CLASS TEST

S.No. : 01 SPME_ABCDEFG_13924

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THEORY OF MACHINES

MECHANICAL ENGINEERING

Date of Test : 13/09/2024

ANSWER KEY

1.	(a)	7.	(d)	13.	(c)	19.	(d)	25.	(a)
2.	(d)	8.	(a)	14.	(b)	20.	(d)	26.	(c)
3.	(b)	9.	(a)	15.	(a)	21.	(c)	27.	(b)
4.	(d)	10.	(d)	16.	(d)	22.	(a)	28.	(c)
5.	(c)	11.	(b)	17.	(a)	23.	(d)	29.	(b)
6.	(c)	12.	(b)	18.	(b)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$12 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{...(i)}$$

$$6 = \frac{1}{2\pi}\sqrt{\frac{k-800}{m}} \quad \text{...(ii)}$$

Divide equation (i) by equation (ii),

$$2 = \sqrt{\frac{k}{k-800}}$$

$$\frac{k}{k-800} = 4$$

$$\Rightarrow \quad 3k = 3200,\ k = \frac{3200}{3}\ \text{N/m}$$

3. (b)

$$\text{Arc of contact} = \frac{\text{path of contact}}{\cos\phi}$$

$$\cos\phi = \frac{25.4}{27} = 0.94074$$

$$\phi = \cos^{-1}(0.94074)$$

$$\phi = 19.8°$$

4. (d)

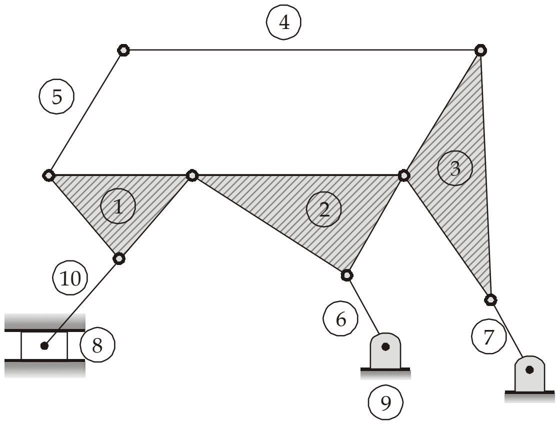
$$\text{Damping coefficient, } c = \frac{F}{v} = \frac{0.05}{0.04} = 1.25\ \text{N/m/s}$$

Critical damping coefficient,

$$c_c = 2\sqrt{mK} = 1.897\ \text{N/m/s}$$

$$\text{Damping ratio, } \xi = \frac{c}{c_c} = \frac{1.25}{1.897} = 0.658 \simeq 0.66$$

5. (c)



$l = 10, \quad h = 0$
$j = 12$

By Grubler's criterion

$$F = 3(l-1) - 2j - h$$
$$= 3(10-1) - 2 \times 12 - 0 = 27 - 24$$
$$F = 3$$

6. (c)

$$\frac{\dfrac{2h\omega}{\psi}}{\dfrac{\pi h\omega}{2\psi}} = \frac{4}{\pi}$$

7. (d)

Given, $\Delta E = 18$ kJ
$N_1 = 100$ rpm
$N_2 = 98$ rpm

We know that, $$\Delta E = \frac{1}{2}\left(I\omega_1^2\right) - \frac{1}{2}\left(I\omega_2^2\right) = \frac{1}{2}I\left(\omega_1^2 - \omega_2^2\right)$$

$$18 \times 10^3 = \frac{I}{2} \times \left[\left(\frac{2\pi\times 100}{60}\right)^2 - \left(\frac{2\pi\times 98}{60}\right)^2\right]$$

$$I = \frac{36\times 10^3 \times 60^2}{4\pi^2\left(100^2 - 98^2\right)}$$

$$I = 8289.915 \text{ kgm}^2$$

Kinetic energy at 140 rpm, $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\times 8289.915 \times \left(\frac{2\pi\times 140}{60}\right)^2 = 890909.088$ J

Kinetic energy at 140 rpm, $E = 890.91$ kJ

8. **(a)**

$$\text{Angular speed, } \omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.944 \text{ rad/s}$$

$$\text{Crank radius, } r = \frac{300}{2} = 150 \text{ mm}$$

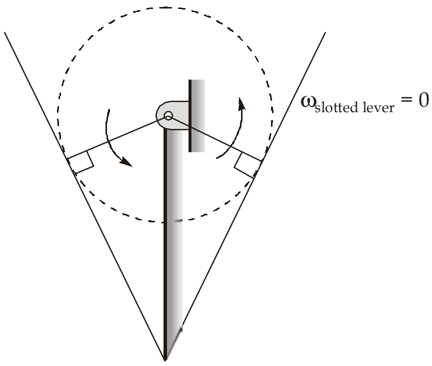
Mass to be balanced at the crank pin = $(c \times m_{reci}) + (m_{rev.})$ = (0.6 × 50) + 60 = 90 kg

