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# FLUID MECHANICS

## MECHANICAL ENGINEERING

Date of Test : 23/08/2024

### ANSWER KEY >

- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (c) | 7. (c)  | 13. (b) | 19. (d) | 25. (b) |
| 2. (c) | 8. (c)  | 14. (a) | 20. (c) | 26. (b) |
| 3. (d) | 9. (c)  | 15. (a) | 21. (c) | 27. (c) |
| 4. (d) | 10. (a) | 16. (c) | 22. (b) | 28. (d) |
| 5. (d) | 11. (a) | 17. (a) | 23. (b) | 29. (b) |
| 6. (b) | 12. (d) | 18. (a) | 24. (b) | 30. (c) |

## DETAILED EXPLANATIONS

1. (c)

$$\text{Vorticity, } \Omega = 2\omega_z = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial x}(4xy) - \frac{\partial}{\partial y}(xy^2)$$

$$\Omega = 4y - 2xy$$

$$\text{At } (1, 2), \quad \Omega_{(1,2)} = 4 \times 2 - 2 \times 1 \times 2 = 8 - 4 = 4\text{s}^{-1}$$

2. (c)

As per given data,

$$\text{Gauge pressure} = 350 \text{ kPa}$$

$$\text{Barometric reading} = 740 \text{ mm Hg}$$

$$\rho_{\text{Hg}} = 13590 \text{ kg/m}^3$$

The atmospheric (or barometric) pressure can be expressed,

$$\begin{aligned} P_{\text{atm}} &= \rho g h = 13590 \times 9.81 \times 740 \times 10^{-3} \\ &= 98.655 \text{ kPa} \end{aligned}$$

Then the absolute pressure in the tank is

$$\begin{aligned} P_{\text{abs}} &= P_{\text{gauge}} + P_{\text{atm}} = 350 \text{ kPa} + 98.655 \text{ kPa} \\ P_{\text{abs}} &= 448.655 \text{ kPa} \end{aligned}$$

3. (d)

$$\text{Continuity equation, } Q_1 = Q_2 + Q_3$$

Now,

$$Q_2 = A_2 V_2 = 0.008 V_2$$

$$\begin{aligned} Q_3 &= A_3 V_3 = 0.004 V_3 = 0.004 \times 2V_2 \\ &= 0.008 V_2 \end{aligned}$$

$$[\text{Given, } V_3 = 2V_2]$$

$$\text{Now, } Q_2 = Q_3$$

$$\therefore \text{Discharge at point 3, } Q_3 = \frac{Q_1}{2} = \frac{0.3}{2} = 0.15 \text{ m}^3/\text{s}$$

4. (d)

By Newton's law of viscosity,

$$\tau = \mu \frac{du}{dy}$$

$$0.5 = \mu \times \frac{1 - 0}{(0.010 - 0)}$$

$$\mu = 0.005 = 5 \times 10^{-3} \text{ Ns/m}^2.$$

5. (d)

Total energy of a flowing fluid can be represented in terms of head which is given by

$$\left( \frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant} \right).$$

Piezometric head is the sum of pressure head and datum head and it is given by  $\left(\frac{P}{\rho g} + Z\right)$ .

The pressure at any point in a static fluid is obtained by hydrostatic law which is given by  $P = -\rho gh$ , where  $h$  is the height of the point from the free surface. As we go down  $h$  is negative so the pressure gets increased and datum gets decreased.

Therefore, Piezometric head remains constant at all points in the liquid.

6. (b)

As for laminar flow,

$$\text{Boundary layer thickness } (\delta) \propto \frac{1}{\sqrt{\text{Re}}}$$

As the free stream velocity,  $\uparrow\uparrow$ ,  $\delta\downarrow\downarrow$   $\therefore \text{Re} = \frac{\rho VD}{\mu}$

For turbulent flow,

$$\text{Boundary layer thickness } (\delta) \propto \frac{1}{(\text{Re})^{1/5}}$$

As the free stream velocity  $\uparrow\uparrow$ ,  $\delta\downarrow\downarrow$  and it also depending on the kinematic viscosity  $\delta\uparrow\uparrow$  as kinematic viscosity ( $\nu$ )  $\uparrow$ .

