CLASS TEST

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ENGINEERING MATHEMATICS

MECHANICAL ENGINEERING

Date of Test : 06/09/2024

ANSWER KEY

1.	(b)	7.	(c)	13.	(b)	19.	(a)	25.	(a)
2.	(b)	8.	(c)	14.	(d)	20.	(c)	26.	(b)
3.	(a)	9.	(b)	15.	(b)	21.	(a)	27.	(b)
4.	(b)	10.	(b)	16.	(d)	22.	(b)	28.	(a)
5.	(d)	11.	(b)	17.	(c)	23.	(c)	29.	(a)
6.	(d)	12.	(a)	18.	(b)	24.	(b)	30.	(b)

DETAILED EXPLANATIONS

1. **(b)**

Given: $x + 2y - 3z = 1$, $(\lambda + 3)z = 3$, $(2\lambda + 1)x + z = 0$.

Given equations are non homogeneous system of equation of the form,

$$AX = B$$

For inconsistent, $\rho(A) \neq \rho(A/B)$

Hence, $$[A/B] = \begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 0 & 0 & \lambda+3 & : & 3 \\ 2\lambda+1 & 0 & 1 & : & 0 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3$$

$\Rightarrow$ $$[A/B] = \begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 2\lambda+1 & 0 & 1 & : & 0 \\ 0 & 0 & \lambda+3 & : & 3 \end{bmatrix}$$

For inconsistent, $\lambda + 3 = 0$

$\Rightarrow$ $\lambda = -3$

2. **(b)**

Given: $$I = \int_0^\infty \frac{dx}{e^x + e^{-x}} = \int_0^\infty \frac{e^x dx}{1 + e^{2x}}$$

Put, $e^x = t$

$e^x dx = dt$

$\therefore$ $$I = \int_1^\infty \frac{dt}{1+t^2} = \left[\tan^{-1} t\right]_1^\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

3. **(a)**

For any given $\vec{F}$,

$$\text{div}\left(\text{cur}\,\vec{F}\right) = 0 \qquad \text{(Always)}$$

Hence, $$\nabla\cdot\left(\nabla \times \vec{F}\right) = 0$$

4. **(b)**

For binomial distribution,

Mean $= np = 9$ ... (i)

Variance $= npq = \sigma^2 = 6$ ... (ii)

From (i) and (ii), $$q = \frac{6}{9} = \frac{2}{3}$$

$$p = 1 - q = \frac{1}{3}$$

$$n \times \frac{1}{3} = 9$$

$$n = 27$$

5. **(d)**

Given,

$$(D^3 - 3D^2 + 3D - 1)y = e^x + 1$$

$$PI = \frac{e^x + 1}{D^3 - 3D^2 + 3D - 1}$$

$$= \frac{e^x}{D^3 - 3D^2 + 3D - 1} + \frac{1.e^0}{D^3 - 3D^2 + 3D - 1}$$

$$= \frac{e^x}{(D-1)^3} + \frac{1.e^0}{(D-1)^3}$$

$$= \frac{xe^x}{3(D-1)^2} + \frac{1.e^0}{(0-1)^3}$$

$$= \frac{xe^x}{3(D-1)^2} - 1$$

$$= \frac{x^2e^x}{6(D-1)} - 1$$

$$= \frac{x^3e^x}{6} - 1$$

6. **(d)**

Given, differential equation is

$$\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

Put $y = vx$

$$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v + \tan(v)$$

$$\Rightarrow \quad \frac{xdv}{dx} = \tan(v)$$

Integrating, $$\int \frac{dv}{\tan v} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \ln(\sin v) = \ln x + \ln c$$

$$\Rightarrow \quad \sin v = cx$$

$$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = cx$$

7. (c)

Given differential equation is

$$\tan\left(\left(\frac{d^2y}{dx^2}\right)^{1/3} + \frac{dy}{du}\right) = 2y$$

$$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)^{1/3} + \left(\frac{dy}{dx}\right) = \tan^{-1}(2y)$$

$$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)^{1/3} = \tan^{-1}(2y) - \left(\frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \left(\tan^{-1}(2y) - \frac{dy}{dx}\right)^3$$

Hence order = 2, degree = 1

8. (c)

$$\because \quad P = \begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$$

And

$$|P| = \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

$$\Rightarrow \quad |P| = -96$$

$$|A| = \begin{vmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{vmatrix} = 2\times2\times2\begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

$$\Rightarrow \quad |A| = 8|P|$$

$$\Rightarrow \quad |A| = 8 \times (-96)$$

$$\Rightarrow \quad |A| = -768$$

9. (b)

