CLASS TEST

S.No.: 03SKECEE_BCDEFG_12924

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Pune | Kolkata

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

SIGNAL & SYSTEM

EC-EE

Date of Test : 12/09/2024

ANSWER KEY

1.	(d)	7.	(c)	13.	(b)	19.	(c)	25.	(c)
2.	(b)	8.	(b)	14.	(d)	20.	(b)	26.	(c)
3.	(d)	9.	(b)	15.	(a)	21.	(c)	27.	(c)
4.	(c)	10.	(d)	16.	(d)	22.	(c)	28.	(a)
5.	(c)	11.	(c)	17.	(b)	23.	(b)	29.	(a)
6.	(a)	12.	(c)	18.	(a)	24.	(a)	30.	(d)

DETAILED EXPLANATIONS

1. (d)

Given, $$y'' - y = -20\delta(t-3)$$

$$L[y''] - L[y] = -20L[\delta(t-3)]$$

$$s^2Y(s) - sy(0) - y'(0) - Y(s) = -20e^{-3s}$$

$$(s^2-1)Y(s) = -20e^{-3s} + s$$

∴ $$Y(s) = \frac{s - 20e^{-3s}}{s^2 - 1}$$

2. (b)

Given system, $$y(t) = u\{x(t)\}$$

Let $$x_1(t) = V(t), \text{ then } y_1(t) = u[V(t)]$$

$$x_2(t) = kV(t), \text{ then } y_2(t) = u[kV(t)] = ku[V(t)] = ky_1(t)$$

For $x(t) = x_1(t) + x_2(t)$,

$$y(t) = u\{x_1(t) + x_2(t)\} \neq u\{x_1(t)\} + u\{x_2(t)\}$$

The system satisfies the homogeneity principle but not the superposition principle.

Hence, the system is non-linear.

$$y_1(t) = u\{V(t)\}$$

$$y_2(t) = u\{V(t-t_0)\} = y_1(t-t_0)$$

Hence, the given system is time-invariant.

Since the response at any time depends only on the excitation at time $t = t_0$ and not on any future values, hence the given system is causal.

3. (d)

Given $$x[n] = \exp\left[\frac{2\pi jn}{3}\right] + \exp\left[\frac{3\pi jn}{4}\right]$$

$$= \exp\left[\frac{j2\pi n}{3}\right] + \exp\left[\frac{2\times j3\pi n}{8}\right]$$

The discrete-time signal $e^{j\omega_0 n}$ is periodic with time-period

$$N = r\cdot\frac{2\pi}{\omega_0}$$

where r is the minimum value of integer such that N is a positive integer.

on comparing, $N_1 = 3, N_2 = 8$

The fundamental period, $N = \text{LCM}(N_1, N_2)$

∴ $$N = \text{LCM}(3, 8) = 24$$

4. (c)

1. $$\cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{3}n\right) \Rightarrow \text{periodic}$$

$$\text{Period} = \frac{2\pi\times 3}{\pi} = 6$$

2. $\cos\left(\frac{1}{2}n\right)+\cos\left(\frac{1}{3}n\right) \Rightarrow$ non-periodic

[∵ A discrete-time sinusoid is periodic if its radian frequency ω is a rational multiple of π]

3. $$\text{Even }\{\cos(4\pi t)u(t)\} = \frac{\cos(4\pi t)\,u(t)+\cos(-4\pi t)u(-t)}{2} = \frac{\cos 4\pi t}{2} \Rightarrow \text{Periodic}$$

4. $$\text{Even }\{\sin(4\pi t)u(t)\} = \frac{\sin(4\pi t)u(t)+\sin(-4\pi t)u(-t)}{2}$$
$$= \frac{1}{2}\sin(4\pi t)\cdot \text{sgn}(t) \Rightarrow \text{non-periodic}$$