7. (c)

We know,

$$\begin{aligned} C_d &= C_c \times C_v \\ &= 0.95 \times \left[\frac{0.180}{0.200}\right]^2 = 0.769 \end{aligned}$$

8. (c)

As we know, the average velocity in fully developed laminar pipe flow is

$$\begin{aligned} V_{\text{avg}} &= \frac{1}{2} V_{\text{max}} \\ V_{\text{max}} &= 2V_{\text{avg}} = 2 \times 2 = 4.0 \text{ m/s} \end{aligned}$$

9. (c)

10. (a)

11. (a)

Applying Bernoulli's equation between the two reservoirs, we get

$$\begin{aligned} 12.5 &= 0.5 \frac{V^2}{2g} + \frac{fLV^2}{2gD} + \frac{V^2}{2g} \\ \Rightarrow 12.5 &= \frac{V^2}{2g} \left[ 1.5 + \frac{fL}{D} \right] \\ \Rightarrow 12.5 &= \frac{V^2}{2 \times 10} \left[ 1.5 + \frac{0.04 \times 1000}{0.5} \right] \end{aligned}$$

$$\Rightarrow 12.5 = \frac{V^2}{20} \times 81.5$$

$$\Rightarrow V = 1.75 \text{ m/s}$$

12. (d)

As, Drag force,  $F = \rho V^2 L^2$

$$\frac{F_m}{F_p} = \frac{\rho_m V_m^2 L_m^2}{\rho_p V_p^2 L_p^2} \quad \dots (i)$$

As,

$$\text{Re}_m = \text{Re}_p$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\frac{V_m}{V_p} = \frac{L_p}{L_m} \quad \dots (ii)$$

Using equation (i) and (ii)

$$\frac{F_m}{F_p} = 1$$

$$\Rightarrow F_p = 300 \text{ N}$$

13. (b)

Volume of the cube,  $V = 125 \text{ mL}$

$$a^3 = 125 \times 10^{-3} \times 10^{-3} \text{ m}^3$$

$$a = 0.05 \text{ m}$$

Pressure at bottom surface,

$$\begin{aligned} P_{\text{bottom}} &= P_{\text{atm}} + (\rho g h)_{\text{oil}} + [\rho g (h + a)]_{\text{water}} \\ &= 101 \times 10^3 + (800 \times 9.81 \times 0.5) + [1000 \times 9.81 \times (0.3 + 0.05)] \end{aligned}$$

$$P_{\text{bottom}} = 108357.5 \text{ Pa}$$

$$\begin{aligned} F_{\text{bottom}} &= P_{\text{bottom}} \times A \\ &= 108357.5 \times 0.05^2 = 270.89 \text{ N} \end{aligned}$$

14. (a)

$$\begin{aligned} \tau &= \mu \frac{du}{dx} = \mu(4 - 4x) \\ &= 2(4 - 4 \times 1) = 0 \text{ N/m}^2 \end{aligned}$$

15. (a)

Pressure gradient  $\left( \frac{\partial p}{\partial x} \right)$ ,

$$\therefore U_{\text{max}} = \frac{-1}{8\mu} \left( \frac{\partial p}{\partial x} \right) \times t^2$$

$$\Rightarrow 3 = \frac{-1}{8 \times 0.02} \left( \frac{\partial p}{\partial x} \right) \times (0.015)^2$$

$$\Rightarrow \left( \frac{\partial p}{\partial x} \right) = \frac{-3 \times 8 \times 0.02}{(0.015)^2} = -2133.33 \text{ N/m}^2/\text{m}$$

16. (c)

Given:  $u = 2xy^2$ ,  $v = 3xyt$ 

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} = 2xy^2 \times 2y^2 + 3xyt \times 4xy + 0$$

$$\text{At } (1, 1) \text{ and } t = 1\text{s}, \quad a_x = 2 \times 2 + 12 = 16 \text{ m/s}^2$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial t} = 2xy^2 \times 3yt + 3xyt \times 3xt + 3xy$$

$$\text{At } (1, 1) \text{ and } t = 1\text{s}, \quad a_y = 6 + 9 + 3 = 18 \text{ m/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{16^2 + 18^2} = 24.08 \text{ m/s}^2 \simeq 24.1 \text{ m/s}^2$$

17. (a)

