CLASS TEST

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 28/08/2024

ANSWER KEY

1.	(c)	7.	(c)	13.	(c)	19.	(b)	25.	(b)
2.	(b)	8.	(b)	14.	(b)	20.	(d)	26.	(b)
3.	(c)	9.	(b)	15.	(a)	21.	(a)	27.	(b)
4.	(b)	10.	(a)	16.	(d)	22.	(a)	28.	(d)
5.	(b)	11.	(c)	17.	(d)	23.	(a)	29.	(a)
6.	(a)	12.	(b)	18.	(d)	24.	(d)	30.	(b)

DETAILED EXPLANATIONS

1. (c)

Given: $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$A^2 = I$

By Cayley Hamilton theorem

$\lambda^2 = 1$

$\Rightarrow \quad \lambda = \pm 1$ are eigen values

$|A| = -1$

$-\alpha^2 - \beta\gamma = -1$

$1 - \alpha^2 - \beta\gamma = 0$

Alternative:

Given: $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$\therefore \quad A^2 = A.A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix}$

Given that $A^2 = I$

$$\begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore \quad \alpha^2 + \beta\gamma = 1$

$1 - \alpha^2 - \beta\gamma = 0$

2. (b)

Given, $4x_4 + 13x_5 = 46$ ...(1)

$2x_1 + 5x_2 + 5x_3 + 2x_4 + 10x_5 = 161$ ...(2)

$2x_3 + 5x_4 + 3x_5 = 61$ ...(3)

$4x_4 + 5x_5 = 30$ ...(4)

$2x_1 + 3x_2 + 2x_3 + 1x_4 + 5x_5 = 81$ ...(5)

Solving (1) and (4) $x_5 = 2$

$x_4 = 5$

Putting in (3) we get

$2x_3 + 25 + 6 = 61$

$x_3 = 15$

Alternative:

The matrix form of the equation is

$$[A|B] = \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 4 & 13 & 46 \\ 2 & 5 & 5 & 2 & 10 & 161 \\ 0 & 0 & 2 & 5 & 3 & 61 \\ 0 & 0 & 0 & 4 & 5 & 30 \\ 2 & 3 & 2 & 1 & 5 & 81 \end{array}\right]$$

Rewriting it as below

$$[A|B] = \left[\begin{array}{ccccc|c} 2 & 5 & 5 & 2 & 10 & 161 \\ 2 & 3 & 2 & 1 & 5 & 81 \\ 0 & 0 & 2 & 5 & 3 & 61 \\ 0 & 0 & 0 & 4 & 13 & 46 \\ 0 & 0 & 0 & 4 & 5 & 30 \end{array}\right]$$

Applying, $R_2 \rightarrow R_1 - R_2$ and $R_5 \rightarrow R_4 - R_5$

$$\left[\begin{array}{ccccc|c} 2 & 5 & 5 & 2 & 10 & 161 \\ 0 & 2 & 3 & 1 & 5 & 80 \\ 0 & 0 & 2 & 5 & 3 & 61 \\ 0 & 0 & 0 & 4 & 13 & 46 \\ 0 & 0 & 0 & 0 & 8 & 16 \end{array}\right]$$

$$\begin{bmatrix} 2 & 5 & 5 & 2 & 10 \\ 0 & 2 & 3 & 1 & 5 \\ 0 & 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 4 & 13 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 161 \\ 80 \\ 61 \\ 46 \\ 16 \end{bmatrix}$$

Now, we get

$$8x_5 = 16$$
$$x_5 = 2$$

and $$4x_4 + 13x_5 = 46$$
$$x_4 = 5$$

Similarly,

$$2x_3 + 5x_4 + 3x_5 = 61$$
$$2x_3 + 25 + 6 = 61$$
$$x_3 = 15$$

3. (c)

$\because$ One of the eigen value is 0,

$\therefore$ Determinant of matrix is equal to 0.

So, $$B_{11} B_{22} - B_{12} B_{21} = 0$$

4. (b)

Here, $$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 2 & 0 \end{array}\right]$$

Applying, $R_3 \to R_3 - 2R_2$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

$\therefore$ Rank[A] = 2 and rank $[A|B] = 3$

Since rank (A) < rank $(A|B)$, the given system of equations is inconsistent, and hence there is no solution.

5. (b)

Statements 1 and 3 are correct.

- For the orthogonal martix $|A| = +1$ or -1.
- For a $n \times n$ matrix, inverse exists only if rank = n.