5. (c)

$$y[n] = x[n] * h[n]$$

$x[n] \Rightarrow$	1	2	3	4			
$h[n] \Rightarrow$	5	6	7	8			
	5	6	7	8			
		10	12	14	16		
			15	18	21	24	
				20	24	28	32
$x[n] * h[n] \Rightarrow$	5	16	34	60	61	52	32

$$y[n] = \{\underset{\uparrow}{5}, 16, 34, 60, 61, 52, 32\}$$

So, $y[3] = 60$

Method-II

$x(n)$ \ $h(n)$	5	6	7	8
1	5	6	7	8
2	10	12	14	16
3	15	18	21	24
4	20	24	28	32

$$y[n] = \{\underset{\uparrow}{5}, 16, 34, 60, 61, 52, 32\}$$

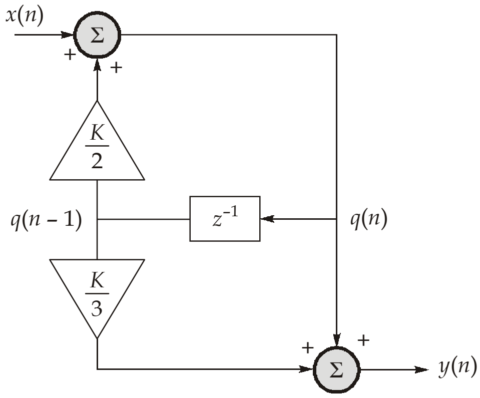
$$y[3] = 60$$

6. (a)

$$a^n x[n] \longleftrightarrow X[z/a]$$

Scaling property of z-transform.

7. (c)



$$q(n) = x(n) + \frac{K}{2}q(n-1)$$

$$y(n) = q(n) + \frac{K}{3}q(n-1)$$

Taking the z-transform of above equations,

$$Q(z) = X(z) + \frac{K}{2}z^{-1}Q(z)$$

$$Q(z)\left(1-\frac{K}{2}z^{-1}\right) = X(z)$$

$$Q(z) = \frac{X(z)}{1-\frac{K}{2}z^{-1}} \qquad \text{...(i)}$$

and $$Y(z) = Q(z) + \frac{K}{3}z^{-1}Q(z) = \left(1+\frac{K}{3}z^{-1}\right)Q(z) \qquad \text{...(ii)}$$

Substituting value of $Q(z)$ from equation (i) in equation (ii),

$$Y(z) = \left(1+\frac{K}{3}z^{-1}\right)\frac{X(z)}{1-\frac{K}{2}z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{K}{3}z^{-1}}{1-\frac{K}{2}z^{-1}} = \frac{z+\frac{K}{3}}{z-\frac{K}{2}} \;;\; \text{ROC}: |z| > \left|\frac{K}{2}\right|$$

System has one zero at $z = \frac{-K}{3}$ and one pole at $z = \frac{K}{2}$ and the ROC is $|z| > \left|\frac{K}{2}\right|$. The system will be BIBO stable if the ROC contains the unit circle, $|z| = 1$. Hence, the system is stable only if $|K| < 2$.

8. (b)

$$X(z) = \frac{1}{(1-az^{-1})^2} = \frac{z^2}{(z-a)^2}; |z|>|a|$$

$$a^n u(n) \longleftrightarrow \frac{z}{z-a} \;;\; \text{ROC: } |z| > |a|$$

$$nx(n) \longleftrightarrow -z\frac{dX(z)}{dz}$$

$$na^n u(n) \longleftrightarrow -z\frac{[z-a-z]}{(z-a)^2} = \frac{az}{(z-a)^2}$$

$$na^{n-1}u(n) \longleftrightarrow \frac{z}{(z-a)^2}; \text{ROC: } |z|>|a| \quad \text{...(i)}$$

Given: $$X(z) = z\left[\frac{z}{(z-a)^2}\right]; \text{ROC:} |z|>|a|$$

applying time shifting property in equation (i), we get

$$(n+1)a^n u(n+1) \longleftrightarrow \frac{z^2}{(z-a)^2} = X(z)$$

$\therefore$ $$x[n] = (n+1)a^n u(n+1)$$

9. (b)

10. (d)

4-point DFT of sequence {2, 1, 0, 3} is given as

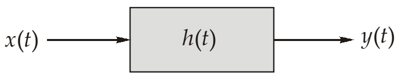
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix} = [6,\ 2+2j,\ -2,\ 2-2j]$$

11. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3+j\omega}$$

and output, $$y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$



We know that, $$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$\therefore$ $$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

By inverse Fourier transform of $X(j\omega)$, we have,

$$x(t) = e^{-4t}u(t)$$

12. (c)

Given, sinusoidal pulse

$$z(t) = \begin{cases} e^{j10t}; & |t|<\pi \\ 0; & |t|>\pi \end{cases}$$

We may express $z(t)$ as the product of a complex sinusoid e^{j10t} and a rectangular pulse $x(t)$.