Given: $$\frac{dy}{dx} = 1 + \tan(y - x)$$

Put, $$y - x = z$$

$$\therefore \quad \frac{dy}{dx} - 1 = \frac{dz}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 + \frac{dz}{dx}$$

$$\therefore \quad 1 + \frac{dz}{dx} = 1 + \tan z$$

$$\Rightarrow \quad \frac{dz}{dx} = \tan z$$

$$\Rightarrow \quad \int \frac{dz}{\tan z} = \int dx$$

$$\Rightarrow \quad \int \frac{\cos z dz}{\sin z} = \int dx$$

$$\Rightarrow \quad \ln(\sin z) = x + C$$

$$\Rightarrow \quad \sin z = e^{x+C}$$

$$\Rightarrow \quad \sin z = Ke^{x}$$

$$\Rightarrow \quad \sin(y - x) = Ke^{x} \quad (\text{where } K = e^{c})$$

10. (b)

Given, D.E. is

$$(D^3 - 2D^2 - 5D + 6)y = e^{3x}$$

$$\text{P.I.} = \frac{e^{3x}}{D^3 - 2D^2 - 5D + 6}$$

At $D = 3$, $D^3 - 2D^2 - 5D + 6 = 0$

$$\therefore \quad \text{P.I.} = \frac{xe^{3x}}{3D^2 - 4D - 5}$$

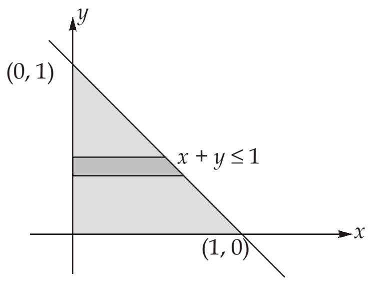
$$\Rightarrow \quad \text{P.I.} = \frac{xe^{3x}}{27 - 12 - 5}$$

$$\Rightarrow \quad \text{P.I.} = \frac{xe^{3x}}{10}$$

11. (b)

Let,

$$I = \iint (x^2 + y^2)\,dxdy$$



$$= \int_{y=0}^{y=1} \int_{x=0}^{x=1-y} (x^2 + y^2)\,dxdy$$

$$= \int_{y=0}^{y=1} \left[\frac{x^3}{3} + y^2 x\right]_0^{1-y} dy$$

$$= \int_{y=0}^{1} \left[\frac{(1-y)^3}{3} + y^2(1-y)\right] dy$$

$$= \int_{y=0}^{1} \left[\frac{(1-y)^3}{3} + (y^2 - y^3)\right] dy$$

$$= \left[\frac{(1-y)^4}{3 \times 4} \times \frac{1}{-1} + \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = \left[\frac{(1-y)^4}{-12} + \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1$$

$$= \frac{1}{-12}[0 - 1] + \frac{1}{3} - \frac{1}{4}$$

$$= \frac{1}{12} + \frac{4-3}{12} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

12. (a)

$$P(2) = 9P(4) + 90P(6)$$

For Poisson's distribution,

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$, where λ is the mean of Poisson's distribution

Hence, $$\frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$$

$\Rightarrow$ $$\frac{e^{-\lambda}\lambda^2}{2} = 9\frac{e^{-\lambda}\lambda^4}{24} + 90\frac{e^{-\lambda}\lambda^6}{720}$$

$\Rightarrow$ $$= \frac{e^{-\lambda}\lambda^6}{8} + \frac{3e^{-\lambda}\lambda^4}{8} - \frac{e^{-\lambda}\lambda^2}{2} = 0$$

$\Rightarrow$ $$\frac{e^{-\lambda}\lambda^2}{2}\left[\frac{\lambda^4}{4} + \frac{3\lambda^2}{4} - 1\right] = 0$$

Given, $\lambda \neq 0$

$\therefore$ $$\left[\frac{\lambda^4}{4} + \frac{3\lambda^2}{4} - 1\right] = 0$$

$\Rightarrow$ $$\lambda^4 + 3\lambda^2 - 4 = 0$$

$\Rightarrow$ $$\lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0$$

$\Rightarrow$ $$\lambda^2(\lambda^2 + 4) - 1(\lambda^2 + 4) = 0$$

$\Rightarrow$ $$(\lambda^2 - 1)(\lambda^2 + 4) = 0$$

$\Rightarrow$ $$\lambda^2 = 1, \lambda^2 + 4 \neq 0$$

$\Rightarrow$ $$\lambda = \pm 1$$

13. (b)