6. (a)

Given,

$$\frac{d^2y}{dx^2} + \frac{6dy}{dx} + 9y = 5e^{3x}$$

$$(D^2 + 6D + 9)y = 5e^{3x}$$

Auxiliary equation is $m^2 + 6m + 9 = 0$

$$(m + 3)^2 = 0$$

$$m = -3, -3$$

$$\text{Complementary function} = (c_1 + c_2x)e^{-3x}$$

$$\text{Particular integral} = \frac{1}{D^2 + 6D + 9}5e^{3x} = \frac{5e^{3x}}{(3)^2 + 6(3) + 9} = \frac{5e^{3x}}{36}$$

The complete solution is,

$$y = (c_1 + c_2x)e^{-3x} + \frac{5e^{3x}}{36}$$

7. (c)

Given equation: $\sin x \frac{dy}{dx} + 2y = \tan^3 \frac{x}{2}$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2}{\sin x}y = \frac{\tan^3 \frac{x}{2}}{\sin x}$$

This is linear form of $\frac{dy}{dx} + Py = Q$

$$\therefore \quad P = \frac{2}{\sin x}$$

$$\text{Integrating factor} = e^{\int P dx} = e^{\int \frac{2}{\sin x} dx}$$
$$= e^{2\int \text{cosec}\, x dx}$$
$$= e^{2 \ln \tan \frac{x}{2}} = \tan^2 \frac{x}{2}$$

8. (b)

Given
$$\frac{dy}{dx} + \frac{x}{y} = 0$$
$$\frac{dy}{dx} = -\frac{x}{y}$$
$$\int y dy = -\int x dx$$
$$\frac{y^2}{2} = -\frac{x^2}{2} + c$$
$$x^2 + y^2 = 2c \qquad \text{Represents family of circles.}$$

9. (b)

If z is function of x alone, the solution will be $z = A\sin x + B\cos x$, where A and B are constants. Since z is a function of x and y, A and B can be arbitrary functions of y. Hence the solution of the given equation is

$$z = f(y)\sin x + \phi(y)\cos x$$
$$\frac{\partial z}{\partial x} = f(y)\cos x - \phi(y)\sin x$$

When $x = 0;\ z = e^y$

$\therefore$ $\phi(y) = e^y$

When $x = 0,\ \frac{\partial z}{\partial x} = 1$

$\therefore$ $f(y) = 1$

Hence the desired solution is,
$$z = \sin x + e^y \cos x.$$

Alternate solution:

$$\frac{\partial^2 z}{\partial x^2} + z = 0$$
$$m^2 + 1 = 0$$
$$m = \pm i$$
$$z = e^0 [A\cos x + B\sin x]$$
$$z(0) = A + 0$$
$$e^y = A$$

$$\frac{\partial z}{\partial x} = -A\sin x + B\cos x$$

At $x = 0$

$$\frac{\partial z}{\partial x} = 1$$

$$B = 1$$

$$z = e^y\cos x + \sin x$$

10. (a)

Given,

$$\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}} = \frac{(\cos 12\theta + i\sin 12\theta)(\cos(-20\theta) + i\sin(-20\theta))}{(\cos 12\theta + i\sin 12\theta)(\cos(-20\theta) + i\sin(-20\theta))}$$

$$= \frac{(\cos\theta + i\sin\theta)^{12}(\cos\theta + i\sin\theta)^{-20}}{(\cos\theta + i\sin\theta)^{12}(\cos\theta + i\sin\theta)^{-20}} = 1$$

11. (c)

Given, $\lim\limits_{x\to 0}\dfrac{\log x}{\cot x}$; $\dfrac{\infty}{\infty}$ Form

Applying L' Hospital's rule.

$$\lim_{x\to 0}\frac{1/x}{-\operatorname{cosec}^2 x} = -\lim_{x\to 0}\frac{\sin^2 x}{x} \quad ; \quad \frac{0}{0} \text{ form}$$

Again applying L' Hospital's rule.

$$= -\lim_{x\to 0}\frac{2\sin x\cos x}{1} = 0$$

12. (b)

Given, $\displaystyle\int_0^a \frac{x^7}{\sqrt{(a^2 - x^2)}}dx$

Put $x = a\sin\theta$

$dx = a\cos\theta d\theta$

Changing limits:

when $x = 0$, $\theta = 0$, where $x = a$, $\theta = \dfrac{\pi}{2}$

$$\therefore \int_0^{\pi/2}\frac{a^7\sin^7\theta}{a\cos\theta}a\cos\theta d\theta = a^7\int_0^{\pi/2}\sin^7\theta d\theta$$

