CLASS TEST

S.No.: 01_JP_CE_02092024

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MECHANICAL ENGINEERING

STRENGTH OF MATERIAL

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

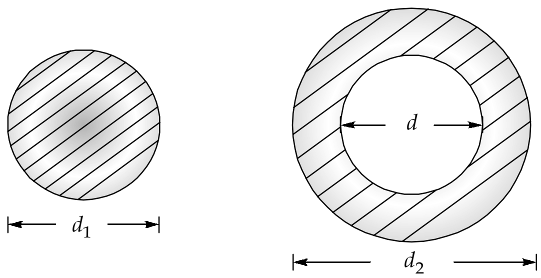
DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 A cube of side 50 mm is subjected to shear stress and a graph between shear stress and shear strain is plotted. It is observed that elastic limit shear stress is 250 N/mm^2. If shear modulus of material is 80 kN/mm^2. Then the proof resilience will be approximately, equals to

(a) 72 N-mm
(b) 56 N-mm
(c) 49 N-m
(d) 70 N-m

Q.2 Cross-sections shown in figure below have equal area. The ratio of section modulus $\frac{Z_2}{Z_1}$ for these section is



(a) $\frac{d_2^2 - 2d_1^2}{d_1 d_2}$

(b) $\frac{2d_2^2 - d_1^2}{d_1 d_2}$

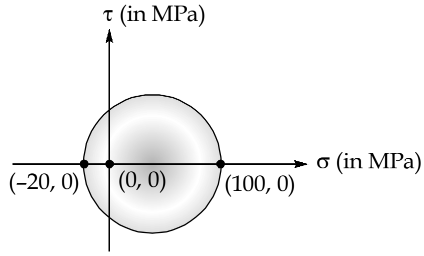
(c) $\frac{2d_1^2 - d_2^2}{d_1 d_2}$

(d) $\frac{d_1^2 - 2d_2^2}{d_1 d_2}$

Q.3 A uniform bar, simply supported at the ends, carries a concentrated point load P at mid-span. If the same load be uniformly distributed over the full length of the bar, the maximum slope decrease by

(a) 25.5%
(b) 41.5%
(c) 37.5%
(d) 33.3%

Q.4 The figure below shows the Mohr circle for a particular loading condition



Absolute maximum value of shear stress is

(a) 50 MPa (b) 60 MPa
(c) 10 MPa (d) 30 MPa

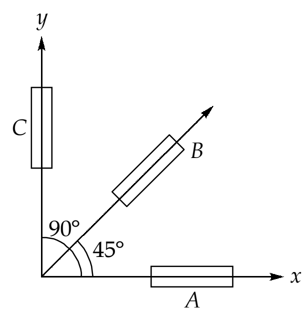
Q.5 A steel rod of 10 m long is at a temperature of 20°C. Upto what temperature the rod is to be heated if permissible stress allowed is 20 MPa and rod is allowed to expand by 4 mm?
[Take E = 200 GPa and $\alpha = 10 \times 10^{-6}$/°C]

(a) 50°C
(b) 70°C
(c) 30°C
(d) None of these

Q.6 In a triangular section maximum shear stress (τ_{max}) is 9 MPa. The shear stress at neutral axis (τ_{NA}) is

(a) 9 MPa (b) 8 MPa
(c) 6 MPa (d) 12 MPa

Q.7 During the static test of an airplane wings, the strain gauge reading from a 45° rosette are as follows: Gauge A, 530×10^{-6}; Gauge B, 420×10^{-6}; Gauge C, -80×10^{-6}. The principle strains are



(a) 530×10^{-6}, -137×10^{-6}
(b) 587×10^{-6}, 137×10^{-6}
(c) 530×10^{-6}, 137×10^{-6}
(d) 587×10^{-6}, -137×10^{-6}

Q.8 Consider the following statements at given point in the case of thick cylinder subjected to internal fluid pressure:

1. Radial stress is compressive
2. Hoop stress is tensile
3. Hoop stress is compressive
4. Longitudinal stress is tensile and it varies along the length
5. Longitudinal stress is tensile and remains constant along the length of the cylinder.

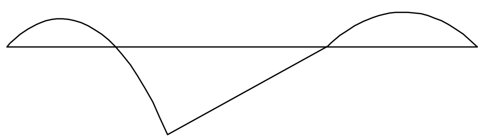
Which of the these statements are correct?

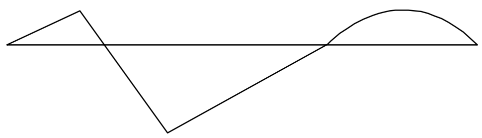
(a) 1, 2 and 4 only
(b) 3 and 4 only
(c) 1, 2 and 5 only
(d) 1, 3 and 5 only

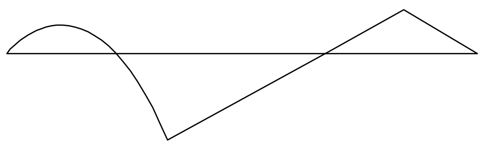
Q.9 The maximum bending stress induced in steel wire of modulus of elasticity 300 kN/mm^2 and diameter 2 mm when wraped on a drum of diameter 2 m equal to, approximately

(a) 300 N/mm^2
(b) 1000 N/mm^2
(c) 200 N/mm^2
(d) 2000 N/mm^2

Q.10 For the loading diagram shown, which of the following represents the correct BMD?