Radius:

$$r = 10 \text{ mm}$$

∴ Diameter:

$$d = 2r = 2 \times 10 = 20 \text{ mm} = 0.02 \text{ m}$$

$$m = 72 \text{ kg/hr} = \frac{72}{3600} = 0.02 \text{ kg/s}$$

$$\mu = 0.002 \text{ kg/ms}$$

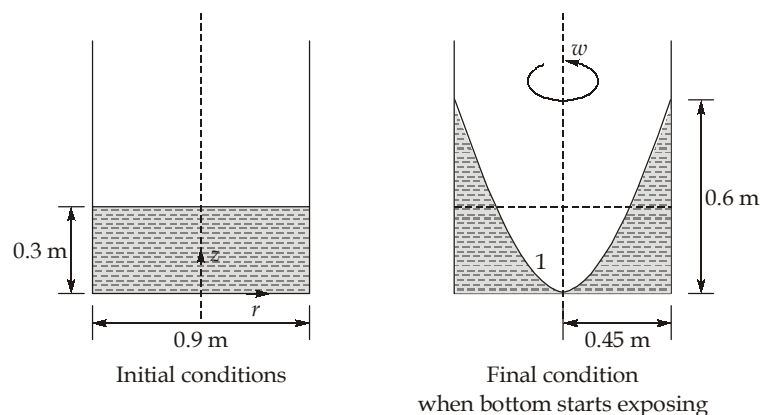
$$\text{Reynolds number: } Re = \frac{\rho V d}{\mu}$$

From continuity equation,  $m = \rho A V$ 

$$\text{or } V = \frac{m}{\rho A} = \frac{m}{\frac{\rho \pi d^2}{4}} = \frac{4m}{\rho \pi d^2}$$

$$\therefore Re = \frac{\rho d}{\mu} \times \frac{4m}{\rho \pi d^2} = \frac{4m}{\pi \mu d} = \frac{4 \times 0.02}{\pi \times 0.002 \times 0.02} = 636.62$$

18. (a)



Apply forced vortex motion equation at points (1) and (2)

$$\frac{P_1}{\rho g} - \frac{(V_1)^2}{2g} + z_1 = \frac{P_2}{\rho g} - \frac{(V_2)^2}{2g} + z_2$$

At point 1,  $P_1 = P_{\text{atm}} \Rightarrow P_{\text{gauge}} = 0$

$$V_1 = \omega R_1 = 0$$

$$z_1 = 0$$

At point,  $P_2 = P_{\text{atm}} \Rightarrow P_{\text{gauge}} = 0$

$$z_2 = 0.6$$

Therefore,  $0 - 0 + 0 = 0 - \frac{(\omega R_2)^2}{2g} + 0.6$

$$\Rightarrow \frac{\omega^2 (0.45)^2}{2 \times (9.81)} = 0.6$$

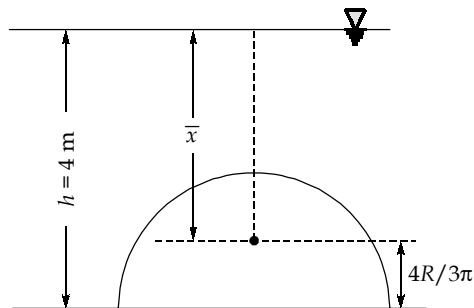
$$\Rightarrow \omega = 7.62 \text{ rad/s}$$

19. (d)

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

20. (c)

Refer figure,



Depth of centre of gravity,

$$\bar{x} = h - \frac{4R}{3\pi} = 4 - \frac{4 \times 1}{3 \times \frac{22}{7}} = 3.576 \text{ m}$$

21. (c)

As

In  $x$ -direction,  $u = \frac{-x}{x^2 + y^2}$

$$\frac{\partial u}{\partial x} = \frac{-(x^2 + y^2) + 2x^2}{(x^2 + y^2)^2} = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

and,

In  $y$ -direction,  $v = \frac{-y}{x^2 + y^2}$

$$\frac{\partial v}{\partial y} = \frac{-(x^2 + y^2) + 2y^2}{(x^2 + y^2)^2} = \frac{(y^2 - x^2)}{(x^2 + y^2)^2}$$

Continuity equation

$$\Rightarrow \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = 0$$

22. (b)

A velocity potential is a scalar function used in potential flow theory.

