CLASS TEST

S.No.: 02SKEC_HIJKLMN_26924

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COMMUNICATIONS

ELECTRONICS ENGINEERING

Date of Test : 26/09/2024

ANSWER KEY

1.	(c)	7.	(d)	13.	(c)	19.	(c)	25.	(d)
2.	(c)	8.	(c)	14.	(c)	20.	(c)	26.	(a)
3.	(d)	9.	(d)	15.	(c)	21.	(a)	27.	(c)
4.	(c)	10.	(c)	16.	(d)	22.	(b)	28.	(c)
5.	(c)	11.	(b)	17.	(a)	23.	(a)	29.	(c)
6.	(a)	12.	(c)	18.	(c)	24.	(b)	30.	(d)

Detailed Explanations

1. **(c)**

Entropy of the source,

$$H(X) = \frac{2}{8}\log_2(8) + \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2 \text{ bits/symbol}$$

$$= \frac{6}{8} + \frac{2}{4} + \frac{1}{2} = \frac{14}{8} = 1.75 \text{ bits/symbol}$$

The entropy of the 2^{nd} order extension of the source will be,

$$H(X^2) = 2H(X) = 2(1.75) = 3.50 \text{ bits/block}$$

2. **(c)**

For zero mean Gaussian random variable, the differential entropy can be given by,

$$H(X) = \frac{1}{2}\log_2(2\pi e\sigma^2)$$

Given that, $\sigma^2 = 2$

So, $$H(X) = \frac{1}{2}\log_2(4\pi e) = \frac{1}{2}\log_2(4) + \frac{1}{2}\log_2(\pi e)$$

$$= 1 + \frac{1}{2}\log_2(\pi e)$$

3. **(d)**

For matched filter,

$$(\text{SNR})_{\max} = \frac{2E_s}{N_0}$$

$$E_s = \text{Energy of the signal } s(t)$$

$$= \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_0^2 (4)^2 dt = 32$$

So, $$(\text{SNR})_{\max} = \frac{2(32)}{N_0} = \frac{64}{N_0}$$

4. **(c)**

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{160-40}{160+40} \times 100 = 60\%$$

5. **(c)**

The amplitude of uniformly distributed is equal to $\frac{1}{K}$.

$\therefore$ $$E[X^{K-1}] = \int_{-\infty}^{\infty} X^{K-1} f_x(x)dx$$

$$= \frac{1}{K}\int_0^k x^{K-1}dx = \frac{1}{K}\left|\frac{x^K}{K}\right|_0^K = (K)^{K-2}$$

6. (a)

Since, $Y = 2x$

Thus, $\frac{dx}{dy} = \frac{1}{2}$

$$f_Y(y) = \frac{dx}{dy} f_X(x) = \frac{dx}{dy} f_X\left(\frac{y}{2}\right)$$

$\therefore$ $K = \frac{1}{2}$

7. (d)

$$(\text{SNR}) \propto (2^n)^2$$

$\therefore$ $$\frac{(\text{SNR})_2}{(\text{SNR})_1} = \frac{(2^{n+1})^2}{(2^n)^2} = 4$$

8. (c)

$$r_b = n \cdot f_s$$
$$= 16 \times 40 \times 10^3 = 640 \times 10^3 \text{ bits/sec}$$

$$\text{Time duration} = \frac{\text{Total bits}}{\text{Bit rate}}$$
$$= \frac{2.304 \times 10^9}{640 \times 10^3} = 3600 \text{ sec}$$
$$= 60 \text{ minutes} = 1 \text{ hour}$$

9. (d)

For $f_l < f_s$

$$f_{si} = f_s - 2(\text{IF})$$
$$= 1000 - 2(300) \text{ kHz}$$
$$= 400 \text{ kHz}$$

10. (c)

As X and Y statistically independent,

$$E[XY] = E[X]E[Y]$$

$$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx = 1$$ $\because$ $f_X(x)$ is symmetric about the point $x = 1$

$$E[Y] = \int_{-\infty}^{\infty} y f_Y(y) dy = 2$$ $\because$ $f_Y(y)$ is symmetric about the point $y = 2$

so, $E[XY] = E[X]E[Y] = (1)(2) = 2$

11. (b)