(a)


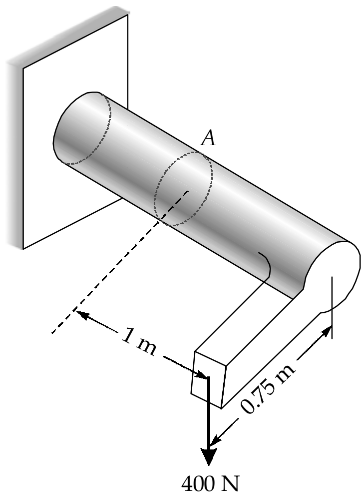
(b)


(c)


(d) None of the above

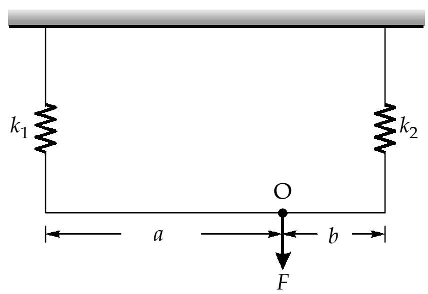
Q.No. 11 to Q.No. 30 carry 2 marks each

Q.11 A 100 mm diameter bar is fixed to the wall and loaded as shown in figure. The principal stresses at the top extremity of the vertical diameter for the section marked A are nearly,



(a) 4.6 N/mm^2, –0.5 N/mm^2
(b) 5.2 N/mm^2, –2.1 N/mm^2
(c) 2.55 N/mm^2, –2.55 N/mm^2
(d) 5.2 N/mm^2, –5.2 N/mm^2

Q.12 In the given figure, the spring constant of two springs are k_1 and k_2. The equivalent spring constant if the force F is applied at point O is



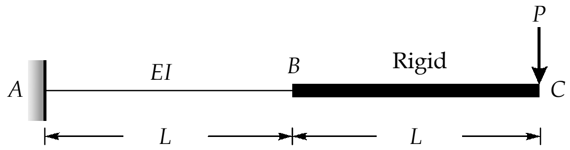
(a) $\dfrac{ab}{\dfrac{a^2}{k_1^2}+\dfrac{b^2}{k_2^2}}$

(b) $\dfrac{ab}{\dfrac{a^2}{k_2^2}+\dfrac{b^2}{k_1^2}}$

(c) $\dfrac{(a+b)^2}{\dfrac{a^2}{k_2}+\dfrac{b^2}{k_1}}$

(d) $\dfrac{(a+b)^2}{\dfrac{a^2}{k_1}+\dfrac{b^2}{k_2}}$

Q.13 A cantilever beam ABC shown below, the segment AB has flexural rigidity EI and the segment BC is rigid.



The deflection of the beam at C is

(a) $\frac{7}{6}\frac{PL^3}{EI}$

(b) $\frac{5}{6}\frac{PL^3}{EI}$

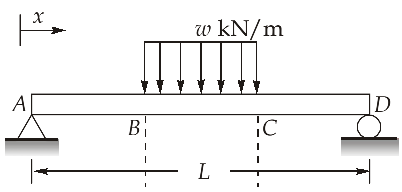
(c) $\frac{7}{3}\frac{PL^3}{EI}$

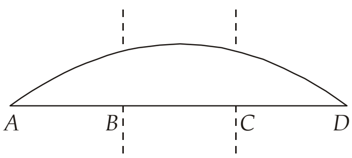
(d) $\frac{8}{6}\frac{PL^3}{EI}$

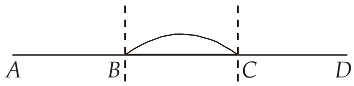
Q.14 If diameter of a long column is reduced by 20% and length is increased by 10% the percentage reduction in Euler's buckling load for the same conditions is nearly,

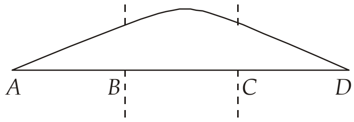
(a) 50%
(b) 45%
(c) 66%
(d) 55%

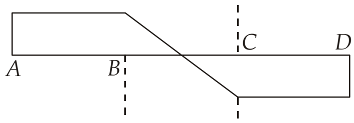
Q.15 Which of the following bending moment diagrams correspond to the simply supported beam shown below?



(a)


(b)


(c)


(d)


Q.16 A piece of material is subjected to two perpendicular tensile stresses of 100 MPa and 60 MPa. The resultant stress on the plane on which the resultant stress has maximum obliquity with the normal is

(a) 77.46 MPa
(b) 75 MPa
(c) 116.61 MPa
(d) 19.36 MPa

Q.17 A beam *AB*, 20 m long is supported on two intermediate supports 12 m apart. It carries a uniformly distributed load of 6 kN/m and two concentrated loads as 30 kN at the left end *A* and 50 kN at the right end *B*. How far away should the first support C be located from the end *A* so that the reactions at both the supports are equal?

(a) 4 m
(b) 5 m
(c) 12 m
(d) 6 m

Q.18 A steel cube of side 1 m is placed at a depth of h m in the sea water. What will be the value of *h*, for which change in volume is 0.05%? ***Take:*** E = 200 GPa and μ = 0.3. Unit weight of sea water = 10.08 kN/m^3.

(a) 8333 m
(b) 8267 m
(c) 1066 m
(d) 1080 m

Q.19 A ring mass of 60 kg encircles a bar and falls through a distance h before checked by a stop fixed to the bottom of the bar which hangs vertically from the rigid support. The bar is of steel which has modulus of elasticity of 2.05×10^5 N/mm^2, and is 40 mm in diameter and 2.5 m long. If the maximum instantaneous extension in the bar is 1.25 mm. What is the value of h?