Let, $$x(t) = \begin{cases} 1 \;; & |t| < \pi \\ 0 \;; & |t| > \pi \end{cases}$$

Fourier transform of $x(t)$ is $X(j\omega)$

$$\therefore \quad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \int_{-\pi}^{\pi} 1\cdot e^{-j\omega t}\,dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\pi}^{\pi}$$

$$= -\frac{1}{j\omega}\left[e^{-j\pi\omega} - e^{+j\omega\pi}\right] = \frac{e^{j\omega\pi} - e^{-j\omega\pi}}{j\omega} = \frac{2}{\omega}\left[\frac{e^{j\omega\pi} - e^{-j\omega\pi}}{2j}\right]$$

$$\therefore \quad X(j\omega) = \frac{2}{\omega}\sin(\omega\pi)$$

By using frequency shift property of Fourier transform, we get,

$$z(t) = e^{j10t}\cdot x(t) \xleftrightarrow{\text{FT}} X(j(\omega - 10))$$

$$\therefore \quad z(t) \xleftrightarrow{\text{FT}} \frac{2}{\omega - 10}\sin((\omega - 10)\pi)$$

13. (b)

Given, $$X(s) = \log(s+2) - \log(s+3)$$

Differentiating both the sides with respect to s

$$\frac{d}{ds}X(s) = \frac{1}{s+2} - \frac{1}{s+3} \quad \text{...(i)}$$

From the properties of Laplace transform, we know that,

$$tx(t) \longleftrightarrow -\frac{d}{ds}X(s)$$

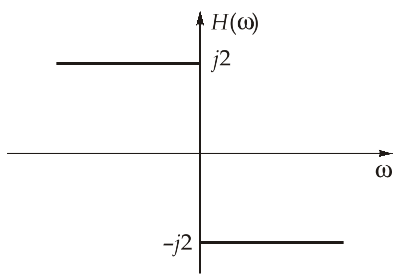
Thus equation (i) can be written as,

$$-tx(t) = [e^{-2t} - e^{-3t}]\,u(t)$$

or, $$x(t) = \left[\frac{e^{-3t} - e^{-2t}}{t}\right]u(t)$$

14. (d)

By redrawing the given frequency response, we get,



We can write $H(\omega) = -j2\,\text{sgn}(\omega)$

We know that,

For $$\text{sgn}(t) \xleftrightarrow{\text{FT}} \frac{2}{j\omega}$$

By duality property

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} 2\pi\,\text{sgn}(-\omega)$$

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} -2\pi\,\text{sgn}(\omega)$$

$$\frac{2}{\pi t} \xleftrightarrow{\text{FT}} -j2\,\text{sgn}(\omega)$$

or $$= 2(\pi t)^{-1}$$

15. (a)

Given $x(t) = \sin(150\pi t)$

Time period, $T = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{150\pi} = \dfrac{1}{75}$ sec

3 time periods $= 3 \times T = 3 \times \dfrac{1}{75} = \dfrac{1}{25}$ sec

$\therefore$ The signal sampled at a rate of five samples is $\dfrac{1}{25}$ sec

So, 1 sample in $\dfrac{1}{125}$ sec $= T_s$ [sampling interval]

$\therefore$ Sampling frequency $= f_s = \dfrac{1}{T_s} = 125$ samples/sec

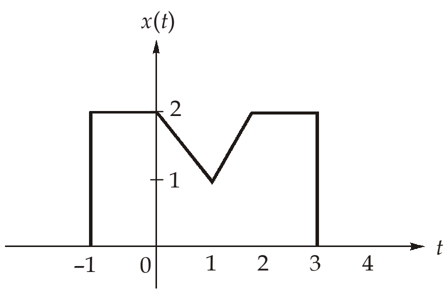
also, Nyquist rate $= f_N = 2f_m = 2 \times 75$ $[\because \omega_m = 150\pi \Rightarrow f_m = 75 \text{ Hz}]$

$= 150$ samples/sec

$\therefore$ The ratio, $\dfrac{f_s}{f_N} = \dfrac{125}{150} = \dfrac{5}{6} = 0.83$

16. (d)