The transmission efficiency of an AM signal can be given by,

$$\eta = \frac{k_a^2 P_m}{1 + k_a^2 P_m}$$

Here, k_a = amplitude sensitivity of the modulator

$= 0.25 \text{ V}^{-1}$

P_m = Power of the message signal

For the given message signal,

$$P_m = A^2 = (2)^2 = 4$$

So, $$\eta = \frac{(0.25)^2(4)}{1+(0.25)^2(4)} = \frac{0.25}{1+0.25} = \frac{1}{5} = 0.20 \text{ (or) } 20\%$$

12. (c)

For Hilbert transform, the impulse response is,

$$h(t) = \frac{1}{\pi t}$$

$$H(\omega) = -j\,\text{sgn}(\omega)$$

$$|H(\omega)|^2 = 1$$

So, the input and output power spectral densities are related as,

$$S_Y(\omega) = |H(\omega)|^2 S_X(\omega) = (1)\, S_X(\omega) = S_X(\omega)$$

Since $S_Y(\omega) = S_X(\omega)$, $R_Y(\tau) = R_X(\tau) \Rightarrow$ Hence, R_1 is correct

$$R_{XY}(\tau) = R_X(\tau) * h(\tau)$$

$$R_{XY}(-\tau) = R_X(-\tau) * h(-\tau)$$

$$h(\tau) = \frac{1}{\pi\tau} \text{ and } h(-\tau) = -h(\tau)$$

$$R_X(-\tau) = R_X(\tau) \qquad \because \text{ ACF of a WSS process is an even function}$$

So, $$R_{XY}(-\tau) = R_X(\tau) * [-h(\tau)] = -[R_X(\tau) * h(\tau)]$$

$$R_{XY}(-\tau) = -R_{XY}(\tau) \Rightarrow \text{Hence, } R_2 \text{ is correct}$$

So, both the given relations are correct.

13. (c)

Given that, $$f_X(x) = \begin{cases} \frac{1}{2}; & -1 < x < 1 \\ 0; & \text{otherwise} \end{cases}$$

$$\bar{X} = E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_{-1}^{1} = 0$$

Covariance of X and Y,

$$C_{XY} = E\left[(X-\bar{X})(Y-\bar{Y})\right] = E[XY] - \bar{X}\bar{Y}$$

$$= E[XY] - (0)\bar{Y} = E[XY]$$

So, $$C_{XY} = E[XY] = E(XX^n) = E[X^{n+1}]$$

$$= \int_{-\infty}^{\infty} x^{n+1} f_X(x)\, dx = \frac{1}{2}\int_{-1}^{1} x^{n+1}\, dx$$

$$= \frac{1}{2}\left[\frac{x^{n+2}}{n+2}\right]_{-1}^{1} = \frac{1}{2}\left[\frac{1}{n+2} - \frac{(-1)^{n+2}}{n+2}\right]$$

$$C_{XY} = \begin{cases} \frac{1}{n+2}; & n = \text{odd} \\ 0; & n = \text{even} \end{cases}$$

14. (c)

$$E_1 = \frac{1}{4}\left[(0)^2 + (2A)^2 + (\sqrt{2}A)^2 + (\sqrt{2}A)^2\right] = 2A^2$$

$$E_2 = \frac{1}{4}\left[(A)^2 + (A)^2 + (A)^2 + (A)^2\right] = A^2$$

So, $E_1 > E_2$

The distance between adjacent symbols is same in both the constellations. So, both the modulation schemes provide same average symbol error under similar circumstances.