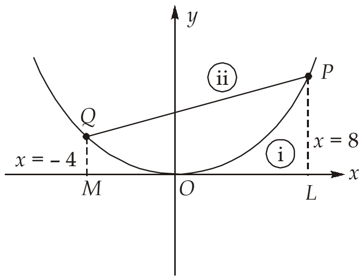
$$= \frac{a^7(n-1)(n-3)....2}{n(n-2)....3}$$

$$= a^7\frac{6\times 4\times 2}{7\times 5\times 3} = \frac{16}{35}a^7$$

NOTE: • When n is odd, $\int_0^{\pi/2} \sin^n x dx = \frac{(n-1)(n-3)(n-5)....2}{n(n-2)(n-4)....3}$

• When n is even, $\int_0^{\pi/2} \sin^n x dx = \frac{(n-1)(n-3)(n-5)....1}{n(n-2)(n-4)....2} \frac{\pi}{2}$

13. (c)



Given, parabola is, $x^2 = 8y$
and the straight line is, $x - 2y + 8 = 0$

$$\text{The required area } POQ = \begin{pmatrix}\text{area bounded by straight line \&} \\ x\text{-axis from } x=-4 \text{ to } x=8\end{pmatrix} - \begin{pmatrix}\text{area bounded by parabola \&} \\ x\text{-axis from } x=-4 \text{ to } x=8\end{pmatrix}$$

$$= \int_{-4}^{8} \frac{x+8}{2} dx - \int_{-4}^{8} \frac{x^2}{8} dx$$

$$= \frac{1}{2}\left|\frac{x^2}{2} + 8x\right|_{-4}^{8} - \frac{1}{8}\left|\frac{x^3}{3}\right|_{-4}^{8}$$

$$= \frac{1}{2}|(32+64)-(-24)| - \frac{1}{24}(512+64)$$

$$= \frac{1}{2}[96+24] - \frac{1}{24}(576) = 36 \text{ square unit}$$

14. (b)

$f(x) = 0$ is the root of the solution.
Clearly the line, $f(x) = 0$ intersects at 4 distinct points in $0 < x < 6$.

15. (a)

By Newton-Raphson method,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x) = x^4 - 3x + 1$$

$$f'(x) = 4x^3 - 3$$

Given, $x_0 = 0$

Therefore, $f(x_0) = 0^4 - 3 \times 0 + 1 = 1$

$$f'(x_0) = 4 \times 0^3 - 3 = -3$$

Hence, $x_1 = 0 - \frac{1}{-3} = \frac{1}{3}$

16. (d)

Bisection, Regula-falsi, Secant and Newton -Raphson methods are used to solve non-linear algebraic and transcendental equations.

17. (d)

The Fourier coefficient $$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx dx = \frac{2}{\pi}\int_{0}^{\pi} x\sin nx dx$$

(x sinnx is an even function on [$-\pi$, π])

$$= \frac{2}{\pi}\left[-x\left(\frac{\cos nx}{n}\right) + \left(\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] = \frac{2}{n}(-1)^{n+1}$$ Put $n = 3$

$$b_3 = \frac{2}{3}(-1)^4 = \frac{2}{3}$$

18. (d)

Taylor series expansion of a function $f(x)$ about $x = 0$ is given by

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$$

$$\text{Coefficient of } x^2 = \frac{f''(0)}{2!} = \frac{f''(0)}{2}$$

Given:

$$f(x) = \cos^2 x$$
$$f'(x) = -\sin(2x)$$
$$f''(x) = -2\cos(2x)$$
$$f''(0) = -2\cos(0) = -2$$

$$\text{Therefore coefficient of } x^2 = \frac{f''(0)}{2} = \frac{-2}{2} = -1$$

19. (b)

The probability that A can solve the problem = $\frac{1}{2}$.

The probability that A cannot solve the problem.

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Similarly the probability that B and C cannot solve the problem are $\left(1 - \frac{3}{4}\right)$ and $\left(1 - \frac{1}{4}\right)$.

The probability that A, B and C cannot solve the problem = $\left(1-\frac{1}{2}\right)\times\left(1-\frac{3}{4}\right)\times\left(1-\frac{1}{4}\right)=\frac{3}{32}$

The probability that the problem will be solved is = $1-\frac{3}{32}=\frac{29}{32}$

20. **(d)**

Here there are three types of families.

Case I: For, zero child family.