Given differential equation is

$$\frac{d^2y}{dx^2} + \frac{7dy}{dx} + 12y = 0$$

$\Rightarrow$ $$m^2 + 7m + 12 = 0$$

$\Rightarrow$ $$(m + 3)(m + 4) = 0$$

$\therefore$ $$m = -3, -4$$

$$\text{C.F. is } y = c_1 e^{-3x} + c_2 e^{-4x}$$

Given, $$y(0) = 1$$

$\Rightarrow$ $$y(0) = 1 = c_1 + c_2$$ ...(i)

$$y'(x) = -3c_1e^{-3x} - 4c_2e^{-4x}$$

$$y'(0) = -3c_1 - 4c_2$$ ...(ii)

From (i) and (ii)

$$c_2 = -3, c_1 = 4$$

Hence, solution is $$y = 4e^{-3x} - 3e^{-4x} = 0 \text{ (given)}$$

14. (d)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x + x^2)\sin(nx)dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x\sin(nx)dx + \int_{-\pi}^{\pi} x^2\sin(nx)dx\right]$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x\sin(nx)dx$$

$$= \frac{2}{\pi}\left[x\left(\frac{-\cos(nx)}{n}\right) - 1\left(\frac{-\sin(nx)}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-(\pi)\frac{\cos n\pi}{n}\right]$$

$$= \frac{-2}{n}(-1)^n = \frac{2}{n}(-1)^{n+1}$$

15. (b)

$\because$ α, β and γ are the roots of the equation $x^3 + px + q = 0$

Then, $\alpha + \beta + \gamma = 0$ and $\alpha^3 + p\alpha + q = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = p$ and $\beta^3 + p\beta + q = 0$

$\alpha\beta\gamma = -q$ and $\gamma^3 + p\gamma + q = 0$

$$\therefore \quad \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = \alpha(\gamma\beta - \alpha^2) - \beta(\beta^2 - \alpha\gamma) + \gamma(\alpha\beta - \gamma^2)$$

$$= \alpha\beta\gamma - \alpha^3 - \beta^3 + \alpha\beta\gamma + \alpha\beta\gamma - \gamma^3$$
$$= 3\alpha\beta\gamma - (\alpha^3 + \beta^3 + \gamma^3)$$
$$= 3(-q) - [-p\alpha - q - p\beta - q - p\gamma - q]$$
$$= -3q + p\alpha + q + p\beta + q + p\gamma + q$$
$$= -3q + p(\alpha + \beta + \gamma) + 3q$$
$$= p(0) = 0$$

16. (d)

Given: $$P = \begin{bmatrix} x & y \\ z & w \end{bmatrix};\; Q = \begin{bmatrix} x^2 + y^2 & xz + yw \\ xz + yw & z^2 + w^2 \end{bmatrix}$$

$$PP^T = \begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix}^T$$

$$= \begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} x & z \\ y & w \end{bmatrix}$$

$$= \begin{bmatrix} x^2 + y^2 & xz + yw \\ xz + yw & z^2 + w^2 \end{bmatrix}$$

$\because$ $Q = PP^T$

and $r(P) = n$

Then, $r(Q) = \min\left\{r(P), r\left(P^T\right)\right\}$

$\Rightarrow$ $r(Q) = \min(n, n) \because \left[r(P) = r\left(P^T\right)\right]$

$\Rightarrow$ $r(Q) = n$

17. (c)

$$|Adj(A)| = |A|^{n-1} \quad .....(1)$$

Where n is order of A,

Now,

$$|A| = 1\begin{vmatrix}5 & 1\\4 & 3\end{vmatrix} + 0 + 1.\begin{vmatrix}2 & 1\\5 & 1\end{vmatrix}$$

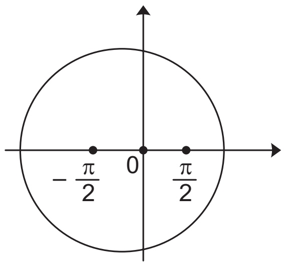
$\Rightarrow$ $|A| = 11 - 3$

$\Rightarrow$ $|A| = 8$

Using (1),

$$|Adj(A)| = 8^{3-1} = 8^2 = 64$$

18. (b)