Hence, $p_1 = p_2$

15. (c)

$$x(t) = m(t) + c(t) = m(t) + \cos(2\pi f_c t)$$

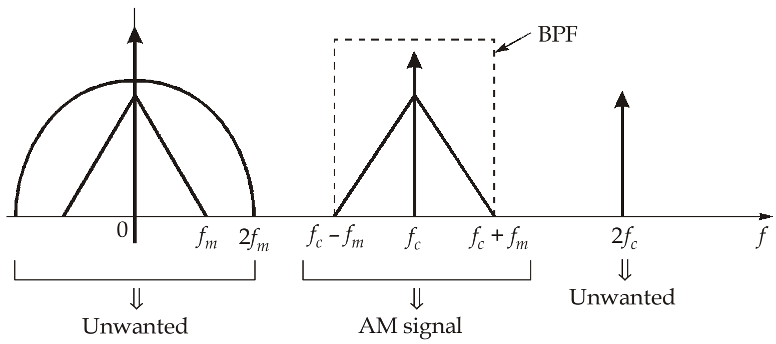
$$y(t) = x(t) + x^2(t)$$

$$= m(t) + \cos(2\pi f_c t) + m^2(t) + 2m(t)\cos(2\pi f_c t) + \cos^2(2\pi f_c t)$$

$$\cos^2(2\pi f_c t) = \frac{1}{2} + \frac{\cos(4\pi f_c t)}{2}$$

So, $$y(t) = \left[\frac{1}{2} + m(t) + m^2(t)\right] + \left[\cos(2\pi f_c t) + 2m(t)\cos(2\pi f_c t)\right] + \left[\frac{\cos(4\pi f_c t)}{2}\right]$$

The spectrum of the signal $y(t)$ can be plotted as follows:



To get the desired AM signal at the output of the filter, the following conditions to be satisfied:

$$f_c + f_m < 2f_c \Rightarrow f_c > f_m \quad \text{...(i)}$$

$$f_c - f_m > 2f_m \Rightarrow f_c > 3f_m \quad \text{...(ii)}$$

From (i) ∩ (ii), the necessary condition to be satisfied is "$f_c > 3f_m$".

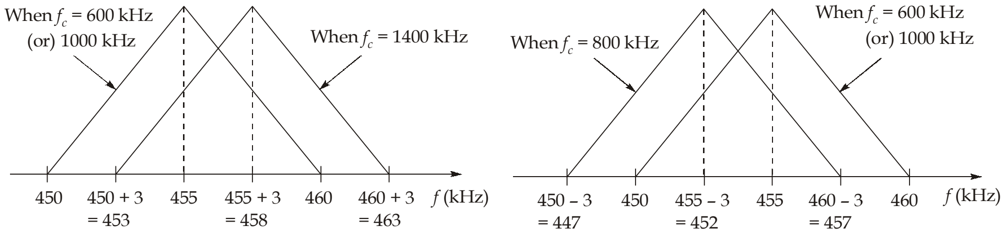
16. (d)

Given that the maximum frequency of the modulating signal is $f_{m\,(\max)} = 5$ kHz. So, the bandwidth of AM modulated signal is 10 kHz.

For ideal tracking (i.e. zero tracking error for any input carrier frequency), the minimum bandwidth required by the IF amplifier is 10 kHz. In this case, the IF amplifier has a flat band from 450 kHz to 460 kHz.

But it is given that the tracking is not ideal. From the given tracking error curve it is clear that only for the carrier frequency 600 kHz and 1000 kHz the tracking error is zero. So, only for these frequencies the spectrum will lie in the range (450 to 460) kHz at the input of the IF amplifier.

From the given tracking error curve, it is clear that, maximum deviation occur from ideality at f_c = 800 kHz, 1400 kHz. The spectrum produced at the input of IF amplifier for these carrier frequencies will be in the range as shown in the figure below.