Now,
$$m_c \times r_c = mr$$
$$90 \times 0.15 = m \times 0.25$$
$$m = 54 \text{ kg}$$

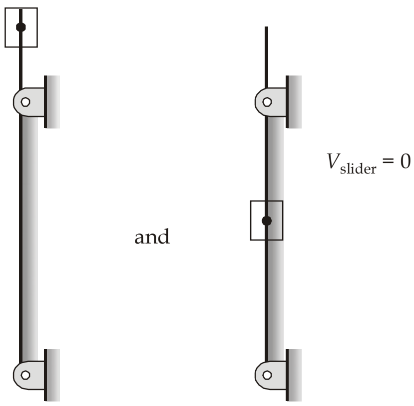
10. **(d)**

Coriolis component of acceleration will only be zero if either angular velocity of slotted lever is zero or the velocity of slider is zero. The possible 4 conditions are

- Two at the extremes of slotted lever.



- Two when the driving crank and slotted lever are vertical because at that position, velocity of slider will be zero.



11. **(b)**

Given:
$$d = 10 \times 16 = 160 \text{ mm}$$
$$D = 10 \times 50 = 500 \text{ mm}$$
$$\phi = 20°$$

$$r_A = \frac{d}{2} + \text{addendum} = 80 + 12 = 92 \text{ mm}$$

$$R_A = \frac{D}{2} + \text{addendum} = 250 + 8 = 258 \text{ mm}$$

$$\text{Path of approach} = \sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi$$

$$\text{Path of approach} = \sqrt{258^2 - (250\cos 20°)^2} - (250\sin 20°) = 21.15 \text{ mm}$$

$$\text{Path of recess} = \sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi$$

$$= \sqrt{92^2 - (80\cos 20°)^2} - (80\sin 20°) = 25.67 \text{ mm}$$

$$\omega_{gear} = \frac{2\pi\times 800}{60} = 83.77 \text{ rad/s}$$

$$\omega_{pinion} = \frac{T_G}{t_p}\times\omega_{gear} = \frac{50}{16}\times 83.77 = 261.799 \text{ rad/s}$$

Maximum sliding velocity = $(\omega_p + \omega_g) \times 25.67$

Maximum velocity of sliding = $(83.77 + 261.799) \times 25.67 = 8870.756$ mm/s = 8.87 m/s

12. (b)

$$T_A = 72$$
$$T_B = 32$$
$$N_{arm} = 18 \text{ rpm}$$
$$r_A = r_C + 2r_B$$
$$T_A = T_C + 2T_B$$
$$72 = 32 + 2T_B$$
$$T_B = \frac{40}{2} = 20$$

Condition	arm	Gear C (32)	Gear B (20)	Gear A (72)
Arm fixed	0	+1	$-\frac{32}{20}$	$-\frac{32}{20}\times\frac{20}{72}$
Gear C rotates by x revolutions	0	x	$-\frac{32}{20}x$	$-\frac{32}{72}x$
add $+y$ revolutions to all	y	$x+y$	$y-\frac{32}{20}x$	$y-\frac{32}{72}x$

$$y = 18 \text{ rpm}$$

Gear A is fixed, $y - \frac{32}{72}x = 0$

$$\Rightarrow \quad y = \frac{32}{72}x$$

$$\Rightarrow \quad x = \frac{72xy}{32}$$

$$\frac{18\times 72}{32} = x$$

$$x = 40.5$$

Speed of 'B' $N_B = y - \frac{32}{20}x$

$= 18 - \frac{32}{20} \times 40.5$

$N_B = -46.8$ rpm

13. (c)

Given, m = 1 tonne = 1000 kg

Logarithmic decrement of n cycles is given by

$$\delta = \frac{1}{n}\log_e \frac{x_0}{x_n}$$

$n = 4$

$$\delta = \frac{1}{4}\log_e \frac{5}{0.128} = 0.916$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad \text{or} \quad 0.916 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\xi = 0.144$$