Pole of $f(z)$ one $= 0, \frac{\pi}{2}, -\frac{\pi}{2}$

$$\frac{i}{4-\pi}\int \frac{dz}{z\cos z} = \frac{i}{4-\pi} \times 2\pi i \quad \text{[Residue]}$$

$$\frac{i}{4-\pi}\int \frac{dz}{z\cos z} = \frac{i}{4-\pi} \times 2\pi i\left[\text{Res}\big|_{z=0} + \text{Res}\big|_{z=\frac{\pi}{2}} + \text{Res}\big|_{z=-\frac{\pi}{2}}\right]$$

$$\text{Res of } f(z)\big|_{z=0} = \lim_{z\to 0}(z-0).\frac{1}{z\cos z} = \lim_{z\to 0}\frac{1}{\cos z} = \frac{1}{\cos z} = 1$$

$$\text{Res of } f(z)\big|_{z=\frac{\pi}{2}} = \lim_{z\to\frac{\pi}{2}}\left(z-\frac{\pi}{2}\right)\frac{1}{z\cos z}\left(\frac{0}{0}\right)$$

by L.H. $$\lim_{z\to\frac{\pi}{2}}\frac{1}{z(-\sin z)+\cos z} = \frac{1}{\frac{\pi}{2}\left(-\sin\frac{\pi}{2}\right)} = \frac{-2}{\pi}$$

$$\text{Res of } f(z)\big|_{z=-\frac{\pi}{2}} = \lim_{z\to\frac{\pi}{2}}\left(z+\frac{\pi}{2}\right)\frac{1}{z\cos z} \quad \frac{0}{0} \text{ form}$$

by L.H.
$$\lim_{z \to \frac{-\pi}{2}} \frac{1}{z(-\sin z) + \cos z} = \frac{1}{\frac{-\pi}{2}\left(-\sin\frac{\pi}{2}\right)} = \frac{-2}{\pi}$$

$$\frac{i}{4-\pi}\int \frac{dz}{z\cos z} = \frac{i}{4-\pi} \times 2\pi i\left[1 - \frac{2}{\pi} - \frac{2}{\pi}\right]$$

$$= \frac{i \times 2\pi i}{4-\pi}\left[\frac{\pi - 4}{\pi}\right] = -2i^2 = 2$$

19. (a)

By Trapezoidal rule

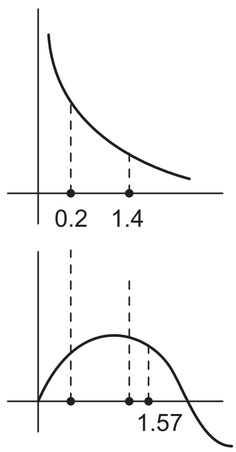
$$T_E = \frac{b-a}{12}h^2 \quad [\text{maximum } f''(x)]$$

$$f(x) = \sin x - \ln x$$

$$f'(x) = \cos x - \frac{1}{x}$$

$$f''(x) = -\sin x + \frac{1}{x^2}$$

$$= \frac{1}{x^2} - \sin x$$



$f''(x) \mid_{\text{max at x = 0.2}}$

$$f''(x)_{max} = \frac{1}{(0.2)^2} - \sin(0.2)$$

= 25 – 0.0034 = 25 – 0 = 25

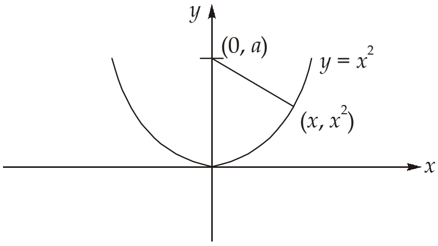
$$T_E = \frac{(1.4-0.2)}{12}\left(\frac{1.4-0.2}{12}\right)^2 \times 25$$

$$= \left(\frac{1.2}{12}\right)\left(\frac{1.2}{12}\right)^2 \times 25$$

= $(0.1)^3 \times 25$

= $10^{-3} \times 25 = 0.025$

20. (c)