(a) 588.61 mm (b) 125.01 mm
(c) 135.53 mm (d) 184.25 mm

Q.20 The solid cylinder of radius r is placed in a sealed container and subjected to pressure p. The stress components $(\sigma_x, \sigma_y, \tau_{xy})$ at any point located on the centreline of the cylinder is given by

(a) $\left(-\frac{p}{\sqrt{2}}, -\frac{p}{\sqrt{2}}, 0\right)$

(b) $\left(0, 0, -\frac{p}{\sqrt{2}}\right)$

(c) $(-p, -p, 0)$

(d) $\left(p, p, -\frac{p}{\sqrt{2}}\right)$

Q.21 A simply supported beam of span L and constant flexural rigidity EI is loaded at the left-hand end by a clockwise couple of moment M. The magnitude of angle of rotation at the left-hand support is

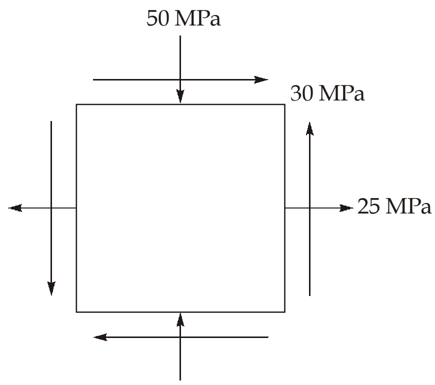
(a) $\frac{ML}{3EI}$

(b) $\frac{ML}{2EI}$

(c) $\frac{ML}{EI}$

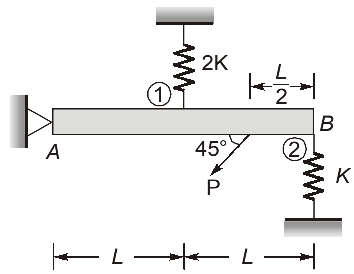
(d) $\frac{3ML}{2EI}$

Q.22 For the state of stress shown below, the maximum in-plane shear stress and the associated average stress at that point respectively are



(a) 25 MPa and –12.5 MPa
(b) 48 MPa and –12.5 MPa
(c) 48 MPa and 37.5 MPa
(d) 25 MPa and 37.5 MPa

Q.23 A massless rigid beam AB of length $2L$ is hinged at A and supported by linear elastic springs (having stiffness $2K$ and K) at point (1) and (2) and an inclined load acts at mid point between (1) and (2) as shown. The force in spring (1) and (2) is



(a) $R_1 = \frac{P}{\sqrt{2}}, R_2 = \frac{P}{2\sqrt{2}}$

(b) $R_1 = \frac{P}{2\sqrt{2}}, R_2 = \frac{P}{\sqrt{2}}$

(c) $R_1 = R_2 = \frac{P}{2\sqrt{2}}$

(d) $R_1 = R_2 = \frac{P}{\sqrt{2}}$

Q.24 A simply supported beam of span L and constant flexural rigidity EI is subjected to a clockwise moment M_0 at a distance $L/3$ from left support and an anticlockwise moment M_0 at distance $L/3$ from right support. The maximum deflection in the beam is

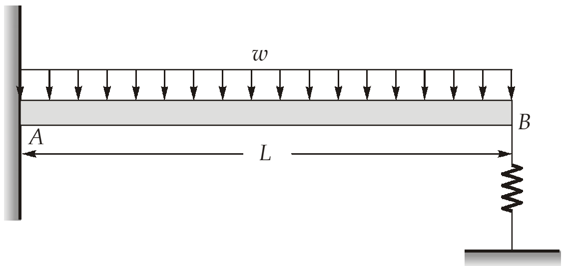
(a) $\dfrac{M_0 L^2}{18EI}$

(b) $\dfrac{5M_0 L^2}{72EI}$

(c) $\dfrac{M_0 L^2}{12EI}$

(d) $\dfrac{7M_0 L^2}{72EI}$

Q.25 A cantilever beam AB of length L and flexural rigidity EI has a fixed support at A and a spring support at B. The spring behaves in a linearly elastic manner with stiffness k. The beam is subjected to uniformly distributed load w per unit length. The force in the spring is given by



(a) $\dfrac{3kwL^4}{24EI + 8kL^3}$

(b) $\dfrac{kwL^4}{8EI + 8kL^3}$

(c) $\dfrac{3kwL^4}{8EI + 3kL^3}$

(d) $\dfrac{kwL^4}{24EI + 8kL^3}$

Q.26 In the case of a prismatic beam with constant flexural rigidily EI, the differential equation of deflection in terms of deflection v and loading intensity $q(x)$ becomes

(a) $v'''' = -\dfrac{q(x)}{EI}$

(b) $v''' = -\dfrac{q(x)}{EI}$

(c) $v'' = -\dfrac{q(x)}{EI}$

(d) $v' = -\dfrac{q(x)}{EI}$

Q.27 Consider three columns each of the same material, length and cross-sectional area. The columns are hinged on both ends and are free to buckle in any direction. The columns have cross sections as follows: (1) a circle, (2) a square, and (3) an equilateral triangle. The ratios of critical loads for these columns are

(a) $1 : \dfrac{\pi}{3} : \dfrac{2\pi\sqrt{3}}{9}$

(b) $1 : \pi : \dfrac{2\pi\sqrt{3}}{3}$

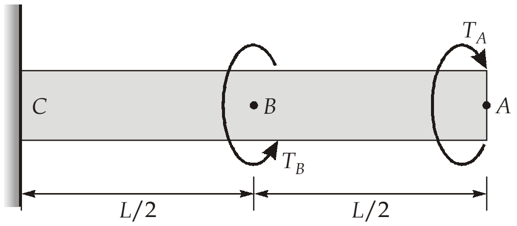
(c) $1 : \dfrac{2\pi}{3} : \dfrac{4\pi\sqrt{3}}{9}$