So, the minimum bandwidth required by the IF amplifier can be given by,

$$BW_{IF(min)} = f_{max} - f_{min}$$

$$f_{max} = f_{IF} + |\text{Max. positive tracking error}| + f_{m(max)}$$

$$= 455 + 3 + 5 = 463 \text{ kHz}$$

$$f_{min} = f_{IF} - |\text{Max negative tracking error}| - f_{m(max)}$$

$$= 455 - 3 - 5 = 447 \text{ kHz}$$

So, $$BW_{IF(min)} = 463 - 447 = 16 \text{ kHz}$$

17. (a)

The signals $x_1(t)$ and $x_2(t)$ can be given by,

$$x_1(t) = m(t)\cos(2\pi f_1 t) = \cos(2\pi f_1 t)\cos(2\pi f_m t)$$

$$= \frac{1}{2}[\cos 2\pi(f_1 + f_m)t + \cos 2\pi(f_1 - f_m)t]$$

$$x_2(t) = m(t)\sin(2\pi f_1 t) = \sin(2\pi f_1 t)\cos(2\pi f_m t)$$

$$= \frac{1}{2}[\sin 2\pi(f_1 + f_m)t + \sin 2\pi(f_1 - f_m)t]$$

$|f_1 - f_m| < f_1$ and $f_1 = 2f_m$. So, after passing $x_1(t)$ and $x_2(t)$ through respective low-pass filters, we get,

$$y_1(t) = \frac{1}{2}\cos 2\pi(f_1 - f_m)t = \frac{1}{2}\cos(2\pi f_m t)$$

$$y_2(t) = \frac{1}{2}\sin 2\pi(f_1 - f_m)t = \frac{1}{2}\sin(2\pi f_m t)$$

The signals $z_1(t)$ and $z_2(t)$ can be given by,

$$z_1(t) = \cos(2\pi f_2 t)\, y_1(t) = \frac{1}{2}\cos(2\pi f_2 t)\cos(2\pi f_m t)$$

$$z_2(t) = \sin(2\pi f_2 t)\, y_2(t) = \frac{1}{2}\sin(2\pi f_2 t)\sin(2\pi f_m t)$$

Now, the modulated signal $s(t)$ can be given by,

$$s(t) = z_1(t) + z_2(t)$$

$$= \frac{1}{2}[\cos(2\pi f_2 t)\cos(2\pi f_m t) + \sin(2\pi f_2 t)\sin(2\pi f_m t)]$$

$$= \frac{1}{2}\cos 2\pi(f_2 - f_m)t \Rightarrow \text{USSB signal}$$

$$= A\cos 2\pi(f_c + f_m)t \Rightarrow \text{USSB signal}$$

From the above equations, it is clear that,

$f_2 - f_m = f_c + f_m \Rightarrow$ $$f_2 = f_c + 2f_m = 1 \text{ MHz} + 2(5 \text{ kHz}) = 1010 \text{ kHz}$$

18. (c)

$$y(t) = 4x(t) + 10x^2(t)$$

$$\therefore \quad y(t) = 4[m(t)+\cos(\omega_c t)]+10[m(t)+\cos(\omega_c t)]^2$$

$$= 4m(t)+4\cos(\omega_c t)+10m^2(t)+\frac{10}{2}+\frac{10}{2}\cos(2\omega_c t)+20m(t)\cos(\omega_c t)$$

$$\therefore \quad y(t) = 4\cos(\omega_c t)+20m(t)\cos(\omega_c t)=4[1+5m(t)]\cos(\omega_c t)$$

Now, $\max\{|m(t)|\} = A_m$

$$\therefore \quad \mu = \max\{5|m(t)|\}$$

$$\mu = 5A_m$$

$$0.8 = 5A_m$$

$$A_c = 0.16$$

19. (c)

$$P_e = 1 - P_c$$

Now, since we are using ML criterion, and the input symbols are equally likely, then we can directly choose the output based on the maximum value of the transmission probabilities.

$$P_c = P(y_1)\,P(y_1 \mid x_1) + P(x_3)\cdot P(y_2 \mid x_3) + P(x_1)\cdot P(y_3 \mid x_1)$$

$$= \frac{1}{3}[0.5+0.5+0.4]=\frac{7}{15}$$

$$\therefore \quad P_e = 1-\frac{7}{15}=\frac{8}{15}=0.533$$

20. (c)

$$s_o(t) = \alpha s_i^2(t)=\alpha A^2\cos^2(\theta) \text{ where } \theta = \omega_c t + \beta\sin\omega_m t$$

$$= \frac{\alpha A^2}{2}[1+\cos 2\theta]=\frac{\alpha A^2}{2}+\frac{\alpha A^2}{2}\cos 2\theta$$