Let there is a point (x, x^2) on the parabola and distance of this point from $(0, a)$ is:

$$D = \sqrt{(x-0)^2 + (x^2-a)^2}$$

$$\Rightarrow \quad D^2 = x^2 + (x^2-a)^2$$

When D is minimum, D^2 will also be minimum

So, $$\frac{d(D^2)}{dx} = 0$$

$$\Rightarrow \quad 2x\left[1+2(x^2-a)\right] = 0$$

$\therefore$ $x = 0$, and $1+2(x^2-a) = 0$

$$\Rightarrow \quad x = 0 \text{ and } x^2 = \left(\frac{2a-1}{2}\right)$$

$\because$ For minimum distance, $\frac{d^2(D^2)}{dx^2}$ should be greater than zero.

$$\therefore \quad \frac{d^2(D^2)}{dx^2} = 2\left[1+2(x^2-a)\right]+2x[4x]$$

$$\frac{d^2(D^2)}{dx^2} = 2+4x^2-4a+8x^2$$

$$\frac{d^2(D^2)}{dx^2} = 2+12x^2-4a$$

At $x = 0$, $$\frac{d^2(D^2)}{dx^2} = 2-4a$$

$\because$ $a > 1$

$$\therefore \quad \frac{d^2(D^2)}{dx^2} < 0$$

So, $x = 0$ is having maxima.

At $x^2 = \frac{2a-1}{2}$, $$\frac{d^2(D^2)}{dx^2} = 2+12\left(\frac{2a-1}{2}\right)-4a$$

$$= 2+6(2a-1)-4a$$

$$= 2+12a-6-4a$$
$$= 8a-4 = 4(2a-1) > 0$$

Hence, $x = \sqrt{\frac{2a-1}{2}}$ is having minima and minimum distance is

$$D_{min} = \sqrt{x^2+(x^2-a)^2}$$
$$= \sqrt{\frac{2a-1}{2}+\left(\frac{2a-1}{2}-a\right)^2} = \sqrt{\frac{2a-1}{2}+\frac{1}{4}}$$
$$= \sqrt{\frac{4a-1}{4}} = \frac{\sqrt{4a-1}}{2}$$

21. **(a)**

Given surfaces are:

Surface-1 = $\phi_1 \Rightarrow ax^2 - byz = (a+2)x$

Normal at surface-1, $\nabla\phi_1 = \frac{\partial}{\partial x}(\phi_1)\hat{i} + \frac{\partial}{\partial y}(\phi_1)\hat{j} + \frac{\partial}{\partial z}(\phi_1)\hat{k}$

$$= [2ax-(a+2)]\hat{i} - bz\hat{j} - by\hat{k}$$

$$\nabla\phi_{1\,(1,\,-1,\,2)} = (2a-a-2)\hat{i} - 2b\hat{j} + b\hat{k}$$
$$= (a-2)\hat{i} - 2b\hat{j} + b\hat{k}$$

Surface-2 = $\phi_2 \Rightarrow 4x^2y + z^3 - 4 = 0$

Normal at surface-2, $\nabla\phi_2 = \frac{\partial}{\partial x}(\phi_2)i + \frac{\partial}{\partial y}(\phi_2)\hat{j} + \frac{\partial}{\partial z}(\phi_2)\hat{k}$

$$\Rightarrow \quad \nabla\phi_2 = 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$

$$\nabla\phi_{2(1,\,-1,\,2)} = -8\hat{i} + 4\hat{j} + 12\hat{k}$$

If surfaces cut orthogonally to each other then their normals also cut orthogonally to each other.

$$\therefore \quad \nabla\phi_1 \cdot \nabla\phi_2 = -8(a-2) - 8b + 12b = 0$$
$$\Rightarrow \quad -8a+16+4b = 0$$
$$\Rightarrow \quad -2a+b+4 = 0 \quad \text{...(i)}$$

Point (1, –1, 2) also lies on both the surfaces

Hence, $a(1)^2 + b(2) = (a+2)$

$\Rightarrow$ $a + 2b = a + 2$

$\Rightarrow$ $2b = 2$

$\Rightarrow$ $b = 1$

Put $b = 1$ in equation (i)

$$-2a+(1)+4 = 0$$
$$\Rightarrow \quad a = \frac{5}{2}$$

22. (b)

$$P = 0.5\% = 0.005,\ n = 5$$
$$m = nP = 5 \times 0.005 = 0.025$$
$$P(X \geq 3) = P(3) + P(4) + P(5)$$
$$= \frac{e^{-0.025}(0.025)^3}{3!} + \frac{e^{-0.025}(0.025)^4}{4!} + \frac{e^{-0.025}(0.025)^5}{5!}$$
$$= \frac{e^{-0.025}(0.025)^3}{3!}\left[1 + \frac{0.025}{4} + \frac{(0.025)^2}{20}\right]$$
$$= 2.56 \times 10^{-6} = 2.56 \times 10^{-4}\%$$