Probability of a family having no child (boys) = 0.2

Case II: For one child family

Boy	Girl
0	1
1	0

In this case probability of a family having no boy = 0.3 × 0.5 = 0.15

Case III:

Boy	Girl
0	2
1	1
2	0

In this case probability of a family having no boy = $0.5\times\frac{1}{3}$ = 0.167

Considering all three cases,

Probability of a family having no boy = 0.2 + 0.15 + 0.167 = 0.517

21. **(a)**

$p = 1\% = 0.01,\ n = 100,\ m = np = 100 \times 0.01 = 1$

$$P(r) = \frac{e^{-m}\cdot(m)^r}{r!} = \frac{e^{-1}(1)^r}{r!} = \frac{e^{-1}}{r!}$$

P(4 or more faulty condensers)

$$= P(4) + P(5) + ... P(100)$$
$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$
$$= 1-\left[\frac{e^{-1}}{0!}+\frac{e^{-1}}{1!}+\frac{e^{-1}}{2!}+\frac{e^{-1}}{3!}\right]$$
$$= 1-e^{-1}\left[1+1+\frac{1}{2}+\frac{1}{6}\right] = 1-\frac{8}{3}e^{-1}$$

22. **(a)**

Given,

$$f(x) = 3x^3 - 7x^2 + 5x + 6$$
$$f'(x) = 9x^2 - 14x + 5$$
$$f''(x) = 18x - 14$$
$$f'(x) = 0$$
$$9x^2 - 14x + 5 = 0$$
$$x = 1,\ 0.55$$

$$\text{For } x = 1, f''(1) = 18 - 14 = 4 > 0 \text{ (local minima)}$$
$$\text{For } x = 0.55$$
$$f''(0.55) = -4.1 < 0 \quad \text{(local maxima)}$$
$$\text{Minimum } \{f(0), f(1), f(2)\}$$
$$\text{Minimum } \{6, 7, 12\} = 6$$

23. (a)

The eigen values of an orthogonal matrix A are real or complex conjucates in pairs and have absolute value 1.

24. (d)

$$\begin{vmatrix} \cos\theta - \lambda & \sin\theta \\ -\sin\theta & \cos\theta - \lambda \end{vmatrix} = 0$$
$$\cos^2\theta + \lambda^2 - 2\lambda\cos\theta + \sin^2\theta = 0$$
$$1 + \lambda^2 - 2\lambda\cos\theta = 0$$
$$\lambda^2 - 2\lambda\cos\theta + 1 = 0$$
$$\lambda = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$
$$= \cos\theta \pm \sqrt{\cos^2\theta - 1}$$
$$= \cos\theta \pm i\sin\theta$$
$$\lambda = e^{i\theta}, e^{-i\theta}$$

Hence, $e^{i\theta}$ and $e^{-i\theta}$ are the eigen values.

25. (b)

Let the roots be a/r, a, ar then the product of the roots

$\Rightarrow$ $a^3 = n$

$\therefore$ $a = (n)^{1/3}$

So, $(n) - l(n)^{2/3} + mn^{1/3} - n = 0$

or $m = ln^{1/3}$

Cubing both sides, we get $m^3 = l^3 n$, which is the required condition.

26. (b)

A is skew symmetric,

$$A = -A^T$$

Now, $(A.A)^T = A^T . A^T$

$$= (-A)(-A) = A.A$$

$\therefore$ $A.A$ is a symmetric matrix.

27. (b)

$$[1 + 2x + 15 \quad 3 + 5x + 3 \quad 2 + x + 2] \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$
$$(1 + 2x + 15) + 2(3 + 5x + 3) + (2 + x + 2)x = 0$$
$$2x + 16 + 10x + 12 + x^2 + 4x = 0$$
$$x^2 + 16x + 28 = 0$$

By solving, we get,

$$x = -2, -14$$

So, $$|x|_{max} = 14$$

28. (d)

The auxiliary equation is

$$D^3 - 2D^2 + 4D - 8 = 0$$

$$(x - 2)(x^2 + 4) = 0$$

$$x = 2, \pm 2i$$

The solutions of eqution is

$$y = C_1e^{2x} + C_2\sin 2x + C_3\cos 2x$$

29. (a)

$$PI = \frac{1}{(D+1)^2}e^{-x}\cos x = e^{-x}\left\{\frac{1}{D^2}\cos x\right\}$$

$$= e^{-x}\left\{\frac{1}{D}\sin x\right\} = e^{-x}\{-\cos x\} = -e^{-x}\cos x$$

30. (b)

$$(x + \log y)dy + ydx = 0$$

It is in the form,

$$Mdx + Ndy = 0$$

The equation is exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = 1, \frac{\partial N}{\partial x} = 1$$

⇒The equation is exact and hence solution is

$$(x + \log y)dy = 0$$

$$xy + y\log y - y + C = 0$$

$y(0) = 1$, $$0 + 0 - 1 + C = 0$$

$$C = 1$$

Hence, the solution is, $y(x - 1 + \log y) + 1 = 0$