Given, T_d = 0.64 seconds

$$\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{0.64} = 9.817 \text{ rad/s}$$

$$\omega_d = \sqrt{1-\xi^2}\ \omega_n$$

$$\omega_n = \frac{9.817}{\sqrt{1-0.144^2}} = 9.92 \text{ rad/s}$$

$$\sqrt{\frac{k}{m}} = 9.92$$

$$k = 9.92^2 \times 1000 = 98406.4 \text{ N/m} = 98.406 \text{ N/mm}$$

14. (b)

$$F = pA - F_I + mg$$

$$= 200\times10^3 \times \frac{\pi}{4}(0.8)^2 - 250\times0.3\times\left(\frac{2\times\pi\times300}{60}\right)^2\left(\cos40° + \frac{\cos80°}{4}\right) + 250 \times 9.81$$

$= 100531 - 59917.6 + 250 \times 9.81$

$= 43065.5$ N

$$F_t = F_c \sin(\theta + \beta) = \frac{F}{\cos\beta}\sin(\theta+\beta)$$

$$\beta = \sin^{-1}\left(\frac{\sin\theta}{n}\right) = \sin^{-1}\left(\frac{\sin40°}{4}\right) = 9.247°$$

$$= \frac{43065.5}{\cos9.247°} \times \sin(40° + 9.247°) = 33053 \text{ N}$$

$T = F_t \times r = 33053 \times 0.3$

$= 9916$ N-m $= 9.916$ kN-m

15. (a)

$$m = 120 \text{ kg}, \quad E = 200 \times 10^9 \text{ N/m}^2$$
$$l = 0.7 \text{ m}, \quad d = 0.04 \text{ m}$$
$$a = 0.25 \text{ m}, \quad b = 0.7 - 0.25 = 0.45 \text{ m}$$

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times (0.04)^4$$
$$= 0.1256 \times 10^{-6} \text{ m}^4$$

$$\Delta = \frac{mga^3b^3}{3EIl^3} = \frac{120 \times 9.81 \times (0.25)^3 \times (0.45)^3}{3 \times 200 \times 10^9 \times 0.1256 \times 10^{-6} \times (0.7)^3}$$
$$= 6.48 \times 10^{-5} \text{ m}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{6.48 \times 10^{-5}}} = 61.90 \text{ Hz}$$

16. (d)

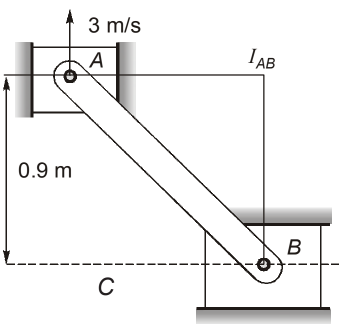
$$\omega = \frac{2\pi \times 240}{60} = 25.13 \text{ rad/s}$$

$$a_{\text{uniform (during ascent)}} = \frac{4h\omega^2}{\psi_a^2} = \frac{4 \times 30 \times (25.13)^2}{\left(70 \times \frac{\pi}{180}\right)^2}$$
$$= 50{,}771 \text{ mm/s}^2 = 50.77 \text{ m/s}^2$$

$$a_{\text{uniform (during decent)}} = \frac{4h\omega^2}{\psi_d^2} = \frac{4 \times 30 \times (25.13)^2}{\left(80 \times \frac{\pi}{180}\right)^2}$$
$$= 38{,}871.5 \text{ mm/s}^2 = 38.871 \text{ m/s}^2$$

$\therefore$ Difference $= 50.77 - 38.871$
$$= 11.89 \approx 11.9 \text{ m/s}^2$$

17. (a)



$$I_{AB} \cdot B = 0.9 \text{ m}$$
$$AB = 1.5 \text{ m}$$

$$I_{AB} \cdot A = \sqrt{(AB)^2 - (I_{AB} B)^2} = \sqrt{(1.5)^2 - (0.9)^2} = \sqrt{2.25 - 0.81} = 1.2 \text{ m}$$

$$\therefore \quad \frac{V_B}{V_A} = \frac{I_{AB}.B}{I_{AB}.A} = \frac{0.9}{1.2}$$

$$V_B = V_A \times \frac{0.9}{1.2} = 3 \times \frac{9}{12} = 2.25 \text{ m/s}$$

18. (b)

$$E_a = E$$
$$E_b = E + 280$$
$$E_L = E + 280 - 600 = E - 320$$
$$E_d = E + 280 - 600 + 100 = E - 220$$
$$E_e = E + 280 - 600 + 100 - 400 = E - 620$$
$$E_f = E + 280 - 600 + 100 - 400 + 890 = E + 270$$
$$E_g = E + 280 - 600 + 100 - 400 + 890 - 270 = E$$

$$\therefore \quad \Delta E = (E+280)-(E-620) = 900\times 750\times\frac{5\pi}{180}$$
$$= 58904 \text{ N-m}$$
$$\Delta E = I\omega^2 C_s$$
$$58904.8 = 60\times 2.4^2\times\left(\frac{2\times\pi\times 1500}{60}\right)^2\times C_s$$
$$\Rightarrow \quad C_s = 0.60\times 10^{-3} \text{ or } 0.69\%$$