(d) None of the above

Q.28 The following readings are recorded by a rectangular strain rosette (the angles are with the x-axis):
$\varepsilon_{0°} = 400 \times 10^{-6}$, $\varepsilon_{45°} = 200 \times 10^{-6}$
and $\varepsilon_{90°} = -100 \times 10^{-6}$.
If E = 210 GPa and Poisson's ratio = 0.3, then principal stresses are

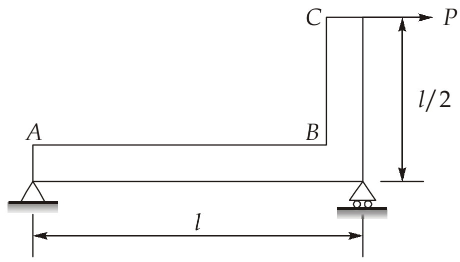
(a) 86.2 MPa, 3.82 MPa
(b) – 104.95 MPa, 404.95 MPa
(c) – 3.82 MPa, 86.2 MPa
(d) 404.95 MPa, 104.95 MPa

Q.29 A solid circular bar of length L = 1.5 m is fixed at one end and free at the other. If the bar is subjected to twisting load T_A and T_B as shonw in the figure, then the strain energy stored in the bar is
(Take T_A = 100 Nm, T_B = 100 Nm, G = 80 GPa and J = 80000 mm^4.)



(a) 585.9 N-mm
(b) 1171.8 N-mm
(c) 1757.7 N-mm
(d) 2343.6 N-mm

Q.30 The bracket ABC of uniform section is subjected to a horizontal load P at point C. The horizontal deflection at C is



(a) $\dfrac{Pl^3}{12EI}$ (b) $\dfrac{Pl^3}{8EI}$

(c) $\dfrac{Pl^3}{6EI}$ (d) $\dfrac{Pl^3}{24EI}$

■■■■

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STRENGTH OF MATERIAL

MECHANICAL ENGINEERING

Date of Test : 02/09/2024

ANSWER KEY

1.	(c)	7.	(d)	13.	(c)	19.	(c)	25.	(a)
2.	(b)	8.	(c)	14.	(c)	20.	(c)	26.	(a)
3.	(d)	9.	(a)	15.	(c)	21.	(a)	27.	(a)
4.	(b)	10.	(a)	16.	(a)	22.	(b)	28.	(a)
5.	(b)	11.	(a)	17.	(b)	23.	(c)	29.	(a)
6.	(b)	12.	(c)	18.	(b)	24.	(b)	30.	(b)

DETAILED EXPLANATIONS

1. (c)

Proof resilience = Total strain energy upto elastic limit

$$= \frac{\tau^2}{2G} \times \text{Volume} = \frac{250^2}{2 \times (80000)} \times 50^3 = 48828.125 \text{ Nmm}$$

$$= 48.828 \text{ N-m}$$

$$\approx 49 \text{ N-m}$$

2. (b)

For equal area,

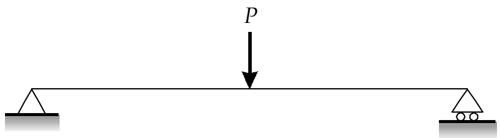
$$\frac{\pi}{4} d_1^2 = \frac{\pi}{4}\left(d_2^2 - d^2\right)$$

$$\Rightarrow \quad d^2 = d_2^2 - d_1^2$$

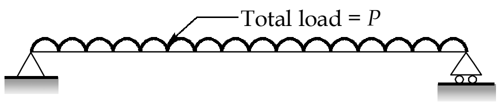
$$\frac{Z_2}{Z_1} = \frac{\left(\dfrac{d_2^4 - d^4}{d_2/2}\right)}{\left(\dfrac{d_1^4}{d_1/2}\right)} = \frac{\left(d_2^2 - d^2\right)\left(d_2^2 + d^2\right)}{d_2 d_1^3}$$

$$= \frac{\left(d_2^2 + d^2\right)}{d_1 d_2} = \frac{\left(d_2^2 + d_2^2 - d_1^2\right)}{d_1 d_2} = \frac{2d_2^2 - d_1^2}{d_1 d_2}$$

3. (d)



$$\theta_{max1} = \frac{PL^2}{16EI}$$



$$\theta_{max2} = \frac{PL^2}{24EI}$$

$$\% \text{ decrease in maximum slope} = \frac{\theta_{max1} - \theta_{max2}}{\theta_{max1}} \times 100 = \frac{\dfrac{1}{16} - \dfrac{1}{24}}{\dfrac{1}{16}} \times 100\% = 33.33\%$$

4. **(b)**

From Mohr circle,

σ_1 = 100 MPa, σ_2 = –20 MPa

$$\text{Absolute } \tau_{max} = \max\left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_1}{2}\right|, \left|\frac{\sigma_2}{2}\right|\right] = \max[60, 50, 10] = 60 \text{ MPa}$$

5. **(b)**

$$4 \text{ mm} = \alpha L \Delta T - \frac{\sigma}{E} L$$

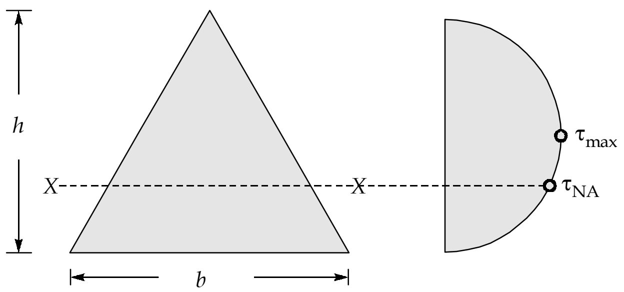
$$4 = \left(10 \times 10^{-6}\right) \times 10^4 \times \Delta T - \frac{20}{200 \times 10^3} \times 10^4$$