Given,



By the definition of Fourier transform,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\cdot e^{j\omega t}\,d\omega$$

at $t = 0$,

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,d\omega$$

$$\therefore \int_{-\infty}^{\infty} X(j\omega)\,d\omega = 2\pi\, x(0) = 2\pi(2) = 4\pi \approx 12.57$$

17. (b)

Given input signal $x(t) = e^{-t}\, u(t)$

Energy of the input signal

$$E_i = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left|e^{-t}u(t)\right|^2 dt$$

$$= \int_{0}^{\infty} e^{-2t}\, dt = \left[\frac{e^{-2t}}{-2}\right]_0^{\infty} = \frac{1}{2} = 0.5 \text{ Joule}$$

now, ESD [Energy spectral density] of the output $y(t)$ is given by

$$E_y(\omega) = |H(\omega)|^2\, E_x(\omega)$$

$\therefore$ input ESD, $$E_x(\omega) = |X(\omega)|^2$$

$$X(\omega) = FT[x(t)] = FT[e^{-t}\, u(t)] = \frac{1}{1+j\omega}$$

$$E_x(\omega) = |X(\omega)|^2 = \frac{1}{1+\omega^2}$$

for a LPF, the square of the transfer function is given is

$$|H(\omega)|^2 = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \text{otherwise} \end{cases}$$

$$E_y(\omega) = |H(\omega)|^2\, E_x(\omega) = \begin{cases} \dfrac{1}{1+\omega^2}, & |\omega| < \omega_c \\ 0, & \text{otherwise} \end{cases}$$

$\therefore$ The total energy of the output signal is E_0

$$E_0 = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_y(\omega)\, d\omega \qquad [\because E_y(\omega) \text{ is an even function of '}\omega\text{'}]$$

$$= \frac{1}{\pi}\int_{0}^{\infty} E_y(\omega)\, d\omega = \frac{1}{\pi}\int_{0}^{\omega_c} \frac{1}{1+\omega^2}\, d\omega$$

$$0.90\left[\frac{1}{2}\right] = \frac{1}{\pi}\tan^{-1}(\omega_c)$$

$$\tan^{-1}\omega_c = 0.90 \times \frac{\pi}{2}$$

$$\omega_c = \tan\left[0.90 \times \frac{\pi}{2}\right]$$

$$\omega_c = 6.314 \text{ rad/sec}$$

18. (a)

$$\because \quad f_{s(\min)} = \frac{2f_2}{m}$$

where m = largest integer less than $\frac{f_2}{B} = \frac{30}{8} = 3.75$

$$\therefore \quad m = 3$$

$$\therefore \quad f_{s(\min)} = 2 \times \frac{30}{3} = 20 \text{ kHz}$$

19. (c)

We know that the Laplace transform of

$$\cos(at)u(t) = \frac{s}{s^2 + a^2}$$

$$\therefore \quad \cos(\pi t)u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function $x(t)$ can be written as,

$$= \cos\pi t[u(t) - u(t-1)]$$

$$= \cos(\pi t)u(t) - \cos\pi t\, u(t-1)$$

$$= \cos\pi t u(t) - \cos\pi(t - 1 + 1)u(t-1)$$

$$= \cos\pi t u(t) - \cos[\pi(t-1) + \pi]u(t-1)$$

$$x(t) = \cos(\pi t)u(t) + \cos[\pi(t-1)]u(t-1)$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2 + \pi^2} + \frac{se^{-s}}{s^2 + \pi^2} \quad [\because x(t - t_0) = X(s) \cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1 + e^{-s}]}{s^2 + \pi^2}$$

20. (b)

Given,
$$x(t) = \frac{\sin(10\pi t)}{\pi t}$$

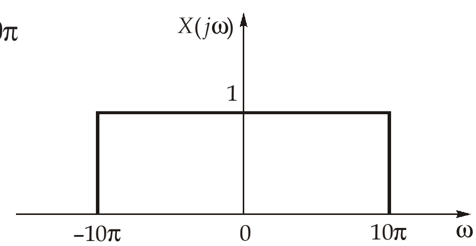
Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 \; ; & |\omega| \le 10\pi \\ 0 \; ; & |\omega| > 10\pi \end{cases}$$

or

$\therefore$ The maximum frequency 'ω_m' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,



$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$$\therefore \quad T_s < \frac{1}{10}$$

21. (c)