23. (c)

Let,

$$y = (\cos(\cos(\cos......x)))$$
$$\Rightarrow \quad y = \cos y$$
As $\quad y - \cos y = f(y)$
$$\Rightarrow \quad f'(y) = 1 + \sin y$$

Using Newton – Raphson's method,
and initial guess value, $x_0 = 1$

$$\Rightarrow \quad f(x_0) = 1 - \cos 1 = 0.4597$$

Now First iteration

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$x_1 = 1 - \frac{0.4597}{1 + \sin 1}$$
$$\Rightarrow \quad x_1 = 0.75036$$
Now, $\quad f(x_1) = 0.0189$

Second Iteration,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$\Rightarrow \quad x_2 = 0.75036 - \frac{0.0189}{0.75036 + \sin(0.75036)}$$
$$\Rightarrow \quad x_2 = 0.7372$$

Now,

$$f(x_2) = 3.153 \times 10^{-3} \simeq 0$$
$$y = x_2 = 0.7372$$
$$\Rightarrow \quad I = \int_0^1 x\cos(\cos(\cos........x))\,dx = \int_0^1 xy\,dx = \int_0^1 0.7372.x\,dx$$
$$\Rightarrow \quad I = 0.7372.\left.\frac{x^2}{2}\right|_0^1$$
$$\Rightarrow \quad I = 0.7372.\frac{1}{2} = 0.3686 \simeq 0.369 \simeq 0.37$$

24. (b)

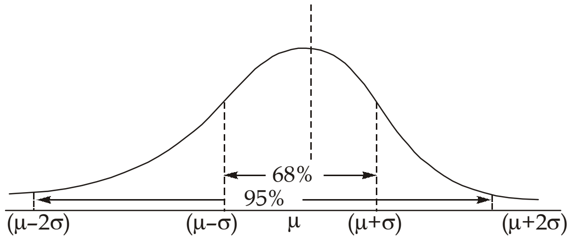
Given,

Mean, $\mu = 1200$

Variance, $\sigma^2 = 9 \times 10^4$

$\Rightarrow$ Standard deviation, $\sigma = \sqrt{9 \times 10^4} = 300$

Using Standard normal curve,



Probability of finding tigers between

$$(\mu - 2\sigma) \;\&\; (\mu + 2\sigma) = 0.95$$

$$\mu - 2\sigma = 1200 - 2 \times 300 = 600$$

$$\mu + 2\sigma = 1200 + 2 \times 300 = 1800$$

i.e. $P(600 \le X \le 1800) = 0.95$

$\Rightarrow P(X \le 600) + P(X \ge 1800) = 0.05$

Since normal curve is symmetric wrt mean value,

So, $P(X \le 600) = P(X \ge 1800)$

$\Rightarrow 2P(X \ge 1800) = 0.05$

$\Rightarrow P(X \ge 1800) = 0.025$

25. (a)

Complete Solution *CS*

$$CS = CF + PI$$

Now Auxilliary equation

$$(D^2 + 4D + 6)y = 0$$

$\Rightarrow m^2 + 4m + 6 = 0$

$\Rightarrow m = -2 \pm \sqrt{2}\, i$

So $C.F \rightarrow \left[c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x\right]$ ....(1)

Now,

$$PI \rightarrow \frac{3^x}{D^2 + 4D + 6} = \frac{e^{x ln 3}}{D^2 + 4D + 6}$$

$$\Rightarrow P.I = \frac{e^{x ln 3}}{(ln 3)^2 + 4.ln 3 + 6} = \frac{e^{x ln 3}}{11.6} = \frac{3^x}{11.6}$$

$$\Rightarrow C.S : y(x) = e^{-\sqrt{2}x}\left[c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x\right] + \frac{3^x}{11.6}$$

26. (b)

Let, P_1, P_2, P_3, P_4 be probability of selection in 1st, 2nd, 3rd & 4th attempt respectively,
Now,

$$P_1 = \frac{1}{24};\ P_2 = \frac{1}{24}[1+0.5]$$

$$P_2 = \frac{1}{24}\times\frac{3}{2}$$

$$P_3 = \frac{1}{24}\times\frac{3}{2}[1+0.5] = \frac{1}{24}\times\left(\frac{3}{2}\right)^2$$

$$P_4 = \frac{1}{24}\times\left(\frac{3}{2}\right)^3$$

Now let A_i be selection in i^{th} attempt & $\overline{A_i}$ be unsuccessful attempt,