$$\Delta T = 50°\text{C}$$

Thus, rod is to be heated upto ΔT + 20 = 70°C

6. **(b)**

In triangular section,



For triangular cross-section,

$$\tau_{max} = \frac{3}{2}\tau_{avg}$$

and,
$$\tau_{NA} = \frac{4}{3}\tau_{avg}$$

⇒
$$\tau_{NA} = \frac{8}{9}\tau_{max} = \frac{8}{9} \times 9 = 8 \text{ MPa}$$

7. **(d)**

$\in_A = \in_{0°} = \in_x = 530 \times 10^{-6}$

$\in_C = \in_{90°} = \in_y = -80 \times 10^{-6}$

$$(\in_n)_\theta = \left(\frac{\in_x + \in_y}{2}\right) + \left(\frac{\in_x - \in_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin\theta$$

$$(\in_n)_{\theta=45°} = \in_B = 225 \times 10^{-6} + 305 \times 10^{-6}\cos 90° + \frac{\gamma_{xy}}{2}(\sin 90°) = 420 \times 10^{-6}$$

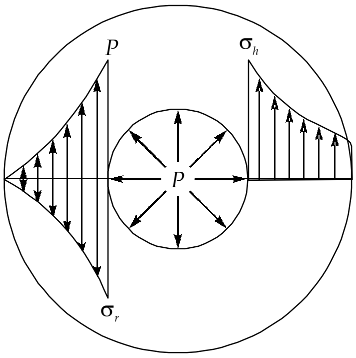
$\gamma_{xy} = 390 \times 10^{-6}$

$$\in_{1,2} = \frac{\in_x + \in_y}{2} \pm \sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{530-80}{2} \pm \sqrt{\left(\frac{530+80}{2}\right)^2 + \left(\frac{390}{2}\right)^2}\right] \times 10^{-6} = [225 \pm 362] \times 10^{-6}$$

$\therefore\ \in_{1,2} = 587 \times 10^{-6}, -137 \times 10^{-6}$

8. (c)



When thick cylinder is subjected to internal pressure 'p', hoop stress (tensile) develops which is maximum at inner radius and minimum at outer radius hyperbolic. The radial stress (σ_r) develope which is compressive in nature and has maximum magnitude at internal radius and varies hyperbolic to zero value at outer radius.

The longitudinal stress developed is tensile in nature and remain constant along the length of cylinder.

9. (a)

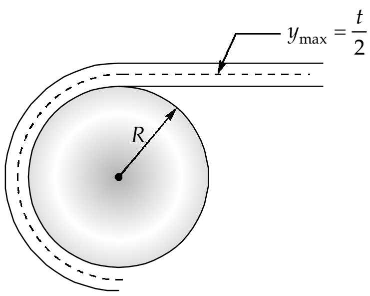
$$E = 300 \times 1000 \text{ N/mm}^2$$

$$y_{max} = \frac{2}{2} = 1 \text{ mm}$$

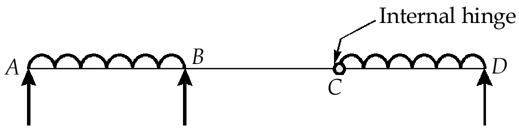
$$R = \frac{2000}{2} = 1000 \text{ mm}$$

$$(\sigma_b)_{max} = \frac{Ey_{max}}{R_1} = \frac{Ey_{max}}{R + \frac{t}{2}} \approx \frac{Ey_{max}}{R}$$

$$= \frac{300 \times 10^3}{1000} = 300 \text{ N/mm}^2$$



10. (a)



- At point C, there is an internal hinge, bending moment, BM = 0 and hence it will become a point of contraflexure.
- Between A and B, beam is subjected to uniformly distributed load, so the variation of BMD will of second degree. Similar BMD will be represented in between C and D.

11. (a)

At section A,

Bending moment , $M = 400 \times 1 = 400$ N-m

Torsion, $(T) = 400 \times 0.75 = 300$ N-m

The principal stresses at top extremity of the vertical diameter, at the section A

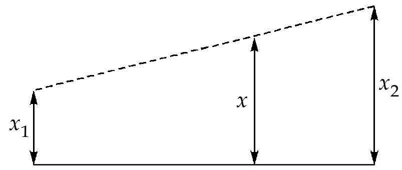
$$\sigma_{1,2} = \frac{16}{\pi d^3}\left(M \pm \sqrt{M^2 + T^2}\right)$$

$$= \frac{16}{\pi (100)^3}\left(400 \pm \sqrt{400^2 + 300^2}\right) \times 10^3$$

$$\sigma_1 = \frac{16}{\pi \times (1000)}(400 + 500) = 4.58 \text{ N/mm}^2$$

$$\sigma_2 = \frac{16}{\pi \times (1000)}(400 - 500) = -0.509 \text{ N/mm}^2$$

12. (c)



$$x_1 = \frac{F \cdot b}{(a+b)k_1}$$

$$x_2 = \frac{F \cdot a}{(a+b)k_2}$$

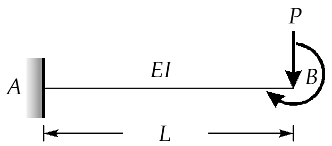
$$x = x_1 + \frac{(x_2 - x_1)}{(a+b)}a = \frac{x_1 b + x_2 a}{a+b}$$

$$= \left(\frac{F}{(a+b)^2}\right)\left(\frac{b^2}{k_1} + \frac{a^2}{k_2}\right)$$

$$k_{eq} = \frac{F}{x} = \left(\frac{(a+b)^2}{\frac{a^2}{k_2} + \frac{b^2}{k_1}}\right)$$

13. (c)