$\because$ The output $s_o(t)$ is passed through a BPF, thus

$$y(t) = \frac{\alpha A^2}{2}\cos 2\theta$$

$$\therefore \quad y(t) = \frac{\alpha A^2}{2}\cos 2(\omega_c t+\beta\sin\omega_m t)$$

21. (a)

$$G = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

$$\therefore \quad P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad H = \left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Hence option 'A'.

22. **(b)**

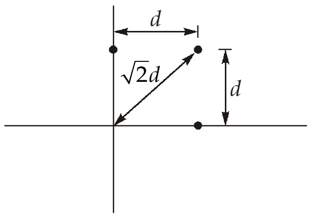
Average energy of constellation 1,

$$E_1 = \sum_{i=1}^{8} E_{Si} P_i$$

$$= R^2$$

Average energy of constellation 2,

$$E_2 = \sum_{i=1}^{8} E_{Si} P_i$$



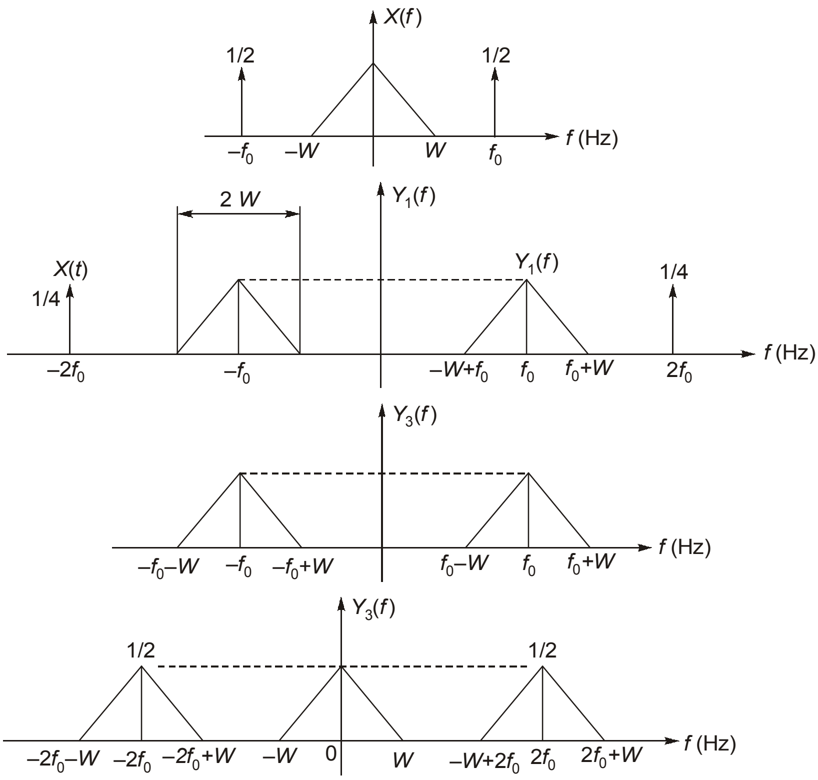
$$E_2 = d^2\left(\frac{1}{8}\right)\times 4 + 2d^2\left(\frac{1}{8}\right)\times 4$$