We know that, unit impulse let $x(t)$,

$$x(t) = \delta(t)$$

for $$\delta(t) \xleftrightarrow{LT} 1$$

for $$\frac{d}{dt}x(t) \xleftrightarrow{LT} sX(s)$$

$$\frac{d}{dt}\delta(t) \xleftrightarrow{LT} s$$

$$\frac{d^2}{dt^2}\delta(t) \xleftrightarrow{LT} s^2$$

22. (c)

23. (b)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nK}$$

$$g[n] = x[n-2]_{\text{mod N}}$$

$$G[k] = e^{-j\frac{2\pi}{N}(2)k} X[k]$$

$$G[1] = e^{-j\frac{2\pi}{4}(2)1} X[1] = e^{-j\pi}X[1]$$

$$G[1] = -X[1] = -7$$

24. (a)

Given, $$X(z) = \frac{10-8z^{-1}}{2-5z^{-1}+2z^{-2}}$$

$$= \frac{2}{(2-z^{-1})} + \frac{4}{(1-2z^{-1})}$$

$$X(z) = \frac{2z}{2z-1} + \frac{4z}{z-2}$$

$$X(z) = \frac{z}{\left(z-\frac{1}{2}\right)} + \frac{4z}{(z-2)}$$

Since, ROC includes unit circle,

$\therefore$ ROC of $X(z)$ is $\frac{1}{2} < |z| < 2$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 4(2^n)u[-n-1]$$

$\therefore$ $$x(1) = \frac{1}{2} = 0.5$$

25. (c)

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} \cdot nk}$$

Considering a four point DFT

Let $x(n) = \{a, b, c, d\}$

$N = 4$

The twiddle matrix is given as

$$[X(k)] = \left[W_4^{nk}\right][x(n)]$$

Similarly again DFT of $X(k)$, will

$$[y(n)] = \frac{1}{4}\left[W_4^{nk}\right][X(k)]$$

$$[y(n)] = \frac{1}{4}\left[W_4^{nk}\right]\left[W_4^{nk}\right][x(n)]$$

$$[y(n)] = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$[y(n)] = \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$[y(n)] = \{a, d, c, b\}$$

$$\sum_{n=0}^{3} x(n) = a + b + c + d$$

$$\sum_{n=0}^{3} y(n) = a + d + c + b$$

$$\sum_{n=0}^{3} x(n) - \sum_{n=0}^{3} y(n) = 0$$

26. (c)

We know that, from the definition of DTFT,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

where, $$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$$

by putting $n = -1$,

$$\therefore \qquad x[-1] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{-j\omega}\,d\omega$$

$$\therefore \qquad \int_{-\pi}^{\pi} X(e^{j\omega})e^{-j\omega}\,d\omega = 2\pi x[-1]$$

$$= 2\pi[-2] = -4\pi$$

$$= -12.57$$

27. (c)

Given, signal $x[n] = (0.5)^{2n}\,u[n] = \left(\frac{1}{2}\right)^{2n} u(n) = \left(\frac{1}{4}\right)^{n} u(n)$

From the definition of DTFT, the signal can be written as,

$$X(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{4}{4-e^{-j\omega}}$$

at $\omega = \pi$, $$X(e^{j\pi}) = \frac{4}{4-\cos\pi} = \frac{4}{4-(-1)} = \frac{4}{5} = 0.8$$

28. (a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!}z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2z}\right)^{n}}{n!}$$

$$X(z) = 1 + \frac{\frac{1}{2z}}{1!} + \frac{\left(\frac{1}{2z}\right)^{2}}{2!} + \frac{\left(\frac{1}{2z}\right)^{3}}{3!} + \ldots$$

$$X(z) = e^{1/2z}$$

$$X(1) = e^{1/2} = \sqrt{e} = 1.648 \approx 1.65$$

29. (a)

$$r_{xx}(k) = \sum_{k=-\infty}^{\infty} x[n]\,x[n-k]$$

$$\sum_{k=-4}^{k=4} r_{xx}(k) = x[n]\,x[n+3] + x[n]\,x[n+2] + x[n]\,x[n+1] + x[n]\,x[n] + x[n]\,x[n-1]$$

$$+ x[n]\,x[n-2] + x[n]\,x[n-3]$$

$$= -2 - 5 + 2 + 10 + 2 - 5 - 2 = 0$$

30. (d)

The given sequence of $x[n]$ is finite duration. Hence, the region of convergence is $0 < |z| < \infty$.

■■■■