Since portion BC is rigid, BC will remain straight



$$\text{Deflection at } B = \frac{PL^3}{3EI} + \frac{PL^3}{2EI} = \frac{5}{6}\frac{PL^3}{EI}$$

$$\text{Slope at } B = \frac{PL^2}{2EI} + \frac{PL^2}{EI} = \frac{3}{2}\frac{PL^2}{EI}$$

Deflection at C = Deflection at B + Slope at $B \times L$

$$= \frac{5}{6}\frac{PL^3}{EI} + \frac{3}{2}\frac{PL^2}{EI} \times L = \frac{7}{3}\frac{PL^3}{EI}$$

14. (c)

$$\frac{P_2}{P_1} = \frac{I_2}{L_2^2} \times \frac{L_1^2}{I_1} = \left(\frac{d_2}{d_1}\right)^4 \times \left(\frac{L_1}{L_2}\right)^2 = \frac{(0.8)^2}{(1.1)^2} = 0.338$$

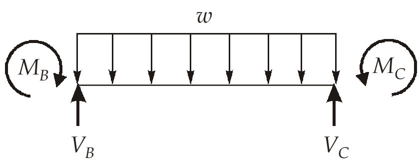
Percentage decrease = (1 – 0.338) × 100 = 66.148% $\simeq$ 66%

15. (c)

For sections *AB* and *CD*, the beam may be modeled as

$M(x)$ is linear with respect to x.

For section *BC*, the beam is modeled as



$M(x)$ is parabolic, reaching a maximum near or at the center.

16. (a)

Given, a biaxial stress system,

$$\sigma_x = 100 \text{ MPa}, \sigma_y = 60 \text{ MPa}$$

For maximum obliquity of the resultant with the normal to a plane is given by

$$\tan\theta = \sqrt{\frac{\sigma_x}{\sigma_y}} = \sqrt{\frac{100}{60}} = 1.29$$

or $\theta = 52.24°$

Direct stress, $\sigma_\theta = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$

$= 100\cos^2 52.24° + 60\sin^2 52.24°$

$= 100 \times 0.375 + 60 \times 0.625 = 37.5 + 37.5 = 75$ MPa

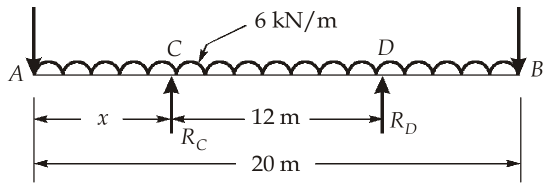
Shear stress, $\tau_\theta = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$

$= -\frac{1}{2}(100 - 60)\sin 104.48° = -19.365$ MPa

Resultant stress, $\sigma_r = \sqrt{\sigma_\theta^2 + \tau_\theta^2} = \sqrt{75^2 + 19.365^2} = 77.46$ MPa

17. (b)

Let the left support C be at a distance x meters from A.



$$
\begin{aligned}
\text{Now,} \quad R_C &= R_D \quad \text{(Given)} \\
\Sigma V &= 0 \\
R_C + R_D - 30 - 6 \times 20 - 50 &= 0 \\
\Rightarrow \quad 2R_C &= 30 + 120 + 50 \\
\Rightarrow \quad R_C &= 100 \text{ kN} \\
\therefore \quad R_D &= 100 \text{ kN} \\
\Sigma M_A &= 0
\end{aligned}
$$

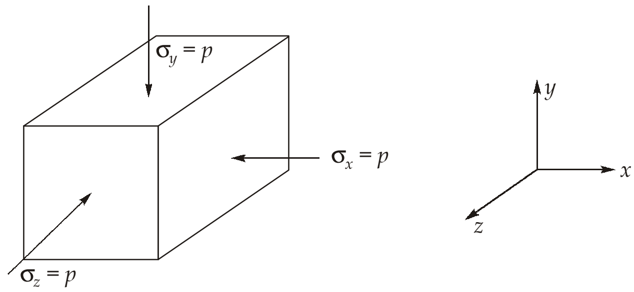
$$100x + 100(12 + x) - 6 \times 20 \times 10 - 50 \times 20 = 0$$

$$
\begin{aligned}
200x &= 1000 \\
x &= 5 \text{ m}
\end{aligned}
$$

18. (b)

Under water, the solid will be subjected to hydrostatic pressure (compressive) of equal magnitude on all sides as shown in figure. In the three principal directions, the strains will be

$$\varepsilon_x = \frac{\sigma}{E}(1-2\mu) = \varepsilon_y = \varepsilon_z$$



Therefore,

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{3p}{E}(1-2\mu)$$

$$\text{Change in volume, } \varepsilon_v = \frac{0.05}{100} = \frac{3p}{E}(1-2\mu)$$

$$p = \frac{0.05}{100} \times \frac{E}{3(1-2\mu)} = \frac{0.05 \times 200{,}000}{100 \times 3(1-2\times 0.3)} = 83.33 \text{ N/mm}^2 \quad \ldots(1)$$

$$\text{Pressure at any depth, } p = wh = 10{,}080h \text{ N/m}^2 \quad \ldots(2)$$

From eq. (1) and (2) we get,

$$
\begin{aligned}
10{,}080h &= 83.33 \times 10^6 \\
\Rightarrow \quad h &= 8267 \text{ m}
\end{aligned}
$$

19. (c)

Impact loading

$$\delta = \text{Max. instantaneous extension} = 1.25 \text{ mm}$$

$$W = Mg = 60 \times 9.81 = 588.6 \text{ N}$$

$$\therefore \quad \sigma = E\frac{\delta}{L} = \frac{2.05\times10^5\times1.25}{2.5\times10^3} = 102.5 \text{ N/mm}^2$$