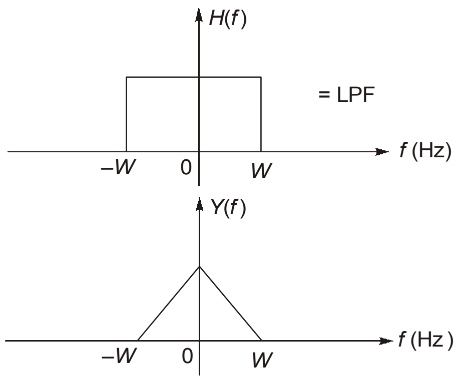
$$E_2 = \frac{3d^2}{2}$$

Since, $E_1 = E_2 \Rightarrow R^2 = \frac{3d^2}{2}$

$\Rightarrow$ $R = 1.22d$

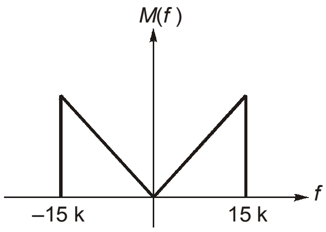
23. **(a)**

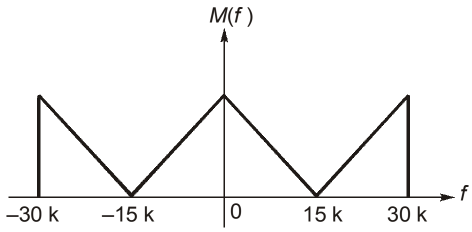




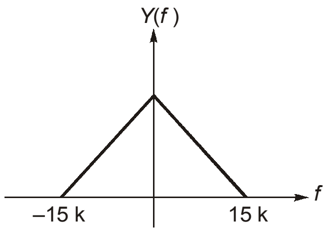
Bandwidth of $y(t) = W$ Hz

24. (b)





After passing through LPF



25. (d)

It is given that "0100011" is a valid code word.

$$\bar{H} = \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & b & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & a \end{array}\right]$$

$$\begin{array}{ccccccc} 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{array} \rightarrow \text{Given codeword}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ b \\ 1 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 0 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$1 \oplus b \oplus 0 = 0 \quad \Rightarrow b = 1$$
$$0 \oplus 1 \oplus a = 0 \quad \Rightarrow a = 1$$

26. (a)

Constellation having 4 points at a distance d from origin and 4 points $\sqrt{2}\,d$ from origin

Energy of a symbol = (Radius from origin to the symbol)^2

$$E_{s\text{ avg}} = \frac{1}{8}\left[4 \times (d)^2 + 4 \times (\sqrt{2}\,d)^2\right]$$
$$= 1.5d^2$$

27. (c)

$$P_e = Q\left(\sqrt{\frac{E_d}{2\eta}}\right) = Q(\sqrt{x})$$

$$E_d = A^2 \frac{T}{2}$$

$$x = \frac{E_d}{2\eta} = \frac{A^2 T}{4\eta} = \frac{(1\times10^{-4})^2 (1\times10^{-6})}{4(2\times10^{-15})} = 1.25$$

28. (c)

$$f_X(x) = \begin{cases} \frac{k}{4}(x+1)\,; & -1 \le x < 3 \\ 0 \quad ; & otherwise \end{cases}$$

$$E[X^2] = \int_{-1}^{3} x^2 \cdot f_X(x)\,dx = \int_{-1}^{3} x^2 \frac{k}{4}(x+1)\,dx$$

$$= \frac{k}{4}\int_{-1}^{3}(x^3 + x^2)\,dx = \frac{k}{4}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{3}$$

$$= \frac{k}{4}\left(\frac{1}{4}(81-1) + \frac{1}{3}(27+1)\right) = \frac{k}{4}\left(\frac{1}{4}\times 80 + \frac{1}{3}\times 28\right)$$

$$= \frac{k}{4}\left(\frac{28}{3} + 20\right) = k\left(\frac{22}{3}\right)$$

$$\int_{-\infty}^{\infty} f_X(x) = 1 = \frac{1}{2}(4)(k) = 1$$

$$k = \frac{1}{2}$$

So, $$E[X^2] = k\left(\frac{22}{3}\right) = \frac{11}{3} = 3.67$$

29. (c)

$|A_C[1 + K_a m(t)]|$

30. (d)

$$m = \frac{A_{\max} - A_{\min}}{A_{max} + A_{\min}}$$

$$0.85 = \frac{x - 0.5}{x + 0.5}$$

$$0.85(x + 0.5) = x - 0.5$$

$$x = 6.17 \text{ cm}$$

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