 6s + 8 = 0$$

On comparing it with standard second order system CE we get

$$\omega_n^2 = 8$$

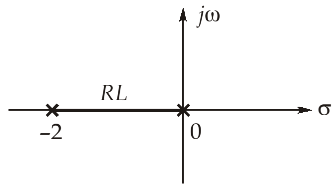
$$2\xi\omega_n = 6$$

$$\xi\omega_n = 3$$

$$\xi = \frac{3}{\sqrt{8}} = 1.06 \quad \text{(overdamped)}$$

**26.** **(b)**

The root locus can be plotted as




$$G(s)H(s) = \frac{K}{s(s+2)(s+1-j)(s+1+j)}$$

$\therefore \quad n = 4$ and $m = 0$

$$\text{Asymptotes} = \theta_q = \frac{(2q+1)\pi}{P-Z} = \frac{(2q+1)\pi}{n-m}$$

$$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ and } \frac{7\pi}{4}$$

$$\text{Centroid} = \sigma = \frac{\Sigma P - \Sigma Z}{n-m} = \frac{-2-1-1-0}{4} = -1$$

$$\text{break point} = \frac{dK}{ds} = 0$$

$$K = -(s)(s+2)(s^2+2s+2)$$

$$\frac{dK}{ds} = \frac{d}{ds}\left[s^4 + 4s^3 + 6s^2 + 4s\right] = 0$$

or $\quad s^3 + 3s^2 + 3s + 1 = 0$

$$(s+1)^3 = 0$$

$\therefore$ actual break points are –1, –1 and –1

$\therefore$ break angle at $s = -1$

is $\quad \pm\frac{\pi}{r} = \pm\frac{180°}{4} = \pm 45°$

The intersection with imaginary axis can be obtained by RH criteria.

as $\quad s^4 + 4s^3 + 6s^2 + 4s + K = 0$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 6 | $K$ |
| $s^3$ | 4 | 4 | 0 |
| $s^2$ | 5 | $K$ | |
| $s^1$ | $\frac{20-4K}{5}$ | 0 | |
| $s^0$ | $K$ | | |

For intersection with $j\omega$ axis,

$$K\text{ mar} = 5$$

$\therefore \quad$ auxiliary equation $= 5s^2 + K = 0$

$$5s^2 + 5 = 0$$

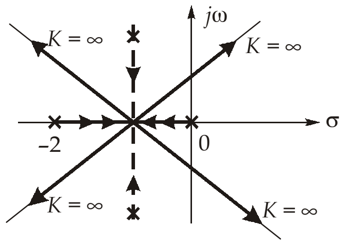
$$s = \pm j1$$

Angle of departure for complex pole $(-1 + j)$ is

$$\theta_d = 180° - (90° + 180° + \phi - \phi)$$

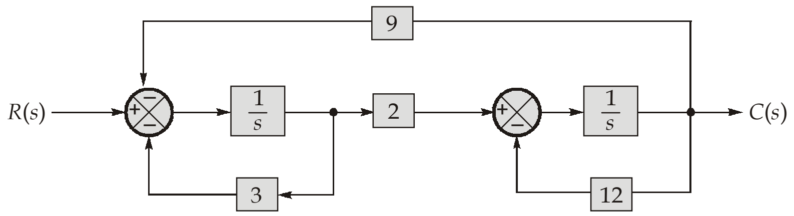
$$= -90°$$

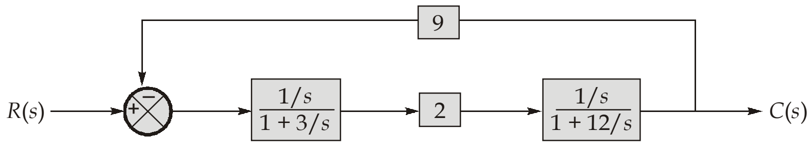
∴ Complete root locus can be plotted as

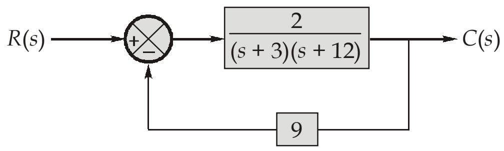



**27.** **(b)**

The block diagram can be solved as




$$\therefore \qquad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\dfrac{2}{(s+3)(s+12)}}{1+\dfrac{2\times 9}{(s+3)(s+12)}} = \frac{2}{s^2+15s+54}$$

On comparing it with standard second order transfer function, we have

$$\omega_n^2 = 54$$

$$2\xi\omega_n = 15$$

or $$\xi = \frac{15}{2\sqrt{54}} = \frac{15}{14.6969} = 1.02 \text{ (overdamped)}$$

**28. (d)**

The characteristic equation is

$$|sI - A| = 0$$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 0 & -K \end{bmatrix} = 0$$

$$\begin{vmatrix} s+2 & -1 \\ 0 & s+K \end{vmatrix} = 0$$

$$(s + 2)(s + K) = 0$$

$$s^2 + (2 + K)s + 2K = 0 \quad \text{...(i)}$$

For having 12% of maximum peak overshoot, the value of $\xi$ can be determined as

$$0.12 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = 0.6749$$

$$0.455(1 - \xi^2) = \xi^2$$

or $$\xi^2 = \frac{0.455}{1+0.455} = 0.3129$$

or $$\xi = 0.559 \simeq 0.56$$

$\because$ $$\tau_s = \frac{4}{\xi\omega_n}$$

$$\xi\omega_n = \frac{4}{\tau_s} = \frac{4}{0.5} = 8$$

$\therefore$ $$\omega_n = \frac{8}{0.56} = 14.30$$

from equation (i)

$$\omega_n = \sqrt{2K}$$

$\therefore$ $$2K = \omega_n^2 = (14.30)^2$$

$$K = 102.25$$

**29. (a)**

The characteristic equation is,

$$1 + \frac{K}{s(s+3)(s^2+s+1)} = 0$$

$$s(s + 3)(s^2 + s + 1) + K = 0$$

$$(s^2 + 3s)(s^2 + s + 1) + K = 0$$

$$s^4 + 4s^3 + 4s^2 + 3s + K = 0$$

Using Routh's tabular form, we get

$$\begin{array}{c|ccc} s^4 & 1 & 4 & K \\ s^3 & 4 & 3 & 0 \\ s^2 & \frac{13}{4} & K & \\ s^1 & \left(\frac{39}{4} - 4K\right)\Big/\frac{13}{4} & & \\ s^0 & K & & \end{array}$$

In order to sustain oscillation

$$\frac{\frac{39}{4} - 4K}{\frac{13}{4}} = 0$$

$$\Rightarrow \qquad K = \frac{39}{16}$$

∴ The subsidiary equation of the third row becomes

$$\frac{13}{4}s^2 + \frac{39}{16} = 0$$

$$s = \pm 0.866j$$

∴ Oscillation frequency is 0.866 rad/sec.

**30. (b)**

$$\phi(t) = L^{-1}[(sI - A)^{-1}]$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & -1 \\ 8 & s+6 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 + 6s + 8}\begin{bmatrix} s+6 & 1 \\ -8 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+6}{(s+2)(s+4)} & \frac{1}{(s+2)(s+4)} \\ \frac{-8}{(s+2)(s+4)} & \frac{s}{(s+2)(s+4)} \end{bmatrix}$$

$$\phi(t) = L^{-1}[(sI - A)^{-1}] = \begin{bmatrix} (2e^{-2t} - e^{-4t}) & \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t}\right) \\ (-4e^{-2t} + 4e^{-4t}) & (-e^{-2t} + 2e^{-4t}) \end{bmatrix}$$

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