Now potential energy lost by weight = Strain energy stored in bar

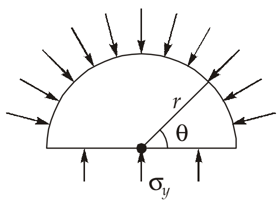
$$\therefore \quad W(h+\delta) = \frac{\sigma^2}{2E}\times V$$

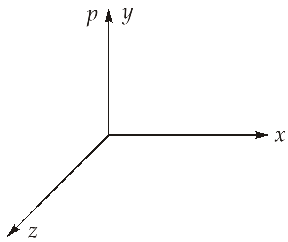
$$\text{or} \quad 588.6(h+1.25) = \frac{(102.5)^2}{2\times2.05\times10^5}\left[\frac{\pi}{4}(40)^2\times2500\right]$$

$$\text{or} \quad h + 1.25 = 136.78$$

$$\Rightarrow \quad h = 136.78 - 1.25 = 135.53 \text{ mm}$$

20. (c)





$$\sigma_y dz(2r) = \int_0^\pi p(rd\theta)dz\sin\theta$$

$$2\sigma_y = p\int_0^\pi \sin\theta d\theta = p[-\cos\theta]_0^\pi$$

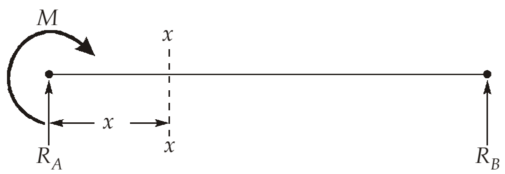
$$\sigma_y = p$$

Due to symmetry, $\sigma_x = \sigma_y = p$,

There will be no shear stress.

$$(\sigma_x, \sigma_y, \tau_{xy}) = (-p, -p, 0)$$

21. (a)



$$R_A = -\frac{M}{L} \qquad R_B = \frac{M}{L}$$

At any section, $$M_x = M - R_A x = M - \frac{M\times x}{L}$$

$$\theta_A = \frac{\partial U}{\partial M} = \int\frac{M}{EI}\frac{\partial M_x}{\partial M}dx \quad \text{(Modified castigliano's theorem)}$$

and, $$\frac{\partial M_x}{\partial M} = \left(1-\frac{x}{L}\right)$$

Hence, $$\theta_A = \int_0^L \frac{M\left(1-\frac{x}{L}\right)\left(1-\frac{x}{L}\right)}{EI}dx = \frac{ML}{3EI}$$

22. **(b)**

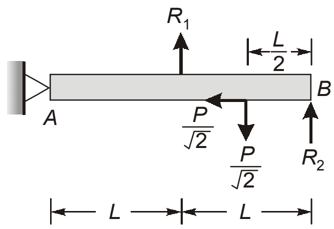
$$\tau_{max}\text{ (in plane)} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \sqrt{\left(\frac{25-(-50)}{2}\right)^2+30^2} = \sqrt{37.5^2+30^2}$$

$$\tau_{max} = 48.02\text{ MPa} \simeq 48\text{ MPa}$$

$$\sigma_{avg.} = \frac{\sigma_x+\sigma_y}{2} = \frac{25+(-50)}{2} = -12.5\text{ MPa}$$

23. **(c)**

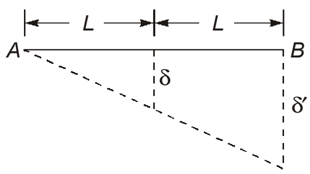
Let forces in spring (1) and (2) be R_1 and R_2



$$\Sigma M_A = 0$$

$$\Rightarrow \quad R_1 \times L + R_2 \times 2L = \frac{P}{\sqrt{2}} \times \frac{3L}{2}$$

$$\Rightarrow \quad R_1 + 2R_2 = \frac{3P}{2\sqrt{2}} \quad \text{...(i)}$$



From similar triangle, $\frac{\delta}{L} = \frac{\delta'}{2L}$

$$\Rightarrow \quad 2\delta = \delta' \quad \text{...(ii)}$$

Also, $\delta' = \frac{R_2}{K}$, $\delta = \frac{R_1}{2K}$

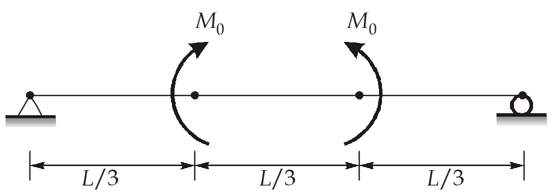
Putting in equation (ii)

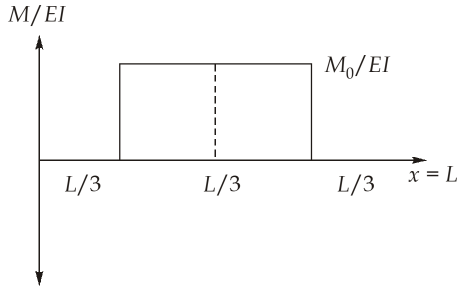
$$R_1 = R_2$$

$$\Rightarrow \quad 3R_1 = \frac{3P}{2\sqrt{2}}$$

$$\Rightarrow \quad R_1 = R_2 = \frac{P}{2\sqrt{2}}$$

24. (b)





Maximum deflection occurs at $x = \frac{L}{2}$

$$\delta_{max} = \text{Moment of area of } \frac{M}{EI} \text{ diagram between } x = 0 \text{ and } x = \frac{L}{2}$$

$$= \left(\frac{M_0}{EI} \times \frac{L}{6}\right) \times \frac{5}{12}L \quad \text{(Moment is calculated about } x = 0)$$

$$= \frac{5}{72}\frac{M_0 L^2}{EI}$$

25. (a)

Deflection of cantilever beam at free end.