19. (d)

20. (d)

$$\omega_{max} = \frac{\omega_1}{\cos\alpha}$$
$$\omega_{min} = \omega_1 \cos\alpha$$

Variation of speed, $\omega_{max} - \omega_{min} = \omega_1\left[\frac{1}{\cos\alpha} - \cos\alpha\right]$

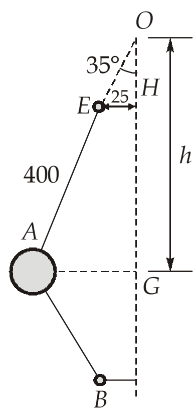
Permissible variation of speed = ±4 % of mean speed

or, $\omega_1\left[\frac{1}{\cos\alpha} - \cos\alpha\right] = 0.08\ \omega_1$

or, $\cos^2\alpha + 0.08\cos\alpha - 1 = 0$

$$\cos\alpha = 0.96$$
$$\Rightarrow \quad \alpha = 16.1°$$

21. (c)



$$h = GO = GH + HO = AE\cos\theta + EH\cot\theta$$
$$h = 400\cos 35° + 25\cot 35° = 363.4 \text{ mm}$$
$$h' = 400\cos 30° + 25\cot 30° = 389.7 \text{ mm}$$

Now, $h = \frac{g}{\omega^2}$ and $h' = \frac{g}{\omega'^2}$

$$\frac{\omega'}{\omega} = \sqrt{\frac{h}{h'}} = \sqrt{\frac{363.4}{389.7}} = 0.966$$

Percengage Decrease in speed = (1 – 0.966) × 100 = 3.44%

22. (a)

As per given data, $I = 1.5$ kg-m^2

The angular velocity of spin of the disc,

$$\omega = \frac{2\pi\times 500}{60} = \frac{100\pi}{6} \text{ rad/s}$$

The angular velocity of precession ,

$$\omega_p = \frac{2\pi}{5} \text{ rad/s}$$

Gyroscopic couple, $T = I\omega\omega_p$

$$= 1.5\times\frac{100\pi}{6}\times\frac{2\pi}{5} = 10\pi^2 = \frac{20\pi^2}{2} \text{ kg-m}^2/\text{s}^2$$

23. (d)

24. (c)

Disturbing force, $F = (1 - c)\ mr\omega^2\cos\theta$

$$= (1 - 0.4) \times 6 \times 0.10 \times 15^2 \times \cos 60 = 40.5 \text{ N}$$

25. (a)

$$l = 200 \text{ mm} = 0.2 \text{ m}$$

Crank length $\quad r = \frac{l}{2} = \frac{200}{2} = 100 \text{ mm} = 0.1 \text{ m}$

$$N = 600 \text{ rpm}, L = 500 \text{ mm} = 0.5 \text{ m}$$

$$m_R = 150 \text{ kg}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times 600}{60} = 20\pi = 62.832 \text{ rad/s}$$

$$n = \frac{L}{r} = \frac{0.5}{0.1} = 5$$

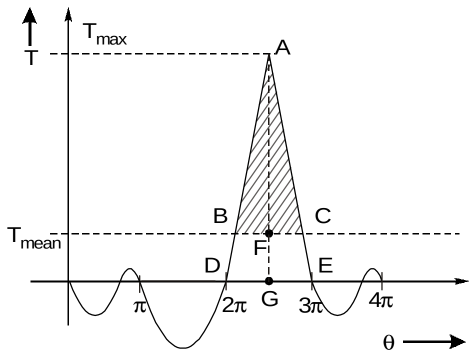
Inertia force, $\quad F_I = m_R\,\omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$= 150\times(62.832)^2\times 0.1\left(\cos 60^\circ + \frac{\cos 120^\circ}{5}\right)$$

$$= 59.2176\times 10^3\left(0.5 - \frac{0.5}{5}\right)$$

$$= 23.687 \times 10^3 \text{ N} = 23.687 \text{ kN}$$

26. (c)



$$P = \frac{2\pi N T_{mean}}{60}$$

$$\Rightarrow \quad T_{mean} = \frac{60 \times 40 \times 10^3}{2 \times \pi \times 130} = 2938.245 \text{ N-m}$$

$$\Rightarrow \quad \text{Energy produced} = T_{mean} \times 4\pi = 36923.076 \text{ N-m}$$