So,

$$P_{selection} = A_1 + \overline{A_1}A_2 + \overline{A_1}\,\overline{A_2}\,A_3 + \overline{A_1}\,\overline{A_2}\,\overline{A_3}\,A_4$$

$$= \frac{1}{24}+\frac{23}{24}\times\frac{1}{24}\times\frac{3}{2}+\frac{23}{24}\left(1-\frac{3}{48}\right)\times\frac{1}{24}\times\left(\frac{3}{2}\right)^2+\frac{23}{24}\left(1-\frac{3}{48}\right)$$

$$\left(1-\frac{9}{96}\right).\frac{1}{24}\times\left(\frac{3}{2}\right)^3 = 0.3$$

27. (b)

To get ABC their are two ways,

(i) $(AB)C$

Now, Number of multiplications in $AB = 2 \times 3 \times 4 = 24$

Now, $ABC = (AB)_{2\times4}\,C_{4\times2}$

Number of multiplication for $(AB)C = 2 \times 4 \times 2 = 16$

$\Rightarrow$ Total multiplication $= 24 + 16 = 40$

(ii) $A(BC)$

Number of multiplication operations in $BC = 3 \times 4 \times 2 = 24$

Now,

$$ABC = A_{2\times3}\,(BC)_{3\times2}$$

Number of multiplication for A(BC) $= 2 \times 3 \times 2 = 12$

$\Rightarrow$ Total multiplication $= 24 + 12 = 36$

$\Rightarrow$ Minimum Number $= 36$

28. (a)

$$\vec{\nabla}\phi = \left(\frac{\partial\phi}{\partial x}\hat{i}+\frac{\partial\phi}{\partial y}\hat{j}+\frac{\partial\phi}{\partial z}\hat{k}\right)\left[3x^2y-4yz^2+6z^2x\right]$$

$$\Rightarrow \quad \vec{\nabla}\phi = (6xy + 6z^2)\,\hat{i} + (3x^2 - 4z^2)\,\hat{j} + (-8yz + 12\,zx)\,\hat{k}$$

Now at (1, 1, 1)

$$\vec{\nabla}\phi = 12\hat{i} - \hat{j} + 4\hat{k} \quad .....(1)$$

Also direction of line is, $\hat{A} = \dfrac{2\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{17}}$ .....(2)

⇒ Directional derivative using (1) & (2)

$$\vec{\nabla}\phi.\hat{A} = \left(12\hat{i} - \hat{j} + 4\hat{k}\right)\left(\frac{2\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{17}}\right)$$

$$= \frac{24-2+12}{\sqrt{17}} = \frac{34}{\sqrt{17}} = 2\sqrt{17}$$

29. (a)

For given PDE,

$$\sin.dx = \cos y.dy = \tan z\, dz$$

⇒ $\sin x\, dx = \cos y\, dy$

⇒ $\int \sin x\, dx = \int \cos y\, dy$

⇒ $-\cos x = \sin y + a$

⇒ $\sin y + \cos x = -a$ .....(1)

& also,

$$\int \sin x\, dx = \int \tan z dz$$

⇒ $-\cos x = \log \sec z + b$

⇒ $\log \cos z - \cos x = b$ .....(2)

from (1) & (2),

ψ ($\sin y + \cos x$, $\log \cos z - \cos x$) = 0 is required solution

30. (b)

Given: $I = \displaystyle\int \frac{\sin x dx}{1-\sin x} = -\int \frac{-\sin x + 1 - 1}{1-\sin x} dx$

$$= -\int \frac{1-\sin x}{1-\sin x} dx + \int \frac{1}{1-\sin x} dx$$

By multiplying (1 + sin x) in 2nd term

$$I = -\int dx + \int \frac{(1+\sin x)}{\{1+(\sin x)\}(1-\sin x)} dx$$

$$= -x + \int \frac{(1+\sin x)}{1-\sin^2 x} dx$$

$$= -x + \int \sec^2 x(1+\sin x)dx$$

$$= -x + \int \sec^2 x dx + \int \sec x \tan x\, dx$$

$$= -x + \tan x + \sec x + C$$

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