1. Due to uniform loading, w

$$\Delta_1 = \frac{wL^4}{8EI}$$

2. Due to a point load, P

$$\Delta_2 = \frac{PL^3}{3EI}$$

Here P is the spring force (F_s)

Net deflection due to superposition, s is

$$s = \Delta_1 - \Delta_2$$

$$\frac{F_s}{k} = \frac{wL^4}{8EI} - \frac{F_s L^3}{3EI}$$

$$F_s = \frac{3kwL^4}{24EI + 8kL^3}$$

26. (a)

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

$$\frac{dM}{dx} = F$$

$$\frac{dF}{dx} = -q$$

From above three relations, we get

$$v'''' = -\frac{q(x)}{EI} \quad \text{or} \quad \frac{d^4v}{dx^4} = -\frac{q(x)}{EI}$$

27. (a)

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Since A, E, L are same.

$$P_1 : P_2 : P_3 = I_1 : I_2 : I_3$$

1. Circle

$$I = \frac{\pi d^4}{64} = \frac{A^2}{4\pi}$$

2. Square

$$I = \frac{a^4}{12} = \frac{A^2}{12}$$

3. Equilateral triangle

$$I = \frac{\sqrt{3}b^4}{96}$$

or
$$I = \frac{A^2\sqrt{3}}{18}$$

$$P_1 : P_2 : P_3 = I_1 : I_2 : I_3 = 1 : \frac{\pi}{3} : \frac{2\pi\sqrt{3}}{9}$$

(Since all 3 cross-sections are symmetric, every centroidal axis has the same moment of Inertia).

28. (a)

For a rectangular strain rosette

$$\varepsilon_x = \varepsilon_{0°} = 400 \times 10^{-6},\ \varepsilon_y = \varepsilon_{90°} = -100 \times 10^{-6}$$

and
$$\gamma_{xy} = 2\varepsilon_{45°} - \varepsilon_x - \varepsilon_y$$

$$\gamma_{xy} = 2 \times 200 \times 10^{-6} - 400 \times 10^{-6} + 100 \times 10^{-6} = 100 \times 10^{-6}$$

Principal strains, $\varepsilon_1, \varepsilon_2$
$$= \frac{1}{2}(\varepsilon_x + \varepsilon_y) \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$= \frac{10^{-6}}{2}\left[(400-100) \pm \sqrt{(400+100)^2 + 100^2}\right]$$

$$= 404.95 \times 10^{-6} \text{ and } -104.95 \times 10^{-6}$$

Principal stresses

$$\sigma_1 = \frac{E(\mu\varepsilon_2 + \varepsilon_1)}{1-\mu^2} = \frac{210000(-0.3\times104.95+404.95)\times10^{-6}}{1-0.3^2} = 86.2 \text{ MPa}$$

$$\sigma_2 = \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1-\mu^2} = \frac{210000(0.3\times404.95-104.95)\times10^{-6}}{1-0.3^2} = 3.82 \text{ MPa}$$

29. (a)

Strain energy due to torsion,

$$U = \frac{T^2 L}{2GJ}$$

Strain energy of portion AB,

$$U_{ab} = \frac{T_A^2\left(\frac{L}{2}\right)}{2GJ}$$

Strain energy for portion BC,

$$U_{bc} = \frac{(T_A - T_B)^2\left(\frac{L}{2}\right)}{2GJ} = 0$$

Total strain energy

$$U = U_{ab} + U_{bc}$$

$$U = \frac{100^2}{2\times 80\times 10^9\times 80000\times 10^{-12}}\left(\frac{1.5}{2}\right) = 0.5859 \text{ N-m}$$

or $\quad U = 585.9$ N-mm

30. (b)

$\Sigma F_x = 0,$

$\Rightarrow \quad H_A = P$

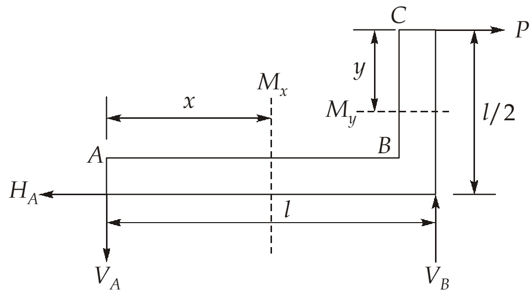
$\Sigma M_A = 0$

$\Rightarrow \quad V_B \times l = P \times \frac{l}{2}$

$$V_B = \frac{P}{2}$$

$\Sigma F_y = 0$

$\Rightarrow \quad V_A = V_B = \frac{P}{2}$



Strain energy stored by the bracket,

$$U = U_{AB} + U_{BC}$$

$$= \int_0^l \frac{M_x^2 dx}{2EI} + \int_0^{l/2} \frac{M_y^2 dy}{2EI} = \int_0^l \frac{\left(-\frac{Px}{2}\right)^2 dx}{2EI} + \int_0^{l/2} \frac{(Py)^2 dy}{2EI}$$

$$= \frac{P^2}{8EI}\left[\frac{x^3}{3}\right]_0^l + \frac{P^2}{2EI}\left[\frac{y^3}{3}\right]_0^{l/2}$$

$$= \frac{P^2}{8EI}\times\frac{l^3}{3} + \frac{P^2(l/2)^3}{6EI} = \frac{P^2 l^3}{24EI} + \frac{P^2 l^3}{48EI}$$

$$U = \frac{P^2 l^3}{16EI}$$

Horizontal deflection at C,

$$\delta_C = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\left(\frac{P^2 l^3}{16EI}\right)$$

$$\delta_C = \frac{2Pl^3}{16EI}$$

$$\delta_C = \frac{Pl^3}{8EI}$$

■■■■