Now, work done during the power stroke

$$= 1.5 \times \text{Energy produced per cycle}$$
$$= 1.5 \times 36923.076$$
$$= 55384.615 \text{ Nm}$$

Now, from similar triangles ABC, ADE;

$$\frac{AF}{AG} = \frac{BC}{DE}$$

$$\frac{1}{2} \times T_{max} \times \pi = 55384.6$$

$$\Rightarrow \quad T_{max} = 35258.93 \text{ Nm} = AG$$

Now, $$\frac{35258.93 - 2938.245}{35258.93} = \frac{BC}{\pi}$$

$$\Rightarrow \quad BC = 2.879 \text{ rad}$$

Now, maximum fluctuation of energy $= \frac{1}{2} \times AF \times BC$

$$= \frac{1}{2} \times (35258.93 - 2938.245) \times 2.879$$

$$= 46525.62 \text{ N-m} \simeq 46.53 \text{ kNm}$$

27. (b)

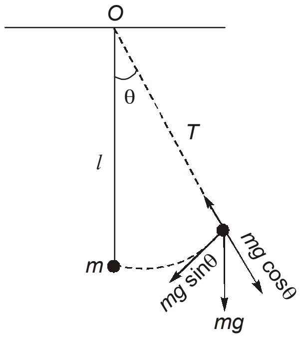
For the Hartnell governor
spring stiffness is given by

$$k = 2\left(\frac{a}{b}\right)^2 \left(\frac{F_1 - F_2}{r_1 - r_2}\right)$$

$$k = 2\left(\frac{a}{b}\right)^2 \left(\frac{1500 - 100}{20 - 15}\right)$$

$$k = 2\left(\frac{1400}{5}\right) = 560 \text{ N/cm} \quad (\because \text{ a and b are same})$$

28. (c)



$$T = -mgl\sin\theta$$

$$T = -mgl\theta \qquad [\because \sin\theta \simeq \theta \text{ as } \theta \text{ is very small}]$$

$$T = I\alpha = -mgl\theta$$

$$I = ml^2$$

$$\alpha = \frac{-g\theta}{l} \qquad \text{...(i)}$$

and we know $\alpha = \ddot{\theta} = -\omega_n^2\theta$ ...(ii)

Comparing (i) and (ii)

$$\omega_n = \sqrt{\frac{g}{l}}$$

So, $\omega_n \propto \sqrt{g}$

$$g_{\text{Moon}} = \frac{g_{\text{Earth}}}{6}$$

$$(\omega_n)_{\text{Moon}} = \sqrt{\frac{g_{\text{Moon}}}{l}}$$

$$(\omega_n)_{\text{Moon}} = \frac{1}{\sqrt{6}}\sqrt{\frac{g_{\text{Earth}}}{l}} = \frac{1}{\sqrt{6}}(\omega_n)_{\text{Earth}}$$

$$(\omega_n)_{\text{Moon}} = 0.4082[\omega_n]_{\text{Earth}}$$

29. (b)

(i) Controlling force, $F = 3r - 60$

At lower extreme radii, $F_1 = 3 \times 120 - 60 = 300$ N

Controlling force at maximum speed, $F_1 = 300 + 30 = 330$ N

Controlling force at minimum speed, $F_2 = 300 - 30 = 270$ N

$$\text{Coefficient of insensitiveness,} = \frac{N_1 - N_2}{N_{\text{mean}}} = \frac{(N_1 - N_2)(N_1 + N_2)}{2 \times N_{\text{mean}}^2}$$

$$= \frac{N_1^2 - N_2^2}{2N_{\text{mean}}^2} = \frac{F_1 - F_2}{2F} = \frac{330 - 270}{2 \times 300} \qquad \{F \propto \omega^2 \propto N^2\}$$

$$\text{Coefficient of insensitiveness} = \frac{60}{600} = 0.1 = 10\%$$

(ii) At upper extreme radii:

$$F = 3r - 60 = 3 \times 190 - 60 = 510 \text{ N}$$

Controlling force at maximum speed, $F_1 = 510 + 30 = 540$ N

Controlling force at minimum speed, $F_2 = 510 - 30 = 480$ N

Coefficient of insensitiveness,$= \dfrac{F_1 - F_2}{2F} = \dfrac{540 - 480}{2 \times 510} = \dfrac{60}{2 \times 510} = 0.0588 = 5.88\%$

Coefficient of insensitiveness at upper extreme radii = 5.88%

Coefficient of insensitiveness at lower extreme radii = 10.00%

30. **(a)**

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{9.81}{0.2 \times 10^{-2}}} = 70 \text{ rad/s}$$

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