23. (b)

$$F_v = \gamma \times \frac{1}{2} \times \frac{\pi D^2}{4} \times 1$$

$$F_v = 10 \times \frac{1}{2} \times \frac{\pi (2)^2}{4} \times 1 = 15.7 \text{ kN/m}$$

24. (b)

$$D_i = 6 \times 10^{-2} \text{ m}$$

$$D_f = 6.9 \times 10^{-2} \text{ m}$$

As soap bubble has two surfaces,

$$\begin{aligned} \text{Therefore total change in surface area} &= 2 \left[ 4\pi (R_f^2 - R_i^2) \right] = 2 \left[ \pi (D_f^2 - D_i^2) \right] \\ &= 2 (0.003647) = 7.294 \times 10^{-3} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Work input required, } W &= \sigma \times \Delta A = 0.039 \times 7.294 \times 10^{-3} \\ &= 2.845 \times 10^{-4} \text{ Joule} \end{aligned}$$

25. (b)

Velocity of air in pitot tube is

$$V = c\sqrt{2gh}$$

$$h = x \left( \frac{S_m}{S} - 1 \right) = 12 \times 10^{-3} \left( \frac{1}{1.2 \times 10^{-3}} - 1 \right)$$

$$h = 9.988 \text{ m}$$

$$V = \sqrt{2 \times 9.81 \times 9.988} \simeq 14 \text{ m/s}$$

26. (b)

Conservation of mass,

$$\dot{m}_{in} - m_{out} = \frac{dm}{dt} \Big|_{\text{tank}}$$

$$\Rightarrow \rho A V_1 - \rho A V_2 = \rho \times \frac{\pi}{4} D^2 \times \frac{dh}{dt}$$

$$\Rightarrow (0.12)^2 \times [2.5 - 1.9] = (0.75)^2 \times \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = 0.01536 \text{ m/s}$$

So, time required to fill remaining tank,

$$t = \frac{1 - 0.3}{0.01536} \text{ s}$$

$$\Rightarrow t = 45.57 \text{ s}$$

27. (c)

Minor loss due to sudden expansion from 6 cm diameter pipe to 12 cm is given by

$$\begin{aligned} (h_f)_{\text{expansion}} &= \frac{V_1^2}{2g} \left[ 1 - \frac{A_1}{A_2} \right]^2 \\ &= \frac{V_1^2}{2g} \times \left[ 1 - \frac{d_1^2}{d_2^2} \right]^2 = \frac{V_1^2}{2g} \times \left( 1 - \left( \frac{1}{2} \right)^2 \right)^2 = \frac{9}{16} \left( \frac{V_1^2}{2g} \right) \end{aligned}$$

28. (d)

$$\begin{aligned} \text{Projected area } (A_p) &= l \times d \\ &= 1 \times 0.05 = 0.05 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Total drag, } F_D &= C_d \times \frac{1}{2} \rho u_0^2 \times A_p \\ &= 1.25 \times \left[ \frac{1}{2} \times 1.2 \times 0.2^2 \right] \times 0.05 \\ &= 1.5 \times 10^{-3} \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Shear drag} &= 0.18 \times \left[ \frac{1}{2} \times 1.2 \times 0.2^2 \right] \times 0.05 \\ &= 0.216 \times 10^{-3} \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Total drag} &= \text{Shear drag} + \text{Pressure drag} \\ 1.5 \times 10^{-3} &= 0.216 \times 10^{-3} + \text{Pressure drag} \end{aligned}$$

$$\text{Pressure drag} = 1.284 \times 10^{-3} \text{ N}$$

29. (b)

The frontal area of a sphere is  $A = \frac{\pi D^2}{4}$ .

The drag force acting on the balloon is

$$F_D = C_D A \frac{\rho V^2}{2} = (0.2) \left[ \frac{\pi(7)^2}{4} \right] \frac{(1.20) \left( \frac{40 \times 5}{18} \right)^2}{2} = 570.14 \text{ N}$$

Acceleration in the direction of the winds

$$a = \frac{F_D}{m} = \frac{570.14}{350} = 1.63 \text{ m/s}^2$$



30. (c)

As per given data:

$$u^* = \frac{u}{U} \text{ and } y^* = \frac{y}{\delta}$$

$$dy^* = \delta^{-1} dy$$

The given parabolic velocity distribution and the expression for the displacement thickness can then be expressed as

$$u^* = 2y^* - y^{*2}, \text{ and } \delta^* = \delta \int_0^1 (1 - u^*) dy^*$$

Combining these equations gives,

$$\delta^* = \delta \int_0^1 (1 - 2y^* + y^{*2}) dy^*$$

$$\delta^* = \delta \left[ y^* - y^{*2} + \frac{1}{3} y^{*3} \right]_0^1$$

$$\delta^* = \frac{1}{3} \delta$$

$$\frac{\delta^*}{\delta} = \frac{1}{3}$$

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